Letter to the Editor

Reduction of Nuclear Collision Problems

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with γ-ray emission, have been solved hitherto by straightforward application of the quantum mechanical method. Now it will be shown that other ~~processes~~ complicated processes can also be reduced in the following way to one body problems so as to make it possible to apply the exact theory of collision ~~in the following way~~.

If only the possibility of the elastic scattering is considered, the stationary state of the whole system will be expressed in the form $u_0(q_0)\chi_0(q_1 q_2 \cdots q_N)$, where u_0 is the superposing plane and ~~the~~ scattered waves ~~eigenfunction~~ for the impinging particle, in ~~the~~ neutron state for instance, ~~consisting of the plane and the scattered wave~~ and χ_0 ~~is the~~ represents the normal state of the nucleus, ~~of~~ with atomic number Z and mass number ~~M~~ N for instance[1]. Now, if the possibilities of the excitation of the nucleus to ~~another~~ other states, χ_1, χ_2 ... say, with energies W_1, ... respectively, and of the disintegration with the emission of another heavy particle, in the proton states for instance, are taken into account, the stationary solution should have the general form

$$\psi(q_0 q_1 \cdots q_N) = u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + u_1(q_0)\chi_1(q_1 q_2 \cdots q_N) + \cdots + v_1(q_0)\varphi_1(q_1 q_2 \cdots q_N) + \cdots \quad (1)$$

where u_1, ~~etc.~~ and v_1, etc. represent ~~the diverging~~ inelastically scattered ~~wave~~ neutron waves and outgoing proton waves respectively and φ_1, ... represent stationary states, with energies W_1, ... respectively, of the nucleus of atomic number $Z-1$ formed by disintegration.

Inserting (1) in the Schrödinger equation

$$(H_1 + H_2 + V)\psi = E\psi \quad (2),$$

where H_1, H_2, V and E denote the Hamiltonians of the impinging

1). The recoil of the nucleus will be neglected throughout, which will be permitted for majority of cases. q's ~~repr~~ should be understood ~~to~~ ~~involve~~ ~~also~~ ~~represent~~ as positions, spins and ~~coordinates~~ ~~quantities~~ variables discriminating neutron and proton states ~~are involved~~ altogether of heavy particle

particle and the nucleus, their interaction energy and the energy of the whole system, and integrating it with respect to $q_1 q_2 \cdots q_N$ after having multiplied both sides by $\tilde{X}_0, \tilde{X}_1, \cdots \tilde{\varphi}_1, \cdots$ in turn, we obtain ~~the system~~ simultaneous equations

$$\left.\begin{aligned} H_1 u_0 + V_{00} u_0 + V_{01} u_1 + \cdots + J_{01} v_1 + \cdots &= (E - W_0) u_0 \\ H_1 u_1 + V_{10} u_0 + V_{11} u_1 + \cdots + J_{11} v_1 + \cdots &= (E - W_1) u_1 \\ \cdots\cdots \\ H_1 v_1 + J_{10} u_0 + \tilde{J}_{11} u_1 + \cdots + U_{11} v_1 + \cdots &= (E - W_1') v_1 \\ \cdots\cdots \end{aligned}\right\} \quad (3)$$

where $W_0, W_1, \cdots W_1', \cdots$ denote the energies of the nucleus in states $X_0, X_1, \cdots \varphi_1, \cdots$ respectively and

$$V_{00} = \int\cdots\int \tilde{X}_0 V X_0 \, dq_1 \cdots dq_N \qquad V_{01} = \int\cdots\int \tilde{X}_0 V X_1 \, dq_1 \cdots dq_N \quad \cdots$$
$$J_{01} = \int\cdots\int \tilde{X}_0 V \varphi_1 \, dq_1 \cdots dq_N \qquad J_{11} = \int\cdots\int \tilde{X}_1 V \varphi_1 \, dq_1 \cdots dq_N \quad \cdots$$
$$U_{11} = \int\cdots\int \tilde{v}_1 V v_1 \, dq_1 \cdots dq_N \qquad \cdots\cdots$$

which depend only on q_0.

The complicated problems of inelastic scattering and ~~the~~ disintegration with particle emission are thus reduced to those of solving simultaneous equations for one particle only, from which the exact expressions of the cross sections of these processes can be obtained by assuming special form for V.[2)]

40 In the above deduction, ~~we~~ it was assumed that the ~~impinging particle in the neutron state is scattered back~~ scattered ~~neutron~~ particle or the emitted ~~proton~~ particle ~~can~~ can be identified with the impinging ~~neutron~~ particle. ~~It can be also be~~ In more complicated case, in which ~~the~~ it is possible that the impinging ~~neutron~~ particle is captured and ~~a~~ one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of ~~the~~ forces (except Coulomb forces, of course,) between heavy particles are so small that they can be ~~replaced by multiples~~

2) Detailed accounts for special cases will be given in forthcoming issue of Proc. Phys.-Math. Soc. Japan.

$$\int\cdots\int \tilde{\chi}_0(q_1 q_2 \cdots)(H_1 + H_2 + V)\{u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + \cdots + v_1(q_1)\varphi_1(q_0 q_2 \cdots)\}\, dq_1 \cdots dq_N$$

$$= H_1 u_0 \quad \cancel{(H_1 + H_2 + V)} + W_0 u_0 + V_{00} u_0 + \cdots$$

$$+ \int \tilde{\chi}_0(q_1 \cdots)(H_1 v_1(q_1))\varphi_1(q_0 q_2 \cdots)\, dq_1 \cdots dq_N + W_1' \int \tilde{\chi}_0(q_1 \cdots q_N) v_1(q_1)\varphi_1(q_0 q_2 \cdots)\, dq_1 \cdots dq_N$$

$$+ \int \tilde{\chi}_0(q_1 q_2 \cdots) V v_1(q_1)\varphi_1(q_0 q_2 \cdots)\, dq_1 \cdots dq_N$$

$$\int T_{01}(q_0 q_1) v_1(q_1)\, dq_1 \qquad T_{01}(q_0, q_1) = \int \tilde{\chi}_0(q_1 \cdots) H_1(1)\varphi_1(q_0 q_2 \cdots)\, dq_2 \cdots dq_N$$

$$W_1' \int \cancel{W_{01}\tilde{\chi}_0(q_1 \cdots q_N)} \int W_{01}(q_0 q_1) v_1(q_1)\, dq_1$$

$$W_{01}(q_0 q_1) = W_1' \int \tilde{\chi}_0(q_1 \cdots q_N)\varphi_1(q_0 q_2 \cdots q_N)\, dq_2 \cdots dq_N$$

T_{01}, W_{01} $\neq 0$ for $0 < q_1 < a$, $0 < q_0 < a$

~~$q_1 > a$~~ $\int T_{01}(q_0 q_1) v_1(q_1)\, dq_1 \cancel{= 0 \text{ for } q_1 > a}$

$$T_{01}(q_0) v_1(q_0) = \int T_{01}(q_0 q_1) v_1(q_1)\, dq_1$$

$$\int\cdots\int \tilde{\varphi}_1(q_1 q_2, \cdots)(H_1 + H_2 + V)(u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + \cdots + v_1(q_1)\varphi_1(q_0 q_2 \cdots)) \times dq_1 \cdots dq_N$$

$$\iint \tilde{\varphi}_1(q_1 \cdots)\,\delta(q_0 q_1)\chi_0(q_1 \cdots)$$
$$= \iint \tilde{\varphi}_1(q_0 \cdots)\chi_0(q_0 \cdots)\, dq_2 \cdots dq_N$$

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sums of ~~multiples of δ-functions~~ expressed by δ-functions.
Namely, inserting the stationary solution of the form

$$\Psi = u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + \cdots + v_1(q_1)\varphi_1(q_0 q_2 \cdots) + \cdots$$

for example in the equation (2) and integrating with respect to $q_1, q_2, \cdots q_N$ after having multiplied both sides by $\tilde{\chi}_0, \cdots \tilde{\varphi}_1, \cdots$ in turn, we obtain the equations

$$\left.\begin{array}{l} H_1 u_0 + V_{01} u_1 + \cdots \quad + K_{01} v_1 = (E - W_0) u_0 \\ \cdots \end{array}\right\} \quad (4)$$

for q_0 only, where

$$K_{01}(q_0) v_1(q_0) = \int \cdots \int \tilde{\chi}_0(q_1 q_2 \cdots q_N) V^* v_1(q_1) \cancel{\chi}\,\varphi_1 (q_0 \cdots q_N)\, dq_1 \cdots d_N,$$

on account of the term in V^* involving $\delta(q_1, q_0)$.[3] Further, by integrating (2) with respect to $q_1 q_2 \cdots q_N$ after having interchanged q_0 and q_1 in (1) and multiplied by $\tilde{\varphi}_1(q_1 q_2 \cdots q_N)$ in turn, we obtain ~~the last~~ the remaining equations

$$\left.\begin{array}{l} H_1 v_1 + K_{10} u_0 + \cdots \quad \cdots + U_{11} v_1 = (E - W_1') v_1, \\ \cdots \end{array}\right\} \quad (4)'$$

which, together with (4), ~~form a complete set of equations for the function u_0 determining~~ just enable us to determine $u_0, u_1, \cdots v_1, \cdots$.

Thus, whatever be the interaction between heavy particles, the nuclear problems ~~can be~~ above considered can be reduced to simple one body problems accessible to exact calculations, only if the range of the

60 interaction is assumed to be very small.

Complete ~~paper will be~~ report, in which ~~especially~~ especially the ~~taking~~ ~~concerning~~ ~~of~~ symmetry property of the eigenfunction is taken into account, will be published in Proc. Phys.-Math. Soc. Japan.

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3) V^* ~~denotes the~~ in the symbol ~~*~~ means the interchange of q_0 and q_1 in V. Other terms in V^* involving $\delta(q_1, q_2)$, for example, ~~vanishes~~ does not contribute to the integral on account of the exclusion principle for the particles in the nucleus.