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Otto Scherzer, Über die Ausstrahlung bei der Bremsung von Protonen und Schnellen Elektronen (Ann. d. Phys. 13, 137, 1932)

§ 1. Vollständigkeit des ~~der~~ benutzten Orthogonalsystems

$$\psi(\alpha,\beta,k) = e^{ikx'} L_{-n}(-ik(r+x'))$$

$$x' = r(\cos\vartheta\cos\alpha + \sin\vartheta\sin\alpha\cos(\varphi-\beta))$$

$$\{e^{-ikx} L_n(ik\xi)\}^* = e^{ikx} L_{-n}(-ik\xi)$$

$$\xi = r + x$$

これを逆に、k の Eigenwert に対する出来れば

$$\psi = \int \sin\alpha\, d\alpha\, d\beta\, \psi(\alpha\beta k) \sum_{l,m} a_{lm} P_l^m(\cos\alpha)\, e^{im\beta} \quad (a)$$

が出来るならば。この事は後で証明を与へられる。。

いづれ

$$\psi(\alpha,\beta,k) = \frac{1}{2\pi i}\int (y+\tfrac{1}{2})^{-n}(y-\tfrac{1}{2})^{n-1}\, e^{ikr(y-\frac{1}{2})} \times e^{ikx'(y+\frac{1}{2})}\, dy$$

$$e^{ikx'(y+\frac{1}{2})} = \sum_{l'} (2l'+1)\, i^{l'} \sqrt{\frac{\pi}{2kr(y+\frac{1}{2})}}\; J_{l'+\frac{1}{2}}(kr(y+\tfrac{1}{2})) \times P_{l'}\left(\frac{x'}{r}\right)$$

$$P_{l'}\left(\frac{x'}{r}\right) = \sum_{m'} \frac{(l'-m')!}{(l'+m')!} P_{l'}^{m'}(\cos\vartheta)\, P_{l'}^{m'}(\cos\alpha)\, e^{im'(\varphi-\beta)}.$$

$$\therefore\ \psi = \sum_{l,m} a_{lm} R(n, ikr) P_l^m(\cos\vartheta)\, e^{im\varphi} \quad (b)$$

$$R(n, ikr) = \sqrt{2\pi}\, i^{l-1} \int (y+\tfrac{1}{2})^{-n}(y-\tfrac{1}{2})^{n-1} \times e^{ikr(y-\frac{1}{2})}\, \frac{J_{l+\frac{1}{2}}(kr(y+\frac{1}{2}))}{\sqrt{kr(y+\frac{1}{2})}}\, dy$$

このRは丁度 polar coord での Eigenfunction の radial part の事である。

故に a_{lm} ~~とおけば~~ $= \delta_{l_0 m_0}^{(lm)}$ とおけば、polar. coord における [illegible] の eigenfunction が出てくる。故に $\psi(\alpha,\beta,k)$ は Vollständig である。

(you $a_{lm} = \delta_{00}(l\,m)$ とおくと

$$\psi = \int \sin\alpha\, d\alpha\, d\beta\; \psi(\alpha\,\beta\,k)$$

は simple spherical wave になる.)

それ故 (b) なる形の式は常に (a) なる形に表せる。

~~(b)~~ ~~が~~ asymptotically に

$$e^{ikz} + \frac{e^{ikr}}{r} f(\theta)$$

なる形になるならば, (a) は

e^{i}

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$$e^{\frac{(l+1)}{2}\pi} \qquad \frac{e^{ikr(y+\frac{1}{2})-\frac{l+1}{2}\pi i}+e^{\cdots}}{2}$$

asymptotic に

$$R(n, ikr) = i^{l-1}\int \frac{2}{kr(y+\frac{1}{2})}\cos\left(kr(y+\tfrac{1}{2})-\frac{l+1}{2}\pi\right) \times e^{ikr(y-\frac{1}{2})}(y+\tfrac{1}{2})^{-n}(y-\tfrac{1}{2})^{n-1}\,dy$$

$$= i^{l-1}\int \quad = \text{indep. of } l$$

ψ

$$\psi = \cancel{R(n,ikr)}\, R'(n,ikr)\, f(\theta,\varphi)$$
$$= \int \psi(\alpha,\beta,k)\, f(\alpha,\beta)\,\sin\alpha\, d\alpha\, d\beta.$$

故に asymptotic に $f(\theta,\varphi)$ なる angular distribution を有してゐることは 故に α,β 方向の wave の amplitude が $|f(\alpha,\beta)|$ なることをあらはす，

$$z' = r\{\cos\theta\cos\alpha + \sin\theta\sin\alpha\cos(\varphi-\beta)\}$$

$$e^{ikz} = \sum_{l=0}^{\infty}(2l+1)\,i^l\,P_l(\cos\theta)\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}}J_{l+\frac{1}{2}}(kr)$$

$$e^{ikz'} = \sum_{l'}(2l'+1)\,i^{l'}\,P_{l'}\left(\frac{z'}{r}\right)\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}}J_{l+\frac{1}{2}}(kr)$$

$$P_{l'}\left(\frac{z'}{r}\right) = \sum_{m'}\frac{(l'-m')!}{(l'+m')!}P_{l'}^{m'}(\cos\theta)\,P_{l'}^{m'}(\cos\alpha)\,e^{im'(\varphi-\beta)}$$

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$$\iint e^{ikz'} \sin\alpha\, d\alpha\, d\beta \sum_{l,m} a_{lm} P_l^m(\cos\alpha)\, e^{im\beta}$$

$$= \iint \sum_{l'} (2l'+1)\, i^{l'} \cancel{\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(kr)\, \sin\alpha\, d\alpha\, d\beta}$$

$$\times \sum_{m'} \frac{(l'-m')!}{(l'+m')!} P_{l'}^{m'}(\cos\vartheta)\, e^{im'\varphi} \sum_{l,m} a_{lm} \iint P_{l'}^{m'}(\cos\alpha)\, e^{-im'\beta}$$

$$\times P_l^m(\cos\alpha)\, e^{im\beta} \sin\alpha\, d\alpha\, d\beta$$

$$= \quad ,\ ,\ \times \delta(ll')\,\delta(mm') \cdot \frac{\cancel{2k}\ 2}{(2l+1)} \frac{(l+m)!}{(l-m)!}$$

$$\times \frac{\cancel{1}}{\cancel{2^{2l}(l!)^2}} (2\pi)$$

$$= \sum_{l,m} (4\pi)\, i^l \left(\frac{\pi}{2kr}\right)^{1/2} J_{l+\frac{1}{2}}(kr)\, a_{lm} P_l^m(\cos\vartheta)\, e^{im\varphi}$$

$a_{lm} = a_l \delta(m,0)$ とすると,

$$\iint e^{ikz'} \sin\alpha\, d\alpha \sum_l a_l P_l(\cos\alpha)$$

$$= \sum_l 4\pi\, i^l \underbrace{\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(kr)}\, a_l P_l(\cos\vartheta).$$

故に

$$e^{ikz} + \int e^{ikz'} \sin\alpha\, d\alpha \overbrace{\sum_l a_l P_l(\cos\alpha)}^{f(\alpha)}$$

$$\rightarrow e^{ikz} + \sum_l \frac{\sin(kr - \frac{l}{2}\pi)}{kr} 4\pi i^l a_l P_l(\cos\vartheta)$$

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$e^{\frac{\pi}{2}li}$

$$\rightarrow e^{ikr} + \frac{2\pi}{i}\frac{e^{ikr}}{kr} f(\vartheta)$$

$$-\frac{2\pi}{i}\frac{e^{-ikr}}{kr} f'(\vartheta). = \sum_l A_l \frac{\sin(kr - \frac{l}{2}\pi + \eta_l)}{kr} P_l(\cos\vartheta)$$

$$f'(\vartheta) = \sum_l (-1)^l a_l P_l(\cos\vartheta) = f(\pi-\vartheta)$$

$$-\frac{2\pi}{i}\frac{e^{-ikr}}{kr} f(\pi-\vartheta)$$

$$\cancel{A_n e^{i\eta_l} - (2l+1)i^l}$$

$$\cancel{= (-1)^l\{A_n e^{i\eta_l} - (2l+1)i^l\}}$$

$$\cancel{\{2\pi a_l - (2l+1)i^l\}} -$$

$$= A_l i^{-l} \frac{e^{ikr}}{kr} P_l(\cos\vartheta) - A_l i^{+l} \frac{e^{-ikr}}{kr} P_l(\cos\vartheta)$$

$$= \cancel{\{(2l+1)i^l + \frac{2\pi}{i}\}} \quad ''$$

$$\cancel{i^{-l}\{A_l - (2l+1)\}} = 2\pi a_l$$

$$\cancel{i^l\{A_l - (2l+1)\}} = 2\pi i a_l$$

$$\cancel{(2l+1)i^l + 4\pi i^l a_l} = \quad A_l \;!!!$$

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$$-e^{i\eta_l}\,\frac{\cancel{2l+1}}{2}\cdot\cancel{\frac{1}{l}}\{(2l+1)-2\pi a_l\}= \quad A_l e^{i\eta_l}$$

$$+e^{i\eta_l}\quad \cancel{\frac{1}{l}}(i)^{2l}\{(2l+1)\pm 2\pi a_l\}=(i)^{2l}A_l e^{-i\eta_l}$$

$$\cancel{A_l}= \quad \cancel{|a_l|^2}= (2l+1)\,2i\sin\eta_l$$

$$=4\pi\cdot(-2)\cos\eta_l\cdot a_l$$

$$a_l=-\frac{(2l+1)}{2\pi}\cdot\tan\eta_l\cdot$$

$$|a_l|^2=\sum_l\frac{(2l+1)^2}{4\pi^2}\tan^2\eta_l\cdot\frac{4\pi}{(2l+1)}$$

$$=\frac{1}{4\pi}\sum_l(2l+1)\tan^2\eta_l$$

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relative coordinate において exact solution の asymptotic は

$$\psi = \sum_l A_l \frac{\sin(kr - \frac{l\pi}{2} + \eta_l)}{kr} P_l(\cos\theta)$$

は書けるとすると.

$$= \sum_l A_l \cos\eta_l \frac{\cancel{\sin(kr-\frac{l\pi}{2})}}{kr} P_l(\cos\theta) + \sum_l A_l \sin\eta_l \frac{\cancel{\sin}\cos(kr-\frac{l\pi}{2})}{kr} P_l$$

$$\iint e^{ikr\cos\omega} \sin\alpha\, d\alpha\, d\beta \sum_{l,m} a_{lm} P_l^m(\cos\alpha)\, e^{im\beta}$$

$$= \sum_{l,m} (4\pi) i^l \left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(kr)\, a_{lm} P_l^m(\cos\vartheta)\, e^{im\varphi}$$

$$\psi(\alpha,\beta,k) = \cancel{e^{ikr\cos\omega}} \frac{1}{2\pi i} e^{ikx'} \int \frac{e^{ik(x+x')(y-\frac{1}{2})}}{y-\frac{1}{2}}\, dy$$

$$= \frac{1}{2\pi i} e^{ikx'}.$$

$$= \int \sin\alpha\, d\alpha\, d\beta\, \psi(\alpha\beta k) \sum_{l,m} a_{lm} P_l^m(\cos\alpha)\, e^{im\beta}.$$

$\psi(\alpha\beta k) =$

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$$\psi(\alpha,\beta,k) = e^{ikr\cos\omega} + \frac{e^{ikr}}{r} f_{\alpha,\beta}(\theta,\varphi) + \frac{e^{-ikr}}{r} g_{\alpha\beta}(\theta,\varphi)$$

$$\psi = \iint \sin\alpha \, d\alpha \, d\beta \, \psi(\alpha,\beta,k) \sum_{l,m} a_{lm} P_l^m(\cos\alpha) \, e^{im\beta}.$$

$$\psi(\alpha,\beta,k) =$$

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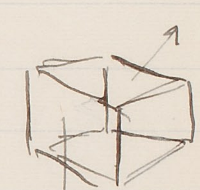
$p\,dp = m\,dE$ $\qquad m\sqrt{2mE}$ $\qquad m^2 v\,dE$

$\frac{dp_x\,dp_y\,dp_z}{h^3} = \frac{m}{h^3}\,dE\;\frac{2\pi}{h^3}\,p^2 dp\,\sin\alpha\,d\alpha$

$$f(\theta) = -\frac{1}{4\pi}\int e^{ik(n_0-n)r}\,U(r)\,dv = -\frac{1}{4\pi}V_{n_0 n}$$

$$\frac{8\pi^2 m}{h^2}V$$



$$\rho\,\frac{h}{2\pi}\,|4\pi f(\theta)|^2 = \frac{4\pi h}{} \cdot \frac{2\pi m^2 v}{h^3}\,|f(\theta)|^2 \sin\alpha\,d\alpha$$

$\sin\alpha\,d\alpha$

$$= \frac{m^2 v}{\pi h^2}\,|f(\theta)|^2 \sin\alpha\,d\alpha$$

$$= 2\pi v\left(\frac{m}{2\pi h^2}\right)|V_{n_0 n}|^2$$

$$\frac{2\pi}{h}\,|2 f(\theta)|^2 \times \frac{m^2 v}{h^3}\,\cancel{\sin\alpha}\,d\omega$$

$$= v\,\boxed{\left(\frac{2\pi m}{h^2}\right)^2 |2 f(\theta)|^2\,d\omega}$$

$$|V_{n_0 n}|^2\,d\omega$$

$$f(\theta) = -\frac{1}{4\pi}\,\frac{8\pi^2 m}{h^2}\,V_{n_0 n} = \frac{2\pi m}{h^2}V_{n_0 n}$$

$$= v\;|f(\theta)|^2\,d\omega$$