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Letter to the Editor

Reduction of Nuclear Collision Problem

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with γ-ray emission, have been solved by straightforward application of the quantum mechanical method. Other processes, such as the disintegration with the emission of another heavy particle or the inelastic scattering with the excitation of the nucleus, have been considered to be so complicated that we should be satisfied only with rough estimation of their cross sections.

10 — Now it will be shown that ~~these cases~~ other processes ~~also~~ can also be reduced to one body problem so as to make it possible to apply the exact theory of collision in the following way.

We consider a ~~nucleus~~ ~~the collision of a heavy particle, a neutron for instance~~ state, with a nucleus of atomic number Z and mass number M initially in a state $X_0(q_1, q_2 \cdots q_M)$ ~~with a heavy particle, a neutron for example~~, where $q_1, q_2, \cdots q_M$ ~~are~~ represent ~~the~~ coordinates and spins of M heavy particles constituting the nucleus. ~~If~~ If the possibilities of the excitation of ~~the nucleus after the collision~~, the ~~latter~~ The ~~deuteron~~ ~~can be excited~~ to a state $X_1(q_1 \cdots q_M)$ say, ~~for example~~ ~~instance~~, and that of the disintegration with the emission of another heavy particle, a proton for example, are taken into account, the stationary state of the system consisting of ~~the two~~ M+1 particles can be expressed approximately in the form

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the stationary state of the nucleus with atomic number Z and ~~mass~~ of mass number N,

If ~~all the possibilities we take~~ only the possibility of ~~only~~, the elastic scattering of the impinging particle ~~take place by collision~~ are considered, the stationary ~~solution~~ state ~~of the~~ for the system of M+1 particles ~~has the ar~~ consisting heavy can be expressed by in ~~an~~ eigenfunction ~~of~~ the form $u_0(q_0)\chi_0(q_1q_2\cdots q_M)$, where q_0 represent coordinates and spin of the impinging particle and u_0 is ~~the~~ the superposition of ~~the scattered~~ that of the impinging particle in initial state plane for instance, which is ~~the incoming plane~~ wave and the ~~diverging~~ scattered wave, ~~of the~~ ~~If~~ Now, if the possibilities of the excitation of the nucleus ~~to a state~~ to another states, χ_1 ~~$(q_1 q_2 \cdots q_M)$~~ etc. say, and ~~that~~ of the disintegration with the ~~emission~~ emission of another heavy particle, in either of a proton ~~states~~ for ~~example~~ instance, are taken into account, the stationary ~~state~~ solution should have the form

$$\psi(q_0q_1\cdots q_M) = u_0(q_0)\chi_0(q_1q_2\cdots q_M)$$
$$+u_1(q_0)\chi_1(q_1q_2\cdots q_M)+v_1(q_0)\varphi_1(q_1q_2\cdots q_M)+\cdots,$$

where u_1, and v_1 represent the diverging waves for neutron and proton respectively and φ_1 ~~denote~~ represent ~~the a~~ stationary states ~~fo~~ of the nucleus of atomic number Z-1 formed after disintegration.

~~Denoting the Hamiltonians of the~~ by H_1, H_2, V, and E, the Hamiltonians of the heavy particle and the nucleus, their mutual interaction energy ~~between~~ and the energy of the system,

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In this case, the
~~the~~ Schrödinger equation

$$(H_1+H_2+V)\,\psi = E\,\psi, \qquad (1)$$

where H_1, H_2, V and E denote the . . —
. . . respectively, can be
transformed into the ~~form following~~ system of equations

$$\left.\begin{aligned} H_1u_0+V_{00}u_0+V_{01}u_1+\cdots+J_{01}v_1+\cdots &= (E-W_0)\,u_0\\ H_1u_1+V_{10}u_0+V_{11}u_1+\cdots+J_{11}v_1+\cdots &= (E-W_1)\,u_1\\ \cdots\cdots\cdots &\\ H_1v_1+J_{10}u_0+\cdots\quad+U_{11}v_1+\cdots &= (E-W_1')\,v_1\\ \cdots\cdots\cdots &\end{aligned}\right\}(2)$$

by ~~multiplying (1)~~ integrating (1) with respect to
$q_1q_2\cdots q_M$ after ~~multiplying~~ having multiplied ~~by~~
both sides by $\chi_0, \chi_1, \cdots \varphi_1, \varphi_2, \cdots$ in turn ~~respectively~~.
In these equations $W_0, W_1, \cdots W_1' \cdots$ denote the energies
of the ~~nuclear~~ nuclear states $\chi_0, \chi_1, \cdots \varphi_1, \cdots$ ~~of the~~ respectively
and

$$V_{00} = \int\!\!\cdots\!\!\int \tilde{u}_0 V u_0\, dq_1\cdots dq_M \qquad V_{01} = \int\!\!\cdots\!\!\int \tilde{u}_0 V u_1\, dq_1\cdots dq_M \text{ etc}$$

~~$V_{10} = \int\cdots\int V$~~

$$J_{01} = \int\!\!\cdots\!\!\int \tilde{u}_0 V v_1\, dq_1\cdots dq_M \text{ etc.}$$

$$U_{11} = \int\!\!\cdots\!\!\int \tilde{v}_1 V v_1\, dq_1\cdots dq_M \text{ etc},$$

which ~~are~~ depend only on q_0.
~~Thus~~ The ~~equat~~ complicated problems of inelastic scattering and the
disintegration with particle emission are thus reduced

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to the solution of ~~the~~ simultaneous equations for one particle only, ~~which can be solve~~ from which the cross sections of these processes can be obtained ~~by~~ by exact expressions of the ~~by assuming~~ certain simplifying assumptions (?)

In the above ~~trans~~ deduction, we assumed ~~how~~ that the impinging particle in the ~~neutron~~ ~~itself~~ state is scattered back ~~or~~ transformed into the outgoing proton. ~~The More general case, We~~ It can be shown, however, that more ~~general~~ ~~process~~ ~~case~~ complicated case, in which ~~other~~ the impinging particle ~~neutron~~ is captured and ~~a parti neutron or a proton~~ ~~or~~ a particle in the nucleus is emitted as a neutron or a proton, can be ~~reduced to the~~ dealed with in the similar ~~by~~ way, only if ~~the~~ the range of the forces between heavy particles are so small ~~have~~ ~~have very short range, so~~ that they can be ~~replaced~~ expressed by δ-functions. appr.

Namely, inserting the ~~stationary~~ solution ~~for this case~~ for this case has the form of the form

$$\psi = u_0(q_0)\,\chi_0(q_1 q_2 \cdots q_m) + \cdots + v_1(q_1)\,\varphi_1(q_0 q_2 \cdots)$$

~~for example~~, ~~for example~~, in the Schrödinger equation (1) and integrating with respect to $q_1 q_2 \cdots q_m$ after having multiplied by $\chi_0(q_1 q_2 \cdots) \cdots$, we ~~have~~ obtain the equations

$$H_1 u_0 + V_{01} u_1 + \cdots \qquad + K_{01} v_1 = (E - W_0) u_0 \qquad (\ )$$

$$\cdots\cdots$$

~~depending~~ for ~~the~~ q_0 only, where

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H_1 $\chi_0(q_1 \cdots q_M)$ $H_1(q_1) + H_1(q_0) + V(q_1, q_0)$ $V(q_1, q_0)$ $\delta(q_1, q_0)$

This relation ~~to~~ hold if V is assumed to ~~have~~ have the term involving $\delta(q_0, q_1)$. ~~The term~~ It should be ~~like~~ noticed that the integral like $\iint \tilde{\chi}_0(q_1 q_2 \cdots) v_1(q_0)\varphi_1(q_0 q_2 \cdots)\, dq_1 \cdots dq_m$ $\delta(q_1 q_2)$ dq_2

$$= \iint \tilde{\chi}_0(q_1 q_0 \cdots)\, v_1(q_0)\, \varphi_1(q_0 q_0 \cdots)\, dq_1 dq_2 \cdots dq_n$$

vanishes on account of ~~the Pauli~~ the exclusion principle for the particles in the nucleus.

$$\cancel{K_{01} = \int \cdots \int \tilde{\chi}_0(q_1 \cdots q_M) V \chi(q_0 \cdots q_M)}$$

$$\cancel{K_{01}(q_0) = \int \cdots \int \tilde{\chi}_0(q_1 \cdots q_M) V\, v_1(q_1)\, \chi(q_0 \cdots q_M)\, dq_1 \cdots dq_M,}$$

$(v_1(q_0))$ $= \delta K_{01}$

~~which~~ assum

~~the~~ $\cancel{K_{01}' = \int \cdots \int \tilde{\chi}_0(q_1 \cdots q_M) V \varphi_1(q_0 \cdots q_M)\, dq_1 \cdots dq_M.}$

$$K_{01}(q_0) v_1(q_0) = \int \cdots \int \tilde{\chi}_0(q_1 \cdots q_M)\, V\, v_1(q_1)\, \chi(q_0 \cdots q_M)\, dq_1 \cdots dq_M, \dagger$$

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~~which can be~~ The last relation hold if V has the term ~~like~~ ~~K~~ involving $\delta(q_0, q_1)$. †

~~Finally, by the wave integrating (1)~~
Further, we obtain, the ~~equation~~ last equation

$$H_1 v_1 + K_{10} u_0 + \cdots + U_{11} v_1 = (E - W_1') v_1,$$

by integrating (1) with respect to $q_1, q_2 \cdots q_M$ after ~~interhaving~~ having interchanged q_0 and q_1 in (1) and multiplied by $\tilde{\varphi}_1(q_1 q_2 \cdots)$

which together with ( ) forms a complete set of equations for the function $u_0, u, \cdots v_1$ ~~for~~ one particle only.

Thus, whatever be the ~~type of~~ interaction between

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heavy particles, the nuclear collision problem can be reduced to simple one body problem only if ~~the~~ the range of the interaction is assume to be very small.

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$$\psi(q_0, q_1, \cdots q_M) = u_0(q_0)\chi_0(q_1 q_2 \cdots q_M)$$
$$+ \cancel{u_1(q_1 q_2 \cdots q_M)} + u_1(q_0)\chi_1(q_1 q_2 \cdots q_M) + v_1(q_0)\varphi_1(q_1 q_2 \cdots q_M),$$

where q_0 ~~den~~ ~~represent~~ denote coordinates and spin of the impinging particle and both u_0 and u_1 ~~den~~ represent ~~mean~~ ~~ty~~ states of the neutron ~~neutron~~ states, ~~while~~ ~~v_1 mean a proton state~~ while φ_1 denote ~~an~~ the state of the nucleus of atomic number $Z-1$ formed after emission of the ~~nuclear~~ proton in a state v_1.

~~The~~ Denoting the Hamiltonians of the ~~neu~~ heavy particle and the nucleus by H_1 and H_2, the interaction potential by V, and the total energy by E, we obtain the