June 23, 1936

Dear Prof. John T. Tate:

A few days ago, I received your letter, in which the disapproval of the publication of my letter on the "Density Matrix in the Theory of the Positron" in the Physical Review was stated. I am very sorry to have troubled you with this matter, but I regret at the same time to find that the opinion of the referee started from a serious misunderstanding of my letter.

Therefore, I feel it necessary to give an answer to the comments of the referee. I shall be much obliged if you will kindly deliver the answer to him. For purposes of reference, my previous letter (rewritten anew with a few insignificant alteration of words) is again enclosed. I hope that the referee will recognize its due value, however small it may be.

Yours truly

Hideki Yukawa

[E22 030 より数式書込 editor]

# Reply to the Comments of the Referee

The wave functions in (5) and (5)' in my letter under discussion can be written in the form

$$\left[\quad \psi_n(x,k) = u_{n,k}\, e^{\frac{i}{\hbar}(\vec{p}_n\vec{r} - W_n t)} \quad\right]$$

for the case $V=0$ and $\vec{A}=0$. In empty space, the summation $\sum_{occ}$ should be made for all the negative energy states ($W_n<0$), while $\sum_{unocc}$ for all the positive energy states ($W_n>0$).

Now, if we take $\psi_n$ instead of $\psi_n^*$, both the energy and the momentum change their signs, so that (5) becomes

$$\left[\quad \frac{1}{2}\Big\{\sum_{W_n>0} \psi_n(x'k')\psi_n^*(xk) - \sum_{W_n>0} \psi_n^*(x'k')\psi_n(xk)\Big\} \qquad (5.1) \quad\right]$$

which is reduced to zero for any set of values of (t r k, t′r′k′), since two summations can be identified by mere relabeling.

On the other hand, (5)′ becomes

$$\left[\quad \frac{1}{2}\Big\{\sum_{W_n>0} \psi_n(x'k')\psi_n^*(xk) - \sum_{W_n>0} \psi_n^*(x'k')\psi_n(xk)\Big\} \qquad (5.2) \quad\right]$$

by the same procedure. The corresponding terms in two summations have time factors

$$\left[\quad e^{-\frac{i}{\hbar}W_n(t'-t)} \quad \text{and} \quad e^{\frac{iW_n}{\hbar}(t'-t)} \quad\right]$$

respectively, so that the limiting value of (5,2) for

$$\left[\quad s^2 = (t'-t)^2 - c^2(\vec{r}\,'-\vec{r})^2 = 0 \quad\right]$$

becomes infinity of order $s^{-2}$, as shown by Dirac in detail, although it vanishes, if we put $t = t'$ at the beginning.

Thus the density matrix for empty space vanishes everywhere in the modified theory in contrast to the usual theory, so that it is clear that they can not be identified by mere relabeling of eigenfunctions. Whether this is trivial or not will be left to the judgement of the referee.

Letter to the Editor of the Physical Review

Density Matrix in the Theory of the Positron

In the present theory of the electron and the positron, only one sort of them is considered at first, the existence of the other being deduced as necassary consequence of the theory. One can proceed, however, on the reverse way, accepting the existence of both at the beginning and introducing afterwards theoretically possible relations between them. The mathematical formulation of the latter method result in a slight modification of the usual theory as follows.

The quantized wave functions (x k) and (x k) of the electron and the positron satisfy Dirac's equations

(1)

respectively, where x denotes both time and position and k takes either of the values 1, 2, 3, 4. If we adopt a representation, in which all matrix elements of 8's are real and those of are pure imaginary, the wave functions and , which are complex conjugate to and respectively, satisfy the same equations as for and respectively, so that if the relations

(2)

are assumed at an instant for all points in space, they will remain forever. These are nothing but the mathematical expressions of the equivalence of the anti-electron and the positron on the one hand and that of the anti-positron and the electron on the other.

Now the density matrix, from which physical quantities such as the resultant charge density

etc. can be derived, takes the form

R(x k, x k )

or by (2)

in contrast to the symmetrical density matrix of Heisenberg

the factor ½ in (3) and (4) being needed on account of the above equivalence.

In Dirac's approximation, in which particles are moving in a common field, it reduces to

in contrast to Dirac's matrix

(1) Heisenberg, Zeits.f.Phys. 90,209,1934; 98,714,1936.
(2) Dirac, Proc.Camb.Phil.Soc. 30,150,1934.

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