18(+16)

H

FRAGMENT H

論文原稿

BOX06

§1 Three dimensional representation of many body problem.

Schrödinger 以来急激ナ發達ヲ遂ゲタ wave mechanics ニヨレバ degree of freedom ガ f ナル mechanical system ハ f-dimensional space ニオケル アル一種ノ波動現象トシテ取扱フコトガ出来ル。

即チ system ノ kinetic energy ヲ system ノ coordinates q_k 及ビ velocity components $\dot{q}_k$ ノ函数トシテアラハシタモノヲ $T(q_k, \dot{q}_k)$ トスル時. 系ニハ line element ガ

$$ds^2 = 2T(q_k, \dot{q}_k)dt^2 \qquad (1)$$

デアラハサレル様ナ f-dimensional space ヲ考ヘ. コノ space ニ於テ. system ハ次ノ equation ニヨツテ characterise サレルトスルノデアル。

$$\mathrm{div}\ \mathrm{grad}\ \psi + \frac{8\pi^2}{h^2}(E-V)\psi = 0 \qquad (2)$$

ガ所謂 wave equation デアツテ. ψ ハ position ノ function トシテ. 全空間デ finite, one valued, continuous トイフ條件ヲ満足スルモノデナケレバナラヌ。

E ハ system ノ total energy, V ハ system ノ potential energy デアル。 div, grad ハ勿論 (1) ナル line element ニ相當スル div, grad ノ operation デアル。

(2) ガ以上ノ條件ヲ満足スル solution ψ ヲ必ズ有スルトハ限ラヌ。 energy E ヲ parameter ト考ヘル時. solution ψ ガ存在スル様ナ E ノ値ガ Eigenwert デアツテ, 之ニ對シテ ψ ガ Eigenfunktion トヨバレル。 之ノ Eigenwert ヲ決定シ, Eigenfunction ヲ求メルコトガ wave mechanics ノ主ナ問題デアル。

扨テ系ガ一ツノ particle 例ヘバ一ツノ electron 丈ヲトツテ考ヘル時ニハ, degree of freedom ガ三デアルカラ三次元空間ニオケル現象トシテ論ジ得ルガ; 二ツ以上ノ particles ヨリナル system ヲトリ扱フニ當ツテハ. 其ノ system ノ degree of freedom ニ相當スル dimension ノ空間ヲ考ヘ. ソノ空間ニオケル現象トシテ. 上ニノベタ (2) ノ equation ヲトクコトニヨツテ問題ヲ解決スルノデアルカラ. ~~現象ヲ~~ 三次元空間ニオケル現象トシテ anschaulich ニ論ズルコトハ

②

~~解ケヌ不可能ナ~~ 困難デアル。

例ヘバ、Schrödinger ノトイタ水素原子ノ問題ハ、nucleus ヲ fix シ、electron ノミ~~運動~~ヲ考ヘルカラ三次元空間ニオケル問題トナリ、eigenfunction ψ ハ

$$-e\psi^2(x)$$

ガ space ノ x ナル一点ニオケル ~~cha~~ electron ノ charge ヲ与ヘルト云フ physical meaning ヲ有シ、従ツテ全空間ニ積分シタ時

$$\int \psi^2(x)\,dv = 1 \qquad (3)$$

ナル normalisation ノ condition ガ出テ来ル。

之ニ反シテ electron ヲ二ツ有スル system 例ヘバ Helium ノ問題ハ Heisenberg ヤ Dirac ノ取扱ツタ所ニヨレバ水素ノ場合ノ如キ Anschaulichkeit ヲ非常ニ失ツテ居ル。

実際複雑ナル問題ヲトクニアタツテハ全ク anschaulich ナ考ヘ方ヲスルコトハ不可能デアルカモシレヌガ、少クトモ不便デアルカモシレヌ。

次ニ、カカル場合ニ於テモ three dimensional space ニオケル現象トシテ ~~取扱~~ 完全ニ解決シ得ルヤ否ヤヲ考ヘルノハ興味アルコトデハナカラウカト思フ。特ニ relativity ノ要求ヲ満足スル four dimensional ナ wave mechanics ヲ完成スル為ニハ、non-relativistic ナ wave mechanics ヲ三次元的ニ取扱フコトガ先決問題デアル。

以下不完全ナガラ many 多クノ particles ヨリナル system ヲ三次元的ニ取扱ツテ見ヨウト思フ。*

~~[illegible]~~ 何レノ場合モ conservative system ヲ考ヘ~~テ居ルノデ~~ルコトトスル。

---

* Jordan, Klein ノ論文ガアルガ q-number ヲツカツテアツテヨクワカラヌ。

吾々ハ先~~ヅ三次元水素原子ノ問題ヲ考ヘテ見ル~~。出発点トシテ hydrogen atom ノ場合ニツイテ考ヘル。~~吾々~~ハ普通ノ wave equation ~~ナカハリニ~~ ノカハリニ特ニ Schrödinger ガ最初ノ假定シタ variation ノ equation

$$(4)\qquad \delta\iiint dx\,dy\,dz\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m}{K^2}\left(E+\frac{e^2}{r}\right)\psi^2\right]=0$$

where $K = \frac{h}{2\pi}$

ヲトツテ、其ノ意味ヲ今一応考ヘテ見ルニ、[illegible] ψ² ヲ charge density ヲアラハス quantity ト考ヘレバ

$$-\iiint\frac{e^2\psi^2}{r}\,dx\,dy\,dz$$

ハ明カニ ~~potenti~~ nucleus ト electron ノ attraction ニヨル potential energy ヲアラハシテ居ルシ、又 $\frac{K^2}{2m}\iiint\left(\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2\right)dx\,dy\,dz$ ハ electron ノ kinetic energy ヲアラハスコトト ~~[illegible]~~ 解釈スルコトガ出来ル。

従ツテ (4) ノ式ハ total energy 即チ ~~ガ~~

$$\iiint\left[\frac{K^2}{2m}\left\{\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2\right\}-\frac{e^2}{r}\psi^2\right]dx\,dy\,dz$$

ガ ~~extrem~~

$$\iiint\psi^2\,dx\,dy\,dz$$

(詳シクハ minimum)

ナル Nebenbedingung ノ下ニ extremum トナル為ノ electric charge ノ distribution ガ実際ニ ~~アラハレ~~ 起ルコトヲアラハス式ト考ヘルコトガ出来ル。

コノ意味ニオイテ E ハ単ニ indeterminate constant トシテトイタ以上ノ physical meaning ヲ考ヘルノハ疑問カモシレヌガ ~~有ツガ、コノ場合ニ於テモ~~ ~~E ハ~~ 先ヅ仮リニ系ノ total energy ニ相当スル一ノ parameter デアルト假定シテオク。

ソコデコノ考ヘヲ多クノ particle ノ場合ニオシヒロゲルト

例ヘバ二ツノ particles ノ場合ニハ、夫々ノ particle ノ electric

③

charge ノ space distribution ヲアラハス ~~アラハス~~ function ヲ ψ_1, ψ_2 ヲ考ヘル.
ソレト (4) ヲ拡張シタ式ハ

$$\delta\iiint dx\,dy\,dz\Big[\frac{h^2}{8\pi^2 m_1}\Big\{\Big(\frac{\partial\psi_1}{\partial x}\Big)^2+\Big(\frac{\partial\psi_1}{\partial y}\Big)^2+\Big(\frac{\partial\psi_1}{\partial z}\Big)^2\Big\}+\frac{h^2}{8\pi^2 m_2}\Big\{\Big(\frac{\partial\psi_2}{\partial x}\Big)^2+\Big(\frac{\partial\psi_2}{\partial y}\Big)^2+\Big(\frac{\partial\psi_2}{\partial z}\Big)^2\Big\}+V_0(e_1\psi_1^2+e_2\psi_2^2)+e_1e_2\psi_2^2\int\frac{\psi_1^2(r')\,dv'}{|r-r'|}+E_1\psi_1^2+E_2\psi_2^2\Big]=0 \tag{5}$$

トナル。
ここニ ~~V_0~~ m_1, m_2 ヲ e_1, e_2 ハ夫々二ツノ particles ノ mass 及ビ electric charge ~~デアリ~~, V_0 ハ external electro-static field ノ potential, E_1, E_2 ハ ~~二ツノ particles ノ系ニ相当スル (energy) parameter デソレハ~~ 最後ノ二項ハ

$$\iiint\psi_1^2\,dx\,dy\,dz=1$$

$$\iiint\psi_2^2\,dx\,dy\,dz=1$$

トイフ nebenbedingung ニツイテ附加サレタ term デ, E_1, E_2 ハ二ツノ parameters (energy parameters) デアラウ。

$\iiint dx\,dy\,dz\; e_1e_2\psi_2^2\iiint\frac{\psi_1^2(r')\,dx'dy'dz'}{|r-r'|}$ ハ ~~[illegible]~~ 二ツノ particles ノ間ノ相互作用 ~~デアラウ~~ ノ potential energy デアル。
之ハ 1, 2 ニツイテ symmetric デ

$$\iiint dx\,dy\,dz\; e_1e_2\psi_1^2\iiint\frac{\psi_2^2(r')\,dx'dy'dz'}{|r-r'|}$$

トシテモ同ジデアル 又

$$\frac{1}{2}\, e_1e_2\Big\{\psi_2^2\iiint\frac{\psi_1^2(r')\,dx'dy'dz'}{|r-r'|}+\psi_1^2\iiint\frac{\psi_2^2(r')\,dx'dy'dz'}{|r-r'|}\Big\} \tag{6}$$

トカイテモヨイ。

~~[illegible]~~ (5) ノ式カラ wave equations ハ次ノ如クシテ出テクル。
即チ

$$\delta\iiint dx\,dy\,dz\,\frac{h^2}{8\pi^2 m_1}\Big\{\Big(\frac{\partial\psi_1}{\partial x}\Big)^2+\Big(\frac{\partial\psi_1}{\partial y}\Big)^2+\Big(\frac{\partial\psi_1}{\partial z}\Big)^2\Big\}$$

$$=\frac{h^2}{8\pi^2 m_1}\iiint\big[\mathrm{div}(\delta\psi_1\,\mathrm{grad}\,\psi_1)-2\delta\psi_1\Delta\psi_1\big]\,dx\,dy\,dz$$

$$=\frac{h^2}{8\pi^2 m_2}\iint_\infty\delta\psi_1\,\mathrm{grad}\,\psi_1\,df-2\frac{h^2}{8\pi^2 m}\iiint\delta\psi_1\Delta\psi_1\,dx\,dy\,dz$$

同様ニ

$$\delta\iiint dx\,dy\,dz\,\frac{h^2}{8\pi^2 m_2}\Big\{\Big(\frac{\partial\psi_2}{\partial x}\Big)^2+\Big(\frac{\partial\psi_2}{\partial y}\Big)^2+\Big(\frac{\partial\psi_2}{\partial z}\Big)^2\Big\}$$

$$=\frac{h^2}{8\pi^2 m_2}\iint_\infty\delta\psi_2\,\mathrm{grad}\,\psi_2\,df-2\frac{h^2}{8\pi^2 m_2}\iiint\delta\psi_2\Delta\psi_2\,dx\,dy\,dz$$

ここニ $\iint_\infty$ ノ df ハ infinity ノ所ニ遠ザカッタ closed surface ニツイテノ surface integral デ。
従ッテ (5) 式ハ

$$\cancel{\frac{h^2}{8\pi^2 m}\Big\{\Big(\frac{\partial\psi_1}{\partial x}\Big)^2}\qquad\iint_\infty(\delta\psi_1\,\mathrm{grad}\,\psi_1+\delta\psi_2\,\mathrm{grad}\,\psi_2)\,df=0$$

~~故ニ~~

$$\cancel{-\frac{h^2}{8\pi^2 m}\delta\psi_1}\Delta\psi_1$$

又

$$\delta\iiint V_0(e_1\psi_1^2+e_2\psi_2^2)\,dx\,dy\,dz=\iiint 2V_0(e_1\delta\psi_1\cdot\psi_1+e_2\delta\psi_2\psi_2)\,dx\,dy\,dz$$

$$\delta\iiint e_1e_2\psi_2^2\,dx\,dy\,dz\iiint\frac{\psi_1^2(r')\,dx'dy'dz'}{|r-r'|}$$

$$=\iiint e_1e_2\cdot 2\psi_2\delta\psi_2\,dx\,dy\,dz\iiint\frac{\psi_1^2(r')\,dx'dy'dz'}{|r-r'|}$$

$$+\iiint e_1e_2\cancel{2}\psi_2\delta\psi_2\,dx\,dy\,dz\iiint\frac{2\psi_1(r')\,\delta\psi_1(r')\,dx'dy'dz'}{|r-r'|}$$

後ノ方ノ項ノ dashed ~~term ト~~ undashed トヲトリカヘテモ ~~同ジ~~ ヨイ. 更ニ integration ノ order ヲカヘテモヨイカラ.

④

$$= \iiint 2e_1e_2\psi_1\delta\psi_1 dxdydz \iiint \frac{\psi_2^2(r')dx'dy'dz'}{|r-r'|}$$

従ッテ (5)ハ

$$\frac{h^2}{8\pi^2 m_1}\iiint \delta\psi_1\left\{\Delta\psi_1 + \frac{8\pi^2 m_1}{h^2}\left(E_1 - V_0 e_1\psi_1 - e_1e_2\iiint\frac{\psi_2^2(r')dx'dy'dz'}{|r-r'|}\right)\right\}$$

$$+ \frac{h^2}{8\pi^2 m_2}\iiint \delta\psi_2\left\{\Delta\psi_2 + \frac{8\pi^2 m_2}{h^2}\left(E_2 - V_0 e_2\psi_2 - e_1e_2\iiint\frac{\psi_1^2(r')dx'dy'dz'}{|r-r'|}\right)\right\}$$

$$+ \frac{h^2}{8\pi^2 m_1}\iint_\infty \delta\psi_1 \operatorname{grad}\psi_1 df + \frac{h^2}{8\pi m_2}\iint_\infty \delta\psi_2 \operatorname{grad}\psi_2 df = 0$$

然ルニ δψ₁, δψ₂ ハ任意ノ variation ニ對シテ成立スルカラ 先ヅ括弧ノ中ハ零 三ツノ式ヲ得ル。

$$(6)\quad \left\{\Delta\psi_1 + \frac{8\pi^2 m_1}{h^2}\left(E_1\psi_1 - e_1V_0\psi_1 - e_1e_2\,\psi_1\iiint\frac{\psi_2^2(r')dx'dy'dz'}{|r-r'|}\right)\right\} \quad ~~dxdydz~~ = 0$$

$$(7)\quad \left\{\Delta\psi_2 + \frac{8\pi^2 m_2}{h^2}\left(E_2\psi_2 - e_2V_0\psi_2 - e_1e_2\,\psi_2\iiint\frac{\psi_1^2(r')dx'dy'dz'}{|r-r'|}\right)\right\} = 0$$

$$(8)\quad \iint\left(\frac{\delta\psi_1}{m_1}\operatorname{grad}\psi_1 + \frac{\delta\psi_2}{m_2}\operatorname{grad}\psi_2\right)df = 0$$

(6)(7) ノ式ヲ見レバ、~~二ツノ式ハ~~ 夫々第三項ガ potential ~~energy~~ × e₁ψ₁ × e₂ψ₂ ヲ含ムコトヲ除ケバ ~~[illegible]~~

~~斯クノ如キ式ハ書キ下シ得ル式ナリ。~~
任意多數ノ particles ノ場合モ容易ニ書キ下シ得。即チ次ノ如シ。

水素分子，
Helium ノ Ionisation potential 等。

$$(\mathrm{I})\quad \Delta\psi_i + \frac{8\pi^2 m_i}{h^2}\left(E_i\psi_i - e_iV_0\psi_i - e_i\psi_i\sum_{k\neq i} e_k\iiint\frac{\psi_k^2(r')dx'dy'dz'}{|r-r'|}\right) = 0$$

$i = 1, 2, \ldots n$

(8)ノ式モ
(II)
$$\iint\sum_i \frac{\delta\psi_i}{m_i}\operatorname{grad}\psi_i\, df = 0.$$

トナルベキデアラウ。

§2. Deduction of many dimensional equation

之ノ式カラ ~~之ヲ~~ many dimensional ~~wave~~ space ニオケル wave equation ガ ~~[illegible]~~ 出セルダラウカ。

簡單ノタメ ~~three dimensional~~ two particles ノ場合ヲ考ヘル。1ノ方ハ $x_1y_1z_1$ coordi デアラハシ 2ノ方ハ $x_2y_2z_2$ ノ coord デアラハサレルトスル。

$$(6)\text{ハ}\quad \Delta_1\underset{(x_1y_1z_1)}{\psi_1} + \frac{8\pi^2 m_1}{h^2}\left(E_1 - e_1\underset{(x_1y_1z_1)}{V_0} - e_1e_2\iiint\frac{\psi_2^2(r_2)dx_2dy_2dz_2}{|r_1-r_2|}\right)\times\underset{(x_1y_1z_1)}{\psi_1} = 0$$

(7)ハ

$$(7')\quad \Delta_2\underset{(x_2,y_2,z_2)}{\psi_2} + \frac{8\pi^2 m_2}{h^2}\left(E_2 - e_2\underset{(x_2y_2z_2)}{V_0} - e_1e_2\iiint\frac{\psi_1^2(r_1)dx_1dy_1dz_1}{|r_2-r_1|}\right)\times\underset{(x_2y_2z_2)}{\psi_2} = 0$$

各々ノ相互作用ヲ除キ

$$(6)\times\frac{1}{2m_1}\psi_2(x_2y_2z_2)$$
$$+(7')\times\frac{1}{2m_2}\psi_1(x_1y_1z_1) \quad \text{ヲ加ヘレバ。}$$

$$\left(\frac{1}{2m_1}\Delta_1 + \frac{1}{2m_2}\Delta_2\right)\psi_1\psi_2 + \frac{8\pi^2}{h^2}\left\{(E_1+E_2)\psi_1\psi_2 - e_1V_0(x_1y_1z_1)\psi_1\psi_2 - e_2V_0(x_2y_2z_2)\psi_1\psi_2 - e_1e_2\left(\iiint\frac{\psi_2^2(r_2)dx_2dy_2dz_2}{|r_1-r_2|} + \iiint\frac{\psi_1^2(r_1)dx_1dy_1dz_1}{|r_2-r_1|}\right)\psi_1\psi_2\right\} = 0$$

$\psi_1\psi_2 = \Psi$ ~~トオキ~~ $\frac{1}{2m_1}\Delta_1 + \frac{1}{2m_2}\Delta_2 = \Delta$ ~~トオケバ~~ $E_1 + E_2 = E$

$$(\text{解})\quad \Delta\Psi + \frac{8\pi^2}{h^2}\left\{E - V - e_1e_2\left(\iiint\frac{\psi_2^2(r_2)dx_2dy_2dz_2}{|r_1-r_2|} + \psi_1^2(r_1)dx_1dy_1dz_1\right)\right\}\times\Psi = 0$$

$$e_1V_0(x_1y_1z_1) + e_2V_0(x_2y_2z_2) = V$$

⑤

2) ~~$\Delta\Psi$~~ ハ即チ (1) ノ line element ヲ有スル space ニオケル div grad ト一致シテ居ル。
又 ~~total~~ external field ニヨル ~~potent~~ system ノ potential ~~energy~~. 粒子ノ間ノ相互作用ノ potential ヲ含ム。
即チ (2) ノ式ハ ~~(?)~~ many dimensional wave equation (2) ト全ク一致シテ居ル。
加之、今迄ノ人々ノ取扱ッタ many dimensional ノ方法デハ potential ノ expression ガ明瞭デナク ~~従ッテ~~ 論ジ難イ点ガアッタガ、(9) ノ式デハ之ヲ全部明瞭ニ示シテ居ル。今迄ノ expression ト異ナッテ居ルガ、此ノ式ノ方ガ正シイト思ハレル。外力ガナイトキハ ~~origin~~ x_0, y_0, z_0 ニアル e_0 ナル point charge ニヨルトキ
~~任意多数ノ particles ノアル場合モ~~

疑ヒガナイ。

$$(9)'\quad \Delta\Psi+\frac{8\pi^2}{h^2}\Big\{E-\frac{e_0e_1}{|r_1-r_0|}-\frac{e_0e_2}{|r_2-r_0|}$$
$$-e_1e_2\iiint\frac{\psi_2^2(r_2)\,dx_2dy_2dz_2+\psi_1^2(r_1)\,dx_1dy_1dz_1}{|r_1-r_{2}|}\Big\}\Psi=0$$

~~(9) ノ式ハ此ノ式ノ first approximation ト考ヘラレル。~~
カラ来ルト
~~任意多数ノ particles ノアル場合ハ~~

$$\Delta\Psi+\frac{8\pi^2}{h^2}\Big\{E-\frac{e_0e_1}{|r_1-r_0|}-\frac{e_0e_2}{|r_2-r_0|}$$
$$(9)''\quad -\frac{e_1e_2}{|r_1-r_2|}\Big\}\Psi=0$$

之ハ (9') ノ first approximation ト考ヘラレル。　此ハ更ニ考ヘヨ。
(但シ此ノ式ノ最後ノ term ハ除カナケレバナラヌカモ知レヌ。之ハ後ニノベル)
任意多数ノ particles ノアル場合ニハ (9) ハ

$$\Delta\Psi+\frac{8\pi^2}{h^2}\Big(E-\cancel{\frac{e_0e_1}{|r_1-r_0|}}-\cancel{\frac{e_0e_2}{|r_2-r_0|}}-\cancel{\frac{e_1e_2}{|r_1-r_2|}}\Big\}\Psi=0$$

又 (9) or (9') ノ式ハ ψ_1, ψ_2 ガ分カッテ居ナイ点ガ不完全デアル。

$$-V-V')\Psi=0$$

where $V\equiv V_1+V_2+\cdots+V_n$
(即チ各 k ノ external field ニヨル potential energy) 及ビ

$$V'=\sum_{i\neq k}e_ie_k\iiint\frac{\psi_i^2\,dx_idy_idz_i+\psi_k^2\,dx_kdy_kdz_k}{|r_i-r_k|}$$

先ニノベタ経内ヲ附添スルニ ~~(?)~~ 次ノ如キ出発点ヲトルコトモ出来ル。即チ total system ノ energy ガ min デアルト云フ

$$\cancel{\delta\int\cdots\int(\text{grad}\,\Psi)^2}$$

式ハ many dimensional wave equation Ψ ニ関スル次ノ式ヲ得ルベシ。

$$(10)\quad \delta\int\cdots\cdots\int\Big\{(\text{grad}\,\Psi)^2+\frac{8\pi^2}{h^2}(E-V)\Psi^2\Big\}dv=0$$

ヨリ

$$(11)\quad \Delta\Psi-\frac{8\pi^2}{h^2}(E-V)\Psi=0$$

而シテ此処デハ ~~やはり~~ V ノ式ハ分カラヌ。

§3. Interpretation of symmetric and antisymmetric eigenfunction of the system with ~~several~~ similar particles

之ハ後述ベシ。

⑥

[9']

(9')ト(9'')ノ相違点ハ inner energy ノ式ノ相違ニアル其ノ差ハ

$$e_1e_2\iiint\frac{\psi_2^2(r_2)dx_2dy_2dz_2}{|r_1-r_2|}+e_1e_2\iiint\frac{\psi_1^2(r_1)dx_1dy_1dz_1}{|r_1-r_2|}\cancel{=}$$

$$-\frac{e_1e_2}{|r_1-r_2|}\left[\cancel{\tfrac{1}{2}}\iiint\psi_1^2(r_1)\,\underset{\underset{1}{\parallel}}{dx_1dy_1dz_1}+\frac{1}{2}\iiint\psi_2^2(r_2)dx_2dy_2dz_2\right]$$

$$=\ e_1e_2\Big[\iiint$$

1st approximation トシテ

$$e_1e_2\iiint\frac{\psi_2^2(r_2)dx_2dy_2dz_2}{|r_1-r_2|}\doteqdot e_1e_2\iiint\frac{\psi_1^2(r_1)dx_1dy_1dz_1}{|r_1-r_2|}$$

$$\doteqdot\frac{e_1e_2}{|r_1-r_2|}$$

トスレバ.

従ツテ ½ノ factor ハイリソウデアルガコノ点ハ解決出来ヌ 元々 many dimensional wave equation デモ解ニアラハサレヌ 之ハ $\Psi=\psi_1\psi_2$ トイフ様ノ形ニナラヌカラデアル。

Helium Spectra ト Broglie Wave

Heisenberg* ハ resonance ナル現象ヲ考ヘルコトニヨッテ、Helium Spectra ノ美事ナ説明ヲシタ。コノ resonance ナル現象ハ wave mechanical ニモ最モ natural ナ説明ガ出来ルガ** symmetric 及ビ antisymmetric ナ eigenfunction ヲ matrix ノ考ヘヲ用ヰズシテ導入スルノハ非常ニ困難デアル。コノ難点ガアルタメニ、many electron problem ニ対シテ、many dimensional wave mechanics ノ方法ガソレ丈デハ不充分デ、observable quantity ノ概念ニヨッテ matrix ヲ考ヘニ入レルノデナケレバ満足ナ解決ガ出来ヌコトヽナッタ。

然ルニ コノ antisymmetric 及ビ symmetric eigenfunction ナルモノハ、次ノ如キ考ヘニヨッテ全ク wave mechanically ニ容易ニ出スコトガ出来ル。次ノ考ヘハコノ他ノ多クノ問題ニモ適用シ得ルモノデアル。而シテ所謂 Pauliverbot ガ之ニヨッテ出テ来ル。

many dimensional wave mechanics ノ fundamental equation ハ

$$\text{div grad}\,\psi+\frac{8\pi^2}{h^2}(E-V)\psi=0$$

デアル。ココニ div, grad ノ operation ハ line element ヲ

$$ds^2=2\overline{T}(q_k,\dot{q}_k)dt^2\qquad \overline{T}:\text{kinetic energy}$$

トセル時ノ div, grad デアル。

デ今 ~~——~~ Helium Atom ニ於テ 二ツノ electron ノ間ノ相互作用ヲ無視スレバ 其レニ対シテ

$$\Delta_1\psi+\frac{8\pi^2}{h^2}\frac{ee'}{r_1}\psi-\frac{4\pi i}{h}\dot{\psi}=0\qquad e':\text{kernladung}$$

(Eigenfunktionen: ψ_k zu E_k)

$$\Delta_2\varphi+\frac{8\pi^2}{h^2}\frac{ee'}{r_2}\varphi-\frac{4\pi i}{h}\dot{\varphi}=0$$

(Eigenfunktionen: φ_l zu F_l)

トスレバ gesamt System ニ対スル equation ハ

$$(\Delta_1+\Delta_2)\Psi+\frac{8\pi^2}{h^2}\left(\frac{ee'}{r_1}+\frac{ee'}{r_2}\right)\Psi-\frac{4\pi i}{h}\dot{\Psi}=0$$

トナル。Eigenwert E_k+F_l ニハ Eigenfunktion $\psi_k\varphi_l$ ガ属シテ居ル。今 electrons ノ相互作用ノ Koppelungglied ヲ入レルト、resonance ノ現象ハ容易ニ説明シ得ル**。而シ gesamt System ニ対スル Eigenfunktion ノ形ハコノ wave equation カラハ出テコナイ。

ソコデ コノ wave equation ノ出テ来タ源ニ溯ッテ考ヘテ見ルニ、Schrödinger ノ erste Mitteilung デハ、Hydrogen Atom ノ場合ニツイテ次ノ Variations-

*

**

gleichung ヲ考ヘヨ。

$$\delta J = \delta\iiint dx\,dy\,dz\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m}{K^2}\left(E+\frac{e^2}{r}\right)\psi^2\right]$$
$$=0$$

ココニ integral ハ凡テ空間ニ就テトルノデアル。

今ノ場合モコレト同様ナ Variations problem トシテ考ヘヨウ。Kopplung ノナイ時ノ gesamt System ノ Eigenwert E = Em + En ニ對スル eigenfunktion ハ m ≠ n ノ時ニハ entartet デ

$$\psi_m\psi_n', \quad \psi_m'\psi_n$$

ノ二ツノ function ヲ考ヘルコトガ出来ル。其ノ linear combination

$$a\psi_m\psi_n' + b\psi_m'\psi_n \qquad (\text{where } a^2+b^2=1.)$$

ハ ~~常ニ~~ ヤハリ Eigenfunktion デアル。

ソコデ a, b ハ何ナル値ヲトルベキカハ wave equation カラハ決定出来ヌカラ、

(I)
$$\iiint dx_1dy_1dz_1\,dx_2dy_2dz_2\left[\left(\frac{\partial\Psi}{\partial x_1}\right)^2+\left(\frac{\partial\Psi}{\partial y_1}\right)^2+\left(\frac{\partial\Psi}{\partial z_1}\right)^2-\frac{2m}{K^2}\frac{ee'}{r_1}\Psi^2\right.$$
$$\left.+\left(\frac{\partial\Psi}{\partial x_2}\right)^2+\left(\frac{\partial\Psi}{\partial y_2}\right)^2+\left(\frac{\partial\Psi}{\partial z_2}\right)^2-\frac{2m}{K^2}\frac{ee'}{r_2}\Psi^2\Big\}-\frac{2m}{K^2}E\Psi^2\right]$$

ガ extremum トナラネバナラヌト假定シヨウ。スルト上ノ wave equation ハ先ヅ成立セネバナラヌ。次ニ

$$\Psi = a\psi_m\psi_n' + b\psi_m'\psi_n$$

ヲ ~~equation~~ integrand ノ中ニ代入シテ integration ヲ實際行フト、

(II)
$$2(a^2+b^2)\left[\int(\mathrm{grad}\,\psi_m)^2dv_1+\int(\mathrm{grad}\,\psi_n)^2dv_1-\Big\{\frac{2mee'}{K^2}\Big(\int\frac{\psi_m^2}{r_1}dv_1\right.$$
$$\left.+\int\frac{\psi_n^2}{r_1}dv_1\Big)\Big\}-\frac{2m}{K^2}E\right]$$

トナッテクル。コノ値ハ $a^2+b^2=1$ デアルカラ、a, b ノ値ニハ無関係デアル。

從ッテコノ ∫ ノ maximum, minimum カラハ a, b ノ値ハキマラヌ。

次ニ Kopplungsglied ヲ考ヘル。ソレ其ノ potential energy ハ

$$\frac{2me^2}{K^2}\cdot\frac{1}{r_{12}}$$

デアルカラ、(I) ノ integrand ノ中ニ $\frac{2me^2}{K^2}\frac{\Psi}{r_{12}}$ ノ term ヲ加ヘネバナラヌ。

スルト (II) ~~値~~ ニモ次ノ term ガ加ハル

即チ
$$\frac{2me^2}{K^2}\int\frac{\Psi^2}{r_{12}}dv_1dv_2 = \frac{2me^2}{K^2}\int\left\{\frac{a^2\psi_m^2\psi_n'^2}{r_{12}}+\frac{b^2\psi_n^2\psi_m'^2}{r_{12}}\right\}dv_1dv_2$$
$$+\frac{4me^2ab}{K^2}\int\frac{(\psi_m\psi_n'^*\psi_n\psi_m')}{r_{12}}dv_1dv_2$$

r_{12} ハ 1, 2 ニツイテ symmetrical デアルカラ、$a^2+b^2=1$ ヲ考慮スルト

(III)
$$=\frac{2me^2}{K^2}\int\frac{\psi_m^2\psi_n'^2}{r_{12}}dv_1dv_2+\frac{4me^2}{K^2}ab\int\frac{(\psi_m\psi_n'^*\psi_n\psi_m')}{r_{12}}dv_1dv_2$$

(II) ト (III) ヲ一緒ニシタモノノ integral ニツイテ a, b ノ値ニ depend スルモノハ最後ノ項丈デアル。ソレデ此ノ term ガ maximum, minimum トナル時ヲ求ムレバヨイ。ソレハ勿論、

$$a = b = \frac{1}{\sqrt{2}}\ ; \quad a = \pm\frac{1}{\sqrt{2}}, \quad b = \mp\frac{1}{\sqrt{2}}$$

ノ二ツノ場合デアル。

而シテ $a=b=\frac{1}{\sqrt{2}}$ ハ maximum ノ場合ニアタリ後ノ二ツノ場合ハ minimum ノ場合ニアタル。從ッテ (I) ノ integral ガ minimum トナルト ~~云フ條件ヲ~~ 如キ amplitude Ψ ノ distribution ガ實際オコルモノデアルト假定スルト symmetrical ノ場合ハ 最モ stable デアル unstable デ antisymmetrical ノ場合ガ stable ナ distribution トナル。

之ハ electrons ガ皆同ジモノデアッテ其ノ間ニ repulsion ガ働クカラ成立スルコトデアルガ、異ナル particles ノ場合ニモ拡張シ得ルシ、又 attraction ガ働ク時ニハ symmetrical ノ場合ト antisymmetrical ノ場合トノ stability ハ逆ニナル。

Lichtquanten (lichtwave) ニツイテ symmetrical ノ方ヲトルノハ light-quanta ノ間ニ外部ニカデアルガ attraction ガ存在スルトシテ説明出来ル。

又 n 個ノ electrons ノ場合ニモ ~~一般~~ 拡張シテ

$$\begin{vmatrix}\psi_m^1 & \psi_n^1 & \cdots \\ \psi_m^2 & \psi_n^2 & \cdots \\ \cdot & \cdot & \cdot\end{vmatrix}$$

ノ形ノ antisymmetrical eigenfunction ヲトルコトモ容易ニ行ハレルデアラウ。

[15]

13.　　Wave packet ニ就テ

以上考ヘタ所ニヨリ更ニ想像シ得ルコトハ. energy E ガアル一定ノ値ヲトルノデハナク. アル ~~energy ノ~~ 小範囲ノ値ヲトリ得ル (or frequency) 其ノ範囲ニ於ケル amplitude ノ Distribution ノ関係ハ如何ナルヤ ソノ内容 ~~モ又同ジ様~~ ナル例ヘバ Schrödinger ノ wave packet ノ問題モ解キ得ル (徹底ニシテ) ノデハナイカト云フコトデアル。

~~E ノ値~~ 今簡単ノ為メ free electron ヲ考ヘル. 其ノ wave equation ハ

$$\Delta\psi+\frac{2m}{K^2}E\psi=0.$$

ソノ variation ノ problem トシテハ

$$\iiint\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m}{K^2}E\psi^2\right]dx\,dy\,dz$$

ガ min トナラネバナラヌト仮定スル。

ψハ實際

ims [13, [14]]

(1)
$$\delta\iiint\left[\frac{1}{2m_1}\left\{\left(\frac{\partial\psi_1}{\partial x}\right)^2+\left(\frac{\partial\psi_1}{\partial y}\right)^2+\left(\frac{\partial\psi_1}{\partial z}\right)^2\right\}+\frac{1}{2m_2}\left\{\left(\frac{\partial\psi_2}{\partial x}\right)^2+\left(\frac{\partial\psi_2}{\partial y}\right)^2+\left(\frac{\partial\psi_2}{\partial z}\right)^2\right\}\right.$$
$$\left.\pm\frac{4\pi^2}{h^2}\left\{V_0(\psi_1^2+\psi_2^2)+e^2\psi_2^2\int\frac{\psi_1^2(r')dv'}{|r-r'|}+e^2\psi_1^2\int\frac{\psi_2^2(r')dv'}{|r-r'|}\right\}\right]dx\,dy\,dz$$
$$=0.$$

Two Body Problem

Helium Atom デハ nucleus ヲ Origin ニ

$$V_0=\frac{2e}{r}$$

~~Coord.~~ many dim. ノ場合

(1) カラ $\delta\iiint\psi_2\frac{1}{2m_1}\left(\frac{\partial\psi_1}{\partial x}\right)^2+\cdots+\frac{1}{2m_2}\left\{\left(\frac{\partial\psi_2}{\partial x}\right)^2\right.$

$\iiint\psi_2^2(x')dx'dy'dz'$

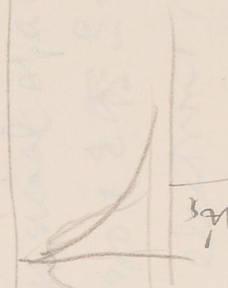
$\iiint\psi_1^2(x')dx'dy'dz'$

$$\psi_2^2(x)\left(\frac{\partial\psi_1}{\partial x}\right)^2+\psi_1^2(x')\left(\frac{\partial\psi_2}{\partial x}\right)^2=\psi_2^2(x)\left(\frac{\partial\psi_1(x')}{\partial x'}\right)^2+\psi_1^2(x')\left(\frac{\partial}{\partial x}\right)$$
$$=\frac{\partial\ \psi_1(x')\psi_2(x)}{\partial x}$$

$AB\equiv\frac{A}{B}$

$\int AB=$

$\iiint\frac{\psi_1^2\psi_2^2\,dx'dy'dz'}{|x-x'|}$



$$\iiiint\left(\text{grad}\,\delta\Psi\right)^2 dx_0\,dx_1\,dx_2\,dx_3$$
$$=0$$

$=0$

$=0$

$=E_{kt})$

$dv'=0$

13.

Wave packet ニ就イテ

以上考ヘタ所ニヨリ更ニ想像セラルルコトハ．energy E ガアル一定ノ値ヲトルノデハナク，アル ~~energy~~ ノ小範囲ノ値ヲトリ得ル（or frequency）其ノ範囲ニ於ケル amplitude ノ distribution ノ関係ハ如何ナルモノカ ~~モヨク同ジ様ナ~~ 即チ Schrödinger ノ wave packet ノ問題モトキ得ル（附加ニシテ）ノデハナイカトイフコトデアル。

~~E ノ値~~ 今簡単ノタメ free electron ヲ考ヘルト．其ノ wave equation ハ

$$\Delta\psi + \frac{2m}{K^2}E\psi = 0.$$

之ヲ variation ノ problem トシテハ

$$\iiint\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m}{K^2}E\psi^2\right]dx\,dy\,dz$$

ガ min トナラネバナラヌト仮定スル。

ψハ實際

§4. Angeregte Zustand des Atoms.

今二ツノ electron ヲ有スル atom ヲ考ヘル。其ノ electron ノ一ツガ normal state カラハナレテ，excited state ニアリトスル時。コノ場合二ツノ electron ガ相互作用ニヨッテイカナル関係ヲヒキオコスカヲ考ヘテ見ル。

先(5)ノ式ヲ擴張シテ

$$\iiint\Big[\frac{h^2}{8\pi^2 m}\Big\{\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\overline{\psi}}{\partial x}\right)+\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial\overline{\psi}}{\partial y}\right)+\left(\frac{\partial\psi}{\partial z}\right)\left(\frac{\partial\overline{\psi}}{\partial z}\right)\Big\}$$

$$+\frac{h^2}{8\pi^2 m}\Big\{\left(\frac{\partial\varphi}{\partial x}\right)\left(\frac{\partial\overline{\varphi}}{\partial x}\right)+\cdots\cdots\Big\}$$

$$+V_0\left(e\psi\overline{\psi}+e\varphi\overline{\varphi}\right)+e^2\varphi\overline{\varphi}\int\frac{\psi\overline{\psi}\,dv'}{|r-r'|}$$

$$+K\iiint\left(\psi\frac{\partial\overline{\psi}}{\partial t}-\overline{\psi}\frac{\partial\psi}{\partial t}\right)+(\quad)\,dv=\min!!$$

$$(1)\quad -K\frac{\partial\psi}{\partial t}+\frac{h^2}{8\pi^2 m}\Delta\psi+eV_0\psi-e^2\psi\int\frac{\varphi\overline{\varphi}}{|r-r'|}dv'=0$$

$$(2)\quad +K\frac{\partial\overline{\psi}}{\partial t}+\frac{h^2}{8\pi^2 m}\Delta\overline{\psi}+eV_0\overline{\psi}-e^2\overline{\psi}\int\frac{\varphi\overline{\varphi}}{|r-r'|}dv'=0$$

（φ，φ̄ ニツイテモ同様ナ式ガ得ラルル筈．）　$K=\frac{h}{2\pi i}$

カク二ツノ wave equation ヲ求メ，particle ニツイテ得ル。

今第二ノ electron (φ) ノ方ガ normal state ニアリ第一ノ electron ハ excite サレテヰルトスル．其ノ eigenfunction ハ一般ニ

$$\psi=\sum c_k u_k e^{\frac{2\pi i}{h}E_k t}\qquad\left(\overline{\psi}=\sum c_k u_k e^{-\frac{2\pi i}{h}E_k t}\right)$$

デ表ハサルル。

之ヲイレルト (1) (2) ハ

$$(3)\quad \cancel{\frac{4\pi i}{h}}E_k u_k+\frac{h^2}{8\pi^2 m}\Delta u_k+eV_0u_k-e^2u_k\int\frac{\varphi\overline{\varphi}}{|r-r'|}dv'=0$$

トナル。

第二ノ electron ハ normal state ニアルカラ，$\varphi\overline{\varphi}$ ハ time ヲフクマヌ。従ッテ

$$\Delta u_k+\left(E_k+eV_0-e^2u_k\int\frac{\varphi\overline{\varphi}}{|r-r'|}dv'\right)u_k=0$$

13.

ins [13, [14])

(1) $$\delta\iiint\Big[\frac{1}{2m_1}\Big\{\Big(\frac{\partial\psi_1}{\partial x}\Big)^2+\Big(\frac{\partial\psi_1}{\partial y}\Big)^2+\Big(\frac{\partial\psi_1}{\partial z}\Big)^2\Big\}+\frac{1}{2m_2}\Big\{\Big(\frac{\partial\psi_2}{\partial x}\Big)^2+\Big(\frac{\partial\psi_2}{\partial y}\Big)^2+\Big(\frac{\partial\psi_3}{\partial z}\Big)^2\Big\}$$
$$\pm\frac{4\pi^2}{h^2}\Big\{\overset{e}{V_0}(\psi_1^2+\psi_2^2)+e^2\psi_2^2\int\frac{\psi_1^2(x')dv'}{|x-x'|}+e^2\psi_1^2\int\frac{\psi_2^2(x')dv'}{|x-x'|}\Big\}\Big]dx\,dy\,dz$$
$$=0.$$

Two Body Problem

Helium Atom デハ nucleus ヲ Origin ト

$$V_0=\frac{2e}{r}$$

~~Coord~~, Many dim, [illegible]

(1) カラ $$\delta\iiint\psi_2\,\frac{1}{2m_1}\Big(\frac{\partial\psi_1}{\partial x}\Big)^2+\cdots\Big\}+\frac{1}{2m_2}\Big\{\Big(\frac{\partial\psi_2}{\partial x}\Big)^2$$

$$\iiint\psi_2^2(x')dx'dy'dz' \qquad \iiint\psi_1^2(x')dx'dy'dz'$$

$$\psi_2^2\Big(\frac{\partial\psi_1}{\partial x}\Big)^2+\psi_1^2(x')\Big(\frac{\partial\psi_2}{\partial x}\Big)^2=\psi_2^2(x)\Big(\frac{\partial\psi_1(x')}{\partial x'}\Big)^2+\psi_1^2(x')\Big(\cdots$$

$$=\frac{\partial\ \psi_1(x')\psi_2(x)}{\partial x}$$

$$A\Delta B\equiv$$

$$\int A\Delta B=-\int\cdots+\int A\int B\,dx$$

$$\iiint\frac{(\psi_1^2+\psi_2^2)\,dx'dy'dz'}{|x-x'||x-x'|}=\cdots$$

$$\delta\iiiint\big\{(\operatorname{grad}\Psi)^2+\lambda\Psi^2\big\}dx_0\,dx\,dx'\cdots=0$$

$$=\iiiint(\operatorname{grad}\Psi)^2\,dx_0\,dx\,dx'\cdots+\iiint\frac{\psi_1^2(x)\psi_2^2(x')\,dx\,dy\,dz\,dx'dy'dz'}{|x-x'|}+\cdots$$

Helium: $$+\frac{1}{2m_1}\Delta\psi_1\pm\frac{4\pi^2}{h^2}\Big(\overset{E_1+}{\cancel{E_1}}\frac{2e^2\psi_1}{r}\pm e^2\psi_1\int\frac{\psi_2^2(x')dv'}{|x-x'|}\Big)=0$$

(1) $$\Delta\psi_1+\frac{8m_1\pi^2}{h^2}\Big(E_1\psi_1+\frac{2e^2\psi_1}{r}-e^2\psi_1\int\frac{\psi_2^2(x')dv'}{|x-x'|}\Big)=0$$

(2) $$\Delta\psi_2+\frac{8m\pi^2}{h^2}\Big(E_2\psi_2+\frac{2e^2\psi_2}{r}-e^2\psi_2\int\frac{\psi_1^2(x')dv'}{|x-x'|}\Big)=0$$

(1) ノ ψ_1 ニ ~~水素核~~ ノ方ヲ ~~[illegible]~~ 他ノ電子ノ perturbation ヲ無視シテ 求メタ物ノ Eigenfunction ヲ入レル、ソレヲ(2) ノ ψ_1 ニ入レテ ψ_2 ヲ計算スル

又ソレヲ (1) ノ ψ_2 ニ入レテ ψ_1 ヲ correct スル、コノ方法ヲ繰返セバヨイ。

[15]

13.

many dimensional wave mechanics ガ大ナル進歩ヲナシ乍、此等ニ多クノ問題ヲトキ得ルニ反シ、wave mechanics ヲ three dimension space ニオケル ~~[illegible]~~ 作用トシテ解シ得ヌ不可能ナ事柄ノ様ニ見エル。只一ツノ particle ノ場合ナラバ degree of freedom ガ三デアルカラ三次元空間ニオケル現象トシテ取扱ヘルガ、二ツ以上ノ particle ノ場合ニハ其ノ degree of freedom ニ相当スル空間ヲ考ヘネバナラヌ。之ヲ三次元的空間ニオケル現象ニ翻訳スルコトハ不可能デナクテモ、己ニ anschaulichkeit ヲ失ッテ居ル。

ソコデ一般ノ many body problem ヲ three dimensional space ニオケル ~~[illegible]~~ [illegible] ~~下~~デ次ノ如ク考ヘテ見ルニ、Schrödinger ノ最初ノ論文デアル式ヲモウ一度見直シテ見ルニ、~~Wass~~ Hydrogen Atom ノ場合ノ electron ノ満足スベキ式トシテ最初ニ得タモノハ

(1) $$\delta\iiint dx\,dy\,dz\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m}{K^2}\left(E+\frac{e^2}{r}\right)\psi^2\right]=0$$

デアル。Schrödinger ノ解釈ニ解釈セル所ニヨレバ $e\psi^2$ ハ electron charge ノ density ヲアラハシテヰル。

$$\iiint\psi^2\,dx\,dy\,dz=1$$

ソレハ分離シテ「

$$\delta\iiint dx\,dy\,dz\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m}{K^2}\varepsilon\frac{e^2}{r}\psi^2\right]=0 \quad (1)'$$

従ッテ(1)式ハ

但シ Nebenbedingung トシテ

$$~~\text{下}~~\iiint\psi^2\,dx\,dy\,dz=1 \qquad (2)$$

ヲ満足セネバナラヌ。」ト考ヘテ ~~(1)式ヲ~~ 出テ来ルモノト解釈シタ。

之ヲ更ニヨク考ヘテ見レバ、$-\iiint\frac{e^2\psi^2}{r}dx\,dy\,dz$ ハ ~~蓄~~ potential energy ノ意味ヲアラハシ、~~[illegible]~~ $\frac{1}{2}\iiint K^2\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2\right]dx\,dy\,dz$ ハ kinetic energy ノ意味ヲアラハスト見ルカラ、(1)′ノ式ハ electric charge ノ（総和）一定ナル時、~~system ノ~~ electric charge ノ distribution ハ Total Energy? ノ値ガ extremum（多クハ minimum）トナル様ニナルコトヲアラハスト解シウル。

以上ノ考ヘヲ多クノ particles ノアル場合ニオシヒロゲルト、例ヘバ二ツノ particle ノ場合ヲ考ヘルニ、夫々ノ particle ノ electric charge ノ distribution ヲアラハス function ψ_1, ψ_2 ヲ夫々同一空間ニ考ヘル。ソレハ(1)′ニ相当スル式ハ

$$\delta\iiint dx\,dy\,dz\left[\frac{1}{2m_1}\left\{\left(\frac{\partial\psi_1}{\partial x}\right)^2+\left(\frac{\partial\psi_1}{\partial y}\right)^2+\left(\frac{\partial\psi_1}{\partial z}\right)^2\right\}+\frac{1}{2m_2}\left\{\left(\frac{\partial\psi_2}{\partial x}\right)^2+\left(\frac{\partial\psi_2}{\partial y}\right)^2+\left(\frac{\partial\psi_2}{\partial z}\right)^2\right\}\right.$$
$$\left.-\frac{4\pi^2}{h^2}\left\{V_0(e_1\psi_1^2+e_2\psi_2^2)+e_1e_2\psi_2^2\int\frac{\psi_1^2(r')dv'}{|r-r'|}+e_1e_2\psi_1^2\int\frac{\psi_2^2(r')dv'}{|r-r'|}\right\}\right]$$
$$=0 \qquad +\frac{1}{2}\{e_1e_2\psi_2^2\int \quad +e_1e_2\psi_1^2\int \quad \cdots\}$$

$dv'=0$

$dv'=0$

$\frac{\varphi\Phi}{|r-r'|}dv'=0$

time

$)=0$

(2)ニ相当スルノハ

$$\iiint \psi_1^2 dxdydz=1 \qquad \iiint \psi_2^2 dxdydz=1.$$

カクシテ得タ式ガ ~~coord~~ many dim.式ト一致スルコトヲ示サネバナラヌ。

(一方又 (1)' ヲ出発点 is many dimensional ノ式ヲ用ヰレバ similar particles ノ場合ノ eigenfunction ガ symm. antisymm. ナ形ヲトルコトハ容易ニ示サレル。)

Qノ様ニ Three dim. ナ式ヲツカフト 種々ノ問題殊ニ原子ノ衝突ノ問題ノ如キモ anschaulich ニトクコトガ出来ル。其ノ他従ッテ many dimension デヤッタ丈ケノ問題ヲ three dimension ニ直セバ ~~夫々~~ 更ニ anschauliche ナ解決ヲ得ル。例ヘバ Helium ニ於ケル ~~electricity~~ density distribution 等ハ先ヅ考ヘル可キ問題デアラウ。

(Jordan, Klein 45 ノ論文ハ矢張リ many body problem ヲ3次元ニ直シタモノ問題トシテトカウトシテヰルガ wave equation カラ出発シテ居ナイカラ要点ガワカラナイ。之レヲ直スニハ q-number ヲ使ッテ居ルガ、カウシテ ~~居~~ anschaulich ナ[illegible]ノ母関数ヲ用ヒルノガ宜シイノデアル。)

2ノ拡張ヲ

1) non-conservative ノ場合 (所謂)

2) relativity ヲ考ヘ、又 magnetic field ノ作用ヲ考ヘル場合、

3) 四次元的ニアラハスコト

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一般ノ Eigenfunction ハ

Eigenfu^n u_K

ニヨツテ Eigenwert E_K ヲ決定スレバ

$$\psi = \sum c_k u_k e^{-\frac{2\pi i}{h}E_k t}$$
$$\bar{\psi} = \sum c_k u_k e^{\frac{2\pi i}{h}E_k t}$$

デアルカラ、従ツテ density ハ

$$\psi\bar{\psi} = \sum c_k c_j u_k u_j e^{\frac{2\pi i}{h}(E_j - E_k)t}$$

トナル、

∴ $\frac{E_j - E_k}{h}$ ナル frequency ノ radiation ガ emit サレル。

次ニ第二ノ electron ノ方モ excite サレテ居ルトスレバ。ソレノ eigenfunction ガ

$$\varphi = \cancel{\Sigma}\, d_1 v_1 e^{-\frac{2\pi i}{h}F_1 t} + d_2 v_2 e^{-\frac{2\pi i}{h}F_2 t}$$

故

$$\bar{\varphi} = d_1 v_1 e^{\frac{2\pi i}{h}F_1 t} + d_2 v_2 e^{\frac{2\pi i}{h}F_2 t}$$

トナツテ居レバ。

$$\varphi\bar{\varphi} = d_1^2 v_1^2 + d_2^2 v_2^2 + 2 d_1 d_2 v_1 v_2 \cos\frac{2\pi(F_1 - F_2)t}{h}$$

従ツテ (3) 式ノ最後ノ項ガ

$$-e^2 \int \frac{\varphi\bar{\varphi}}{|r - r'|} dv' \cdot u_k$$

ハ time ヲフクンデ居ツテ、其ノ形ハ

$$A \cos\frac{2\pi(F_1 - F_2)t}{h}$$

デアル。即チ $F_1 - F_2$ ナル frequency ノ periodic ~~vibration~~ disturbance ガオコツテ居ルコトニナル。従ツテ第一ノ electron ハ resonance ヲオコシ ~~其ノ frequency ハ~~ eigenfunction ハ

$$\cancel{\psi = \psi = \sum c_k u_n(x) e^{\frac{2\pi i E_k}{h}t} + \frac{1}{2}\sum a_{kn} u_n(x)}$$

$$\psi = \sum_k \left[c_k u_k e^{\frac{2\pi i E_k t}{h}} + \cancel{\tfrac{1}{2}} \sum_n a_{kn} u_n \left(\frac{e^{\frac{2\pi i}{h}(E_k + F_1 - F_2)}}{E_k - E_n + F_1 - F_2} + \frac{e^{\frac{2\pi i t}{h}(E_k - F_1 + F_2)}}{E_k - E_n - F_1 + F_2} \right) \right]$$

トナル。従ツテ density

$$\psi\bar{\psi} = \sum c_k c_j u_k u_j e^{\frac{2\pi i}{h}(E_j - E_k)t}$$

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$$+ a_{jm} u_m c_k u_k \left\{ \frac{e^{\frac{2\pi i}{h}(E_j - E_k + F_1 - F_2)}}{-E_j + E_m + F_1 - F_2} + \frac{\quad}{-E_j + E_m - F_1 + F_2} \right\}$$

$$+ \sum_k \sum_n \overset{\sum_j \sum_m}{} a_{kn} u_n c_j u_j \left\{ \frac{e^{\frac{2\pi i}{h}(E_k - E_j + F_1 - F_2)}}{E_k - E_n + F_1 - F_2} + \frac{e^{\frac{2\pi i}{h}(E_k - E_j - F_1 + F_2)}}{E_k - E_n - F_1 + F_2} \right\}$$

ここニ

$$a_{kn} = \frac{1}{2}\int A \cdot u_k u_n dv.$$

従ツテ $E_k - E_j$ ナル普通ノ frequency ノ radiation ノ他ニ

$$(E_k - E_j + F_1 - F_2)/h$$

トイフ新ラシイ frequency ノ radiation モ emit サレル

之所謂 Alkali earth spectra ニ於ケル P' term デアル。

其ノ ~~intensity ハ~~ density ノ式ノコレニ相当スル term ハ

收[illegible] ?

$$\sum_{nm} \left[\left\{ \frac{a_{kn} u_n c_j u_j}{E_k - E_n + F_1 - F_2} + \frac{a_{jm} u_m c_k u_k}{-E_j + E_m - F_1 + F_2} \right\} e^{E_k - E_j + F_1 - F_2} + \left\{ \frac{a_{jm} u_m c_k u_k}{E_j - E_m + F_1 - F_2} + \frac{a_{kn} u_n c_j u_j}{-E_k + E_n - F_1 + F_2} \right\} e^{E_j - E_k - F_1 + F_2} \right]$$

$$= \sum_{nm} 2 \left\{ \frac{a_{kn} u_n c_j u_j}{E_k - E_n + F_1 - F_2} + \frac{a_{jm} u_m c_k u_k}{E_j - E_m + F_1 - F_2} \right\} \times \overset{\cos ?}{\sin}\frac{2\pi}{h}(E_k - E_j + F_1 - F_2)t$$

従ツテ ~~intensit amplitu~~ electric moment ハ

$$\int 2\left[\qquad \right] e x\, dv$$

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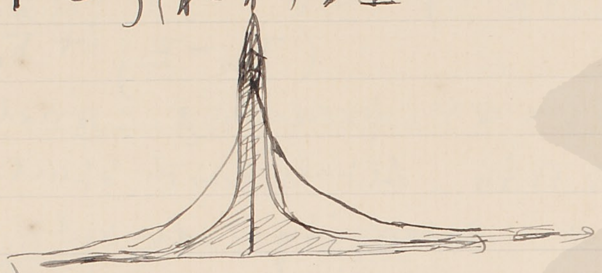
§ Two Body Problem (Hydrogen Atom)

$$\Psi \quad \frac{1}{2m}\left\{\left(\frac{\partial\Psi}{\partial x}\right)^2 + (\quad) + (\quad)\right\}$$

$$\begin{cases} \Delta\psi + \frac{8\pi^2 m}{h^2}\left(E + e^2\int\frac{\Psi_1^2 dv'}{|r-r'|}\right)\psi = 0 \\ \Delta\Psi + \frac{8\pi^2 M}{h^2}\left(E + e^2\int\frac{\psi^2 dv'}{|r-r'|}\right)\Psi = 0 \end{cases}$$

$$\iiint \Psi^2 dv = 1$$



$$\Delta\Psi + \frac{8\pi^2 m}{h^2}\left(E + \lambda \underset{\downarrow \atop 0}{V(r)}\right)\psi = 0$$

$$\Psi = e^{\cdots}$$

$$\frac{eV}{c}\rho + e(A\cdot X) + mc\,\square = 0$$

$$\frac{1}{c}\frac{\partial V}{\partial t} + \operatorname{div} A = 0$$

grad

$$V\square V + A\square A$$

$$\boxed{\beta = -1 + \sqrt{1+\gamma C}}$$

$$\beta(\beta+2) = \gamma^2$$

$$\frac{2\pi a e^2}{hc} = \gamma$$

$$-aC + \beta b = 0$$

$$A a + \gamma a = 0 \qquad bC + (\beta+2)a = 0$$

$$+B b + \qquad P = a r^\beta$$

$$Q = b r^\beta$$

$$\frac{\operatorname{grad}\operatorname{div}\Psi}{\frac{W}{c}+\frac{eV}{c}-mc} - \frac{\operatorname{div}\Psi\cdot\frac{e}{c}\operatorname{grad}V}{(\quad)^2} + \frac{4\pi^2}{h^2}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\Psi = 0$$

$$\operatorname{grad}\frac{1}{r^2}\frac{d(r^2\Psi_r)}{dr} - \frac{}{(\quad)} \qquad c = \frac{2\pi a e^2}{hc}$$

$$-\left(A+\frac{B}{r}\right)P(r) - \lambda Q(r) + Q'(r) = 0$$

$$\left(B+\frac{C}{r}\right)Q(r) - \lambda P(r) + P'(r) + \frac{2P(r)}{r}$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\Psi_r + \frac{\partial X_0}{\partial r} = 0$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-mc\right)X_0 + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Psi_r) = 0$$

$$\Psi_r = P(r)e^{-\lambda r} \qquad X_0 = Q(r)e^{-\lambda r}$$

$$P(r) = i\,p(r)$$

$$\begin{cases} \frac{2\pi i}{h}\left(\frac{W}{c}+\frac{e^2}{cr}+mc\right)P(r) - \lambda Q(r) + Q'(r) = 0 \\ \frac{2\pi i}{h}\left(\frac{W}{c}+\frac{e^2}{cr}-mc\right)Q(r) - \lambda P(r) + \frac{1}{r^2}\frac{d}{dr}(r^2 P(r)) \\ \qquad + P'(r) + \frac{2P(r)}{r} \end{cases}$$

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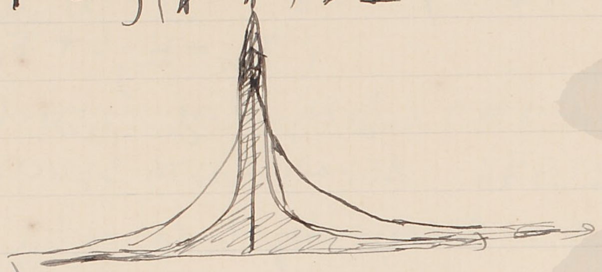
§ Two Body Problem (Hydrogen Atom)

Ψ $\frac{1}{2m}\left\{\left(\frac{\partial\psi}{\partial x}\right)^2+(\quad)+(\quad)\right\}$

$$\begin{cases}\Delta\psi+\frac{8\pi^2m}{h^2}\left(E+e^2\int\frac{\Psi_1^2\,dv'}{|r-r'|}\right)\psi=0\\ \Delta\Psi+\frac{8\pi^2M}{h^2}\left(E+e^2\int\frac{\psi^2\,dv'}{|r-r'|}\right)\Psi=0\end{cases}$$

$$\iiint\Psi^2\,dv=1$$



$$\Delta\Psi+\frac{8\pi^2m}{h^2}\left(E+\lambda\underset{\substack{\downarrow\\0}}{V(r)}\right)\psi=0$$

$$\Psi=e^{\cdots}$$

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Hydrogen ノ問題

ins ([17],[18])

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\mathbf{Y}+\operatorname{grad}X_0=0$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-mc\right)X_0+\operatorname{div}\mathbf{Y}=0$$

$$\operatorname{grad}\left(\frac{\operatorname{div}\mathbf{Y}}{\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-mc\right)}\right)-\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\mathbf{Y}=0$$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}j=0 \qquad \frac{e}{c}\Big[\cancel{2}X_0\underline{\frac{\partial X_0}{\partial t}}-\cancel{X_0}\frac{\partial Y_1}{\partial x}+\underline{\mathbf{X}}\frac{\partial\mathbf{X}}{\partial t}+Y_0\underline{\frac{\partial Y_0}{\partial t}}+\mathbf{Y}\frac{\partial\mathbf{Y}}{\partial t}\Big]$$

$$+\;Y_3\frac{\partial X_2}{\partial x}+Y_1\frac{\partial X_3}{\partial x}+Y_2\frac{\partial X_1}{\partial x}\quad+\underline{\operatorname{div}\mathbf{X}}\cdot Y_0+\underline{\mathbf{X}}\frac{\partial Y_0}{\partial t}+$$

$$-\;\underline{X_3\frac{\partial Y_2}{\partial x}-X_2\frac{\partial Y_3}{\partial x}}\qquad-\underline{\operatorname{div}\mathbf{Y}}\,X_0+\underline{\mathbf{Y}}\frac{\partial X_0}{\partial t}+$$

$$\left\{\begin{matrix}Y_3\frac{\partial X_2}{\partial x}\\ Y_1\frac{\partial X_3}{\partial x}\\ Y_2\frac{\partial X_1}{\partial x}\end{matrix}\right.$$

$$=\left(\frac{W}{c}-mc\right)Y_0-A\mathbf{X}\Big)Y_0+\left(\frac{W}{c}+mc\right)X_0+A\mathbf{Y}\Big)X_0$$

$$\left(\frac{W}{c}+mc\right)\cancel{\mathbf{X}}+[A\,\mathbf{Y}]\Big)\mathbf{X}+\Big(\qquad\Big)\mathbf{Y}+[A\,\mathbf{X}]\Big)\mathbf{Y}$$

$$+\left(\tfrac{W}{c}\right)Y_0\,\mathbf{X}$$

$$Y_0\,(p_0+mc)Y_0+Y_0(p,\mathbf{X})$$

$$\mathbf{Y}\,(p_0+mc)\mathbf{Y}-\mathbf{Y}\operatorname{grad}X_0-\mathbf{Y}[p\,\mathbf{X}]$$

$$X_0\,(p_0-mc)X_0+X_0(p,\mathbf{Y})$$

$$\mathbf{X}\,(p_0-mc)\mathbf{X}+\mathbf{X}\,p\,Y_0+\mathbf{X}[p\,\mathbf{Y}]$$

$$\frac{eV}{c}\rho+(A\cdot\mathbf{X})+mc\bigcirc=0$$

$$\frac{1}{c}\frac{\partial V}{\partial t}+\operatorname{div}A=0$$

$$\frac{V}{c}\square V+A\,\square A \quad \text{grad}$$

$$\frac{2\pi e^2}{hc}=\gamma$$

$$-aC+\beta b=0$$

$$\lambda b=0$$

$$A\,a+B\,b+\lambda a=0$$

$$\rho=\mathfrak{a}r^{\beta} \qquad bC+(\beta+2)a=0$$

$$Q=br^{\beta}$$

$$\boxed{\beta=-1+\sqrt{1+\gamma C}}$$

$$\beta(\beta+2)=\gamma^2$$

$$\frac{\operatorname{grad}\operatorname{div}\mathbf{Y}}{\frac{W}{c}+\frac{eV}{c}-mc}-\frac{\operatorname{div}\mathbf{Y}\cdot\frac{e}{c}\operatorname{grad}V}{(\qquad)^2}+\frac{4\pi^2}{h^2}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\mathbf{Y}=0$$

$$Q'(r)=0$$

$$\Big(\frac{B}{r}\Big)P(r)-\lambda Q(r)+$$

$$-\left(A+\frac{\mathfrak{a}}{r}\right)P(r)-\lambda Q(r)$$

$$\left(\cancel{B}+\frac{C}{r}\right)Q(r)-\lambda P(r)+P'(r)+\frac{2P(r)}{r}$$

$$\operatorname{grad}\frac{1}{r^2}\frac{d(r^2Y_r)}{dr}\quad-\quad\frac{\qquad}{(\qquad)}\quad C=\frac{2\pi e^2}{hc}$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)Y_r+\frac{\partial X_0}{\partial r}=0$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-mc\right)X_0+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2Y_r)=0$$

$$P(r)=i\,P(r)$$

$$\left(\frac{2\pi}{h}\qquad\right)$$

$$Y_r=P(r)e^{-\lambda r}\qquad X_0=Q(r)e^{-\lambda r}$$

$$\begin{cases}\dfrac{2\pi i}{h}\left(\dfrac{W}{c}+\dfrac{e^2}{cr}+mc\right)P(r)-\lambda Q(r)+Q'(r)=0\\[2ex]\dfrac{2\pi i}{h}\left(\dfrac{W}{c}+\dfrac{e^2}{cr}-mc\right)Q(r)-\lambda\cancel{Q(r)}\,\cancel{\tfrac{d}{dr}(r^2P(r))}\;\dfrac{1}{r^2}\dfrac{d}{dr}(r^2P(r))\\\qquad+P'(r)+\dfrac{2P(r)}{r}\end{cases}$$

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$$\mathrm{div\,grad}\,X_0 + \frac{1}{A+U}\,\mathrm{div}$$

$$\mathrm{div}\,\frac{a_x i + a_y j + a_z k}{A}\;\frac{X_0}{(\quad)}\qquad X_0 \qquad \mathrm{div}$$

$$= \frac{\partial}{\partial x}\left(\frac{a_x}{A}\right) + \cdots$$

$$= \frac{1}{A}\,\mathrm{div}\,a_x - \frac{a_x \frac{\partial A}{\partial x}}{A^2} + \frac{\partial A}{\partial x}$$

$$\cancel{\mathrm{div\,grad}\,X_0 + (A+U)}\;\partial$$

$$\frac{\partial}{\partial x}\;\frac{1}{r^2}\frac{\partial}{\partial r}\left\{\frac{r^2\frac{\partial X_0}{\partial r}}{\left(\frac{W}{c}+\frac{eV}{c}+mc\right)}\right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left\{\frac{\frac{1}{r\sin\theta}\frac{\partial X_0}{\partial\varphi}}{(\quad)}\right\}$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left\{\frac{\frac{\sin\theta}{r}\cdot\frac{\partial X_0}{\partial\theta}}{(\quad)}\right\}$$

$$= \frac{1}{(\,)}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial X_0}{\partial r}\right) + \frac{\partial X_0}{\partial r}\frac{\partial}{\partial r}\left\{\frac{1}{(\quad)}\right\} + \frac{1}{(\,)}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\frac{1}{r\sin\theta}\frac{\partial X_0}{\partial\varphi}\right)$$

$$+ \frac{1}{(\,)\,r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left\{\frac{1}{(\quad)}\right\} + \frac{1}{(\,)\,r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{r}\frac{\partial X_0}{\partial\theta}\right)$$

$$+ \frac{1}{(\,)}\cdots\frac{\partial}{\partial\theta}\frac{}{(\quad)}$$

$$(\quad)\;\frac{\partial}{\partial x}\qquad \frac{\partial}{\partial r}(\quad)$$

$$[p, X] =$$

$$\begin{cases} \mathrm{div\,grad}\,X_0 - \frac{2\pi i}{h}\,\overset{\mathrm{div}}{}\left(\frac{W}{c} + \frac{eV}{c} + 2mc\right)Y = 0 \\ \mathrm{grad}\quad X_0 + \mathrm{div}\,Y = 0 \end{cases}$$

$$\mathrm{div\,grad}\,X_0 + \qquad X_0 + \qquad Y = 0$$

$$Y(r)\,Y(\theta)\,Y(\varphi)$$

$$X_0 =$$

$$\underset{FGH}{\mathrm{div\,grad}\,X_0} + \frac{4\pi^2}{h^2}(\quad)(\quad)X_0 + \frac{2\pi i}{h}Y_r\frac{\partial V}{\partial r} = 0.$$

$$X_0 = X_r P_{pq}\qquad Y_r = F(r)P_{mn}\qquad Y_\varphi = H(r) - \frac{\partial X_0}{\partial r}$$

$$Y_\theta = G(r)\,P_{pq}$$

申

$$\frac{\Delta_1 F}{F} + \frac{\Delta_2 G}{G} \cancel{+} \frac{\cancel{\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)}}{A+\frac{e^2}{cr_1}+\frac{e^2}{cr_2}} \quad \frac{1}{r_1^2 F}\frac{\partial F}{\partial r_1} + \frac{1}{r_2^2 G}\frac{\partial G}{\partial r_2}$$

$$+\frac{1}{r_1^2}$$

$$+\frac{4\pi^2}{h^2}\left(AB + B\left(\frac{e^2}{cr_1}+\frac{e^2}{cr_2}\right)\right) = 0$$

$$\frac{1}{A}\left(A - \frac{e^2}{Acr_1} - \frac{e^2}{Acr_2}\right)$$

$$A + \frac{1}{r_1} + \frac{1}{r_2} \doteqdot A_1 + \frac{e^2}{A c r_1}\frac{\partial F}{\partial r_1} + \frac{e^2}{A^2 c r_1^3}\frac{\partial F}{\partial r_1}$$

$$\Delta_1 F - \frac{1}{r_1^2 A}\frac{\partial F}{\partial r_1} + \frac{4\pi^2}{h^2}\left(D_1 + B\frac{e^2}{cr_1}\right)F = 0$$

$$\Delta_1 G - \frac{1}{r_2^2 A}\frac{\partial G}{\partial r_2} + \frac{4\pi^2}{h^2}\left(D_2 + B\frac{e^2}{cr_2}\right)G = 0$$

$$D_1 + D_2 = A\cdot B q + \text{const.}$$

$$\begin{cases} \Delta_1 F - \left(\frac{1}{r_1^2 A} - \cancel{\frac{e^2}{A^2 c r_1^3}}\right)\frac{\partial F}{\partial r_1} + \frac{4\pi^2}{h^2}\left(D_1 + B\frac{e^2}{cr_1}\right)F = 0 \\ \Delta_1 G - \left(\frac{1}{r_2^2 A} - \cancel{\frac{e^2}{A c r_2^3}}\right)\frac{\partial G}{\partial r_2} + \frac{4\pi^2}{h^2}\left(D_2 + \frac{Be^2}{cr_2}\right)G = 0 \end{cases}$$

$$\Delta_1 F - \frac{1}{r_1^2 A}\frac{\partial F}{\partial r_1} + \frac{4\pi^2}{h^2}\left(D_1 + B\frac{e^2}{cr_1}\right)F = 0$$

$$\frac{1}{F}\frac{\partial F}{\partial r}$$

$$F = R(r)\cdot\Phi(\varphi,\theta)$$

$$\frac{\partial^2 R}{\partial r^2} + \left(\frac{2}{r}\frac{\partial R}{\partial r} + \frac{4\pi^2}{h^2}\left(D_1 + B\frac{e^2}{cr_1}\right)R - \frac{j(j+1)}{r^2}R = 0\right.$$

$$\left(-\frac{1}{r^2 A}\right)$$

$$R = r^{-\frac{1}{A}}$$

$$\frac{\partial V}{\partial r}$$

$$\boxed{\operatorname{div}\operatorname{grad}X_0 + \cancel{\frac{e^2}{cr_1}}\ \frac{1}{A}\left(\frac{\partial U}{\partial r_1} + \frac{\partial U}{\partial r_2}\right)\frac{\partial X_0}{\partial r_1} + \frac{\partial U}{\partial r_2}\frac{\partial X_0}{\partial r_2}}$$

$$\boxed{+\frac{4\pi}{h^2}\left(AB + B\,U\right)X_0} = 0$$

$$\operatorname{grad}\left\{\frac{\operatorname{div}Y}{\frac{W}{c}+\frac{eV}{c}-2mc}\right\} + \left(\frac{W}{c}+\frac{eV}{c}+2mc\right)Y = 0$$

$$\operatorname{grad}\operatorname{div}Y - \frac{\frac{e}{c}\cdot\operatorname{grad}V\cdot\operatorname{div}Y}{\frac{W}{c}+\frac{eV}{c}-2mc} + \left(\frac{W}{c}+\frac{eV}{c}+2mc\right)\left(\frac{W}{c}+\frac{eV}{c}-2mc\right)Y = 0$$

$$Y_r \qquad Y_\varphi = Y_\theta = 0$$

$$\frac{e}{c}\operatorname{grad}V\cdot\operatorname{div}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Y)\Big\} + (\quad)Y = 0$$

$$\operatorname{grad}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Y)\right\} + \cdots$$

$$\operatorname{grad}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Y)\right\} + \frac{e}{c}\operatorname{grad}\frac{1}{r^2}\frac{\partial}{\partial r}r^2 Y$$

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Helium:

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\Psi_0 = \mathrm{Div}_1 \Psi + \mathrm{Div}_2 \Psi$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)\Psi \mp \mathrm{grad}_1 X_0 + \mathrm{grad}_2 X_0 \mp \mathrm{curl}_1 X \mp \mathrm{curl}_2 X = 0$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-mc\right)X_0 \mp \mathrm{Div}_1 Y + \mathrm{Div}_2 Y = 0$$

$$-\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-mc\right)X + \mathrm{grad}_1 Y_0 + \mathrm{grad}_2 Y_0 \mp \mathrm{curl}_1 Y \mp \mathrm{curl}_2 Y = 0$$

$$\cancel{\psi_1 \psi_2} \qquad \cancel{X = X_1^{(1)} X_2^{(2)}}$$

$$\cancel{= X^{(2)}\mathrm{Div}_1 X_1^{(1)} + X^{(1)}\mathrm{Div}_2 X^{(2)} \frac{h}{2\pi i}}$$

$$\cancel{X = 0 \quad Y_0 = 0} \qquad [p, X] = [p, Y] = 0 \;!!$$

$$\cancel{\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}\right.}$$

$$\mathrm{div}\begin{cases}\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+2mc\right)Y + \mathrm{grad}\, X_0 = 0 \quad -\mathrm{curl}\, X \\ \frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-2mc\right)X_0 + \mathrm{div}\, Y = 0\end{cases}$$

$$\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}-2mc\right)X_0 \pm \mathrm{div}\left\{\frac{\mathrm{grad}\, X_0}{\frac{2\pi i}{h}\left(\frac{W}{c}+\frac{eV}{c}+mc\right)}\right\} = 0$$

$$\frac{W}{c}+2mc = A, \quad \frac{W}{c}-2mc = B. \qquad \frac{eV}{c} = U$$

$$\frac{4\pi^2}{h^2} B X_0 + \mathrm{div}\left\{\frac{\mathrm{grad}\, X_0}{A+U}\right\} = 0$$

$$\frac{\mathrm{div}\,\mathrm{grad}\, X_0}{A+U} - \mathrm{div}\left(\frac{1}{A+U}\right)\cdot \mathrm{grad}\, X_0 + \frac{4\pi^2}{h^2} B X_0 = 0$$

$$\mathrm{div}\,\mathrm{grad}\, X_0 + \frac{\mathrm{grad}\, U \cdot \mathrm{grad}\, X_0}{A+U} + \frac{4\pi^2}{h^2}(AB + BU) X_0 = 0$$

$$X_0 = F(r_1) G(r_2) \qquad U = \frac{e^2}{cr_1} + \frac{e^2}{cr_2}$$

$$G\Delta_1 F + F\Delta_2 G + \frac{\left(\mathrm{grad}_1 \frac{1}{r_1} + \mathrm{grad}_2 \frac{1}{r_2}\right)\left(\mathrm{grad}_1 \frac{F}{F} + \mathrm{grad}\, G\right)}{A + \frac{1}{r_1} + \frac{1}{r_2}}$$

$$+ \frac{4\pi^2}{h^2}\left(AB + B\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)FG = 0$$

[18]

⑪

§ Diracノ理論ノ擴張.

$$\int \frac{1}{c}\frac{\partial\rho}{\partial t} + \operatorname{div} I + \qquad \psi_1\psi_1^* + \psi_2\psi_2^* - \psi_3\psi_3^* - \psi_4\psi_4^*$$

$$\int \Big\{ \frac{h}{4\pi i}\Big(\frac{1}{c}\frac{\partial\rho}{\partial t} + \operatorname{div} I\Big) + mc\, \overset{\shortparallel}{X_{ik}X^{ik}}$$

$$+ \frac{e}{c}\Big(\rho\cdot V + \frac{I}{c}A\Big) - \frac{1}{8\pi c}\underset{H^2-E^2}{\underset{\shortparallel}{F^{ik}F_{ik}}}\Big\}\, dv$$

or $\int$

many body problem:

$$H\, W = \int \Big\{\frac{h}{4\pi i}\Big(\frac{1}{c}\frac{\partial}{\partial t}\sum\rho_k + \operatorname{div}\sum I_k\Big) + \overset{mk}{\cancel{m}c}\sum_k X_{(k)ij}X^{ij}_{(k)}$$

$$+ \frac{1}{c}\Big(\sum e_k\rho_k V_0 + \sum e I_k\cdot A_0\Big) - \frac{1}{8\pi c}(H^2 - E^2)\Big\}\, dv$$

$$+ \sum_{i\neq k}\int\cdots\int \frac{e_i e_k}{c}\,\rho_i^{(s)}\rho_k^{(s')}\Big/ s_{ik}^2\; d\tau\, dv' + \sum\int\cdots\int \frac{e_i e_k}{c}\,\rho_i\, I_i^{(s)} I_k^{(s')}\Big/ s_{ik}^2\; dv\, dv'$$

從ッテ wave equation ハ

$$\Big\{\Big(p_0 + \frac{e_i}{c}\sum_{i\neq k}\int \frac{e_k\rho_k^{(s')}}{s_{ik}^2}dv'\Big) + \sum\alpha\Big(p + \sum_{i\neq k}\frac{e_i}{c}\int\frac{e_k I_k^{(s')}}{s_{ik}^2}dv'\Big)$$

$$+ \alpha_4 mc\Big\}\,\psi = 0.$$

トナル。

Helium Problem:

$$\frac{2e}{r} \pm \int\frac{\rho}{|r - r'|}dv \cong \frac{e}{r} + e\Big(\frac{1}{r} + \frac{1}{a}\Big)e^{-\frac{2r}{a}}$$

$$W + \frac{e}{c}\int\frac{\rho(r')}{|r - r'|}dv'$$

$$\Big(+\frac{2e}{r}\Big)$$

$$\Big\{A^2 + \gamma\Big(\frac{1}{r} + \Big(\frac{1}{r} + \frac{1}{a}\Big)e^{-\frac{2r}{a}}\Big)\Big\}F + \frac{dG}{dr} - \frac{\kappa}{r}G = 0$$

$$\Big\{B^2 - \gamma\Big(\frac{1}{r} + \Big(\frac{1}{r} + \frac{1}{a}\Big)e^{-\frac{2r}{a}}\Big)\Big\}G + \frac{dF}{dr} + \frac{\kappa+2}{r}F = 0$$

$$F = e^{-\frac{r}{aN}}(\ ,\ ) \qquad G = e^{-\frac{r}{aN}}(\ ..\ ) + e^{-\frac{2r}{aN} + \frac{r}{aN}}(\ ,\ .)$$

$$+ e^{-\frac{2r}{a} - \frac{r}{aN}}(\ ..\ ) + e^{-2\frac{2r}{a} - \frac{r}{aN}}\cdots + \cdots\cdots$$

$$F = P\, e^{-\frac{r}{aN}}\sum_{n=0}^{\infty}P_n(r)\, e^{-\frac{2r}{a}n}.$$

$$G = e^{-\frac{r}{aN}}\sum_{n=0}^{\infty}Q_n(r)\, e^{-\frac{2r}{a}n}.$$

$$\Big\{A^2 + \gamma\Big(\frac{1}{r} + \Big(\frac{1}{r} + \frac{1}{a}\Big)e^{-\frac{2r}{a}}\Big)\Big\} \qquad -\frac{4}{3} + 2 = \frac{2^6}{3}\quad \frac{2^2}{}$$

$$e^{-\frac{2r}{a}} = 1 - \frac{2}{a}r + \frac{2^2}{2a^2}r^2 - \frac{2^3}{2\cdot 3a^3}r^3 + \cdots + (-1)^n\frac{2^n}{n!\,a^n}r^n$$

$$\frac{1}{r}(\qquad) = \frac{1}{r} - \frac{2}{a} + \frac{2}{a^2}r - \frac{2^2}{3a^3}r^2 + \cdots$$

$$\frac{1}{a}(\qquad) = \frac{1}{a} - \frac{2}{a^2}r + \frac{2^2}{2a^3}r^2 + \cdots$$

$$\frac{1}{r} - \frac{1}{a} + \frac{2r^2}{3a^3} - \frac{2}{3}\frac{r^3}{a^4} + \frac{2}{5}\frac{r^4}{a^4} - \frac{8}{45}\frac{r^5}{a^6}$$

$$(-1)^{n+1}\frac{2^{n+1}}{(n+1)!\,a^{n+1}}r^n$$

$$+ (-1)^n\frac{2^n}{n!\,a^{n+1}}r^n$$

$$= (-1)^n\frac{(n-1)2^n}{(n+1)!\,a^{n+1}}r^n$$

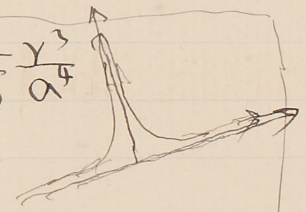
$n=2$ $n=3$

$$\frac{2\cdot 2^3\cdot r^3}{2\cdot 3\cdot 4\,a^4}\qquad \frac{3\cdot 2^4}{2\cdot 3\cdot 4\cdot 5}$$

$n=4$

$$\frac{4\cdot 2^5}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot a^6} \qquad \frac{8}{45}$$

$$\frac{1}{r} - \frac{1}{a} + \frac{2r^2}{3a^3} - \frac{2}{3}\frac{r^3}{a^4}$$



$$F = e^{-\frac{r}{aN}}\cdot P(r)$$

$$G = e^{-\frac{r}{aN}}\, Q(r) \qquad \frac{dG}{dr} - \frac{\kappa}{r}G$$

$$\{P + \qquad Q$$

$$\frac{1}{r} - \frac{1}{a} + \frac{2r^2}{3a^3} - \frac{2}{3}\frac{r^3}{a^4}$$

[21]

$\frac{\partial X_1}{\partial x}\frac{\partial X_2}{\partial y}\frac{\partial X_3}{\partial z}\frac{\partial X_4}{\partial t}$ $h^{\mu}_{n}X_{n\mu}$ $- mcX$

$h^{\mu}_{n}X_{n\mu}$ $h^{\mu}_{m}X_{n\mu}$ (p_0+mc)

$\gamma^{\mu}\cdot\frac{\partial X}{\partial x^{\mu}} + \dots = 0$ X

$h^{\mu}_{m}X_{n\mu}$ $h^{\mu}_{m}X_{n\mu}$ $\frac{\partial}{\partial x_1}$

$\boxed{h^{\mu}_{n}X_{n\mu}}$ γ_{μ} $\gamma^{\mu}\frac{\partial X}{\partial x^{\mu}} = \{\ \}^{\mu}\frac{\partial X}{\partial x^{\mu}}$

ノ Einstein ノ 新シキ 理論 ト Dirac ノ Elektrontheorie

原 $h^{\mu}_{n}\frac{\partial X_n}{\partial x_{\mu}}$ $h^{\mu}_{mn}\frac{\partial X_n}{\partial x^{\mu}}$

際 $h^{1}_{1}\frac{\partial X_1}{\partial x_1} + h^{2}_{1}\frac{\partial X_1}{\partial x_2} + h \dots$ $h^{\nu}_{\nu a}\frac{\partial X_a}{\partial x^{\nu}}$

係 $\boxed{g_{\mu\nu} = h_{\mu a}h_{\nu a}}$

div

系 ハバ rot X

$\sum \frac{\partial X_{\mu}}{\partial x^{\nu}} \pm \frac{\partial X_{\nu}}{\partial x^{\mu}} + \frac{\partial X_{\lambda}}{\partial x^{\lambda}}$

象

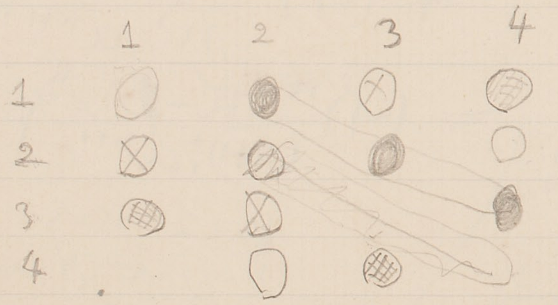
$\boxed{\text{generalised Fieldequation}}$

大 ge

大 (λ, μ, ν) $\boxed{\frac{\partial X_{\mu}}{\partial x^{\nu}} + \frac{\partial X_{\nu}}{\partial x^{\lambda}} \pm \frac{\partial X_{\lambda}}{\partial x^{\mu}}}$

係 1, 2, 3, 4.

General



curl X
$= [pX]$
$= pX$

[22]

$[XY]_1 = \boxed{X_2Y_3 - X_3Y_2 + X_0Y_1 - X_1Y_0}$
$= \boxed{X_1Y_1}$

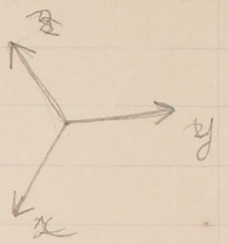
[EH]

$\psi_1\psi_2\psi_3\psi_4$

$\rho = X_0^2 + Y_0^2 + X_1^2 + \dots + Y_0^2 + Y_1^2 + \dots$

$\mathrm{I\!I}_x = [XY]$ $\mathrm{I\!I}$

$L = E^2 + X^2 - Y^2 + X_0^2 - Y_0^2$



0 1 2 3 4

$\begin{Bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{Bmatrix}$ $\begin{Bmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&+1&0\end{Bmatrix}$ $\begin{Bmatrix}0&0&+1&0\\0&0&0&-1\\-1&0&0&0\\0&+1&0&0\end{Bmatrix}$ $\begin{Bmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\1&0&0&0\end{Bmatrix}$

$Z_0 \to -Z_0$
$\frac{\partial Z_{\nu}}{\partial x_{\nu}} = k\overline{Z_0}$

$\begin{Bmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{Bmatrix}$ $\sigma_1 = \begin{Bmatrix}0&+i&0&0\\-i&0&0&0\\0&0&0&-i\\0&0&+i&0\end{Bmatrix}$

$\frac{\partial Z_{\mu}}{\partial x_{\nu}} - \frac{\partial Z_{\nu}}{\partial x_{\mu}} - \frac{\partial Z_0}{\partial x_{\lambda}} - \frac{\partial Z_{\lambda}}{\partial x_0}$
$= k\overline{Z_{\lambda}}$

(λ, μ, ν)

$\begin{Bmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{Bmatrix}$ $\sigma_2 = \begin{Bmatrix}0&0&+i&0\\0&0&0&i\\-i&0&0&0\\0&-i&0&0\end{Bmatrix}$

$-k\overline{Z_0}$
$\frac{\partial Z_{\nu}}{\partial x_{\nu}} = -imc\overline{Z_0}$

$\psi\sigma_1\psi^*$ $\frac{\partial Z_{\nu}}{}$ $\frac{\partial Z}{\partial x_0}$

$\sigma_3 = \begin{Bmatrix}0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0\end{Bmatrix}$

$\frac{\partial Z_{\mu}}{\partial x_{\nu}} - \frac{\partial Z_{\nu}}{\partial x_{\mu}} + \frac{\partial Z_0}{\partial x_{\lambda}} - \frac{\partial Z_{\lambda}}{\partial x_0} = imc\overline{Z_{\lambda}}$
$= k\overline{Z_{\lambda}}$

$(p_0 + mc)$

$\sigma_0 = \begin{Bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{Bmatrix}$

$\psi_1\psi_1^* + \psi_2\psi_2^* + \psi_3\psi_3^* + \psi_4\psi_4^*$
$i\{\psi_1\psi_2^* - \psi_2\psi_1^* - \psi_3\psi_4^* + \psi_4\psi_3^*\}$
$i\{\psi_1\psi_3^* + \psi_2\psi_4^* - \psi_3\psi_1^* - \psi_4\psi_2^*\}$
$i\{\psi_1\psi_4 - \psi_2\psi_3^* + \psi_3\psi_2^* - \psi_1\psi_4^*\}$

$-(p_0 + mc)Y_0 = (p, X)$
$(p_0 + mc)Y - pX_0 = [p, X]$
i $(p_0 - mc)X_0 = (p, Y)$
$-i$ $(p_0 - mc)X + pY_0 = -[p, Y]$

$i(X_0 + iY_0)$ X

$ip_0 Z_0 - imc\overline{Z_0} = (p, Z)$
$-ip_0 Z + imc\overline{Z} - pZ_0 = [p, Z]$

$ip_0 \to p_0$
$-ip_0 \to p_0$

$(p, Z) -ip_0Z_0 = -imc\overline{Z_0}$
$[p, Z] + ip_0 Z + pZ_0 = imc\overline{Z}$

$\frac{\partial Z_{\nu}}{\partial x_{\nu}} = -imc\overline{Z_0}$
$\frac{\partial Z_{\mu}}{\partial x_{\nu}} - \frac{\partial Z_{\nu}}{\partial x_{\mu}} + \frac{\partial Z_0}{\partial x_{\lambda}} - \frac{\partial Z_{\lambda}}{\partial x_0}$ $= imc\overline{Z_{\lambda}}$

(13)

$\frac{\partial}{}$

central $\left\{\begin{matrix}\text{field}\\ ~~\text{Force}~~\end{matrix}\right.$ $Z_r, Z_\theta, Z_\varphi, Z_0$

$$\frac{\partial Z_0}{\partial x_0} + k\cancel{\varphi}(r) Z_0 + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Z_r) + \frac{1}{r\sin\theta}\frac{\partial Z_\varphi}{\partial\varphi} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta Z_\theta)}{\partial\theta} = k_2\overline{Z_0}$$

$$\frac{1}{r\sin\theta}\frac{\partial Z_\theta}{\partial\varphi} - \frac{1}{r\sin\theta}\frac{\partial Z_\varphi}{\partial\theta} - \frac{\partial Z_0}{\partial r} - \frac{\partial Z_r}{\partial x_0} - k_1V(r)Z_r = k_2\overline{Z_r}$$

$$\frac{1}{r}\frac{\partial Z_r}{\partial\theta} - \frac{1}{r}\frac{\partial(rZ_\theta)}{\partial r} \pm \frac{1}{r\sin\theta}\frac{\partial Z_0}{\partial\varphi} - \frac{\partial Z_\varphi}{\partial x_0} - k_1V(r)Z_\varphi = k_2\overline{Z_\varphi}$$

$$\frac{1}{r}\frac{\partial(rZ_\varphi)}{\partial r} - \frac{1}{r\sin\theta}\frac{\partial Z_r}{\partial\varphi} - \frac{1}{r}\frac{\partial Z_0}{\partial\theta} - \frac{\partial Z_\theta}{\partial x_0} - k_1V(r)Z_\theta = k_3\overline{Z_\theta}$$

$Z_0 = R_0\cdot\Phi_0\,\Theta_0$

$Z_i = R_i\,\Phi_i\,\Theta_i$

Z_r const

$Z_\theta = Z_\varphi = 0$, const

$X_1 = X_2 = X_3 = 0$

X_0,

$Y_0 = 0$, Y_r!!

$$(k_3W + k_1V(r))Z_0 + \frac{1}{r^2}\frac{\partial(r^2Z_r)}{\partial r} = k_2\overline{Z_0}$$

$$\left\{\begin{aligned} &\frac{\partial Z_0}{\partial r} + (k_3W + k_1V(r))Z_r + k_2\overline{Z_r} = 0 \text{ !!}\\ &\frac{1}{r}\frac{\partial Z_r}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial Z_0}{\partial\varphi} ~~Z_\varphi~~ - k_1V(r)Z_\varphi = k_2\overline{Z_\varphi}\end{aligned}\right.$$

$$-\frac{1}{r\sin\theta}\frac{\partial Z_r}{\partial\varphi} - \frac{1}{r}\frac{\partial Z_0}{\partial\theta} ~~=0~~ - k_1V(r)Z_\theta = k_2\overline{Z_\theta}$$

$$\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial Z_r}{\partial\theta}\right\} + \frac{\partial}{\partial\varphi}\left\{\frac{1}{\sin\theta}\frac{\partial Z_r}{\partial\varphi}\right\} = 0 \text{ !!}$$

$$\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial Z_0}{\partial\theta}\right\} + \frac{\partial}{\partial\varphi}\left\{\frac{1}{\sin\theta}\frac{\partial Z_0}{\partial\varphi}\right\} = 0$$

Z_μ div $\mathbf{P} = \mathbf{E}$

$\overline{Z_0}$ $\quad \frac{\partial Z_\nu}{\partial x_\nu} = k\overline{Z_0}$

$-\overline{Z_\lambda}$ $\quad \frac{\partial Z_\mu}{\partial x_\nu} - \frac{\partial Z_\nu}{\partial x_\mu} - \frac{\partial Z_0}{\partial x_\lambda} - \frac{\partial Z_\lambda}{\partial x_0} = k\overline{Z_\lambda}$

$$\frac{\partial}{\partial x_0}(\overline{Z_0}Z_0 + Z_\lambda\overline{Z_\lambda}) + \frac{\partial}{\partial x_1}($$

$$= 2k(Z_0^2 Z_0 + \overline{Z_0}^2 + Z_1^2 + \overline{Z_1}^2 + Z_2^2 + \overline{Z_2}^2 + Z_3^2 + \overline{Z_3}^2)$$

$$=$$

$$j_1 = ce((Y_3 + iY_0)(iX_1 - X_2) + (Y_1 \pm iY_2)(iX_3 + X_0)$$
$$+ (Y_1 + iY_2)(X_0 - iX_3) + (Y_3 - iY_0)(-iX_1 - X_2))$$
$$= ce(X_0Y_1 + X_3Y_2 - X_2Y_3 - X_1Y_0)$$

$$j_2 = ce(-i(Y_3 + iY_0)(iX_1 - X_2) + i(Y_1 - iY_2)(X_0 + iX_3)$$
$$- i(Y_1 + iY_2)(X_0 - iX_3) + i(Y_3 - iY_0)(-iX_1 - X_2))$$
$$= ce(X_1Y_3 - X_2Y_0 \mp X_0Y_2 - X_3Y_1)$$

$$j_3 = ce((Y_3 + iY_0)(X_0 + iX_3) - (Y_1 - iY_2)(iX_1 - X_2)$$
$$+ \{~~Y_1 + iY_2~~\}(-iX_3 + X_0) - (-iX_1 - X_2)(Y_1 + iY_2)$$
$$(Y_3 - iY_0)$$
$$= ce(Y_3X_0 - Y_0X_3 + Y_1X_2 - Y_2X_1)$$

[illegible]

$$j_1 = -2ce(X_2Y_3 - X_3Y_2 + X_1Y_0 - X_0Y_1)$$
$$j_2 = -2ce(X_3Y_1 - X_1Y_3 + X_2Y_0 - X_0Y_2)$$
$$j_3 = -2ce(X_1Y_2 - X_2Y_1 + X_3Y_0 - X_0Y_3)$$

$$\rho = -e(X_0^2 + X_1^2 + X_2^2 + X_3^2 + Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2)$$

$X_0 = Y_0 = 0$ $\quad \rho = m$ $\quad p = m[XY]$

$m(X^2 - Y^2)$

$(E^2 - H^2)$

$= T_{\mu\nu}T^{\mu\nu}$

$$\rho = -e(X^2 + Y^2) \qquad \mathbf{j} = -2ce[\mathbf{XY}]$$

$$W = \frac{1}{8\pi}(\mathbf{E}^2 + \mathbf{H}^2) \qquad \mathbf{S} = \frac{c}{4\pi}[\mathbf{EH}]$$

$p = -2ce[\mathbf{EH}]$

Energie - Impulssatz der Materiefeld.

Schrö: $\frac{\partial}{\partial x_\sigma}(T_{\rho\sigma} + S_{\rho\sigma}) = 0$

$$S_{\rho\sigma} = \overline{\Psi}_\rho \frac{\partial L_m}{\partial \overline{\Psi}_\sigma} + \psi_\rho \frac{\partial L_m}{\partial \psi_\sigma} + \varphi_\rho \frac{\partial L_m}{\partial \varphi_\sigma} - \delta_{\rho\sigma} L_m$$

$$T_{\rho\sigma} = f_{\rho\alpha} f_{\sigma\alpha} - \delta_{\rho\sigma} L_e$$

$$L_m = (\psi_\alpha + i\varphi_\alpha \psi)(\overline{\Psi}_\alpha - i\varphi_\alpha \overline{\Psi}) + k^2 \psi \overline{\Psi}$$

$$L_e = \frac{1}{4} f_{\alpha\beta} f_{\alpha\beta}$$

$S_{\rho\sigma} =$

$$J^\mu_{\cdot\nu} + kJ^\mu \varphi_\nu$$

$$= \frac{\partial J^\mu}{\partial x_\nu} + \{\varepsilon\nu, \mu\} J^\varepsilon + k\varphi_\nu J^\mu$$

General Relativistic Theorie

$$= \overset{\varepsilon=\mu}{\{\mu\nu, \mu\}} + k\varphi_\nu$$

$$= \boxed{\frac{1}{2g}\frac{\partial g}{\partial x_\nu} + k\varphi_\nu}$$

$$= \frac{\partial}{\partial x_\nu}\{\log\sqrt{g} + k\varphi\}$$

(β, X)

$$\left(J^\mu_{\cdot\mu} + kJ^\mu \int J^\mu_{(k)} G \xi d\tau\right.$$

$A_{\mu\nu\sigma}$

$$J^\mu_{\cdot\mu} = \frac{\partial A^\mu}{\partial x_\mu} + \{\varepsilon\mu, \mu\} A^\varepsilon + k\varphi^\mu A^\mu$$

$$\{\varepsilon\mu, \mu\} + k\varphi^\mu =$$

$$\frac{1}{2} g^{\mu\lambda}\left(\frac{\partial g_{\varepsilon\lambda}}{\partial x_\mu} + \frac{\partial g_{\mu\lambda}}{\partial x_\varepsilon} - \frac{\partial g_{\varepsilon\nu}}{\partial x_\lambda}\right) J^\varepsilon + k\varphi^\mu J^\mu$$

$$\{\mu\nu, \mu\} = \frac{1}{2} g^{\mu\lambda}\left(\frac{\partial g_{\mu\lambda}}{\partial x_\nu} + \frac{\partial g_{\nu\lambda}}{\partial x_\mu} - \frac{\partial g_{\mu\nu}}{\partial x_\lambda}\right)$$

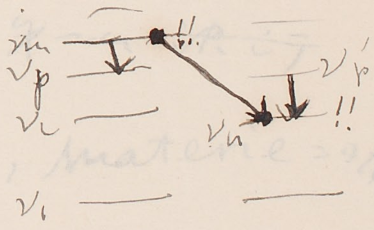
$$= \frac{1}{2} g^{\mu\lambda} \frac{\partial g_{\mu\lambda}}{\partial x_\nu} = \frac{1}{2g}\frac{\partial g}{\partial x_\nu}$$

二ツノelectronsアル時ニdensity, currentハ次ノ如クアラハサレルト考ヘテ居ル

$$\left\{\begin{array}{l} \rho = \Sigma(X_0 + X_0')^2 + (Y_0 + Y_0')^2 + \Sigma(X + X')^2 + (Y + Y')^2 \\ \Pi = [X + X, Y + Y'] + XY_0 - X_0Y \end{array}\right.$$

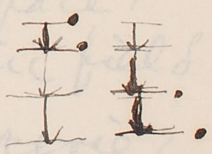
$$\rho = \Sigma A_n^2 + \Sigma 2A_mA_n \cos(\nu_m - \nu_n)t$$

$$X_0 = \Sigma A_n \cos \nu t \qquad Y_0 = \Sigma A_n \sin \nu t.$$



$$X_0 + X_0' = \Sigma A_n \cos \nu t + \Sigma A_n' \cos(\nu' t + \delta)$$

$$(\quad)^2 = \Sigma A_n^2 \cos^2 \nu t + 2\Sigma A_n A_n \cos \nu t \cos \nu_m t + \Sigma\Sigma A_m A_n' \cos \nu_m t \cos(\nu_n' t + \delta)$$



$$(X_0 + X_0')^2 + (Y_0 + Y_0')^2 = \Sigma A_n^2 + 2\Sigma \underline{A_m A_n} \cos \boxed{\nu_m - \nu_n} t + \boxed{\nu_m' - \nu_n'}$$

$$+ \Sigma \underline{\underline{A_m}}\,\underline{\underline{A_n'}} \cos \boxed{(\nu_m - \nu_n')t \mp \delta}$$

$$\nu_m - \nu_n' = \nu_m - \nu_p + \nu_p' - \nu_n'$$

$\nu_m - \nu_m'$

$\boxed{X_0 + X'}$

$$(X_0 + X_0' * X_0'')^2 + \cdot$$

$$X^2 + X_1^2 + X_2^2 + X_3^2 \quad *$$

$$X_0 = \psi_3 + \psi_3^*$$

$$0 = \psi_1 \psi_1^* \quad \psi_1$$

$$= \psi_2 \psi_2^* \quad \psi_2$$

$$(\psi_3 + \psi_3^* + \psi_3' + \psi_3'^*)^2 = 2\psi_3\psi_3^* + \psi_3^2 + \psi_3^{*2} + \cdots$$

$$-(\psi_4 - \psi_4^*)^2 = +2\psi_4\psi_4^* \pm \psi_4^2 \pm \psi_4^{*2}$$

$$\left(\frac{-1}{\gamma} + mc\right) Y_r$$

$$(\quad) Y_r = 0$$

$$) = 0$$

$$= 0$$

Energie - Impulssatz der Materiefeld.

Schrö: $\frac{\partial}{\partial x_\sigma}(T_{\rho\sigma} + S_{\rho\sigma}) = 0$

$$S_{\rho\sigma} = \overline{\Psi}_\rho \frac{\partial L_m}{\partial \overline{\Psi}_\sigma} + \Psi_\rho \frac{\partial L_m}{\partial \Psi_\sigma} + \varphi_\rho \frac{\partial L_m}{\partial \varphi_\sigma} - \delta_{\rho\sigma} L_m$$

$$T_{\rho\sigma} = f_{\rho\alpha} f_{\sigma\alpha} - \delta_{\rho\sigma} L_e$$

$$L_m = (\Psi_\alpha + i\varphi_\alpha \Psi)(\overline{\Psi}_\alpha - i\varphi_\alpha \overline{\Psi}) + k^2 \Psi \overline{\Psi}$$

$$L_e = \frac{1}{4} f_{\alpha\beta} f_{\alpha\beta}$$

$S_{\rho\sigma} =$

$$J^\mu{}_{;\nu} + k J^\mu \varphi_{;\nu}$$

$$= \frac{\partial J^\mu}{\partial x_\nu} + \{\varepsilon\nu, \mu\} J^\varepsilon + k\varphi_\nu J^\mu$$

General Relativistic Theorie

$$= \overset{\varepsilon = \mu}{\{\mu\nu, \mu\}} + k\varphi_\nu$$

$$= \boxed{\frac{1}{2g}\frac{\partial g}{\partial x_\nu} + k\varphi_\nu}$$

$$= \frac{\partial}{\partial x_\nu}\{\log\sqrt{g} + k\varphi\}$$

(p, X)

$$\int (J^\mu{}_{;\mu} + k J^\mu_{(k)} \ldots) G \ldots d\tau$$

A_μ ~~$\nu\sigma$~~

$$J^\mu{}_{;\mu} = \frac{\partial J^\mu}{\partial x_\mu} + \{\varepsilon\mu, \mu\} J^\varepsilon + k\varphi^\mu J^\mu$$

$$\{\varepsilon\mu, \mu\} + k\varphi^\mu =$$

$$\frac{1}{2} g^{\mu\lambda}\left(\frac{\partial g_{\varepsilon\lambda}}{\partial x_\mu} + \frac{\partial g_{\mu\lambda}}{\partial x_\varepsilon} - \frac{\partial g_{\varepsilon\nu}}{\partial x_\lambda}\right) J^\varepsilon + k\varphi^\mu (J^\mu)$$

$$\{\mu\nu, \mu\} = \frac{1}{2} g^{\mu\lambda}\left(\frac{\partial g_{\mu\lambda}}{\partial x_\nu} + \frac{\partial g_{\nu\lambda}}{\partial x_\mu} - \frac{\partial g_{\mu\nu}}{\partial x_\lambda}\right)$$

$$= \frac{1}{2} g^{\mu\lambda} \frac{\partial g_{\mu\lambda}}{\partial x_\nu} = \frac{1}{2g}\frac{\partial g}{\partial x_\nu}$$

$\cancel{X}_0$ $\qquad Y_0 = 0$

$X = 0 \qquad Y,$

$$(p_0 + mc) Y_0 = (p, X)$$

$$0 = 0$$

$$(p_0 + mc) Y + p X_0 = 0$$

$$(p_0 - mc) X_0 + (p, Y) = 0$$

$$[p, Y] = 0$$

$$\left\{\begin{aligned} &\frac{2\pi i}{h}\left(\frac{W}{c} \overset{+\frac{eV}{c}}{} + mc\right) Y + \text{grad}\, X_0 = 0 \\ &\frac{2\pi i}{h}\left(\frac{W}{c} \underset{+\frac{eV}{c}}{} - mc\right) X_0 + \text{div}\, Y = 0. \end{aligned}\right.$$

$\underline{\text{curl}\, Y = 0}$

$[p, Y] = 0$

$X_0 : (r, \theta, \varphi)$

$Y_\varphi (r, \theta, \varphi)$

$$Y_\varphi = Y_\theta = 0.$$

X_0 ハ r ダケノ函

Y ハ r ダケノ函

$$\frac{2\pi i}{h}\left(\frac{W}{c} + \frac{eV}{c} + mc\right) Y_r + \frac{\partial X_0}{\partial r} = 0$$

$$\frac{\partial X_0}{\partial \varphi} = \frac{\partial X_0}{\partial \theta} = 0.$$

$$\frac{2\pi i}{h}\left(\frac{W}{c} + \frac{eV}{c} - mc\right) X_0 + \frac{\partial Y_r}{\partial r} + \frac{2}{r} Y_r = 0$$

$$\frac{2\pi i}{h}\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} + mc\right) Y_r + \frac{\partial X_0}{\partial r} = 0$$

$$\frac{2\pi i}{h}\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} - mc\right) X_0 + \frac{\partial Y_r}{\partial r} + \frac{2}{r} Y_r = 0$$

$\frac{\partial}{\partial r}$

$$\frac{\partial}{\partial r}\left\{\frac{\frac{\partial Y_r}{\partial r} + \frac{2}{r} Y_r}{\frac{2\pi i}{h}\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} - mc\right)}\right\} - \frac{2\pi i}{h}\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} + mc\right) Y_r = 0$$

$$\frac{\frac{\partial^2 Y_r}{\partial r^2} + \frac{2}{r}\frac{\partial Y_r}{\partial r} \pm \frac{2}{r^2} Y_r}{(\quad)} + \frac{\left(\frac{\partial Y_r}{\partial r} + \frac{2}{r} Y_r\right)\frac{e^2}{c}\frac{1}{r^2}}{(\quad)^2} - \frac{2\pi i}{h}\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} + mc\right) Y_r = 0$$

$$\frac{\partial^2 Y_r}{\partial r^2} + \frac{2}{r}\frac{\partial Y_r}{\partial r} + \frac{4\pi^2}{h^2}\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} - mc\right)\left(\frac{W}{c} + \frac{e^2}{c}\frac{1}{r} + mc\right) Y_r + \frac{\left(\frac{\partial Y_r}{\partial r} + \frac{2}{r} Y_r\right)\frac{e^2}{c}\frac{1}{r^2}}{(\quad)} = 0.$$

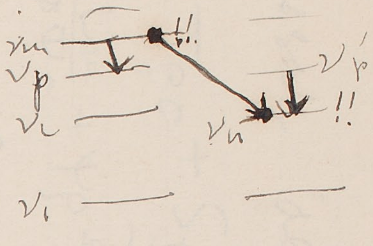
二ツノelectrons アル時ニ density, current ハ次ノ如ク アラハサレルト考ヘテ見ル

$$\rho = \sum(X_0+X_0')^2 + (Y_0+Y_0')^2 + \sum(X+X')^2 + (Y+Y')^2$$

$$\mathrm{II} = [X+X,\ Y+Y'] + XY_0 - X_0Y$$

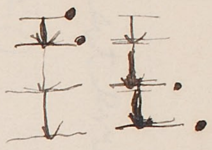
$$\rho = \sum A_n^2 \cancel{\cos^2\nu t} + \sum 2A_mA_n\cos(\nu_m-\nu_n)t$$

$$X_0 = \sum A_n\cos\nu t \qquad Y_0 = \sum B_n\sin\nu t,$$



$$X_0 + X_0' = \sum A_n\cos\nu t + \sum A_n'\cos(\nu' t+\delta)$$

$$(\quad)^2 = \sum A_n^2\cos^2\nu t + 2\sum A_mA_n\cos\nu_n t\cos\nu_m t + \sum A_mA_n'\cos\nu_m t\cos(\nu_n' t+\delta)$$



$$(X_0+X_0')^2 + (Y_0+Y_0')^2 = \sum A_n^2 + 2\sum A_mA_n\cos\boxed{\nu_m-\nu_n}\,t + \sum A_mA_n'\cos\boxed{\nu_m-\nu_n'}\,t\mp\delta \qquad \boxed{\nu_m'-\nu_n'}$$

$$\nu_m-\nu_n' = \nu_m-\nu_p+\nu_p'-\nu_n'$$

$$\nu_m-\nu_m'$$

$$\boxed{X_0+X'} \qquad (X_0+X_0'+X_0'')^2+\cdots$$

$$X^2+X_1^2+X_2^2+X_3^2$$

$$X_0 = \psi_3+\psi_3^*$$

$$0 = \psi_1\psi_1^* \quad \psi_1^2$$

$$= \psi_2\psi_2^* \quad \psi_2^2$$

$$(\psi_3+\psi_3^*+\psi_3'+\psi_3'^*)^2 = 2\psi_3\psi_3^* + \psi_3^2 + \psi_3^{*2} + \cdots$$

$$-(\psi_4-\psi_4^*\quad)^2 = +2\psi_4\psi_4^* - \psi_4^2 - \psi_4^{*2}$$

$$X^2+Y^2+Z^2 \ !!$$

Electron, Nucleus ノ charge ノ distribution ガ先ヅ 1st approximation トシテ point ニ集中シテヰルトスレバ. 其レノ charge ノ distrib ハ ~~[illegible]~~ ρ, II ハ

$\cos(\nu_m-\nu_n)t \quad \cos(\nu_m'-\nu_n')t$
$\cos(\nu_m-\nu_n')t$

ノ term ヲフクム. $\cos(\nu_m-\nu_n)t$ ハ普通ノ term
$\nu_m'-\nu_n'$ ハ非常ニ小, $\nu_m\ \nu_n'$ ハ非常ニ大.

2ノ electron ガ 1ノ electron ニツイテ [illegible] freq ノ radiation resonance ヲオコシ, combination line ヲ生ズル. コレガ更ニ 3ノ electron ニ作用シテ モウ一ツノ term ノ comb. ノ line ガ出テクル. コノ時 negative combination モアリウル。更ニ四ツ以上ノ combination モオコリウルガ 夫々 intensity ハ減ズル.
一般ニ electron ノ density 位相ガ 2ノ combination ~~line~~ ノ freq ノ term ヲ与ヘルモノト考ヘルノデアル。
Emission ノ mechanism モカク考ヘレバ 非常ニ Anschaulich デアル.

Schrödinger ノ考ヘル Eigenfeld ノ概念ヲ除クニハ次ノ如ク考ヘレバヨイ。即チ
material field ト material field トガ直接相互作用ヲナスノデアッテ、
electromagnetic field ナルモノハ其ノ中間デ補助的ニ考ヘル量ニスギヌ。
~~ニ附随~~
~~決セ~~テ 従ッテ ~~一~~ツノ material 外ヘハ electron 自身ニハ ~~決テ~~
Eigenfeld ハ影響セヌコトモ理解サレル。
其ノ第一ノ materie ニ作用スル ~~field~~ force ハ 他ノ凡テノ materie ニ外
force ノ総和デアリ。~~第~~ 他ノ 第二 materie ~~ニ~~ 又第一ノ materie 基
第二以下ノ materie 全体カラ force ヲウケル。従ッテ space ノ
一点ヲ考ヘテモソコニハタラク force 即チ electromagnetic field
ノ intensity ハ一般的ニハキマラヌ。~~ソコニ存在スル materie~~ ノ
ドノ materie ヲ考ヘテ居ルカニヨッテ異ナッテクル。
実際今日ノ理論デモ空間ノ一点ニ各種ノ materie ガ
superpose サレテ居ル。~~ナガ~~ ~~Feldgleich~~ ヲハ invariant integral
$\int H d\Omega$ ヲ min ニスル様ナ 設ノ materie density 及 current ノ distribution ヲスル
デアル。

一ツノ electron ガ作ル ~~ニヨッテ生ジ~~ total magnetic moment ハ Ⅱ カラ求メ
ラレ、即チ一点 r' ニ対シテハ Ⅱ

$$\int \mathrm{II}(r, r')\,dv \qquad \frac{\mathrm{II}}{|r-r'|}$$

origine ニツイテ
デアルカラ、之デ Hydrogen Atom ノ場合ニ計算スレバ、

$$\int \mathrm{II}(r\cancel{-r'})\,dv =$$

$I_r =$

[27]

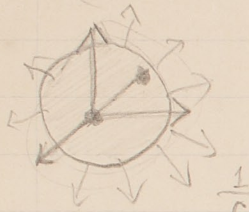
之ハ$(n,0,0)$ノstateニ於テハ、之等ニ分レテヰ

$$j = -X_0 Y \equiv \text{const.}$$ デナケレバナラヌ

デアキラカニ r ノ方向ニ向ッテル magnetic Moment ハ 0

$(\because \quad [r \cdot j] = 0)$

j : pure imaginary



~~$X_0 Y$ Density 0.~~

$$\frac{1}{c}\frac{\partial \rho}{\partial t} + \operatorname{div} j = 0.$$

$\rho = \text{const}$ $\quad \underline{\operatorname{div} j = 0}$

之デハ $\operatorname{div} j = 0$ トハナラヌ.

∴ $Y_0 \neq 0$, $X_r \neq 0$ トスルト

$$(p_0 + mc) Y_0 - \frac{1}{r^2}\frac{\partial (r^2 X_r)}{\partial r} = 0$$

$$(p_0 - mc) X_r - \frac{\partial}{\partial r}(Y_0) = 0$$

$$\begin{cases} Y_0 = kX_0 \\ X_r = kY_r \end{cases}$$

$\underline{\underline{j = 0}}$

$$V = \Sigma A_n \sin(\lambda_n x + \mu_n y + \nu_n z + \delta)$$

~~V~~=

$$(p_0 + mc) Y + p X_0 = 0$$

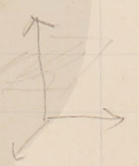
$$(p_0 - mc) X_0 + p Y = 0$$

$$[p, Y] = 0$$

$$(p_0 + mc) Y_x + \frac{\partial X_0}{\partial x} = 0 \quad \frac{\partial X_z}{\partial y} - \frac{\partial X_y}{\partial z}$$

$$(p_0 - mc) X_0 + \frac{\partial Y_x}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Y_z}{\partial z} = 0$$

$$(p_0 + mc) Y_0 - \frac{\partial X_x}{\partial x} - \frac{\partial X_y}{\partial y} - \frac{\partial X_z}{\partial z} = 0$$



Chrystal 内ニ於ケル Electron ノ運動.

[28]

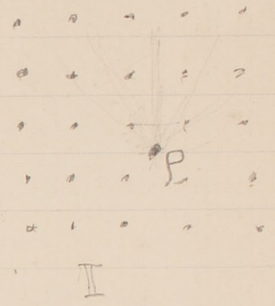
$$(p_0 - mc) X_x - \frac{\partial Y_0}{\partial x} + \frac{\partial Y_z}{\partial y} - \frac{\partial Y_y}{\partial z} = 0$$

$$\underline{V = A \sin(\lambda x + \mu y + \nu z + \delta)}$$

$Y_0 =$

ρ

j_x



current ハ 大体 x-direction ニ向ッテヰル.

∴ XY ガ大キクナレバ、 j_y, j_z モ 漸次マス筈.

即 X_0, Y_x ガ $\neq 0$ $\quad Y_y, Y_z$ ハ small トシテオクト 1st approximation トシテ

$$(p_0 + mc) Y_x + \frac{\partial X_0}{\partial x} = 0.$$

$$(p_0 - mc) X_0 + \frac{\partial Y_x}{\partial x} + \cdots = 0 \qquad -\frac{W^2}{c^2} + m^2c^2 + p^2 = 0$$

$$X_0 = C \sin(wx - pt)$$

$$~~(p_0 + mc)~~ \quad \frac{\partial Y_z}{\partial y} - \frac{\partial Y_y}{\partial z} = 0.$$

$$\frac{\partial Y_x}{\partial z} - \frac{\partial Y_z}{\partial x} = 0 \qquad \frac{4\pi^2}{h^2}\frac{1}{c^2}\frac{\partial^2 X_0}{\partial t^2} + m^2c^2 X_0$$

$$\frac{\partial Y_y}{\partial x} - \frac{\partial Y_x}{\partial y} = 0 \qquad - \frac{4\pi^2}{h^2}\frac{\partial^2 X_0}{\partial x^2} = 0.$$

$$\frac{2\pi i}{ch}\frac{\partial Y_x}{\partial t}$$

$$\begin{cases} \left\{\frac{W}{c} + \frac{e}{c} A \sin(\lambda x + \mu y + \nu z + \delta) + mc\right\} Y_x + \frac{h}{2\pi i}\frac{\partial X_0}{\partial x} = 0 \\ \left\{~~\frac{W}{c}~~ + p_0 - mc \right. \qquad - mc\} X_0 + \frac{h}{2\pi i}\frac{\partial Y_x}{\partial x} = 0 \end{cases}$$

1st approxim.

$$\begin{cases} Y_x = B \sin(px + \delta - Wt) \\ X_0 = B \sin(px - Wt + \delta) \end{cases}$$

$$\frac{2\pi i}{h}\frac{\partial}{\partial x}\frac{\partial}{\partial x} \quad \left(\frac{W}{c} + mc\right) Y_x + \frac{\partial X_0}{\partial x} = 0$$

$$\left(\frac{W}{c} + mc\right) \quad \left(\frac{W}{c} - mc\right) X_0 + \frac{\partial Y_x}{\partial x} = 0$$

$$Y_x = -C\sqrt{\frac{\frac{W}{c} - mc}{\frac{W}{c} + mc}} \cos \sqrt{\ } x$$

$$\left(\frac{W}{c} + mc\right)\left(\frac{W}{c} - mc\right) X_0 + \frac{4\pi^2}{h^2}\frac{\partial^2 X_0}{\partial x^2} = 0.$$

$$X_0 = C \sin\left\{\frac{2\pi}{h}\sqrt{\left(\frac{W}{c} + mc\right)\left(\frac{W}{c} - mc\right)}\, x + \delta\right\}$$

$$\rho = C^2\{1 + \sqrt{\ }\} \doteqdot C \sin\left(m v x \frac{v^2}{c}\right)$$

⑯

[29]

$$Y_x = \bigcirc + Y_x^{(1)}$$

$$\begin{cases} \left(\frac{h}{2\pi i}\frac{\partial}{\partial t} + mc\right)Y_x^{(1)} + \frac{h}{2\pi i}\frac{\partial X_0^{(1)}}{\partial x} = -\frac{e}{c}A\sin(\lambda x+\mu y+\nu z)Y_x^{(0)} \\ \left(\frac{h}{2\pi i}\frac{\partial}{\partial t} - mc\right)X_0^{(1)} + \frac{h}{2\pi i}\frac{\partial Y_x^{(1)}}{\partial x} = -\frac{e}{c}A\sin(\quad)X_0^{(0)} \end{cases}$$

$$\begin{cases} Y_x^{(0)} = B\sin\frac{2\pi}{h}(px - Wt)\,e^{\frac{2\pi i}{h}(px-W} \\ X_0^{(0)} = C\sin\frac{2\pi}{h}(px - Wt)\,e^{\frac{2\pi i}{h}(px-Wt)} \end{cases}$$

$$+e^{-} \qquad +e^{-\cdots}$$

$$\frac{h}{2\pi i}\frac{\partial Y_y}{\partial y} + \frac{h}{2\pi i}\frac{\partial Y_z}{\partial z}$$

$$-(W+mc)e^{-}$$

$$\begin{cases} \left(\frac{h}{2\pi i}\frac{\partial}{\partial t} - mc\right)\left(\frac{h}{2\pi i}\frac{\partial}{\partial t} + mc\right)Y_x^{(1)} + \frac{h^2}{4\pi^2}\frac{\partial^2 Y_x^{(1)}}{\partial x^2} = -\frac{e}{c}A\sin(\quad)\frac{\partial}{\partial t}\,B\cdot\frac{h}{2\pi i}(-W+mc)e \\ (\qquad)(\qquad)X_0^{(1)} + \frac{h^2}{4\pi^2}\partial\cdots = -\,\cdots\;B(W+mc)e^{-} \end{cases}$$

$$+(-W+mc)e^{-}$$

↑

又ハ magnetic field ガ作用スル場.

$$\frac{1}{2M}\left\{\left(\frac{\partial\Phi}{\partial X_1}\right)^2 + \cdots + \right\} + \frac{1}{2m}\left\{\left(\frac{\partial\Phi}{\partial x_1}\right)^2 + \cdots + \cdots\right\}$$

$$= \frac{1}{2(M+m)}\left\{\left(\frac{\partial\Phi}{\partial X_1'}\right)^2 + \cdots\right\} + \left(\frac{1}{2M} + \frac{1}{2m}\right)\left\{\left(\frac{\partial\Phi}{\partial x_1'}\right)^2 + \cdots\right\}$$

]

$$dX_1 \cdots dz_1 = \frac{\partial(X_1 \cdots z_1)}{\partial(X_1' \cdots z_1')}\,dX_1' \cdots dz_1'$$

$$= dX_1' \cdots dz_1' \qquad E = Ze,$$

$$\frac{1}{2(M+m)}\left(\frac{\partial^2\Phi_1}{\partial X_1'^2} + \frac{\partial^2\Phi_1}{\partial Y_1'^2} + \frac{\partial^2\Phi_1}{\partial Z_1'^2}\right) + \left(\frac{1}{2M} + \frac{1}{2m}\right)\left(\frac{\partial^2\Phi_1}{\partial x_1'^2} + \frac{\partial^2\Phi_1}{\partial y_1'^2} + \frac{\partial^2\Phi_1}{\partial z_1'^2}\right) + \frac{4\pi^2}{h^2}\left(W_1 + \frac{eE}{r_1'}\right)\Phi_1 = 0$$

$$\Phi_1 = \frac{1}{\sqrt{2\pi}}\sqrt{2k\frac{1}{2}}\cdot\sqrt{\left(\frac{2Z}{na_0}\right)^3\frac{1}{2}}$$

$n=1,\ m=0$
$k=0$

$$\times e^{-\frac{x}{2}}\cdot L_{1+k}^{1}(x)$$

$$= \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\frac{Zr_1'}{a_0}}\cdot e^{\frac{2\pi i}{h}(PR_1')}$$

$$\begin{cases} -W_1' = \frac{-2\pi^2 m' Z^2 e^4}{h^2} \\ W_1' = \frac{2\pi^2\cdot P^2}{M' h^2} \end{cases}$$

$$W_1' + W_1' = W_1$$

$$e^{-ax} = \frac{1}{1 + ax + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + \cdots}$$

$$= \frac{a}{x} + \frac{b}{x^2} + \cdots$$

$$= e^{-x}$$

$$\theta = \int^r\left\{c - \frac{2}{h^2}\int^r f(r)\,dr - \frac{1}{r^2}\right\}^{-\frac{1}{2}}\frac{dr}{r^2}$$

$$= \int^r\left\{c - \frac{Ae^{-ar}}{h^2} - \frac{1}{r^2}\right\}^{-\frac{1}{2}}\frac{dr}{r^2}$$

$$= \int_\infty^r\left[c - \frac{1}{2}\left(\frac{Ae^{-ar}}{h^2} - \frac{1}{r^2}\right)\right]\frac{dr}{r^2}$$

$$= \frac{W + e^2}{c}$$

[29]

$$Y_x = \bigcirc + Y_x^{(1)}$$

$$\begin{cases} \left(\frac{h}{2\pi i}\frac{\partial}{\partial t} + mc\right) Y_x^{(1)} + \frac{h}{2\pi i}\frac{\partial X_0^{(1)}}{\partial x} = -\frac{e}{c} A \sin(\lambda x + \mu y + \nu z) Y_x^{(0)} \\ \left(\frac{h}{2\pi i}\frac{\partial}{\partial t} - mc\right) X_0^{(1)} + \frac{h}{2\pi i}\frac{\partial Y_x^{(1)}}{\partial x} = -\frac{e}{c} A \sin(\quad) X_0^{(0)} \end{cases}$$

$$\begin{cases} Y_x^{(0)} = B \sin\frac{2\pi}{h}(px - Wt)\ e^{\frac{2\pi i}{h}(px+W}\ + e^{-} \\ X_0^{(0)} = C \sin\frac{2\pi}{h}(px - Wt)\ e^{\frac{2\pi i}{h}(px-Wt)}\ + e^{-\ -} \end{cases}$$

$$\frac{h}{2\pi i}\frac{\partial Y_y}{\partial y} + \frac{h}{2\pi i}\frac{\partial Y_z}{\partial z}$$

$$(W+mc)e^{-}$$

$$\begin{cases} \left(\frac{h}{2\pi i}\frac{\partial}{\partial t} - mc\right)\left(\frac{h}{2\pi i}\frac{\partial}{\partial t} + mc\right) Y_x^{(1)} + \frac{h^2}{4\pi^2}\frac{\partial^2 Y_x^{(1)}}{\partial x^2} = -\frac{e}{c} A \sin(\quad) \frac{\partial}{\ } \\ (\quad)(\quad) X_0^{(1)} + \frac{h^2}{4\pi^2}\partial \cdot = \end{cases}$$

$$B \cdot \frac{h}{2\pi i}(-W+mc)e$$

$$B(W+mc)e^{-}$$

$$(W+mc)e^{-}$$

$\underline{\uparrow}$

𝔥 = magnetic field, が存在すれば.

[30]

▨ $\underline{g_{\mu\nu}}$

$$\left(\frac{1}{c}\frac{\partial W}{\partial t} + \frac{e}{c}V\right)^2 \pm \Delta W - m^2c^2 = 0$$

$$\Delta\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \mp \frac{4\pi i e}{hc}\frac{V}{c}\frac{\partial\psi}{\partial t}$$
$$+ \frac{4\pi^2 e^2}{h^2c^2}\left(V^2 - \frac{m^2c^4}{e^2}\right)\psi = 0$$

$$\Delta\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \overset{\mp}{\underset{+}{\cancel{\pm}}} \frac{4\pi i e}{hc}\frac{V}{c}\frac{\partial\psi}{\partial t}$$
$$+ \cancel{4\pi^2e^2} - \frac{4\pi^2}{h^2} m^2c^2\,\psi = 0$$

$$\psi = \varphi\, e^{\frac{2\pi}{h} imc^2}$$

$$\frac{\partial\psi}{\partial t} = \frac{\partial\varphi}{\partial t} e^{-} + \frac{2\pi i}{h}mc^2 \varphi e^{-}$$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\varphi}{\partial t^2} e^{-} + \frac{4\pi i}{h} mc\frac{\partial\varphi}{\partial t} e^{-} - \frac{4\pi^2}{h^2} m^2c^4 \varphi e^{-}$$

$$\Delta\varphi - \cancel{\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}} + \frac{8\pi^2 mV}{h^2}\varphi = 0$$

$$+ \cancel{\frac{4\pi i}{h}mc^2 - \frac{4\pi^2}{(h)^2}} \quad \frac{4\pi i m}{h}\frac{\partial\varphi}{\partial t}$$

$\bigcirc$

$$\Delta\varphi \pm \frac{4\pi i m}{h}\frac{\partial\varphi}{\partial t} + \frac{8\pi^2 m}{h^2}V\varphi = 0$$

$$V = V_0 e^{i(\lambda x + \mu y + \nu z)}$$

$$\Delta\varphi \pm \frac{4\pi i m}{h}\frac{\partial\varphi}{\partial t} + \frac{8\pi^2 m}{h^2} V_0 e^{i(\lambda x + \mu y + \nu z)}\varphi = 0$$

$$\Delta\varphi \pm \frac{4\pi i m}{h}\frac{\partial\varphi}{\partial t} + \frac{8\pi^2 m}{h^2} \underset{e^{-a(z-z_0)}}{\underset{\parallel}{V_0(z)}}\, e^{i(\lambda x + \mu y)} \varphi = 0$$

$$\varphi = \int f(p_x p_y p_z)\, e^{\frac{2\pi i}{h}(p_x x + p_y y + p_z z - Wt)}\, dp_x dp_y dp_z$$

$$\int -\frac{4\pi^2}{h^2}p^2 f(p_x, p_y, p_z) \pm \frac{8\pi^2 m}{h^2}W + \frac{8\pi^2 m}{h^2}V_0 e^{i(\lambda x+\mu y+\nu z)}$$
$$e^{-}\ dp_x dp_y dp_z = 0.$$

⑰

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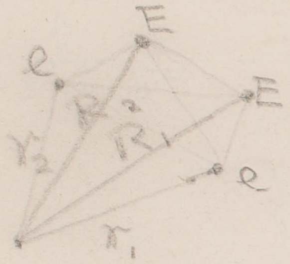
$$\mathbb{H}=\int\cdots\int\frac{1}{2M}\left\{\left(\frac{\partial\Phi}{\partial X_1}\right)^2+\left(\frac{\partial\Phi}{\partial Y_1}\right)^2+\left(\frac{\partial\Phi}{\partial Z_1}\right)^2\right\}+\frac{1}{2m}\left\{\left(\frac{\partial\Phi}{\partial x_1}\right)^2+\left(\frac{\partial\Phi}{\partial y_1}\right)^2+\left(\frac{\partial\Phi}{\partial z_1}\right)^2\right\}+\frac{1}{2M}\left\{\left(\frac{\partial\Phi}{\partial X_2}\right)^2+\cdots+\cdots\right\}+\frac{1}{2m}\left\{\left(\frac{\partial\Phi}{\partial x_2}\right)^2+(\quad)^2+(\quad)^2\right\}\ \frac{\pm}{\frac{4\pi^2}{h^2}}\left\{\frac{eE}{|R_1-r_1|}\mp\frac{eE}{|r_2-R_2|}-\frac{E^2}{|R_2-R_1|}-\frac{e^2}{|r_1-r_2|}+\frac{eE}{|R_1-r_2|}+\frac{eE}{|R_2-r_1|}\right\}\Phi^2$$



$\times\, dX_1 \cdots\cdots dz_2$

$= \text{minimum}.$

$$\int\cdots\int\Phi^2 dx_1\cdots\cdots dz_2=1.$$

$$\mathbb{H}-W\int\cdots\int\Phi^2 dx\cdots dz_2=\mathbb{H}-W$$

$= \text{extremum}.$

$$\therefore\ \Delta\Phi+\frac{4\pi^2}{h^2}\left(W+\frac{eE}{|R_1-r_1|}+\cdots\right)\Phi=0.$$

$|R_1-R_2| \gg |R_1-r_1|$
$\gg |R_2-r_2|$　then, $\Phi=\Phi_1\Phi_2$

$$\frac{1}{2M}\left(\frac{\partial^2\Phi_1}{\partial X_1^2}+\frac{\partial^2\Phi_1}{\partial Y_1^2}+\frac{\partial^2\Phi_1}{\partial Z_1^2}\right)+\frac{1}{2m}\left(\frac{\partial^2\Phi_1}{\partial x_1^2}+\frac{\partial^2\Phi_1}{\partial y_1^2}+\frac{\partial^2\Phi_1}{\partial z_1^2}\right)+\frac{4\pi^2}{h^2}\left(~~E_1~~\,W_1+\frac{eE}{|R_1-r_1|}\right)\Phi_1=0.$$

~~W~~

$$MX_1+mx_1=(M+m)X_1'$$
$$x_1-X_1=x_1'$$

$$\frac{\partial}{\partial X_1}=\frac{M}{M+m}\frac{\partial}{\partial X_1'}-\frac{\partial}{\partial x_1'}$$
$$\frac{\partial}{\partial x_1}=\frac{m}{M+m}\frac{\partial}{\partial X_1'}+\frac{\partial}{\partial x_1'}$$

$$\begin{vmatrix}\frac{M}{M+m} & \frac{m}{M+m}\\ -1 & 1\end{vmatrix}=\frac{M+m}{(M+m)}=1$$

ins([31],[32])-

[30]

$$\frac{1}{2M}\left(\frac{\partial\Phi}{\partial X_1}\right)^2+\cdots+\quad\}+\frac{1}{2m}\left\{\left(\frac{\partial\Phi}{\partial x_1}\right)^2+\cdots+\cdots\right\}$$
$$=\frac{1}{2(M+m)}\left\{\left(\frac{\partial\Phi}{\partial X_1'}\right)^2+\cdots\quad\right\}+\left(\frac{1}{2M}+\frac{1}{2m}\right)\left\{\left(\frac{\partial\Phi}{\partial x_1'}\right)^2+\cdots\right\}$$

$$dX_1\cdots dz_1=\frac{\partial(X_1\cdots\cdots z_1)}{\partial(X_1'\cdots z_1')}dx_1'\cdots dz_1'$$
$$= dx_1'\cdots dz_1'$$

$E = Ze,$

$$\frac{1}{2(M+m)}\left(\frac{\partial^2\Phi_1}{\partial X_1'^2}+\frac{\partial^2\Phi_1}{\partial Y_1'^2}+\frac{\partial^2\Phi_1}{\partial Z_1'^2}\right)+\overset{\left(\frac{1}{2M}+\right.}{\frac{1}{2m}}\left(\frac{\partial^2\Phi_1}{\partial x_1'^2}+\frac{\partial^2\Phi_1}{\partial y_1'^2}+\frac{\partial^2\Phi_1}{\partial z_1'^2}\right)+\frac{4\pi^2}{h^2}\left(W_1+\frac{eE}{r_1'}\right)\Phi_1=0$$

$$\Phi_1=\frac{1}{\sqrt{2\pi}}\sqrt{2k\frac{1}{2}}.\qquad\sqrt{\left(\frac{2Z}{na_0}\right)^3\frac{1}{2}}$$

$n=1,\ m=0$
$k=0$

$$\times e^{-\frac{x}{2}}\cdot L^{1}_{1+k}(x)$$

$$=\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac32}e^{-\frac{Zr_1'}{a_0}}\cdot e^{\frac{2\pi i}{h}(PR_1')}$$

$$\begin{cases}W_1=\dfrac{-2\pi^2m'Z^2e^4}{h^2}\\[2ex] W_1'=\dfrac{2\pi^2\cdot P^2}{M'h^2}\end{cases}$$

$$W_1+W_1'=W_1$$

$$e^{-\alpha x}=\frac{1}{1+\alpha x+\frac{(\alpha x)^2}{2!}+\frac{(\alpha x)^3}{3!}+\cdots}$$
$$=\frac{a}{x}+\frac{b}{x^2}+\cdots$$
$$=e^{-x}$$

$$\varphi=\int^r\left\{c-\frac{2}{r}\int^r f(r)dr-\frac{1}{r^2}\right\}^{-\frac12}\frac{dr}{r^2}$$
$$=\int^r\left\{c-\frac{Ae^{-ar}}{r^2}-\frac{1}{r^2}\right\}^{-\frac12}\frac{dr}{r^2}$$
$$=\int_\infty^r\left[c-\frac12\left(\frac{Ae^{-ar}}{r^2}-\frac{1}{r^2}\right)\right]\frac{dr}{r^2}$$
$$=\frac{c}{W+e^2}$$

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$$\frac{1}{2M}\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}+\frac{\partial^2\psi}{\partial Z^2}\right)+\frac{1}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)$$
$$+\frac{4\pi^2}{h^2}\left(E+\frac{e^2}{|\mathbb{R}-\mathfrak{r}|}\right)\psi=0.$$

$$\{p_0+P_0+V+\alpha_1(p_1+P_1)+\alpha_2(p_2+P_2)+\alpha_3(p_3+P_3)$$
$$+\alpha_4(mc+Mc)\}\psi=0$$

$$\left\{\frac{W+eV}{c}+mc+Mc\right\}\psi_1+$$

$$p_1=\frac{h}{2\pi i}\frac{\partial}{\partial x}\qquad P_1=\frac{h}{2\pi i}\frac{\partial}{\partial X}$$

$$mx+MX=\frac{x_0}{(M+m)}\qquad \frac{\partial}{\partial x}=\frac{m}{M+m}\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}$$
$$x-X=x_2$$
$$x-x_0=x_1\qquad \frac{M}{M+m}x-\frac{M}{M+m}X=x_1\qquad \frac{\partial}{\partial X}=\frac{M}{M+m}\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}$$
$$X-x_0=X_1\qquad \frac{m}{M+m}X-\frac{mX}{M+m}=X_1$$
$$\frac{1}{m}\frac{\partial}{\partial x}+\frac{\partial}{\partial X}=\frac{\partial}{\partial x_1}$$
$$=\frac{\partial}{\partial x_1}$$
$$p_1+P_1=\frac{h}{2\pi i}\frac{1}{M+m}\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x}$$
$$+\left(-\frac{1}{M}+\frac{1}{m}\right)\frac{\partial}{\partial x_2}$$

$$p_1=\frac{h}{2\pi i}\frac{1}{\sqrt{m}}\frac{\partial}{\partial x}\qquad P_1=\frac{h}{2\pi i}\frac{1}{\sqrt{M}}\frac{\partial}{\partial X}$$

$$p_1+P_1=\frac{h}{2\pi i}\left\{(\sqrt{m}+\sqrt{M})\frac{\partial}{\partial x_1}+\left(\frac{1}{\sqrt{m}}-\frac{1}{\sqrt{M}}\right)\frac{\partial}{\partial x_2}\right.$$

$$=$$
$$-p_0^2+\ldots+p^2$$
$$(p_0+P_0+\ldots)^2\qquad \sqrt{m}\sqrt{m}\qquad \sqrt{M}$$
$$\frac{p_0}{\sqrt{m}}+\frac{P_0}{\sqrt{M}}+V$$

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$$M\quad M$$
$$\frac{e^2}{V}$$
$$=S$$

$$k\log\psi=S.$$

$$p_0+P_0+V+\alpha_1$$

$$\psi=e^{p_x x_1+P_x X}$$
$$\frac{\partial\psi}{\partial x}=p.$$
$$\frac{\partial\psi}{\partial X}=P.$$

[30]

1

$$\frac{h}{2\pi i}\frac{\partial Y_0}{\partial t}+\Sigma mcY_0=$$
$$+\frac{2\pi i}{h}\frac{e}{c}VY_0$$

$$\frac{1}{c}\frac{\partial Y_0}{\partial t}+\frac{2\pi i}{h}\Sigma mcY_0=\mathrm{div}\,\mathbb{X}+\frac{2\pi i}{h}\frac{e}{c}(A\mathbb{X})$$

$$\frac{1}{c}\frac{\partial \mathbb{Y}}{\partial t}+\frac{2\pi i}{h}\Sigma mc\mathbb{Y}-\mathrm{grad}\,X_0=\mathrm{curl}\,\mathbb{X}+\frac{2\pi i}{h}\frac{e}{c}[A\mathbb{X}]$$

$$\frac{1}{c}\frac{\partial X_0}{\partial t}-\frac{2\pi i}{h}\Sigma mcX_0=\mathrm{div}\,\mathbb{Y}+\frac{2\pi i}{h}\frac{e}{c}(A\mathbb{X})$$

$$\frac{1}{c}\frac{\partial \mathbb{X}}{\partial t}-\frac{2\pi i}{h}\Sigma mc\mathbb{X}-\mathrm{grad}\,Y_0=-\mathrm{curl}\,\mathbb{Y}$$
$$-\frac{2\pi i}{h}\frac{e}{c}[A\mathbb{Y}]$$

$m=0,\ e=0,$

$$\frac{1}{c}\frac{\partial E_0}{\partial t}=\mathrm{div}\,\mathbb{E}$$

$$\frac{1}{c}\frac{\partial \mathbb{H}}{\partial t}-\mathrm{grad}\,E_0=\mathrm{curl}\,\mathbb{E}$$

$$\frac{1}{c}\frac{\partial E_0}{\partial t}=\mathrm{div}\,\mathbb{H}$$

$$\frac{1}{c}\frac{\partial \mathbb{E}}{\partial t}-\mathrm{grad}\,H_0=-\mathrm{curl}\,\mathbb{H}$$

$$\mathbb{E}=$$
$$E_x=E\cos\nu t\qquad \ldots=0\quad \ldots=0$$
$$H_y=H\cos\nu t\qquad \ldots=0\quad \ldots=0$$

$$+\varphi e$$
$$=0$$
$$\varphi=0$$
$$\psi_0 e^{i(\ldots)}$$

$$A_0 = -\frac{2e^2}{Rr_2}\Big\{ \frac{a_0^2}{2} e^{-\frac{2|R-r_2|}{a_0}} - \frac{a_0^2}{2} e^{-\frac{2|R+r_2|}{a_0}}$$

$$+ a_0 |R-r_2| e^{-\frac{2|R-r_2|}{a_0}} - a_0 (R+r_2) e^{-\frac{2(R+r_2)}{a}}$$

$$+ \frac{1}{2} |R-r_2|^2 e^{-\frac{2|R-r_2|}{a_0}} - \frac{1}{2} (R+r_2)^2 e^{-\frac{2(R+r_2)}{a_0}}$$

$$+ \frac{|R-r_2|^3}{a_0} e^{-\frac{2|R-r_2|}{a_0}} - \frac{(R+r_2)^3}{a_0} e^{-\frac{2(R+r_2)}{a_0}} \Big\}$$

$$\int_0^\infty A_0 \Phi_2^2 \, r_2^2 dr_2 \, \cancel{\sin}\,\theta \cdot 2\pi =$$

$\varphi = 0$

$\varphi = 0$

$$r_2 e^{-\frac{4r_2}{a_0}}\left(-\frac{a_0}{4}\right) - \int e^{-\frac{4r_2}{a_0}}\left(-\frac{a_0}{4}\right)$$

$$- e\left(\frac{a_0}{4}\right)^2$$

$$\int \frac{1}{2m_1}\left[\left(\frac{\partial\psi_1}{\partial x}\right)^2+\left(\frac{\partial\psi_1}{\partial y}\right)^2+\left(\frac{\partial\psi_1}{\partial z}\right)^2\right]$$

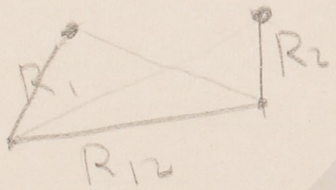
$$+\frac{1}{2m_2}\left[\left(\frac{\partial\psi_2}{\partial x}\right)^2+\left(\frac{\partial\psi_2}{\partial y}\right)^2+\left(\frac{\partial\psi_2}{\partial z}\right)^2\right]$$

$$+\frac{4\pi^2}{h^2} e_1 e_2 \int \frac{\psi_1^2\psi_2^2}{r_{12}}\,dx\,dy\,dz$$

$$+ \; E_1 e_1 \frac{\psi_1^2}{R_1} + E_2 e_2 \frac{\psi_2^2}{R_2}$$

$$- E_1 e_1 \frac{\psi_1^2}{R_1} + - E_2 e_2 \psi$$

$$- \frac{E_1 E_2}{R_{12}}$$



$$\int \delta\psi\,\Delta\psi$$

$$\iiint \frac{1}{2m_1}\left\{\mathrm{div}\left(\delta\psi_1 \,\mathrm{grad}\,\psi_1\right) - \underline{\delta\psi_1}\,\Delta\psi_1\right\}dv$$

$$+\iiint \frac{1}{2m_2}\left\{\mathrm{div}\left(\delta\psi_2 \,\mathrm{grad}\,\psi_2\right) - \underline{\delta\psi_2}\,\Delta\psi_2\right\}dv$$

$$-\iiint \frac{4\pi^2}{h^2}\left\{V_0\left(e_1\psi_1\underline{\delta\psi_1} + e_2\psi_2\delta\psi_2\right) + e_1 e_2 \psi_2\delta\psi_2 \int\frac{\psi_1^2 dv}{|x-x'|}\right.$$

$$+\iiint e_1 e_2 \psi_2^2 \int \frac{\psi_1(x)\,\delta\psi_1\,dv}{|x-x'|}\Big\}\,dv = 0,$$

$$-\frac{1}{2m_1}\Delta\psi_1 - \frac{4\pi^2}{h^2}\left(V_0 e_1 \psi_1 + e_1 e_2 \psi_1(x)\int\frac{\psi_2^2(x')\,dv}{|x-x'|}\right)$$

$$=0$$

$$\int e_1 e_2 \psi_2^2(x) \int \frac{\psi_1(x)\,\delta\psi_1\,dv'}{|x-x'|}\,dv$$

$$= \int e_1 e_2 \psi_2^2(x')\int\frac{\psi_1(x)\,\delta\psi_1\,dv}{|x-x'|}\,dv'$$

$$= \int e_1 e_2 \psi_1(x)\,\delta\psi_1\,dv \int\frac{\psi_2^2(x')\,dv'}{|x-x'|}$$

$$-\frac{1}{2m_2}\Delta\psi_2 - \frac{4\pi^2}{h^2}\left(V_0 e_2\psi_2 + e_1 e_2 \psi_2 \int\frac{\psi_1^2(x')\,dv}{|x-x'|}\right)$$

$$=0$$

$$\int\left(\frac{1}{2m_1}\delta\psi_1\,\mathrm{grad}\,\psi_1 + \frac{1}{2m_2}\delta\psi_2\,\mathrm{grad}\,\psi_2\right)dv = 0$$

ins([31][32])-2

$$A_0 = \int_{-1}^{+1} A\,dx = 2\cancel{-}e^2\int_{-1}^{+1}\left(1 - \frac{2|R+r_2|}{a_0} + \frac{2|R+r_2|^2}{a_0^2}\right)e^{-\frac{2|R+r_2|}{a_0}}dx$$

$$\sqrt{R^2 - 2xRr_2 + r_2^2} = y$$

$$dy = \frac{-Rr_2\,dx}{y}$$

$$= 2e^2\int_{|R-r_2|}^{|R+r_2|}\left(1 - \frac{2y}{a_0} + \frac{2y^2}{a_0^2}\right)e^{-\frac{2y}{a_0}}\left(\frac{-y\,dy}{Rr_2}\right)$$

$$= \frac{-2e^2}{Rr_2}\int_{|R-r_2|}^{R+r_2}\left[ye^{-\frac{2y}{a_0}} - \frac{2y^2}{a_0}e^{-\frac{2y}{a_0}} + \frac{2y^3}{a_0^2}e^{-\frac{2y}{a_0}}\right]dy.$$

$$\int_{|R-r_2|}^{R+r_2} ye^{-\frac{2y}{a_0}}dy = \left[ye^{-\frac{2y}{a_0}}\cdot\frac{-a_0}{2}\right]_{|R-r_2|}^{R+r_2} \cancel{+}\left[e^{-\frac{2y}{a_0}}\frac{a_0^2}{4}\right]_{|R-r_2|}^{R+r_2}$$

$$= -\frac{a_0}{2}(R+r_2)e^{-\frac{2(R+r_2)}{a_0}} + |R-r_2|\frac{a_0}{2}e^{-\frac{2|R-r_2|}{a_0}}$$

$$\pm\frac{a_0^2}{4}\cancel{R+r_2}\,e^{-\frac{2(R+r_2)}{a_0}} + \frac{a_0^2}{4}e^{-\frac{2|R-r_2|}{a_0}}$$

$$\frac{2}{a_0}\int_{|R-r_2|}^{R+r_2} y^2e^{-\frac{2y}{a_0}}dy = \frac{2}{a_0}\left\{\left[\cancel{2y}\,y^2e^{-\frac{2y}{a_0}}\frac{-a_0}{2}\right]_{|R-r_2|}^{R+r_2} \cancel{-4y^2e^{-2}}\right.$$

$$\left.-\left[2ye^{-\frac{2y}{a_0}}\frac{a_0^2}{4}\right]_{|R-r_2|}^{R+r_2} \pm\left[2e^{-\frac{2y}{a_0}}\frac{a_0^3}{8}\right]\right\}$$

$$= -(R+r_2)^2e^{-\frac{2(R+r_2)}{a_0}} + |R-r_2|^2e^{-\frac{2|R-r_2|}{a_0}} - a_0(R+r_2)e^{-\frac{2(R+r_2)}{a_0}}$$

$$+ a_0|R-r_2|e^{-\frac{2|R-r_2|}{a_0}} - \frac{a_0^2}{2}e^{-\frac{2(R+r_2)}{a_0}} + \frac{a_0^2}{2}e^{-\frac{2|R-r_2|}{a_0}}$$

$$\int\frac{2y^3}{a_0^2}e^{-\frac{2y}{a_0}}dy = \frac{2}{a_0^2}\left\{\left[y^3e^{-\frac{2y}{a_0}}\cdot\frac{-a_0}{2}\right]_{|R-r_2|}^{R+r_2} \pm\left[3y^2e^{-\frac{2y}{a_0}}\cdot\frac{a_0^2}{4}\right]\right.$$

$$\left.-\left[6ye^{-\frac{2y}{a_0}}\cdot\frac{a_0^3}{8}\right] - \left[6e^{-\frac{2y}{a_0}}\frac{a_0^4}{16}\right]\right\}$$

$$= -\frac{(R+r_2)^3}{a_0}e^{-\frac{2(R+r_2)}{a_0}} + \frac{|R-r_2|^3}{a_0}e^{-\frac{2|R-r_2|}{a_0}}$$

$$-\frac{3}{2}(R+r_2)^2e^{-\frac{2(R+r_2)}{a_0}} + \frac{3}{2}|R-r_2|^2e^{-\frac{2|R-r_2|}{a_0}}$$

$$-\frac{3}{2}a_0(R+r_2)e^{-\frac{2(R+r_2)}{a_0}} + \frac{3}{2}a_0|R-r_2|e^{-\frac{2|R-r_2|}{a_0}} - \frac{3}{4}a_0^2e^{-\frac{2(R+r_2)}{a_0}} + \frac{3}{4}a_0^2e^{-\frac{2|R-r_2|}{a_0}}$$

$\varphi = 0$

$\rho = 0$

$$\{p_0 + p_0' + V + \alpha p + \alpha' p' + ~~2\alpha_4 mc~~ + (\alpha_4 + \alpha_4') mc\} \times \psi = 0$$

$$V = e^2\left\{\frac{2}{r} + \frac{2}{r'} - \frac{1}{r''}\right\}$$

$p_0 + p_0$

$$\left\{\frac{W + e^2 \cdot V}{c} + \alpha(p + p') + 2\alpha_4 mc\right\}\psi = 0$$

$x + x' = 2x_0$

$x - x' = x_1$

$$\frac{2(r + r')r'' - rr'}{rr'r''} = \frac{}{rr'r''}$$

$$\sqrt{\Sigma\left(x_0 + \frac{x_1}{2}\right)^2}$$

$$\frac{W + e^2V}{c} + \frac{h}{2\pi i}\left(\alpha_1\frac{\partial}{\partial x_0} + \alpha_2\frac{\partial}{\partial y_0} + \alpha_3\frac{\partial}{\partial z_0}\right)\psi + 2\alpha_4 mc\psi = 0.$$

$$\frac{2\pi i}{h}\left(\frac{W + eV}{c} + 2mc\right)\psi_1 + \left(\frac{\partial}{\partial x_0} - i\frac{\partial}{\partial y_0}\right)\psi_4 + \frac{\partial}{\partial z_0}\psi_3 = 0$$

$$\frac{2\pi i}{h}\left(\frac{W + eV}{c} + 2mc\right)\psi_2 + \left(\frac{\partial}{\partial x_0} + i\frac{\partial}{\partial y_0}\right)\psi_3 - \frac{\partial}{\partial z_0}\psi_4 = 0$$

$$\frac{2\pi i}{h}\left(\frac{W + eV}{c} - 2mc\right)\psi_3 + \left(\frac{\partial}{\partial x_0} - i\frac{\partial}{\partial y_0}\right)\psi_2 + \frac{\partial}{\partial z_0}\psi_1 = 0$$

$$\frac{2\pi i}{h}\left(\frac{W + eV}{c} - 2mc\right)\psi_4 + \left(\frac{\partial}{\partial x_0} + i\frac{\partial}{\partial y_0}\right)\psi_1 - \frac{\partial}{\partial z_0}\psi_2 = 0$$

$W \quad \frac{W}{c}$

$$\left\{\frac{p_0 - p_0' + e^2\left(\frac{2}{r} - \frac{2}{r'}\right)}{c} + \alpha(p ~~+ \alpha'~~ p')\right\}\psi = 0$$

$$\frac{2\pi i}{h} \qquad \left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial y_1}\right)\psi_4 + \frac{\partial}{\partial z_1}$$

$$\left\{\left(\frac{h}{2\pi i}\frac{\partial}{\partial t} - mc\right)\right.$$

$($

↑

$h =$

φe^{-}

$= 0$

$\varphi = 0$

$V_0 e^{i(\lambda x + \mu y + \nu z)}$

[illegible]

$$\int \Big(p^2 - 2\underset{\wedge}{\overset{m}{}} W - 2meV_0 e^{-i(\lambda x+\mu y+\nu z)}\Big) \underset{f(p_x p_y p_z)}{e^{\frac{2\pi i}{h}(pr \cancel{-Wt})}} dp_x dp_y dp_z$$

$$= 0$$

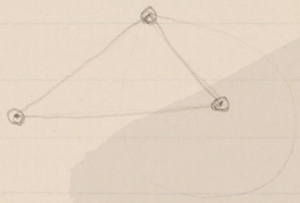
$$\int \Big\{p^2 - 2mW - 2meV_0 e^{\frac{2\pi i}{h}(\lambda x+\mu y+\nu z)}\Big\} \underset{f(p_x p_y p_z)}{e^{\frac{2\pi i}{h}(pr)}} dp_x dp_y dp_z$$

$$= \int \Big\{p^2 - 2mW - 2meV_0 \underset{\nu}{e^{\frac{2\pi i}{h}(\lambda x+\mu y+\nu z)}}\Big\} f(p_x p_y p_z) e^{\frac{2\pi i}{h}(pr)} \times dp_x dp_y dp_z$$

$$= \int \{$$

$$\int p_x^2 f(\quad) e^{\frac{2\pi i}{h}(pr)} dp_x dp_y dp_z$$

$$= p_x$$



§ Three dimensional representation of many body problem.

Schrödinger ノ方法ニヨレバ、degree of freedom ガ f ナル mechanical system ハ f-dimensional space ニオケル、アル一種ノ波動現象トシテ取扱フコトガ出来ル。従ッテ一ツノ particle 例ヘバ一ツノ electron 丈ヲトッテ論ズル際ニハ degree of freedom ガ三デアルカラ普通ノ三次元空間ニオケル現象トシテ全ク直観的ニ頭ニ画クコトガ出来ルノデアルガ、二ツ以上ノ particles ヲ同時ニトリ、其ノ間ノ相互作用ヲモ考慮スル時ニハ之ヲ三次元空間ニオケル現象トシテ取扱フノハ困難デアル。今日ノ量子論デハ大抵、many dimensional space ニオケル問題トシテ論ジテ居ル。

~~即チ此ノ場合ニ於テ、~~コノ many dimensional space ~~ノ~~ ノ line element ハ、

$$ds^2 = 2T(q_k, \dot{q}_k)\,dt^2$$

デ與ヘラレルト假定スル。コノ T ハ system ノ total kinetic energy ヲ ~~[illegible]~~ system ノ coordinates q_k 及ビ velocity $\dot{q}_k$ ノ函数デアラハシタモノデアル。wave mechanics ノ fundamental equation ハ、

$$\mathrm{div}\,\mathrm{grad}\,\psi + \frac{8\pi^2}{h^2}(E-V)\psi = 0$$

~~ニヨッテ定義~~ デアル。

ソンナワケデ、