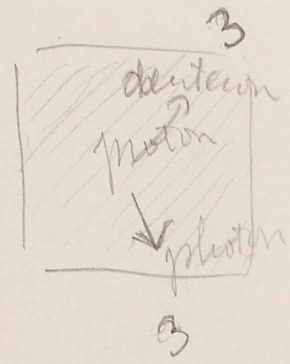
Cross Sections of the Capture of ~~the~~ neutrons by the proton and of the photoelectric effect of the deuteron.

i)

neutron $v \rightarrow$



$$\sigma_c = \frac{2\pi}{h} \cdot \frac{\rho_2}{v} \cdot |V_{12}|^2$$

ρ_2: unit energy range について の final state の 数.

photon の state は $h\nu$, $h(\nu+d\nu)$ の間に $\frac{8\pi\nu^2 d(h\nu)}{hc^3}$ ~~個~~ あり.

deuteron の state は $\frac{4\pi M^3 V^2 dV}{h^3} = \frac{2\pi M^2 V}{h^3} d(MV^2) = \frac{\pi M^2 V}{h^3} d(MV^2)$

$$\rho_2 = \frac{8\pi^2 M^2 V \nu^2}{h^4 c^3}$$

$$V_{12} = -\frac{e}{c}\int \Psi A \Psi$$

$$\sigma_c = \frac{M^2\nu^2}{\cancel{8}\pi^2 h^5 c^3}|V_{12}|^2$$

$\vec{r}_1 - \vec{r}_2 = \vec{r}$

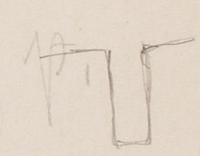
$\frac{\vec{r}_1 + \vec{r}_2}{2} = \vec{R}$

$z_1 = Z + \frac{z}{2}$

neutron wave function; $e^{ikz_1} = e^{i(kZ + k\frac{z}{2})}$

proton wave function: $\psi_P(|\vec{r}_1 - \vec{r}_2|) \doteq 1$ for $|\vec{r}_2 - \vec{r}_1| > a$

deuteron wave function $\cong e^{i\vec{k}\vec{Z}} \psi_D(r)$



photon wave function (vector)

$$\vec{A} = \vec{A}_n e^{\cancel{\frac{i\nu}{c}(\vec{n}\vec{r}_1)} - i\nu t}$$

$\sin(\frac{\nu}{c}(\vec{n}\vec{r}_1) + \nu t)\ n_y A_{zy}$

$\vec{A}_n \vec{n} = 0.$

$$\vec{E} = -\frac{1}{c}\dot{\vec{A}} = \frac{-2\pi\nu i}{c}\vec{A}_n \cos(\cdots)\ e^{\frac{2\pi\nu i}{c}} \qquad \vec{H} = \frac{\nu}{c}[\vec{n}\vec{A}_n]\cos(\cdots)$$

$$\frac{1}{8\pi}(\vec{E}^2 + \vec{H}^2) = \frac{4\pi^2\nu^2}{8\pi c^2}\cancel{\cos^2}(\cdots)\cdot 2A_n^2$$

$$\frac{1}{8\pi}\int(\vec{E}^2 + \vec{H}^2)dv = \frac{\pi\nu^2}{\cancel{8}\pi c^2}A_n^2 = h\nu$$

$A_n^2 = \frac{4c^2 h}{\nu}$

$$|A_n| = 2c\sqrt{\frac{h}{\nu}}$$

$$\frac{e}{c}\vec{A}(\vec{r}_2) = 4\pi e\sqrt{\frac{\hbar}{\nu}}\cdot\vec{e}\cdot\sin\left(\frac{\nu}{c}\vec{n}\vec{r}_2+\delta\right)$$

$$\vec{e}\,\vec{n}=0 \qquad \vec{e}^{\,2}=1.$$

$$\cancel{V_{12} = -4\pi e\sqrt{\frac{\hbar}{\nu}}\cdot\vec{e}\int e^{\frac{ikz}{2}}\left(\frac{-\vec{p}_1}{M}\right)\psi_D(r)\sin\left(\frac{\nu}{c}\vec{n}\left(\vec{R}-\frac{\vec{r}}{2}\right)+\delta\right)}$$

$$\cancel{= 4\pi e\sqrt{\frac{\hbar}{\nu}}\cdot\vec{e}\int e^{\frac{ikz}{2}}\sin}$$

$$\frac{\partial}{\partial\vec{r}_2} = \frac{\partial\vec{r}}{\partial\vec{r}_2}\frac{\partial}{\partial\vec{r}} + \frac{\partial\vec{R}}{\partial\vec{r}_2}\frac{\partial}{\partial\vec{R}}$$

$$= -\frac{\partial}{\partial\vec{r}} + \frac{1}{2}\frac{\partial}{\partial\vec{R}}$$

$$V_{12} = -4\pi e\sqrt{\frac{\hbar}{\nu}}\,\vec{e}\,\frac{i\hbar}{M}\iint e^{i(kZ+k\frac{z}{2})}\psi_P(\cancel{r_1 r_2})\left(\frac{\partial}{\partial\vec{r}}+\frac{1}{2}\frac{\partial}{\partial\vec{R}}\right)$$

$$\times\, e^{-ik'z}\psi_D(r)\sin\left(\frac{\nu}{c}\vec{n}\left(\vec{R}-\frac{\vec{r}}{2}\right)+\delta\right)dv\,dV$$

$$\int\cdots\int e^{i(kZ+k\frac{z}{2})}\psi_P(r_1 r_2)\cdot e^{-ik'z}\psi_D(r)\,dv\,dV$$

$$=0.$$

$$V_{12} \cong -\frac{4\pi i\hbar e}{M}\sqrt{\frac{\hbar}{\nu}}\int\cdots\int\left\{\frac{\vec{e}\,\vec{r}}{r}\frac{1}{2}\sin\delta\cdot e^{\frac{ikz}{2}}\frac{d\psi_D}{dr}dv\right.$$

$$\left.+(-ie_z k')\frac{\sin\delta}{4}\psi_D e^{\frac{ikz}{2}}dv\right\}$$

$$\frac{d}{dr}$$

$$|V_{12}^2| \cong \frac{4\pi^2\hbar^3e^2}{M^2\nu}\frac{\sin^2\delta}{4}\left|\iiint\left(\frac{\vec{e}\,\vec{r}}{r}e^{\frac{ikz}{2}}\frac{d\psi_D}{dr} - \frac{ie_z k}{2}e^{\frac{ikz}{2}}\psi_D\right)dv\right|^2$$

$$\overline{|V_{12}|^2} \cong \frac{2\pi^2\hbar^3e^2}{M^2\nu}\left|\iiint(\vec{e}\,\mathrm{grad})\left(e^{\frac{ikz}{2}}\psi_D\right)dv\right|^2$$

$$\overline{|V_{12}^2|} = \frac{2\pi^2\hbar^3 e^2}{M^2\nu}\,\frac{1}{3}\iiint \cos\theta \cdot e^{\frac{ikr\cos\theta}{2}}\frac{d\psi_D}{dr} - \frac{ik}{2}e^{\frac{ikr\cos\theta}{2}}\psi_D)$$

$$2\pi\cdot r^2 dr \sin\theta\, d\theta)$$

$$e^{\frac{ikz}{2}} = \frac{\sin\frac{kr}{2}}{\frac{kr}{2}} + 3i\left(\frac{\pi}{kr}\right)^{\frac{1}{2}} J_{\frac{3}{2}}\left(\frac{kr}{2}\right)\cos\theta + \cdots$$

$$\psi_D = \left(\frac{\alpha}{2\pi(1+a\alpha)}\right)^{\frac{1}{2}}\left(\frac{V_0-W}{V_0}\right)^{\frac{1}{2}} e^{-\alpha(r-a)} = Ce^{-\alpha r}$$

$$\alpha = \frac{(MW)^{\frac{1}{2}}}{\hbar}$$

$$\frac{d\psi_D}{dr} = -\alpha C e^{-\alpha r}$$

$$J_{\frac{3}{2}}\left(\frac{kr}{2}\right) = \sqrt{\frac{4}{\pi kr}}\left(\frac{\sin\frac{kr}{2}}{\frac{kr}{2}} - \cos\frac{kr}{2}\right)\cos\theta$$

$$\overline{|V_{12}^2|} = \frac{\cancel{8\pi^2}\ 2496\pi^4}{\cancel{8}\,M^2\nu}\,\hbar^3 e^2\, C^2 \iiint \frac{1}{k^2 r}\left(\cos\theta + \frac{i\alpha k}{2}\right)\left(\frac{\sin\frac{kr}{2}}{\frac{kr}{2}} - \cos\frac{kr}{2}\right)\cos\theta \times e^{-\alpha r}\, r^2 dr \sin\theta\, d\theta\Big|^2$$

$$3 \quad 3^2 \qquad \frac{(\cos\theta)^3}{3} \quad \hbar^2 e^2 \quad \frac{2}{3} \qquad \frac{1}{3} \qquad \cancel{\frac{kr}{2}} \;1 - \frac{1}{3}\left(\frac{kr}{2}\right)^2 \quad " \;1 - \frac{1}{2}\left(\frac{kr}{2}\right)^2$$

$$\frac{x^3 e^{-\alpha x}}{a} + \frac{3}{a}\left(\frac{x^2 e^{\alpha x}}{a} + \frac{2}{a}\left(\frac{x e^{\alpha x}}{a} + \frac{e^{\alpha x}}{a}\right)\right)$$

$$= \frac{128\pi^4 C^2}{3M^2\nu}\left|\int_a \frac{r\,dr}{k}\, e^{-\alpha r}\left(\frac{\sin\frac{kr}{2}}{\frac{kr}{2}} - \cos\frac{kr}{2}\right)\right|^2$$

$$= \hbar^2 e^2\,\frac{128\pi^4 C^2}{3M^2\nu}\left|\frac{1}{12}k\int_a^\infty r^3 e^{-\alpha r}dr\right|^2$$

$$= \frac{a^3 e^{-\alpha a}}{\alpha} + \frac{3a^2 e^{-\alpha a}}{\alpha^2} + \frac{6a e^{-\alpha a}}{\alpha^3} + \frac{6e^{-\alpha a}}{\alpha^4}$$

$$= \frac{128\pi^4 C^2}{3M^2\nu}\left|\frac{k}{2}e^{-\alpha a}\frac{\alpha}{\alpha^4}\left(\frac{1+\alpha a}{6} + \frac{3(\alpha a)^2}{\cancel{2}} + \cancel{(\alpha a)^3}\right)\right|^2$$

$$\int_0^\infty \frac{\cancel{X}\,dr}{k} e^{-\alpha r}\left(\sin\frac{kr}{2} - \frac{kr}{2}\cos\frac{kr}{2}\right)$$

$$= \frac{1}{k}\,\mathrm{Re}\int_0^\infty e^{-\alpha r}\left(-i - \frac{kr}{2}\right)e^{+i\frac{kr}{2}}\,dr$$

$$= \frac{1}{k}\,\mathrm{Re}\left(\frac{-i}{(\alpha - i\frac{k}{2})} - \frac{k}{2}\frac{1}{(\alpha - \frac{ik}{2})^2}\right)$$

$$= \frac{1}{2}\left\{\frac{1}{\alpha^2 + (\frac{k}{2})^2} - \frac{\alpha^2 - (\frac{k}{2})^2}{(\alpha^2 + (\frac{k}{2})^2)^2}\right\}$$

$$\int_0 r e^{-\alpha r}\,dr = \cancel{0} + \frac{1}{\alpha}$$

$$\frac{k}{2} \qquad (\alpha + \frac{ik}{2})^2 = \alpha^2$$

$$= \frac{1}{\cancel{4}}\,\frac{k^2}{(\alpha^2 + (\frac{k}{2})^2)^2}$$

$$|V_{12}|^2 = \frac{8\pi^4 c^2 \hbar e^2}{3M^2\nu}\,\frac{k^4}{(\alpha^2 + (\frac{k}{2})^2)^4}$$

$$\sigma_c = \frac{8\pi^3 \nu e^2}{3\hbar^2 c}\,\frac{k^4}{(\alpha^2 + (\frac{k}{2})^2)^4}$$

$$V_{12} = \frac{e}{c}\vec{A}_n \int\!\!\int dv_1 dv_2\, \tilde{\Psi}_1(\vec{r}_1\vec{r}_2)\, \sin\left(\frac{\nu}{c}(\vec{n}\vec{r}_2)+\delta\right) \underbrace{\Psi_2(\vec{r}_1\vec{r}_2)}_{\frac{\vec{p}_2}{M}}$$

$$\cancel{\frac{\partial}{\partial \vec{r}_2}\, \Psi_1(\vec{r}_1\vec{r}_2)\sin\left(\frac{\nu}{c}\vec{n}\vec{r}_2+\delta\right)\Psi_2(\vec{r}_1\vec{r}_2)}$$

$$\vec{p}_2\Psi_2 = -i\hbar\left(\frac{1}{2}\frac{\partial}{\partial R} - \frac{\partial}{\partial \vec{r}}\right)\Psi_2$$

$$= -i\hbar\left(\frac{ik}{2}\Psi_2 - \alpha\frac{\vec{r}}{r}\right)$$

$$= -i\hbar\left(\frac{i\sqrt{2ME}}{2\hbar}\Psi_2 - \sqrt{\frac{MW}{\hbar}}\frac{\vec{r}}{r}\right)$$

$$\vec{I}_x\vec{p}_2\Psi_2 = -i\hbar\left(-\sqrt{\frac{MW}{\hbar}}\frac{x}{r}\right)$$

$$\vec{I}_y\vec{p}_2\Psi_2 = -i\hbar\left(\cancel{\sqrt{\frac{MW}{2}}}\ \frac{y}{r}\right)$$

$$\vec{I}_z\vec{p}_2\Psi_2 = -i\hbar\left(i\sqrt{\frac{2ME}{\hbar}} - \sqrt{\frac{MW}{\hbar}}\frac{z}{r}\right)$$

$e\vec{r}_2 = e\vec{r}_1 - \frac{e\vec{r}}{2}$ $\frac{\vec{r}_1+\vec{r}_2}{2}$ $\vec{r}_1 - \vec{r}_2$

$$\frac{1}{\nu}\cdot\frac{8\pi\nu^2 d(h\nu)}{hc^3}\cdot\frac{2\pi M^2 V}{h^3}\cdot\left|\frac{e}{2c}\frac{\hbar c}{\nu}\, e^{i\vec{k}\vec{r}}\, \sqrt{2}\cdot\sin\nu(\vec{n}\vec{r})\, \psi_D(\vec{r})\right|^2$$

$$= \frac{1}{\nu}\,\frac{e^2 \cancel{2}\nu\, d\nu\, M^2 V}{4hc^3}\left|\int e^{i\vec{k}\vec{r}} \sin\frac{\nu}{c}(\vec{n}\vec{r})\, \psi_D(\vec{r})\right|^2$$

$$\cancel{\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}\cdot\frac{1}{2\pi}\cdot\frac{\sin\frac{\nu}{c}r}{r}}$$

$$= \frac{e^2 M^2 d\nu}{4hc^3}\,\Big|$$

$$\psi_D(\vec{r}) = \qquad \cancel{\frac{\sin kr}{r}}\cdot\left(\frac{4\pi}{3}\right)^{\frac{1}{2}}\frac{1}{2\pi}\frac{\nu}{c}\cdot\frac{}{r}$$

$$e^{i\vec{k}\vec{r}} =$$

$$e^{ikz} = \frac{\sin kr}{kr} + 3i\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{\frac{3}{2}}(kr)\cos\theta + \cdots$$

$$\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}\cdot\frac{1}{2\pi}\cdot\frac{\sin\frac{\nu}{c}r}{r}$$

$$\frac{1}{3} \qquad \cos^2\theta$$

$$4\pi\int\frac{\sin^2 kr}{(kr)^2}\, r^2 dr$$

$$= \frac{4\pi}{k^3}\int_0^{ka}\sin^2 x\, dx$$

$$= \frac{4\pi}{4k^3}\int_0^{2ka}(1-\cos x)\, dx$$

$$= \frac{2\pi a}{k^2} = \frac{2\pi}{k^2}\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}$$

$\frac{1}{2}(\cos 2x \quad 1-\cos 2x)$

$= \cancel{1-}\, 2\sin^2 x$

$\frac{4\pi}{3}a^3 = 1$

$a = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}}$