Note on the ~~Scattering~~ Cross Section of Scattering of Neutrons by Protons.

Assuming a Rectangular hole of radius a and depth V_0, we obtain

~~Wigner's suggestion~~

$$\psi = b \sin k'r \qquad \text{for } r < a$$

$$\psi = c \sin(kr + \delta_0) \qquad \text{for } r > a$$

$$\cot \cancel{\tan}(ka + \delta_0) = \frac{k}{k'} \cancel{\tan} k'a$$

$$= \frac{k'}{k} \cot k'a$$

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$$\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0 = \frac{4\pi}{k^2(1 + \cot^2 \delta_0)}$$

$$\cot(k_0 a + \delta_0^{(0)}) = 0 \quad : \quad k_0 a + \delta_0^{(0)} = \frac{\pi}{2}.$$

$$\sigma_0 = \frac{4\pi}{k_0^2} \cos^2 k_0 a \simeq \frac{4\pi}{k_0^2}$$

$$\cancel{\cot(ka + \delta_0)} = \quad k' = \frac{\sqrt{M(V_0 + \frac{E_0}{2})}}{\hbar},$$

$$k = \frac{\sqrt{\frac{ME_0}{2}}}{\hbar}.$$

$$k_0' a = \frac{\sqrt{M(V_0 + \frac{E_0}{2})}}{\hbar} \cdot a = \frac{\pi}{2}.$$

$$\tan(ka + \delta_0) = \frac{\tan ka + \tan \delta_0}{1 - \tan ka \tan \delta_0}$$

$$\cot(ka + \delta_0) = \frac{\cancel{\cot ka}\ \cot ka \cot \delta_0 - 1}{\cot \delta_0 + \cot ka} = \frac{\cot ka \cot \delta_0 - 1}{\cot \delta_0 + \cot ka} = \frac{k'}{k} \cot k'a.$$

$$\cot \delta_0 = \frac{\frac{k'}{k} \cot k'a + \cot ka}{\frac{k'}{k} \cot k'a \cot ka - 1} = \frac{k' \cot k'a + k \cot ka}{k' \cot k'a \cdot \cot ka - k}$$

$$1 + \cot^2 \delta_0 = \frac{(k' \cot k'a \cot ka)^2 + (k' \cot k'a)^2 + (k \cot ka)^2 + k^2}{(k' \cot k'a \cot ka - k)^2}$$

$$= \frac{(\cot^2 ka + 1)(k'^2 \cot^2 k'a + k^2)}{(k' \cot k'a \cot ka - k)^2}$$

$$\sigma = \frac{4\pi}{k^2} \cdot \frac{(k' \cot k'a \cot ka - k)^2}{(\cot^2 ka + 1)(k'^2 \cot^2 k'a + k^2)}$$

$$= \frac{4\pi}{k^2} \cdot \frac{(k' \cos k'a \cos ka - k \sin k'a \sin ka)^2}{k'^2 \cos^2 k'a + k^2 \sin^2 k'a}$$

$$k'a = \frac{\pi}{2} - \varepsilon \qquad \cos k'a = \varepsilon \qquad \sin k'a = 1 - \frac{\varepsilon^2}{2}$$

$$\sigma \cong \frac{4\pi}{k^2} \cdot \frac{\left(k' \varepsilon \cdot \left(1 - \frac{(ka)^2}{2}\right) - k\left(1 - \frac{\varepsilon^2}{2}\right) ka\right)^2}{k'^2 \varepsilon^2 + k^2 (1 - \varepsilon^2)}$$

$$\cong \frac{4\pi}{k^2} \frac{(k'\varepsilon - k^2 a)^2}{k'^2 \varepsilon^2 + k^2} = \frac{4\pi}{k^2} \cdot \frac{\{(k'a)\varepsilon - (ka)^2\}^2}{(k'a)^2 \varepsilon^2 + (ka)^2}$$

$$\cong \frac{4\pi}{k^2} \frac{\left\{\frac{\pi}{2}\varepsilon - \varepsilon^2 - (ka)^2\right\}^2}{\left(\frac{\pi}{2}\right)^2 \varepsilon^2 +}$$

$$\cot \delta_0 = \frac{-\left(1 + \frac{k'}{k} \cot k'a \cot ka\right)}{\frac{k'}{k} \cot k'a - \cot ka}$$

$$= \frac{-(k' \cot k'a \cot ka + k)}{k' \cot k'a - k \cot ka}$$

$$1 + \cot^2 \delta_0 = \frac{(k' \cot k'a)^2 + (k \cot ka)^2 + (k' \cot k'a \cot ka)^2 + k^2}{(k' \cot k'a - k \cot ka)^2}$$

$$= \frac{(\cot^2 ka + 1)(k'^2 \cot^2 k'a + k^2)}{(k' \cot k'a - k \cot ka)^2}$$

$$= \frac{\cancel{\cot^2 ka}\; k'^2 \cos^2 k'a + k^2 \sin^2 k'a}{(k' \cos k'a \sin ka - k \sin k'a \cos ka)^2}$$

$$\sigma = \frac{4\pi}{k^2} \frac{(k' \cos k'a \sin ka - k \sin k'a \cos ka)^2}{k'^2 \cos^2 k'a + k^2 \sin^2 k'a}$$

$$k'a = \frac{\pi}{2} - \varepsilon. \qquad \cos k'a = \varepsilon \qquad \sin k'a = 1 - \frac{\varepsilon^2}{2}$$

$$\sigma \cong 4\pi a^2 \frac{\{k'a \cdot \varepsilon - ka(1-\frac{\varepsilon^2}{2})(1-\frac{(ka)^2}{2})\}^2}{k'^2 a^2 \cdot \varepsilon^2 + k^2 a^2 (1-\varepsilon^2)}$$

$$\cong 4\pi \cdot \frac{1}{k'^2 \varepsilon^2 + k^2}$$

$$\frac{\sqrt{M(V_0+\frac{E}{2})}}{\hbar} \cdot a = \frac{\pi}{2} - \varepsilon,$$

$$\sqrt{M(V_0+\frac{E_0}{2})}\left(1 - \frac{E_0 - E}{2(V_0+\frac{E_0}{2})}\right) \cong \sqrt{M_0(V_0+\frac{E_0}{2})}\left\{1 - \frac{E_0-E}{4(V_0+\frac{E_0}{2})}\right\}$$

$$\frac{\sqrt{M_0(V_0+\frac{E_0}{2})}}{\hbar} \cdot a = \frac{\pi}{2}$$

$$(k'a)^2 = (\frac{\pi}{2} - \varepsilon)^2$$

$$\frac{\pi}{2} \frac{(E_0 - E)}{4(V_0+\frac{E_0}{2})} = \varepsilon,$$

$$\cong 4\pi a^2 \frac{1}{(\frac{\pi}{2})^2 \cdot (\frac{\pi}{2})^2 \cdot \left\{\frac{E_0 - E}{4(V_0+\frac{E_0}{2})}\right\}^2}$$

$$\cong \frac{4^5}{\pi^3} a^2 \cancel{\frac{E_0}{V_0}} \cdot \left(\frac{V_0}{E_0}\right)^2$$

$$a^2 \cong \frac{\hbar^2}{M_0 V_0} \cancel{(\tfrac{2}{\pi})^2} (\frac{\pi}{2})^2 \qquad V_0 = \frac{\hbar^2}{M_0 a^2}(\frac{\pi}{2})^2$$

$$\sigma \cong \cancel{\frac{4^5\, 2 \times 256}{\pi^3}} \cdot \frac{4^4}{\pi} \cdot \frac{\hbar^2 (MV_0)}{(ME_0)^2}$$

$$\cong 4^3 \pi \quad \frac{\hbar^4}{M_0^2 a^2} \frac{1}{E_0^2}$$

$$(\sigma_B = 4\pi \hbar^2 \cdot / ME_0 \qquad \frac{\sigma}{\sigma_B} = \frac{16\hbar^2}{M a^2 E_0} = \frac{8\lambda_0^2}{a^2}$$

$\cancel{\frac{\hbar^2}{4 M E_0}} = 10^{-54}$ $\quad 6.55 \quad E_0 \, \text{leg} = \varepsilon_0 \, \text{MEV} \quad a = 2\times 10^{-13}$

$$\sigma \cong \frac{4 \cdot \hbar^4}{\pi^3 \cdot M^2 \cdot \varepsilon_0^2 \quad a^2} = \cancel{\frac{16\times(6.55)^4\times(10^{-27})^4\times 3^4\times 10^{40}\times 10^{26}}{\pi^3\times(1.84)^2\times 10^6\times \varepsilon_0^2\times 4}}$$

$$= \cancel{\frac{4\times(6.55)^4\times 3^4}{\pi^3\times(1.84)^2\times\varepsilon_0^2}}\times 10^{-}$$

$$= \frac{1}{\pi^3}\frac{\hbar^4}{M^2 m^2 c^4 \varepsilon_0^2 a^2} \qquad 10^{82} \quad 10^{-8} \quad \begin{array}{r} 27 \\ 3 \\ \hline 81 \\ -82 \end{array}$$

$$= \frac{(6.55)^4\times(10^{-27})^4\times 10^{26}}{\pi^3\times(1.66)^2\times(10^{-24})^2\times(0.9)^2\times(10^{-27})^2\times 9^2\times 10^{-40}\,\varepsilon_0^2\times 4}$$

$$= \frac{(6.55)^4}{(1.66)^2\times\pi^3\times(0.9)^2\times 9^2\times 4}\times 10^{-20}\frac{1}{\varepsilon_0^2}$$

$\left(\frac{5}{3}\right)^2$

$$= \frac{(6.55)^4}{100\times\pi^3\times(0.9)^2\times 9}\times 10^{-20}\frac{1}{\varepsilon_0^2}$$

$$= \frac{(6.55)^4}{\pi\times(0.9)^3}\times 10^{-24}\times\frac{1}{\varepsilon_0^2}$$

$$= \frac{1840}{3.14\times 0.81\times 0.9}$$

$$= \frac{2\times 10^3\times 10^{-24}}{3.14\times 0.81}\times\frac{1}{\varepsilon_0^2}$$

$$= 4\times 10^{-24}\boxed{\frac{10^3}{5.1\,\varepsilon_0^2}}$$

$$\varepsilon_0 = \frac{10^5}{10^{-1}\cdot 100} \quad \frac{10^5}{5.1}$$

$$\begin{array}{r} 6.28 \\ 0.81 \\ \hline 628 \\ 5.044 \end{array} \qquad \begin{array}{r} 6.55 \\ 6.55 \\ \hline 3275 \\ 3275 \\ 3930 \\ \hline 42.9025 \end{array} \qquad \begin{array}{r} 42.9 \\ 42.9 \\ \hline 3861 \\ 858 \\ 1716 \\ \hline 1840.41 \end{array}$$

$$\sigma = 4\pi \frac{\left(k'a \cos k'a \frac{\sin ka}{ka} - \sin k'a \cos ka\right)^2}{k'^2 \cos^2 k'a + k^2 \sin^2 k'a}$$

$ka \ll 1,$
$k'a \simeq \frac{\pi}{2}.$
$k' \gg k$

$$\simeq 4\pi \frac{\sin^2 k'a}{k'^2 \cos^2 k'a \cancel{+ k^2 \sin^2 k'a}}$$

$$= \frac{4\pi}{k'^2} \cancel{\sin^2 k'a} \tan^2(k'a)$$

$$k' \cong \frac{\sqrt{MV_0}}{\hbar}$$

$$\frac{MV_0}{\hbar^2} \cong \frac{1}{a^2}\left(\frac{\pi}{2}\right)^2$$

$$\sigma \cong \frac{a^2}{\pi^3} \tan^2(k'a)$$

$$\frac{35 \times 10^{-24}}{4 \times 10^{-26}} \times \pi^3 = \tan^2 k'a,$$

$10^2 \times 10 \times 3.14$

$2.7 \times 10^4 = \tan^2 k'a$

$$\tan k'a \cong 1.6 \times 10^2$$

3.14
35
1570
942
4 | 109.90
27.

1.6
1.6
96
16
256

$k'a = \frac{\pi}{2} - \varepsilon.$　$\sin k'a = 1 - \frac{\varepsilon^2}{2}$
$\cos k'a = \varepsilon,$

$$\boxed{\varepsilon \cong \cancel{1.6 \times 10^2}\ \frac{1}{1.6 \times 10^2}}$$

$$\frac{\pi}{2} \frac{(E_0 - E)}{4 V_0} = \varepsilon$$

$$E_0 = \frac{8 V_0}{\pi \times 1.6 \times 10^2} = \frac{V_0}{20 \times \pi} = \frac{V_0}{63}$$

$\frac{30 \times 10^6}{60} = 5 \times 10^5$

$$= \frac{\pi^2}{a} \frac{8 V_0}{\pi} \cdot \cancel{=} \varepsilon$$

$$= \frac{8 V_0}{\pi} \cdot \frac{a}{\pi^{\frac{3}{2}} \sigma^{\frac{1}{2}}} = V_0 \frac{8 a}{\pi^{\frac{5}{2}} \sigma^{\frac{1}{2}}}$$

$$\frac{11.8 \times 2 \times 10^{-13}}{1.7 \times 10 \times 6 \times 10^{-12} \times 2}$$

$\sigma = 35 \times 10^{-24}$

$$\cong \cancel{\frac{V_0 a}{\sigma^{\frac{1}{2}}}}$$

$$\cong \frac{V_0}{100} \cdot 0.4 \quad \cong 10^5$$

$2 \sim 3 \times 10^5$