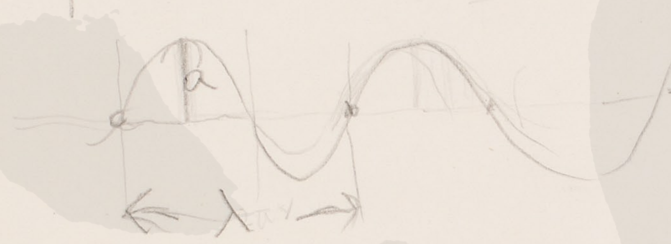
It should be observed that there is a difference between a light-wave and the de Broglie or Schrödinger wave associated with the light quanta. Firstly, the light-wave is always real, while the de Broglie wave associated with a light-quantum moving in a definite direction must be taken to involve an imaginary exponential. A more important difference is that their intensities are to be interpreted in different ways. The number of light-quanta per unit volume associated with a monochromatic light-wave equals the energy per unit volume of the wave divided by the energy $(2\pi h)\nu$ of a single light-quantum. On the other hand a monochromatic de Broglie wave of amplitude a (multiplied into ~~quanta~~ the imaginary exponential factor) must be interpreted as representing a^2 light-quanta per unit volume for all frequencies. This is a special case of the general rule for interpreting the matrix analysis, according to which, if (ξ'/α') or $\psi_{\alpha'}(\xi_k)$ is the eigenfunction in the variables ξ_k of the state α' of an atomic system (or simple particle), $|\psi_{\alpha'}(\xi_k')|^2$ is the probability of each ξ_k having the value ξ_k', [or $|\psi_{\alpha'}(\xi_k')|^2 d\xi_1' d\xi_2' \ldots$ is the probability of each ξ_k lying between the values ξ_k' and $\xi_k' + d\xi_k'$, when the ξ_k have continuous ranges of characteristic values] on the assumption that all phases of the system are equally probable. The wave whose intensity is to be interpreted in the first of these two ways appears in the theory only when one is dealing with an assembly of the associated particles satisfying the Einstein-Bose statistics. There is thus no such wave associated with electrons.

(Dirac; The Quantum Theory of The Emission and Absorption of Radiation. Roy. Soc. Proc. 114 243)



$$\frac{\nu}{c}\int_0^{\infty\,1/\nu} a^2 \sin^2 2\pi\nu\, d\theta$$

$$\lambda = \frac{c}{\nu}$$

$$=$$

$$\sin^2 2\pi\nu = \frac{1}{2}(1-\cos 2\pi\nu)$$

$$\frac{\nu}{c}\frac{a^2}{2} = \frac{a^2}{2c}\frac{1}{2\pi\nu}$$

number of light quanta per unit volume

$$\frac{a^2}{2c}\cdot\frac{1}{h\nu} = \boxed{\frac{a^2}{2ch\nu}}$$

$$\frac{\nu}{c}\int_0^{1/\nu} a^2 e^{4\pi i\nu\theta}d\theta = \frac{\nu\cdot a^2}{c\cdot 4\pi i\nu}$$

$$a^2 e^{\frac{4\pi i}{h}px}$$

$$\int_0^{L} a^2 \sin^2\frac{4\pi px}{h}dx$$

$$\frac{a^2}{2c\,h\nu}$$

system ノ assembly ガ互ニ independent デソレゾレ別ニ perturbe サレタトキ system ガアル state ニアル probability number ヲ求メテミヨウ.

$$i\hbar \frac{\partial \psi}{\partial t} = (H_0 + V)\psi$$

ソレデ perturbed system ノ eigen函数ガ与ヘラレ.

$\psi = \Sigma_r a_r \psi_r$ ガアル initial condition ヲ満足スル solution トシ ψ_r ハ unperturbed system ノ r 番ノ state ニアル時ノ eigen函数, a_r ハ time ノ函数

$|a_r|^2$ ハ ~~state~~ ニアル probability. a_r's ハ加減ニ (system ガ r ノ状態ニアル)

normalize シ, 更ニ normalize サレテヰルトスル。

ソコデ N similar independent system カラナル assembly ニ apply スル. a_r = 夫々 $N^{\frac{1}{2}}$ ヲ掛ケテ

$$\Sigma_r |a_r|^2 = N$$

(コレニヨリ) $|a_r|^2$ ハ state r ニアル system ノ probable number ヲ

$$i\hbar \dot{a}_r = \Sigma_s V_{rs} a_s \quad (4)$$

ガ a's change ヲ支配スル式デアル, V_{rs} ハ V ヲ現ハス matrix

conjugate imaginary equation ハ

$$-i\hbar \dot{a}_r^* = \Sigma_s V_{rs}^* a_s^* = \Sigma_s a_s^* V_{sr}. \quad (4)'$$

(4), (4)' ハ a_r, $i\hbar a_r^*$ ヲ canonical conjugate トシテ $F_1 = \Sigma_{rs} a_r^* V_{rs} a_s$ ヲ Hamiltonian function トスル Hamiltonian form トナル

$$\frac{da_r}{dt} = \frac{1}{i\hbar}\frac{\partial F_1}{\partial a_r^*} \qquad i\hbar \frac{da_r^*}{dt} = -\frac{\partial F_1}{\partial a_r}$$

contact transformation ニヨリ N_r, ϕ_r ナル canonical variable ニ移ル.

$$a_r = N_r^{\frac{1}{2}} e^{-i\phi_r/\hbar} \qquad a_r^* = N_r^{\frac{1}{2}} e^{i\phi_r/\hbar}$$

N_r, ϕ_r ハ real トナリ　$N_r = a_r a_r^* = |a_r|^2$

$\phi_r/\hbar$ ハ eigen函数ノ phase.

$$F_1 = \Sigma_{rs} V_{rs} N_r^{\frac{1}{2}} N_s^{\frac{1}{2}} e^{i(\phi_r - \phi_s)/\hbar}$$

$$\dot N_r = -\frac{\partial F_1}{\partial \phi_r}, \quad \dot\phi_r = \frac{\partial F_1}{\partial N_r}.$$

之ガ transition ニ於ケル rate ヲサダムル式ナリ

又　$b_r = a_r e^{-iW_r t/\hbar}$, $b_r^* = a_r^* e^{iW_r t/\hbar}$

W_r ハ r state ノ energy.　$|b_r|^2 = |a_r|^2$

$$i\hbar \dot b_r = W_r b_r + i\hbar \dot a_r e^{-iW_r t/\hbar}$$

$$= W_r b_r + \Sigma_s V_{rs} b_s e^{i(W_s - W_r)t/\hbar}$$

$V_{rs} = v_{rs} e^{i(W_r - W_s)t/\hbar}$ トオケバ V ガ time ヲ explicite ニフクマヌ時ハ v_{rs} ハ const デ

$$i\hbar \dot b_r = W_r b_r + \Sigma v_{rs} b_s \qquad (5)$$

$$= \Sigma_s H_{rs} b_s$$

$H_{rs} = W_r \delta_{rs}$ ハ total Hamiltonian $H = H_0 + V$ ノ matrix element. (但シ time $e^{i(W_r - W_t)t/\hbar}$ ヲノゾク)

H ガ time ヲ explicit ニフクマヌ時ハ H_{rs} ハ constant ナリ.

(5) ハ (4) ト同ジ形トナル. 従ッテ同ジ形ニ Hamilton ノ form ニナル.

(5) ハ Schrödinger ノ wave equation ナリ, ソウモアル.

サテ, b_r ト $i\hbar b_r^*$ ヲ canonically conjugate variable　(7)

ニトレバ, H ハ　$F = \Sigma_{rs} b_r^* H_{rs} b_s$

トナル,　$b_r = N_r^{\frac{1}{2}} e^{-i\theta_r/\hbar}$　$b_r^* = N_r^{\frac{1}{2}} e^{i\theta_r/\hbar}$　(8)

トナレバ,　$F = \Sigma_{rs} H_{rs} N_r^{\frac{1}{2}} N_s^{\frac{1}{2}} e^{i(\theta_r - \theta_s)/\hbar}$

$$\dot N_r = -\frac{\partial F}{\partial \theta_r}, \quad \dot\theta_r = \frac{\partial F}{\partial N_r}$$

$$F = \Sigma_r W_r N_r + \Sigma_{rs} V_{rs} N_r^{\frac{1}{2}} N_s^{\frac{1}{2}} e^{i(\theta_r - \theta_s)/h}$$

$\Sigma_r W_r N_r$: total proper energy.

phase θ_r ハ perturbation ガナイトキ time ト共ニ linearly ニマス. 故ニ phase ϕ_r ハコノ時ハ constant デアル。

§3. Einstein-Bose ノ Statistics ニ従フ assembly ノ perturbation

b_r, ihb_r ヲ q-number トシ canonical

$$b_r ihb_r^* - ihb_r^* b_r = ih,$$

$$b_r b_r^* - b_r^* b_r = 1$$

$$b_r b_s - b_s b_r = 1 \qquad b_r^* b_s^* - b_s^* b_r^* = 0$$

$$b_r^* b_s^* - b_s^* b_r = 0 \quad (s \neq r)$$

(8) ノ quantum form
$N_r \theta_r$ が canonical ナラバ

$$b_r = (N_r + 1)^{\frac{1}{2}} e^{-i\theta_r/h} = e^{-i\theta_r/h} N_r^{\frac{1}{2}}$$

$$b_r^* = N_r^{\frac{1}{2}} e^{i\theta_r/h} = e^{i\theta_r/h} (N_r + 1)^{\frac{1}{2}}$$

ノ equation ハ N_r ハ 0 ヨリ小ナラヌ integral ナ characteristic value ヲトリ得ヌコトヲ示ス。之ハ variables ガ q-number ナリトノ仮定ヲ justifies. コトナル system ニ於ル system ハ何ハカク. ［illegible］

q-number デアル。
カクシテ Hamiltonian (7) ハ

$$F = \Sigma_{rs} b_r^* H_{rs} b_s = \Sigma_{rs} N_r^{\frac{1}{2}} e^{i\theta_r/h} H_{rs}$$

$$\times (N_s + 1)^{\frac{1}{2}} e^{-i\theta_s/h}$$

$$= \Sigma_{rs} H_{rs} N_r^{\frac{1}{2}} (N_s + 1 - \delta_{rs})^{\frac{1}{2}} e^{i(\theta_r - \theta_s)/h}$$

H_{rs} ハ矢張リ c-number