Letter to the Editor

Reduction of Nuclear Collision Problems

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with γ-ray emission, have been solved hitherto by straightforward application of the quantum mechanical method. Now it will be shown that other complicated processes can also be reduced to one body problems in the following way so as to make it possible to apply the exact theory of collision to them.

First, if the possibility of the elastic scattering only is considered, the stationary state of the whole system will be expressed in the form $u_0(q_0)\chi_0(q_1 q_2 \cdots q_N)$, where u_0 is the superposition of plane and scattered waves for the impinging particle, in the neutron state for instance, and χ_0 represents the normal state with energy W_0 of the nucleus with atomic number Z and mass number N.[1] Next, if the possibilities of the excitation of the nucleus to one of the states $\chi_1, \ldots$, with energies $W_1, \ldots$ respectively, and of the disintegration with the emission of a heavy particle, in the proton state for instance, are taken into account, the stationary solution should have the general form

$$\psi(q_0 q_1 q_2 \cdots q_N) = u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + u_1(q_0)\chi_1(q_1 q_2 \cdots q_N) + \cdots$$
$$+ v_1(q_0)\varphi_1(q_1 q_2 \cdots q_N) + \cdots, \qquad (1)$$

where $u_1, \ldots$ and $v_1, \ldots$ represent inelastically scattered neutron waves and outgoing proton waves respectively and $\varphi_1, \ldots$ represent stationary

---

1) The recoil of the nucleus will be neglected throughout, which will be permitted for majority of cases. It should be understood that positions, spins, and the variables discriminating neutron and proton states are all involved in q's.

states, with energies W_1',.... respectively, of the nucleus of atomic number Z-1 formed by disintegration.

Inserting (1) into the Schrödinger equation

$$(H_1 + H_2 + V)\psi = E\psi, \qquad (2)$$

where H_1, H_2, V and E denote the Hamiltonians of the impinging particle and the nucleus, their interaction energy and the energy of the whole system, and integrating it with respect to $q_1 q_2 \dots q_N$ after having multiplied both sides by $\tilde{\chi}_0, \tilde{\chi}_1, \dots, \tilde{\varphi}_1, \dots$ in turn, we obtain simultaneous equations

$$\left.\begin{array}{l} H_1 u_0 + V_{00} u_0 + V_{01} u_1 + \dots + J_{01} v_1 + \dots = (E - W_0) u_0 \\ H_1 u_1 + V_{10} u_0 + V_{11} u_1 + \dots + J_{11} v_1 + \dots = (E - W_1) u_1 \\ \dots\dots \\ H_1 v_1 + J_{10} u_0 + \tilde{J}_{11} u_1 + \dots + U_{11} v_1 + \dots = (E - W_1') v_1 \\ \dots\dots, \end{array}\right\} \qquad (3)$$

where

$$V_{00} = \int\cdots\int \tilde{\chi}_0 V \chi_0 dq_1 \dots dq_N, \quad V_{01} = \int\cdots\int \tilde{\chi}_0 V \chi_1 dq_1 \dots dq_N, \quad \dots\dots$$

$$J_{01} = \int\cdots\int \tilde{\chi}_0 V \varphi_1 dq_1 \dots dq_N, \quad J_{11} = \int\cdots\int \tilde{\chi}_1 V \varphi_1 dq_1 \dots dq_N, \quad \dots\dots$$

$$U_{11} = \int\cdots\int \tilde{\varphi}_1 V \varphi_1 dq_1 \dots dq_N, \quad \dots\dots,$$ [2]

~~which depend only on q_0.~~

The complicated problems of inelastic scattering and disintegration with particle emission are thus reduced to those of solving simultaneous equations (3) for one particle only, enabling us to obtain exact expressions of the cross sections of these processes by assuming special form for V.[3]

---

3) Detailed accounts for special cases will be given in forthcoming issue of Proc. Phys.-Math. Soc. Japan.

2) These are operators diagonal with respect to the position $\vec{r}_0$ of the impinging particle.

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more complicated case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb forces, of course,) between heavy particles are so small that they can be replaced by δ-functions.

Namely, ~~inserting~~ assuming the stationary solution of the form

$$\psi = u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + \cdots + v_1(q_1)\varphi_1(q_0 q_2 \cdots q_N) + \cdots$$

~~in~~ for the equation (2) and integrating with respect to $q_1 q_2 \cdots q_N$ after having multiplied both sides by $\tilde{\chi}_0, \cdots, \tilde{\varphi}_1, \cdots$ in turn, we obtain the equations

$$\left.\begin{array}{l} H_1 u_0 + V_{01} u_1 + \cdots + K_{01} v_1 + \cdots = (E - W_0) u_0 \\ \cdots\cdots \end{array}\right\} \quad (4)$$

involving q_0 only, where ~~K_{01} is the operator defined by~~

$$K_{01} v_1(q_0) = \int \cdots \int \tilde{\chi}_0(q_1 q_2 \cdots q_N) V^* v_1(q_1) \varphi_1(q_0 q_2 \cdots q_N) dq_1 dq_2 \cdots dq_N, \quad \cdots\ ^{3)},$$

the symbol * denoting the interchange of q_0 and q_1 ~~$^{3)}$~~, ~~$V^{3)}$~~, ~~the par~~ [illegible]
~~on account of the term in V^* involving $\delta(q_1 q_0)$. The operation on~~

Further, by integrating (2) with respect to $q_1 q_2 \cdots q_N$ after having interchanged q_0 and q_1 in (2) and multiplied by $\tilde{\varphi}_1(q_1 q_2 \cdots q_N)$, $\cdots$ in turn, we obtain the remaining equations

$$\left.\begin{array}{l} H_1 v_1 + K_{10} u_0 + \cdots + U_{11} v_1 + \cdots = (E - W_1') v_1 \\ \cdots\cdots, \end{array}\right\} \quad (4)'$$

~~3) The symbol * means the interchange of q_0 and q_1 in V. Other terms in V^*, involving $\delta(q_1, q_2)$ for example, do not contribute to the integral on account of the exclusion principle for the particles in the nucleus.~~

3) K_{01} ~~is an operator a matrix operating on the spin, the position~~ again becomes an operator diagonal with respect to $q_1 q_2 \ldots \vec{r}_0$ ~~where~~ on account of the ~~facto~~ term in V^* involving $\delta(\vec{r}_1, \vec{r}_0)$, other terms ~~such as~~ involving $\delta(q, \vec{r}_1, \vec{r}_2)$, ~~etc.~~ $\cdots$ respectively having no contribution because of the exclusion principle for the particles in the nucleus.

which, together with (4), are enough to determine u_0, u_1, ..., v_1, ...

Thus, whatever be the interaction between heavy particles, the nuclear collision problems above considered can be reduced to simple one body problems accessible to exact calculations, only if the range of the interaction is assumed to be very small.

Complete discussions of the subject, especially of the symmetry property of the eigenfunction, will be made in Proc. Phys.-Math. Soc. Japan.

Hideki Yukawa

Department of Physics,

Osaka Imperial University,

February 7, 1936.

states, with energies W ,.... respectively, of the nucleus of atomic number Z-1 formed by disintegration.

Inserting (1) into the Schrödinger equation

$$(H + H + V) \quad = E \quad , \qquad (2)$$

where H , H , V and E denote the Hamiltonians of the impinging particle and the nucleus, their interaction energy and the energy of the whole system, and integrating with respect to $q\ q\ \ldots q$ after having multiplied both sides by in turn, we obtain simultaneous equations

$$
\begin{aligned}
&H\,u + V\,u + V\,u + \ldots + J\,v + \ldots = (E-W\ )u \\
&H\,u + V\,u + V\,u + \ldots + J\,v + \ldots = (E-W\ )\bar{u} \\
&\quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \qquad (3) \\
&H\,v + J\,u + J\,u + \ldots + U\,v + \ldots = (E-W\ )v \\
&H\,v + \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ ,
\end{aligned}
$$

where

$$V = \quad V\ dq\ \ldots dq\ ,\ \ldots\ldots$$

are operators all diagonal with respect to the position r of the impinging particle.

The complicated problems of inelastic scattering and disintegration with particle emission are thus reduced to those of solving simultaneous equations (3) for one particle only, enabling us to obtain exact expressions of the cross sections of these processes by assuming suitable forms for V.

---

2) Results of calculations for special cases will be published in forthcoming issue of Proc. Phys.-Math. Soc. Japan.

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more general case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb force, of course,) between heavy particles are so small that they can be replaced by -functions.

Namely, assuming a stationary solution of the form, for example,

$$= u (q) (q q \ldots q) + \ldots + v (q) (q q \ldots q) + \ldots$$

for the equation (2) and integrating with respect to q q ...q after having