Methoden der

Math. Physik.

$$\int_{-1}^{1} P_n(x)\, x^m dx = \frac{1}{2^n n!}\int_{-1}^{1} u_n^{(n)}(x)\, x^m dx$$

$$= \left[x^m u_n^{(n-1)}(x) \right]_{-1}^{1}$$

$$\int (\lambda f + g)^2 dx \geq 0.$$

$$\lambda^2 (f, f) + 2\lambda (f g) + (g g) \geq 0$$

$$(\lambda (f, f) - (f g))^2 + (\quad)$$

$$\varphi_2' = -(\varphi, v_2)\, \varphi_1 + v_2$$

$$(\varphi_1 \varphi_2') = -(\varphi_1 v_2)(\varphi_1 \varphi_2') + (\varphi_1 v_2)$$

$$= (\varphi_1 v_2)\, (1 - (\varphi_1 \varphi_2'))$$

$$\varphi_1 \varphi_2' = c_1 \varphi_1 + c_2 v_2$$

$$= c_1 +$$

$$f = \sum c_\nu \varphi_\nu$$

$$\int (f - \sum \gamma_\nu \varphi_\nu)\, \varphi_\nu\, dx = 0$$

$$= (f \varphi_\nu) - \gamma_\nu = 0$$

P.38. (11) $\sum_{k=1}^{\infty} a_{ik} a_{jk} = \sum_k (\varphi_i \psi_k)(\varphi_j \psi_k)$

$$= \sum_k \int (\varphi_i \underbrace{\psi_k}_{\delta_{ki}}) dx \cdot \int (\varphi_j \psi_k) dx$$

$$= a_{ik} \psi$$

$a_{ik} a_{jk}$

$$= \sum_k a_{ik} (\varphi_j \psi_k)$$

$$\sum_k a_{ki} a_{kj} = \sum_k (\varphi_k \psi_i)(\varphi_k \psi_j)$$

$$=$$

(11) $\sum_{k=1}^{\infty} a_{ik} a_{jk} = \sum_k a_{ik} b_{kj} =$

$$\varphi_i = \sum_k a_{ik} \psi_k$$

$$= \sum_k a_{ik'} b_{kj} \varphi_j$$

$$= \sum_k a_{ik} a_{jk} \varphi_j$$

(11) $\psi_i = \sum b_{ik} b_{jk} \psi_j$

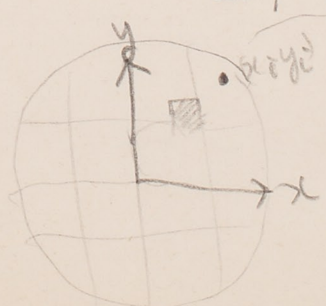
$$= \sum a_{ki} a_{kj} \psi_j$$

P.34 §2

$$\lim_{\substack{n\to\infty \\ m\to\infty}} N(\mathfrak{v}_n - \mathfrak{v}_m) = \lim_{\substack{n\to\infty \\ m\to\infty}} \sum_{i=1}^{k} (a_{ni} - a_{mi})^2$$

$$=$$

$$\sum |a_{ik}|^2 < \infty$$



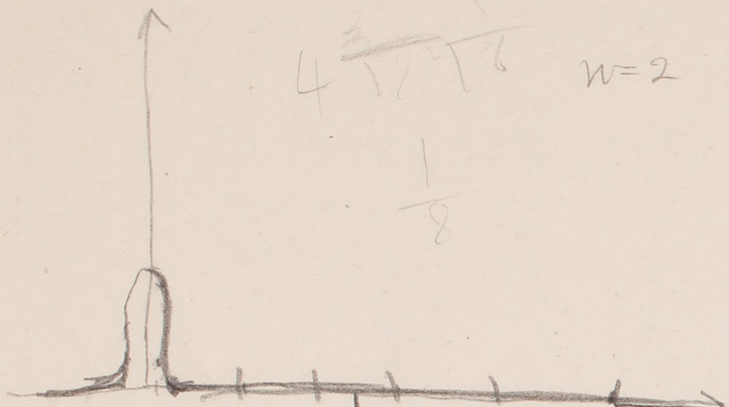
$$|x_i|^2 + |y_i|^2 < M.$$

?

$f_n(x) = 1 - n^2x^2$ für $x^2 \le \frac{1}{n^2}$

$= 0$ für $x^2 \ge \frac{1}{n^2}$

$n=2$



$$\lim_{n\to\infty}\int_{-\frac{1}{n}}^{\frac{1}{n}}(1-n^2x^2)^2\,dx = \lim_{n\to\infty}\int(1-2n^2x^2+n^4x^4)\,dx$$

$$= \lim_{n\to\infty}\left[x - \frac{2}{3}n^2x^3 + \frac{n^4}{5}x^5\right]$$

$$= 0.$$

$f_1, f_2 \ldots\ldots$

$$f_n(x_1) - f_n(x_2)$$

$$= (1-n^2x_1^2 - 1 + n^2x_2^2)$$

$$= n^2\boxed{(x_2^2-x_1^2)}.$$

中

$\frac{\partial}{\partial x}$

$$|g(x) - g(x_h)| \le \underline{|g(x) - g_m(x_h)|} + \underline{|g_m(x_h) - g_m(x_h)|}$$

S.10

$\sum a_{pq} x_p x_q$

$x_p = \sum_{r,q=1}^{n} l_{pr} y_r$

$= \sum a_{pq} l_{pr} y_r \, l_{qs} y_s$

?

$= \sum_{rs} \sum_{pq} (a_{pq} l_{pr} l_{qs}) y_r y_s$

$b_{rs} = a_{pq} l_{pr} l_{qs}$

$b_{pq} = a_{rs} l_{rp} l_{sq}$

$L = (l_{pq})$

$L^{-1} = (l'_{pq})$

$L \cdot L^{-1} = (l_{pr} l'_{rq}) = 1$

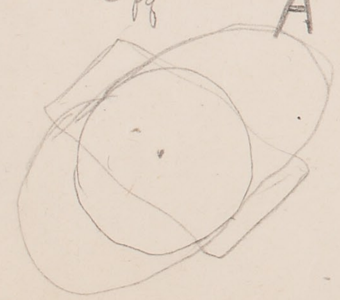
$l_{p1} l'_{1q} + l_{p2} l'_{2q} + \cdots = 1$

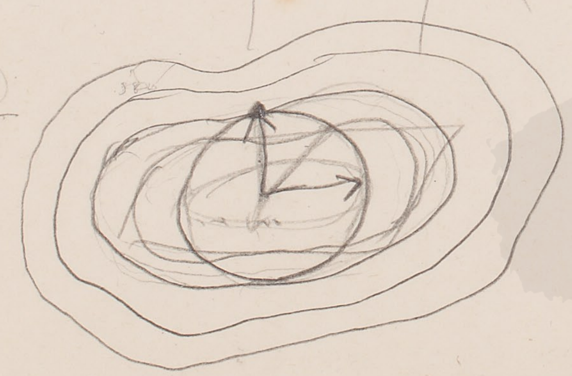
$\sum l_{pr} l'_{rq} = 1 \quad p = q$

$= 0 \quad p \neq q$

$l'_{rq} =$

$l'_{pq} = \frac{-l_{pq}}{A}$





S.13.

$\sum_{p=1}^{n} x_p y_p^2 = \sum_{p=1}^{n} x'_p y'^2_p$

$\sum_{p=1}^{n} y'^2_p = \sum_{p=1}^{n} y_p^2$

$y_1 = 1 \quad y_q = 0, \quad q \neq 1$

$1 = \sum_{p=1}^{n} y_p^2$

$x_p = \boxed{\sum_{p=1}^{n} x'_p y'^2_p}$

$x'^2_p -$

$c_1^{(h)} = x_1^h + \cdots + x_n^h$

$\cancel{\sqrt[h]{c_1^{(h)}}} = \sum x_n^h$

$x_1^h + 2$

$\cancel{x_1 =}$

$\lim_{h \to \infty} \left| \sqrt[h]{c_1^{(h)}} \right| = |x_1| \sqrt[h]{1 + \left(\frac{x_2}{x_1}\right)^h + \cdots \left(\frac{x_n}{x_1}\right)^h}$

$\lim_{h\to\infty} \longrightarrow 0$

$= x_1$

$\lim_{h \to \infty} \left| \sqrt[h]{\frac{c_2^{(h)}}{c_1^{(h)}}} \right| = L \sqrt[h]{\frac{x_1^h x_2^h}{x_1^h + \cdots}}$

$= L |x_2| \sqrt[h]{\frac{1 + \left(\frac{x_3}{x_1}\right)^h + \cdots + \left(\frac{x_3 x_4}{x_1 x_2}\right)^h + \cdots}{1 + \left(\frac{x_2}{x_1}\right)^h + \cdots}}$

$L \left| \sqrt[h]{\frac{c_3^{(h)}}{c_2^{(h)}}} \right| = |x_3|$

$= \frac{|x_2 x_3| \sqrt{\quad}}{|x_2| \sqrt{\cdots}}$

S.15

$$\lim_{h\to\infty} \frac{C_1^{(h)}}{C_1^{(h-1)}} = \lim \frac{x_1^h + \cdots + x_n^h}{x_1^{h-1} + \cdots + x_n^{h-1}} = \lim x_1 \cdot \frac{1 + \left(\frac{x_2}{x_1}\right)^h + \cdots + \left(\frac{x_n}{x_1}\right)^h}{1 + \left(\frac{x_2}{x_1}\right)^{h-1} + \cdots + \left(\frac{x_n}{x_1}\right)^{h-1}} = x_1$$

$$\lim_{h\to\infty} \frac{C_2^{(h)}}{C_2^{(h-1)}} =$$

$$P = \begin{vmatrix} \sum_{\nu=1}^{n} v_{1\nu} v_{1\nu} & \sum v_{1\nu} v_{2\nu} & \cdots \\ & \sum v_{m\nu} v_{x\nu} & \\ \sum v_m & \sum v_{m\nu} v_{p\nu} & \end{vmatrix}$$

$$= \overbrace{\begin{vmatrix} \sum v_{1\nu} & v_{12} & \cdots & v_{1m} \\ \vdots & & & \\ v_{m1} & & & \end{vmatrix}}^{m} \begin{vmatrix} v_{11} & & 0 & 0 \\ v_{12} & & & \\ \vdots & & & \\ v_{mn} & & & \end{vmatrix}$$

$$= \sum \left| \right|^2$$

$$\begin{pmatrix} \frac{\lambda_p - \lambda}{\lambda_p} & & \\ & \ddots & \\ & & \end{pmatrix} \begin{pmatrix} l & 0 & 0 \\ 0 & l_{pq} & \\ 0 & & \end{pmatrix} = 1$$

$$\frac{\lambda_p - \lambda}{\lambda_p} \cdot l_{pq} = 1 \quad p = q$$
$$= 0 \quad p \neq q,$$

$pp \cdot pq$

Tensor: $\begin{pmatrix} a_{\kappa\tau} & a \\ & \end{pmatrix}$

P20.

$x = \sum c_i e_i$
$x' = \sum c_i' e_i$
$xx' = \sum c_i c_i'$

$$\begin{pmatrix} X_x & X_y & X_z \\ Y_x & & \\ Z_x & & \end{pmatrix}$$

$$\mathfrak{S}_x = X_x(i\, S_0) + Y_x(j\, S_0) + Z_x(k\, S_0)$$

S

$$\begin{pmatrix} & \end{pmatrix} S_0 = i\{X_x(i\, S_0) + \cdots\} + j(\quad) + k(\quad)$$

S.23.

$u_q - \lambda_p x_q u_q = 0 \qquad (q = 1 \cdots n)$

S.28

$$\mathcal{L}^{-1} S \mathcal{L} = (\Sigma l'_{mr} S_{rs} l_{sn})$$
$$= (\sum_{rs} S_{rs} l'_{mr} l_{sn})$$
$$= (\sum_{rs} S_{rs} l_{rm} l_{sn})$$
$$= ($$

S.29.

$b'_{ik} = \varepsilon \beta_{ik} \qquad a_{ik} = \varepsilon \rho \alpha_{ik}$

$$\sum_{k=1}^{n} \{ \varepsilon \beta_{ik} - (\rho_h - \varepsilon \rho_h \alpha_{hh} + \varepsilon \beta_{hh})$$
$$\varepsilon \alpha_{ik} \} x'_{lk} = 0$$

$\sum_k$

§8.

S.66.

$$\varphi_{n+1} = \frac{1}{\sqrt{N v_{n+1}}} \left[v_{n+1} - \sum_{h=1}^{n} \varphi_h (\varphi_h v_{n+1}) \right]$$

$$\varphi_0 = 1/2$$

$$\varphi_1 = \frac{1}{\sqrt{\frac{2}{3}}} \left[x - \frac{1}{2} \cdot \cancel{\frac{1}{2}} x \right]$$
$$= \sqrt{\frac{3}{2}} \cdot x$$

$\frac{1}{2}\int x^2 dx \quad x^4 dx \quad \frac{1}{5} \quad \frac{2}{2 \cdot 3}$

$$\varphi_2 = \frac{1}{\sqrt{\frac{2}{5}}} \left[x^2 - \frac{1}{2} \cdot \frac{1}{3} \right]$$

S.70

$$|f(x) - S_n(x)| < \frac{\delta}{2}$$

$\int (1 - v^2) dv = 1 - \frac{2v^3}{3}$

$$J_n = \int_0^1 (1 - v^2)^n dv$$
$$= [v \cdot (1 - v^2)^n]_0^1 + \int_0^1 2n v^2 (1 - v^2)^{n-1} dv$$
$$= 2n \int_0^1 (1 - v^2)^{n-1} dv - 2n \int_0^1 (1 - v^2)^{n-1} dv$$

$$J_n = \frac{2n}{2n+1} J_{n-1}$$

$$J_1 = \frac{2}{3}$$

$$J_n = \frac{2 \cdot 4 \cdot 6 \cdots\cdots}{3 \cdot 5 \cdot 7 \cdots\cdots}$$

S.49,

$$\int_\delta^{2\pi-\delta}\left(\frac{\sin n\frac{t}{2}}{\sin\frac{t}{2}}\right)^2 dt$$

$$< \frac{1}{\sin^2\frac{\delta}{2}}\int_0^{2\pi} dt,$$

$$-\int_0^\delta$$

$$\sin\frac{\delta}{2}$$

$$\frac{1}{\sin^2\frac{\delta}{2}}\int_0^\pi \left(\sin n\frac{t}{2}\right)^2 dt$$

S.74

$$(1-x^2)y''-xy'+n^2y=0$$

$$P_n(x) = \cos(n\arccos x)$$

$$P_n'(x) = +\sin(n\arccos x)\, n\cdot\frac{1}{\sqrt{1-x^2}}$$

$$P_n''(x) = -\cos(n\arccos x)\cdot n^2\frac{1}{1-x^2}$$

$$+\sin(n\arccos x)\, n\cdot\frac{x}{(1-x^2)\sqrt{1-x^2}}$$

$$M\{(1-x^2)y''-xy'\} =$$

$$M(1-x^2)\quad \sqrt{1-x^2}\, y''-\frac{xy'}{\sqrt{1-x^2}}$$

$$\{\sqrt{1-x^2}\,y'\}'+\frac{n^2}{\sqrt{1-x^2}}y=0.$$

P_n

S.72.

$$|P_n(x)-f(x)|$$

$$P_n(x) = \frac{\int_{\alpha_1}^{\beta_1}}{(2J_n)^m}$$

$$\frac{\int_{\alpha_m}^{\beta_m} f(u_1\cdots u_m)E(\ \ ) \to du_m}{\int_{-1}^{1}(1-u^2)^n du}$$

$$= \quad P_n -$$

$$P_n(x)\iiint\left(2f(x)(J_n-J_n^*)+2\varepsilon J_n+MJ_n^*+MJ_n^*\right)$$
$$(u_1\cdots x_n)$$

$$< \quad f(x)+\iint$$

Forsyth
P. 264.

$$\frac{d^2y}{dx^2}+\frac{M}{T}\frac{dy}{dx}+\frac{N}{T^2}y=0$$

T: poly of x of any degree ~~to~~ n having no equal roots

M: Polyns of x of degrees not greater than $n-1$

N: 〃 〃 $2n-2$

S. 75.

$$e^{x^2}e^{-(t-x)^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n$$

$$H_n(x) = \left(\frac{d^n \psi(x,t)}{dt^n}\right)_{t=0}$$

$$= e^{-t^2} \cdot e^{2tx}$$

$$= \frac{d^n e^{-t^2}}{dt^n} \cdot e^{2tx} + 2n x \frac{d^{n-1} e^{-t^2}}{dt^{n-1}} e^{2tx}$$

$t=0$

$$= \left(\frac{d^n e^{-t^2}}{dt^n}\right)_{t=0} + 2nx \frac{d^{n-1} e^{-t^2}}{dt^{n-1}} \cdots$$

$$\frac{d^n e^{-x^2}}{dx^n} = e^{-x^2} \frac{d^n e^{x^2}}{dx^n} + \frac{d^{n-1} e^{-x^2}}{dx^{n-1}}$$

$$e^{x^2}\left(\frac{d^n e^{-(t-x)^2}}{dt^n}\right)_{t=0}$$

$$t - x = y$$

$$e^{x^2}\left(\frac{d^n e^{-y^2}}{dy^n}\right)_{y=-x}$$

$$\frac{\partial \psi(x,t)}{\partial t} + 2(t-x)\psi(x,t) = 0$$

$$H_{n+1} = \left(\frac{\partial^{n+1} \psi(x,t)}{\partial t^{n+1}}\right)_{t=0} = \left(\frac{\partial^n}{\partial t^n} 2x\,\psi(x,t)\right)_{t=0}$$

$$\int_{-\infty}^{+\infty} H_m(x) \frac{d^n e^{-x^2}}{dx^n} dx = \left\{ H_m \frac{d^{n-1} e^{-x^2}}{dx^{n-1}} \right\} - \cdots$$

S. 77

$$L_n(x) = e^x \frac{d^n}{dx^n}(x^n e^{-x})$$

$$= \sum_{k=0}^{n} (-1)^k \binom{n}{k} n(n-1)\cdots(k+1)\, x^k \qquad \frac{1}{(1-t)^{k+1}} =$$

$$\frac{t^k}{(1-t)^{k+1}} = t^k\left\{1 + \binom{k+1}{1} t + \cdots\right\}$$

$$= t^k + \binom{k+1}{1} t^{k+1} + \cdots$$

$$k + m = p$$

$$= \binom{p}{m} t^{p-m+1}$$

$$= \frac{t}{\left(1-\frac{1}{t}\right)^{k+1}} \qquad t^k \sum \binom{k+1}{m} t^m$$

$=$

$$\sum \binom{n}{k} t^n = t^k \cdot \sum \binom{n}{k} t^{n-k}$$

$$= \sum \binom{n}{n-k} t^{n-k}$$

$$= \qquad n - k = m.$$

$$= \binom{m+k}{m} t^m$$

$$\frac{(1-t^k)-1}{(1-t)^{k+1}} = \frac{1+t+t^2+\cdots \quad -1}{(1-t)^k}$$

$$= \{t + t^2 + \cdots + \cdots\} \left\{\binom{k}{m} t^m\right\}$$

$$= \sum_{m=1}^{\infty} t^m \sum_{n=p}^{\infty} \binom{k}{m} t^n =$$

$f(s) - \sum_{\nu=1}^{n} c_\nu \varphi_\nu(s) = \quad f(s) - \sum \int f(s)\varphi_\nu(s)\,ds$

φ

$c_\nu = \int$.

$\sum_{\nu=1}^{\infty} \int f(s)\varphi_\nu(s)\,ds\, \varphi_\nu(s)$

M

$$f(s) = \lim_{n\to\infty} \int f(t) \sum_{\nu=1}^{n} \varphi_\nu(s)\varphi_\nu(t)\,dt.$$

$\sum_{\nu=1}^{\infty} \varphi_\nu(s)\varphi_\nu(t)$ gleichmäßig konvergieren

なら. lim ト $\int$ ヲ トリカヘル.

$t = s$, $\int \varphi_\nu^2(s)\,ds = 1$

$$\varphi_\nu(s) = \sum_{n=1}^{\infty} \int \varphi_\nu(t)\,\varphi_n(t)\,dt$$

$$= \sum_{n=1}^{\infty} \varphi_\nu(s) \int \varphi_n^2(t)\,dt$$

$$= \sum_{n=1}^{\infty} \varphi_\nu(s)\,\lambda_\nu \int \frac{\varphi_\nu(t)\varphi_n(t)}{\lambda_n}\,dt$$

$$= \sum_{n=1}^{\infty} \lambda_n \int \varphi_n(t) \frac{\varphi_\nu(s)\varphi_\nu(t)}{\lambda_n}\,dt$$

$$\varphi_\nu(s) = \sum_{n=1}^{\infty} \varphi_n(s) \int \varphi_\nu(t)\varphi_n(t)\,dt$$

$$= \sum_{\substack{n=1\\ n=\nu}}^{\infty} \lambda_n \int \varphi_\nu(t) \frac{\varphi_n(s)\varphi_n(t)}{\lambda_n}\,dt$$

$$= \int$$

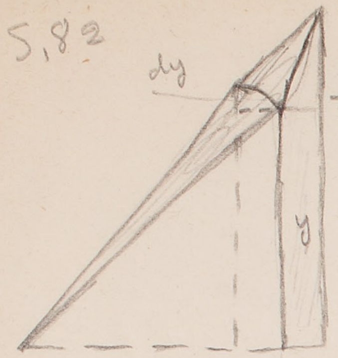
$$\sum_{\nu=1}^{\infty} \frac{\varphi_\nu(s)\varphi_\nu(t)}{\lambda_\nu} = \frac{K(s,t)}{\varphi_n(t)}$$

$$\sum \int \varphi_n(t) \frac{\varphi_\nu(s)\varphi_\nu(t)}{\lambda_\nu}\,dt = \int K(s,t)\,\varphi_n(t)\,dt$$

$\nu = n$.

$$\int \frac{\varphi_n(t)}{\lambda_n} =$$

S. 8. 2



$\frac{1}{2}(x+dx)(y+dy) - \frac{1}{2}xy - y\,dx - \frac{1}{2}dy\,dx$
$= \frac{1}{2}(x\,dy - y\,dx)$

$\int_0^1 \cot\pi(u-t)\,du \left[\int_0^1 g(u)\cot\pi(t-u)\,du\right] = -\int_0^1 g(u)\,du \int_0^1 \cot^2\pi(t$

$=$

$B_n(1) = B_n(0)$
$B_n(1) = (-1)^n B_n(0)$

$\int_x^{x+1} B_n(z)\,dz = \frac{B_n(x+1) - B_n(x)}{n+1}$

$B'_{n+1} = (n+1) B_n$

$\frac{B_n(x+1) - B_n(x)}{n+1}$

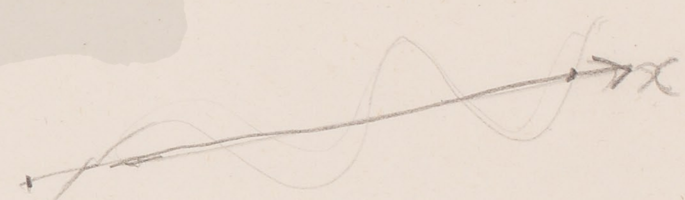
$2\left(\frac{L}{2\pi}\right)^2 = 2\,\frac{1}{\pi}\int_0^{2\pi}\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\}dt$

$=$

$\sqrt{\pi}$

$N\left(\frac{dx}{dt}\right) = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{dx}{dt}\right)^2 dt =$

$=$



S. 8. 3

$f(x) = \sum_{k=0}^{n} \frac{f(x_k)}{\varphi'(x_k)}\,\frac{\varphi(x)}{x - x_k} + \varphi(x)\,\psi(x)$

$x = x_k$ $\qquad \varphi(x) = 0$

$= f(x_k)$

$\int_{-1}^{+1}(a_0x^5 + a_1x^4 + \cdots + a_5)\,dx = \sum 2a_5 +$

$= \frac{10}{18}(a_1x_0^4 + a_3x_0^2 + a_5) + \frac{4}{9}a_5 +$

$\int_0^1 f(t)\cot\pi(u-t)\,dt = \int_0^1 f(t)\,dt \int_0^1 g(u)\cot\pi(t-u)\,du$

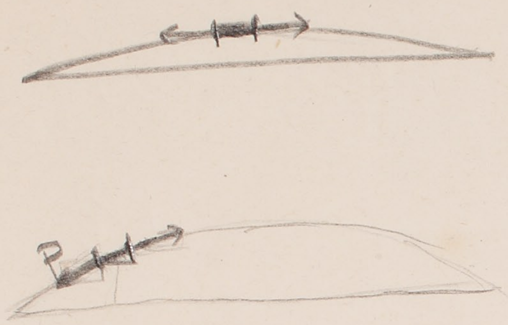
$= \int_0^1 g(u)\,du \int_0^1 f(t)\cot\pi(t-u)\,dt$

$= \int (g(u)+1)\,du \int$

mth, Math, Physik,

S. 237.

$(p u_x)_x = \rho u_{tt}$



δx

$p\frac{dy}{dx}$ $\quad P = p(x)$

$(p+\delta p)\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}\delta x\right) - p\frac{dy}{dx} = \left(p\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{dp}{dx}\right)\delta x$

$\rho\,\delta x\,\frac{d^2y}{dt^2} = \left(p\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{dp}{dx}\right)\delta x$

$\qquad = \frac{d}{dx}\left(p\frac{dy}{dx}\right)$

$\xi = \int \frac{dx}{p(x)}$

$\frac{dy}{dx} = \frac{dy}{d\xi}\frac{1}{p(x)}$ $\qquad \frac{d^2y}{dx^2} = \frac{d^2y}{d\xi^2}\frac{1}{p(x)} \ldots \frac{dy}{d\xi}\frac{1}{p(x)}$

$y_2 \mid (p y_1')' - q y_1 + \lambda_1 \rho y_1 = 0$

$y_1 \mid (p y_2')' - q y_2 + \lambda_2 \rho y_2 = 0$

$\int_0^\pi \{(p y_1')' y_2 - y_1 (p y_2')'\} dx + (\lambda_1 - \lambda_2)\int_0^\pi \rho y_1 y_2 \, dx = 0$

$\frac{1}{r^2}\frac{d}{dr}\left\{r^2\frac{dX}{dr}\right\} + \left(\frac{2mE}{\hbar^2} + \frac{2me^2}{\hbar^2 r} - \frac{n(n+1)}{r^2}\right)X = 0$

E

$L[y] - \lambda^* r y + E^* \rho y$

$= L(u_k(x) + \lambda v_k(x)) - \lambda_k r (u_k(x) + \lambda_k v_k(x))$

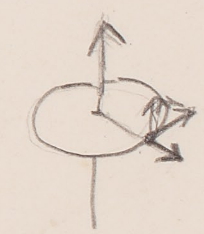
$+ (E_k + \lambda_k \varepsilon_k)\rho y$

$= \lambda L[v_k] - \lambda r u_k + \cdots$

$v_k = \sum \gamma_{ki} u_i(x)$

$\int \rho u_k^{*2} dx = \int \rho (u_k + \lambda \gamma_k)^2 dx$

$L[u_k^*(x)] + \lambda r u_k^*(x) + E_k^* \rho u_k^* = 0$



$\frac{d}{dt}(m v) = \frac{e^2 r}{r^3} + [H v]$

IV.
S. 100.

$$\lambda^2 \int K(s,t)^2 dt \geqq \sum C_\nu^2$$

$$\cancel{c_\nu} =$$

$$K(s,t) = \sum c_\nu \varphi_\nu(t)$$

$$\left(\int K(s,t) \varphi(t) dt\right) = c_\nu$$