

FRAGMENT M

Schrödinger  
Wellenmechanik

In der Tat beruht unsere gewöhnliche Beschreibung der Natur ersch. letzten Endes auf der Voraussetzung, daß in Rede stehenden Phänomene beobachtet werden können, ohne sie wesentlich zu beeinflussen.

Einstein: Jede Beobachtung oder Messung schließlich auf dem Zusammenfallen zweier unabhängigen Ereignisse im selben Raum-Zeitpunkt.

Quantenpostulat: Jede Beob. atomarer Phänomene eine nicht zu vernachlässigende Wechselwirkung mit dem Messungsmittel fordert, und daß also weder Phän. noch sein Beob. mittel eine selbständige phys. Redl. im gewöhnlichen Sinne zugeschrieben werden kann.

Oben dieses Zusammenfallen wird nicht berührt durch den Unterschied, den die Raum-Zeit-Beschreibung verschiedener Beobachter im Übrigen aufweisen mag.  
Superpositionsprinzip — Erhaltungssatz.

BOX06

stetige Übergang

S. 58.

$$\psi = \sum_{n=0}^{\infty} \left(\frac{A}{2}\right)^n \frac{\psi_n}{n!}$$

$$= e^{\pi i \nu t} \sum_{n=0}^{\infty} \left(\frac{A}{2} e^{2\pi i \nu t}\right)^n \frac{1}{n!} e^{-\frac{x^2}{4t}}$$

Beziehung zur Quantenmechanik

S. 68.

$$\int p(x) u_k(x) [P, u_l(x)] dx$$

$$= \int p(x) u_k(x) \left( f \cdot k \frac{\partial}{\partial x} \dots u_l \right) dx$$

$$= - \int \left( \frac{\partial}{\partial x} k^2 f p u_k(x) \right) ( \dots ) dx$$

S. 69

(13)  $q_e^{ik} = \int q_e p(x) u_i(x) u_k(x) dx$

$p_e^{ik} = k \int p(x) u_i(x) \frac{\partial u_k(x)}{\partial q_e} dx$

$q_e^{ij} p_e^{ik} = k \int q_e p(x) u_i(x) u_j(x) dx$   
 $\times \int p(x) u_k(x) \frac{\partial u_l(x)}{\partial q_e} dx$

$p_e^{ij} q_e^{ik} = k \int q_e p(x) u_j(x) u_k(x) dx$   
 $\times \int p(x) u_l(x) \frac{\partial u_m(x)}{\partial q_e} dx$

$= - \dots$

$\int q_e dx \dots = \int p(x) u_i(x) u_j(x) dx$

$k q_e$

$$q_e^{ik} p_e^{kj} - p_e^{ik} q_e^{kj} = \int q_e p(x) u_i(x) u_k(x) dx$$

$$\times k \int p(x) u_k(x) \frac{\partial u_j(x)}{\partial q_e} dx$$

$$- k \int p(x) u_i(x) \frac{\partial u_k(x)}{\partial q_e} dx \int q_e p(x) u_k(x) u_j(x) dx$$

$$= k \int \left[ q_e p(x) u_i(x) \right] \left[ p(x) \frac{\partial u_j(x)}{\partial q_e} \right] dx$$

$$+ k \int \left[ q_e p(x) u_j(x) \right] \left[ p(x) \frac{\partial u_i(x)}{\partial q_e} \right] dx$$

$$= k \int q_e p(x) \left( u_i(x) \frac{\partial u_j(x)}{\partial q_e} - u_j(x) \frac{\partial u_i(x)}{\partial q_e} \right) dx$$

$$\frac{1}{2} (p_k^2 q_k + q_k p_k) = \frac{1}{2} (p_k p_k - q_k p_k) - \frac{1}{2} (p_k p_k) + \frac{1}{2} q_k p_k$$

BRUNN

S. 16.

$$\delta \int d\tau \left\{ \kappa^2 \mathcal{I}(q, \frac{\partial \psi}{\partial q}) + \psi^2 V \right\} = 0,$$

$$\int d\tau \left\{ \mathcal{I}(q, \frac{\partial \psi}{\partial q}) + V(q) \right\}$$

$s = \kappa \log \psi$        $\frac{\partial s}{\partial q} = \frac{\kappa}{\psi} \frac{\partial \psi}{\partial q}$

$p = f(q)$

$\mathcal{I} = \sum a_{ik} p_i p_k = \sum b_{ik} p_i p_k$

$q = f(q')$

$$\delta \int d\tau \left\{ \kappa^2 \mathcal{I}(q, \frac{\partial \psi}{\partial q}) + \psi^2 V \right\} = 0$$

$\left( a_{ik} \frac{\partial \psi}{\partial q} \frac{\partial \psi}{\partial q} \right)$        $\frac{a_{ik} p_i p_k}{\sqrt{a_{ik}}}$

$b_{ik} \frac{\partial \psi}{\partial q} \frac{\partial \psi}{\partial q}$

$d\tau = \dots dt$

$\Rightarrow \delta \int L dt = 0$

$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0$

$\left( \begin{matrix} \text{grad } V \cdot \mathbf{e} \\ \text{grad } V \cdot \frac{\mathbf{e}}{|\mathbf{e}|} + V \frac{\text{grad } \mathbf{e}}{|\mathbf{e}|} \end{matrix} \right) d\tau$

$\Delta_p^{-\frac{1}{2}} \frac{\partial}{\partial q_k} \left( \Delta_p^{-\frac{1}{2}} p_k T_{pk} \right)$

$\text{grad } V = 0$

$[H, \psi] = \left( \frac{1}{2} T(q_k, p_k) - V(q_k) \right) \psi$

$- V(q_k) \psi$

$\frac{1}{2} (\text{grad } V)^2 d\tau$

$= \frac{1}{2} \sum_k p_k T_{pk} (q_k, p_k) \psi$

$\left( \frac{1}{2} \sum_k p_k \frac{\partial \psi}{\partial q_k} \right)$

$0 = \int P(x) dx$

$P(x)$

$g_l = \int (u_i(x) u_k(x)) dx$

$\int g_l g_l = \int u_i u_j dx$

$\int u_j u_k dx$

$$\text{Raum dichte} = \psi \frac{\partial \psi}{\partial t}$$

$$= \sum C_k u_k e^{\frac{2\pi i \sqrt{E_k} t}{h}}$$

$$\times \sum -C_m u_m \frac{2\pi i \sqrt{E_m}}{h} e^{-\frac{2\pi i \sqrt{E_m} t}{h}}$$

$$= \sum C_k C_m u_k u_m \frac{2\pi i}{h}$$

$$E_m e^{\frac{2\pi i}{h}(E_k - E_m)}$$

$$+ i \sin(E_k - E_m)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4\pi^2 E^2}{h^2} \psi$$

$$\Delta \psi - \frac{E^2}{E-V} \psi = 0$$

$$\Delta \psi - \frac{2(E-V)}{E} \frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\Delta \psi + \frac{\psi \cdot h^2}{\frac{\partial \psi}{\partial t^2} h} \frac{\partial^2 \psi}{\partial t^2}$$

$$\frac{\partial \psi}{\partial t} = \frac{2\pi i}{h} \psi$$

$$2(E-V) = 0$$

$$\Delta \psi + \frac{8\pi^2}{h^2} \psi (E-V) = 0$$

$$E = \frac{h^2}{2\pi^2} \left( \frac{\partial \psi}{\partial t} \right)^2 / \psi^2$$

$$\left( \Delta - \frac{8\pi^2}{h^2} V \right) \psi^2 = \frac{16\pi^2}{h^2} \frac{\partial \psi}{\partial t} \frac{\partial \psi}{\partial t} = 0$$

$$\left( \Delta \psi - \frac{8\pi^2}{h^2} V \psi \right)^2 = (\Delta \psi)^2 - 2 \frac{8\pi^2}{h^2} V \psi \Delta \psi + \left( \frac{8\pi^2}{h^2} V \psi \right)^2$$

$\Delta^2$

$$\Delta(\psi \psi) = \psi \Delta \psi + (\Delta \psi)^2$$

$$\Delta \psi + \frac{8\pi^2}{h^2} \left( \frac{\partial \psi}{\partial t} \right) \left( \frac{\partial \psi}{\partial t} \right) \left( \frac{\partial \psi}{\partial t} \right) \left( \frac{\partial \psi}{\partial t} \right)$$

$$= 0$$

$$= \Delta \psi - 0 \Delta \psi - \Delta 0 \psi + 0 \psi$$

$$\Delta \psi - \frac{8\pi^2}{h^2} V \psi =$$

$$\psi \sim e^{\pm \frac{2\pi i E t}{h}}$$

$$\frac{\partial \psi}{\partial t} = \psi \cdot \frac{2\pi i E}{h} + \psi \frac{2\pi i}{h} \frac{\partial E}{\partial t} \cdot t$$

$$\frac{\partial^2 \psi}{\partial t^2} = \psi \left( -\frac{4\pi^2 E^2}{h^2} \right) + \psi \left( \frac{8\pi^2 E}{h^2} \frac{\partial E}{\partial t} \right) + \psi \left( \frac{8\pi^2}{h^2} \frac{\partial E}{\partial t} \right)$$

$$\Delta \left( \frac{4\pi^2}{h^2} \psi \right) = \frac{\partial}{\partial t} (\Delta \psi)$$

$$= \frac{\partial}{\partial t} \left( \frac{2(E-V)}{h^2} \frac{\partial \psi}{\partial t} \right)$$

$$\frac{\Delta^2}{2(E-V)} \psi = -\frac{4\pi^2 E^2}{h^2} \psi$$

$$\left( \Delta \psi - \frac{8\pi^2}{h^2} V \psi \right) \psi = \frac{-8\pi^2 E^2}{h^2} \psi$$

$$\left( \Delta - \frac{8\pi^2}{h^2} V \right) \psi = -\frac{8\pi^2 E^2}{h^2} \psi$$

$$= -\frac{16\pi^2 E^2}{h^2} \psi$$

$$\Delta \psi - \frac{8\pi^2}{h^2} (V_0 + A \cos 2\pi \nu t) \psi = \frac{8\pi^2 E^2}{h^2} \psi = 0$$

$$\psi = \psi_0 e^{\pm \frac{2\pi i E t}{h}} + B$$

$$A \cos 2\pi \nu t \psi_0 = 0$$

$$+ \Delta \psi = 0$$

$$\psi = R \cdot e^{2\pi i v t}$$

$$\Delta \psi = \Delta (R \cdot e^{2\pi i v t}) + R \cdot (2\pi i v)^2 \psi$$

$$\frac{1}{u} \frac{\partial \psi}{\partial t^2} = \frac{1}{u} R \cdot \frac{\partial^2 e^{2\pi i v t}}{\partial t^2}$$

$$\Delta R - \frac{4\pi^2 v^2}{u} R = \psi$$

$$\Delta \psi - \frac{4\pi^2 v^2}{u} \psi$$

$$\frac{\partial \psi}{\partial x^2} - \frac{1}{u(x,t)} \frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\psi = R e^{2\pi i v t}$$

$$\frac{\Delta R}{u} + \frac{4\pi^2 v^2}{u} R = 0$$

$$\psi = \sum_{n=-\infty}^{+\infty} R_n e^{i n \frac{2\pi i E_n t}{\hbar}}$$

$$(\Delta - \frac{8\pi^2}{\hbar} V) \psi = \sum_n \frac{2\pi i E_n^2}{\hbar} R_n e^{i n \frac{2\pi i E_n t}{\hbar}} - V \psi$$

$$(\Delta - \frac{8\pi^2}{\hbar} V) \psi = R_n - \Delta$$

$$(\Delta - \frac{8\pi^2}{\hbar} V)^2 \psi - \frac{64\pi^4}{\hbar^4} E^2 \psi$$

$$(\Delta - \frac{8\pi^2}{\hbar} V) \psi$$

$$\Delta \psi - \frac{2(E-V)}{\hbar^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4\pi^2 E^2}{\hbar^2} \psi$$

$$(\Delta + \frac{8\pi^2 (E-V)}{\hbar^2}) \psi = 0$$

$$(\Delta - \frac{8\pi^2}{\hbar^2} V) \psi = -\frac{8\pi^2 E}{\hbar^2} \psi$$

$$\frac{\partial \psi}{\partial t} = \frac{2\pi i E}{\hbar} \psi$$

$$(\Delta - \frac{8\pi^2}{\hbar^2} V) \psi - \frac{4\pi i}{\hbar} \frac{\partial \psi}{\partial t} = 0$$

$$\psi = \sum R_n e^{\frac{2\pi i n E t}{h}}$$

$$\left(\Delta - \frac{8\pi^2}{h^2} V\right) \psi + \frac{16\pi^2}{h^2} \frac{\partial \psi}{\partial t}$$
$$= \left(\right) \frac{8\pi^2}{h^2} E \sum R_n e^{\frac{2\pi i n E t}{h}} + \frac{16\pi^2}{h^2} \frac{\partial \psi}{\partial t} \frac{4\pi i E}{h} \psi$$

$$\cancel{0} R_n \frac{8\pi^2}{h^2} (E - V) R_n = 0$$

$$= 0 \quad (\dots)$$

$$H = T(p, q) + V(q, t)$$

$$\dot{p} = -\frac{\partial H}{\partial q} = -\frac{\partial T}{\partial q} + \frac{\partial V}{\partial q}$$

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{\partial T}{\partial p}$$

$$\frac{dH}{dt} - \frac{\partial H}{\partial t} = \frac{dT}{dt} + \frac{dV}{dt} - \frac{\partial V}{\partial t} = 0$$

$$\frac{dH}{dt} - \frac{\partial V}{\partial t} = 0$$

$$-\frac{\partial T}{\partial t}$$

$$\frac{\partial \psi}{\partial t} = \pm \frac{2\pi i}{h} E \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{8\pi^2}{h^2} E \psi$$

$$\Delta \psi + 2(E-V) \frac{4\pi i}{h} \psi = 0$$

$$\Delta \psi - \frac{8\pi^2}{h^2} V \psi \pm \frac{4\pi i}{h} \frac{\partial \psi}{\partial t} = 0$$

$$\Delta(X+iY) - \frac{8\pi^2}{h^2} V(X+iY) + \frac{4\pi i}{h} \frac{\partial X}{\partial t} - \frac{4\pi i}{h} \frac{\partial Y}{\partial t} = 0$$

$$\Delta X - \frac{8\pi^2}{h^2} V X + \frac{4\pi i}{h} \frac{\partial X}{\partial t} = 0$$

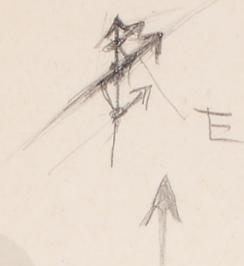
$$\Delta Y - \frac{8\pi^2}{h^2} V Y + \frac{4\pi i}{h} \frac{\partial Y}{\partial t} = 0$$

$$(1+i) \left( \Delta - \frac{8\pi^2}{h^2} V - \frac{4\pi i}{h} \frac{\partial}{\partial t} \right) \psi$$

$$= \left( \Delta - \frac{8\pi^2}{h^2} V \right) \psi - \frac{4\pi i}{h} \frac{\partial \psi}{\partial t}$$

$$= \Delta \psi + 0$$

$$\frac{\partial(V\psi)}{\partial x} = \frac{\partial A}{\partial x} \cos \pi i t$$



$$\frac{\partial A}{\partial x} = E_x \cdot e_i$$

$$-A = \int E_x dx = \int (\sum E_x e_i) \cdot dS$$

$$= \int \sum E \cdot e_i ds$$

$$= F \cdot \sum e_i dz_i$$

$$w = w_+ e^{\frac{2\pi i t}{h}(E_k + h\nu)} + w_- e^{\frac{2\pi i t}{h}(E_k - h\nu)}$$

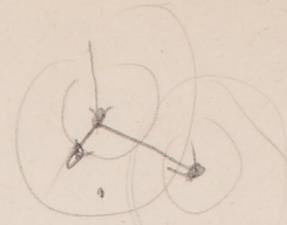
$$\Delta w = e^- \Delta w_+ + e^- \Delta w_-$$

$$-\frac{8\pi^2}{h^2} V_0 w = 0$$

$$-\frac{2\pi i}{h} \frac{\partial w}{\partial t} = \frac{2\pi i}{h} (E_k + h\nu) w_+ e^- + w_- e^-$$

S. 145

$$\begin{aligned}
 W &= W_+(x) e^{\frac{2\pi i t}{h}(E_k + h\nu)} \\
 &\quad + W_-(x) e^{\frac{2\pi i t}{h}(E_k - h\nu)} \\
 (11) \rightarrow e^{\frac{2\pi i t}{h}(E_k + h\nu)} \Delta W_+ - \frac{8\pi^2}{h^2} V_0 W_+ e^{-\frac{4\pi i t}{h} \frac{2\pi i}{h}(E_k + h\nu)} \\
 &\quad + e^{\frac{2\pi i t}{h}(E_k - h\nu)} \Delta W_- - \frac{8\pi^2}{h^2} V_0 W_- e^{-\frac{4\pi i t}{h} \frac{2\pi i}{h}(E_k - h\nu)} \\
 &= \frac{4\pi^2}{h^2} A u_k (e^{\frac{2\pi i t}{h}(E_k + h\nu)} + e^{\frac{2\pi i t}{h}(E_k - h\nu)}) \\
 \Delta W_+ + \frac{8\pi^2}{h^2} V_0 (E_k + h\nu - V_0) W_+ \\
 &+ \\
 &= \frac{4\pi^2}{h^2} A u_k.
 \end{aligned}$$



S. 148  $\psi\bar{\psi} =$

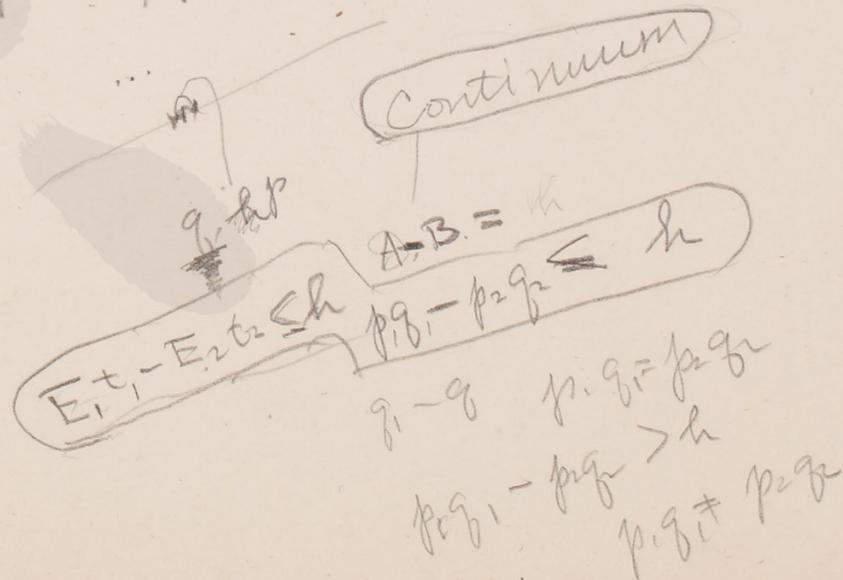
S. 145

$$\begin{aligned}
 W &= W_+(x, t) e^{\frac{2\pi i t}{h}(E_k + h\nu)} + W_-(x, t) e^{\frac{2\pi i t}{h}(E_k - h\nu)} \\
 W &= W_+(x) e^{\frac{2\pi i t}{h}(E_k + h\nu)} + W_-(x) e^{\frac{2\pi i t}{h}(E_k - h\nu)}
 \end{aligned}$$

$$\Delta W_{\pm} + \frac{8\pi^2}{h^2} (E_k \pm h\nu - V_0) W_{\pm} = 0$$

$$\frac{1}{p_1} \sum_i \left( \frac{q_i^{(1)} q_i^{(2)}}{p_2 p_{10}} \right)$$

$$\left| \frac{p_1 q_1}{p_2 q_2} \right|$$



$$\dot{c}_l = \frac{2\pi i}{\hbar} \sum_k \epsilon_{kl} c_k \quad (l=1, 2, \dots, \alpha)$$

$$\begin{aligned} \frac{d}{dt} (\sum_l q_l c_l^*) &= \frac{2\pi i}{\hbar} \sum_{k,l} \epsilon_{kl} (c_k c_l^* + c_k^* c_l) \\ &= \frac{2\pi i}{\hbar} \sum_{k,l} \epsilon_{kl} (c_l c_k^* + c_k c_l^*) \end{aligned}$$

86 = 3x<sup>2</sup>  
255