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Letter to the Editor

Reduction of Nuclear Collision Problems

Among various processes caused by the collision of the heavy particles with the nucleus, only two cases, namely the elastic scattering and the capture with γ -ray emission, have been solved hitherto by straightforward application of the quantum mechanical method. Now it will be shown that other complicated processes can also be reduced to one body problems in the following way so as to make it possible to apply the exact theory of collision to them.

First, if the possibility of the elastic scattering only is considered, the stationary state of the whole system will be expressed in the form $u(q)(qq \dots q)$, where u is the superposition of plane and scattered waves for the impinging particle, in the neutron state for instance, and represents the normal state with energy W of the nucleus with atomic number Z and mass number N . Next, if the possibilities of the excitation of the nucleus to one of the states $W, \dots$, with energies $W, \dots$ respectively, and of the disintegration with the emission of a heavy particle, in the proton state for instance, are taken into account, the stationary solution should have the general form

$$(qqq \dots q) = u(q)(qq \dots q) + u(q)(qq \dots q) + \dots + v(q)(qq \dots q) + \dots, \quad (1)$$

where $u, \dots$ and $v, \dots$ represent inelastically scattered neutron waves and outgoing proton waves respectively and $\dots$ represent stationary

1) The recoil of the nucleus will be neglected throughout, which will be permitted for majority of cases. It should be understood that positions, spins, and the variables discriminating neutron and proton states are involved in qs .

states, with energies $W, \dots$ respectively, of the nucleus of atomic number $Z-1$ formed by disintegration.

Inserting (1) into the Schrödinger equation

$$(H + H + V) = E, \quad (2)$$

where H, H, V and E denote the Hamiltonians of the impinging particle and the nucleus, their interaction energy and the energy of the whole system, and integrating it with respect to $q \dots q$ after having multiplied both sides by

in turn, we obtain simultaneous equations

$$\begin{aligned} H u + V u + V u + \dots + J v + \dots &= (E - W) u \\ H u + V u + V u + \dots + J v + \dots &= (E - W) u \\ \dots \dots \dots \\ H v + J u + J u + \dots + U v + \dots &= (E - W) v \end{aligned} \quad (3)$$

where

$$\begin{aligned} V &= \int V dq \dots dq, & V &= \int V dq \dots dq, & \dots \\ J &= \int V dq \dots dq, & J &= \int V dq \dots dq, & \dots \\ U &= \int V dq \dots dq, & \dots & \dots, & \end{aligned}$$

which depend only on q .

The complicated problems of inelastic scattering and disintegration with particle emission are thus reduced to those of solving simultaneous equations (3) for one particle only, enabling us to obtain exact expressions of the cross sections of these processes by assuming special form for V .

2) Detailed accounts for special cases will be given in forthcoming issue of Proc. Phys.-Math. Soc. Japan.

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more complicated case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb forces, of course,) between heavy particles are so small that they can be replaced by δ -functions.

Namely, inserting the stationary solution of the form

$$\psi = u(q) \chi(q_1, q_2, \dots, q_n) + \dots + v(q_1) \varphi_1(q_2, \dots, q_n) + \dots$$

in the equation (2) and integrating with respect to $q_1, \dots, q_n$ after having multiplied both sides by $\chi^*, \dots, \varphi_1^*, \dots$ in turn, we obtain the equations

$$H u + V u + \dots + K v + \dots = (E - W) u \quad (4)$$

involving q only, where

$$K(q) v_1(q) = \int \chi^*(q, q_1, \dots, q_n) V v_1(q_1) \varphi_1(q_2, \dots, q_n) dq_1 dq_2 \dots dq_n, \dots$$

on account of the term in V^* involving $\delta(q, q_1)$.

Further, by integrating (2) with respect to $q_1, q_2, \dots, q_n$ after having interchanged q and q_1 in (2) and multiplied by $\varphi_1^*(q, q_2, \dots, q_n)$, in turn, we obtain the remaining equations

$$H v_1 + K u + \dots + U v_1 + \dots = (E - W_1) v_1 \quad (4')$$

* The symbol $*$ means the interchange of q and q_1 in V . Other terms in V , involving $\delta(q, q_1)$ for example, do not contribute to the integral on account of the exclusion principle for the particles in the nucleus.

which, together with (4), are enough to determine u , u , u , ..., v ,

Thus, whatever be the interaction between heavy particles, the nuclear collision problems above considered can be reduced to simple one body problems accessible to exact calculations, only if the range of the interaction is assumed to be very small.

Complete discussions, of the subject, especially of the symmetry property of the eigenfunction, will be made in Proc. Phys.-Math. Soc. Japan.

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