Heisenberg 38 Jun 16 12日着

Department of Physics,
Osaka Imperial University,
Nakanosima, Osaka, Japan,
August 6, 1938.

Dear Prof. W. Heisenberg,

I believe that you have received my previous letter, in which I stated that the mean lifetime of the heavy quantum in the theory equivalent to that of Konopinski and Uhlenbeck had the same order of magnitude as in that of Fermi. This conclusion was just the contrary to our intuitive expectation that the life time in the former case ~~should~~ was to be much shorter than in the latter case, as you had pointed out. Since then we reexamined the procedure which lead us to the above conclusion and found that our method of quantization was not legitimate. I hope ~~that~~ you will pardon me for having overlooked this point in spite of your kind advice. Detailed discussions of the subject will be made in a subsequent paper (Proc. Phys.-Math. Soc. Japan im press). Here, I should like to give a brief account of the point at issue.

We start from the Lagrangian

$$\bar{L} = \iiint L\, dv$$

$$L = L_s + L_\ell + L_u$$

where $L_s$, $L_\ell$ are the Lagrange functions for the heavy and light particles respectively and

$$L_u = \frac{1}{4\pi}(\tilde{F}F - \tilde{G}G + \tilde{U}_0 U_0 - \tilde{U}U)$$
$$+ \frac{g_1}{\varkappa}(\tilde{U}M - \tilde{U}_0 M_0 + U\tilde{M} - U_0\tilde{M}_0)$$
$$+ \frac{g'}{\varkappa}(\tilde{U}M' - \tilde{U}_0 M'_0 + U\tilde{M}' - U_0\tilde{M}'_0),$$

notations being the same as in our previous paper III. $\vec{F}, \vec{G}$ ~~etc. were~~ are considered as defined by other variables by the relations

$$\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \operatorname{grad} U_0 + \varkappa \vec{F} = 4\pi g_2 \vec{T} + 4\pi g' \vec{T}'$$
$$\operatorname{curl} \vec{U} - \varkappa \vec{G} = -4\pi g_2 \vec{S} - 4\pi g' \vec{S}'.$$

According to III, the interaction between the heavy quantum and the light particle is given by

$$M_0' = \tilde{\psi}\phi \qquad \vec{M}' = \tilde{\psi}\rho_1\vec{\sigma}\phi$$
$$\vec{S}' = \tilde{\psi}\rho_3\vec{\sigma}\phi \qquad \vec{T}' = -\tilde{\psi}\rho_2\vec{\sigma}\phi,$$

where $\psi, \phi$ is the wave functions for the electron and the neutrino respectively. This assumption results in the energy distribution of $\beta$-ray of Fermi type.

Now, the most general form for the interaction involving the first derivatives of the neutrino wave functions is

$$M_0' = \tilde{\psi}\left\{\lambda_1 - i\lambda_2 \frac{\hbar}{mc}\rho_2\vec{\sigma}\operatorname{grad} + i\lambda_3\frac{\hbar}{mc}\rho_3\frac{\partial}{c\,\partial t}\right\}\phi$$
$$\vec{M}' = \tilde{\psi}\left\{\lambda_1\rho_1\vec{\sigma} + i\lambda_2\frac{\hbar}{mc}\left(\rho_2\vec{\sigma}\frac{1}{c}\frac{\partial}{\partial t} + \rho_3\vec{\sigma}\times\operatorname{grad}\right) - i\lambda_3\frac{\hbar}{mc}\rho_3\operatorname{grad}\right\}\phi$$
$$\vec{S}' = \tilde{\psi}\left\{\mu_1\rho_3\vec{\sigma} - i\mu_2\frac{\hbar}{mc}\rho_1\vec{\sigma}\times\operatorname{grad} + i\mu_3\frac{\hbar}{mc}\left(\vec{\sigma}\frac{1}{c}\frac{\partial}{\partial t} + \rho_1\operatorname{grad}\right)\right\}\phi$$
$$\vec{T}' = \tilde{\psi}\left\{-\mu_1\rho_2\vec{\sigma} - i\mu_2\frac{\hbar}{mc}\left(\rho_1\vec{\sigma}\frac{1}{c}\frac{\partial}{\partial t} + \operatorname{grad}\right) + i\mu_3\frac{\hbar}{mc}\vec{\sigma}\times\operatorname{grad}\right\}\phi$$

where $\lambda$'s and $\mu$'s are numerical constants of the order of 1. We introduce canonical variables and construct the Hamiltonian as usual. If we want to go over into the quantum theory, we meet with the difficulty that the canonical conjugate ~~$\phi$~~ to $\phi$ is not equal to $i\hbar\tilde{\psi}$, so that the quantization for the light part~~ø~~icle can be performed only in the first approximation ~~n~~

neglecting the quantities of the second and higher orders with respect to g'. The situation is essentially the same as in the ordinary theory of K.-U., which was discussed by Fierz (Helv. Phys. 10, 123, 284, 1937). Thus, in the other theory of K.-U. type correct in the first approximation, the probability of annihilation of the heavy quantum turns out to be larger by a factor about $(m_u/m)^2 \sim 10^4$ than that in the theory equivalent to Fermi, as you already predicted, so that it is impossible to reconcile the cosmic ray phenomena with the theory of K.-U. type, which has, in addition, the formal difficulties connected with the quantization of the light particle wave. (At present, however, it ... find ... plausible interpretation of the asymmetry in the distribution of the β-ray in our scheme.)

On the other hand, a cloud chamber photograph obtained by Anderson and Neddermeyer very recently (Phys. Rev. 54, July 1, 1938) can probably be considered as a further evidence for the transformation of the heavy quantum into the light particles. Under these circumstances, it seems to me reasonable to retain the above general scheme of the theory of the β-decay and to search for the origin of the asymmetry of distribution of the β-ray not in the above way of interpreting the β-disintegration other than the introduction of the derivatives of the neutrino wave functions, although no good idea occurs to us for the time being.

I remain,

Yours very truly,

Hideki Yukawa.

the main feature of our theory of β-decay