Furry and Oppenheimer: On the Theory of the Electron and Positive (Phys. Rev. 45, 245, 1934; 45, 343, 1934;)

(1) Free electron の state は二つに分けられる.

即ち. $i\hbar\dot{\psi} = H_0\psi$ $\qquad H_0 = -\rho mc^2 - c(\alpha p)$

の solution は

$H_0\psi = E\psi \qquad E \geq mc^2$ pos. kin. energy (a)

$H_0\psi = E\psi \qquad E \leq -mc^2$ neg. 〃 〃 (b)

の二種の solution の superposition で与へられる. (a) 型の solution のみの superposition よりなる state を pos. state, (b) 〃 neg. state といふ.

dynamical variables p, x, α 等の性質の一を matrix であらはしたとき pos. と neg. とに refer する二つの matrix element が 0 となるものは operator (dyn. vari) を even といふ.

すると even operator に correspond する量のみを, neg. 及び pos. electron particle の数をかへずに決定出来る.

electron の state は凡て pos. state の superposition
positron 〃 〃

と出来ると考へる.

一つの particle の dynamical variable としては even operator 丈が意味がある. (かゝるものに関する. を測定しようとすれば particle の数が indet. になる. (即ち even でないもの ~~partic~~ 量を測定した後に ~~ele~~ particle の数を測定して見ると, 結果は一定でない)

configuration space
quantized wave

(2) 以上の考察を position の determination の問題に apply する. 但し, こゝでは $\hbar/mc$ にのみ関係しており, e^2/mc^2 には関係してゐない.
critical length
$x_0 = \alpha_0 \hbar/mc$

γ-ray microscope では electron の position を arbitrary precision をもって決定出来ることはよく知られてゐる. つまり large angle に scatter された γ-ray の wave length は x_0 の order になるからである.

electron initially at rest.

$$h\nu' = \frac{h\nu}{1+\frac{2h\nu}{mc^2}\sin^2\frac{\theta}{2}}$$

$$h\nu + \frac{mc^2\beta}{\sqrt{1-\beta^2}}\cos\varphi = h\nu'\cos\theta + \frac{mc^2\beta'}{\sqrt{1-\beta'^2}}\cos\varphi'$$

$$h\nu + \frac{mc^2}{\sqrt{1-\beta^2}} = h\nu' + \frac{mc^2}{\sqrt{1-\beta'^2}}$$

$$\cancel{h\nu}\ \frac{mc^2\beta}{\sqrt{1-\beta^2}}\sin\varphi = -h\nu'\sin\theta + \frac{mc^2\beta'}{\sqrt{1-\beta'^2}}\sin\varphi'$$

$$\left(h\nu + \frac{mc^2\beta}{\sqrt{1-\beta^2}}\cos\varphi\right)^2 + \left(\frac{mc^2\beta}{\sqrt{1-\beta^2}}\sin\varphi\right)^2 = (h\nu')^2$$

$$\left(-\frac{mc^2\beta'}{\sqrt{1-\beta'^2}}\cos\varphi'\right. \qquad \left(-\frac{mc^2\beta'}{\sqrt{1-\beta'^2}}\sin\varphi'\right.$$

$$= \left(h\nu + \frac{mc^2}{\sqrt{1-\beta^2}} - \frac{mc^2}{\sqrt{1-\beta'^2}}\right)^2$$

$$2h\nu\,\cancel{\frac{mc^2}{\sqrt{1-}}}\left(\frac{mc^2}{\sqrt{1-\beta^2}}\right)(1-\beta\cos\varphi) - \frac{mc^2}{\sqrt{1-\beta'^2}}(1-\beta'\cos\varphi')$$

$$= \frac{mc^2\beta}{\sqrt{1-\beta^2}}\sin\varphi \cdot \frac{mc^2\beta'}{\sqrt{1-\beta'^2}}\sin\varphi'$$

$$\left(h\nu + \frac{mc^2\beta}{\sqrt{1-\beta^2}}\cos\varphi - h\nu'\cos\theta\right)^2 + \left(\frac{mc^2\beta}{\sqrt{1-\beta^2}}\sin\varphi + h\nu'\sin\theta\right)^2$$

$$= \left(\frac{mc^2\beta'}{\sqrt{1-\beta'^2}}\right)^2 = \left(\frac{mc^2}{\sqrt{1-\beta'^2}}\right)^2 - (mc^2)^2$$

$$= \left(h\nu + \frac{mc^2}{\sqrt{1-\beta^2}} - h\nu'\right)^2 - m^2c^4$$

$2h\nu\frac{mc^2\beta}{\sqrt{1-\beta^2}}$

$$-2h\nu\,\cancel{\frac{mc^2}{\sqrt{1-}}}\,h\nu'\cos\theta - 2\frac{mc^2\beta}{\sqrt{1-\beta^2}}h\nu'(\cos\theta - \sin\theta)$$

$2h\nu\frac{mc^2}{\sqrt{1-\beta^2}}$

$$= -2h\nu h\nu' - 2\frac{mc^2}{\sqrt{1-\beta^2}}h\nu' - (mc^2)^2$$

$\theta = \frac{\pi}{2}$

$$\frac{mc^2\beta}{\sqrt{1-\beta^2}}(h\nu + h\nu') = \frac{mc^2}{\sqrt{1-\beta^2}}(h\nu - h\nu') - (mc^2)^2$$

$$h\nu'\left(\frac{1+\beta}{\sqrt{1-\beta^2}}\right) = h\nu\left(\frac{1-\beta}{\sqrt{1-\beta^2}}\right) - mc^2 \quad h\nu' = h\nu\frac{1-\beta}{1+\beta} - mc^2\frac{\sqrt{1-\beta^2}}{1+\beta}$$

$$\left(h\nu+\frac{mc^2\beta}{\sqrt{1-\beta^2}}\cos\varphi-h\nu'\cos\theta\right)^2+\left(\frac{mc^2\beta}{\sqrt{1-\beta^2}}\sin\varphi+h\nu'\sin\theta\right)^2$$

$$=\left(h\nu+\frac{mc^2}{\sqrt{1-\beta^2}}-h\nu'\right)^2-m^2c^4$$

$$h^2\nu^2+\left(\frac{mc^2}{\sqrt{1-\beta^2}}\right)^2-(mc^2)^2+h^2\nu'^2+2h\nu\left(\frac{mc^2\beta}{\sqrt{1-\beta^2}}\cos\varphi-h\nu'\cos\theta\right)$$

$$-2\frac{mc^2\beta}{\sqrt{1-\beta^2}}h\nu'(\cos\varphi\cos\theta-\sin\varphi\sin\theta)$$

$$=h^2\nu^2+\left(\frac{mc^2}{\sqrt{1-\beta^2}}\right)^2-(mc^2)^2+h^2\nu'^2$$
$$+2h\nu\left(\frac{mc^2}{\sqrt{1-\beta^2}}-h\nu'\right)-2\frac{mc^2}{\sqrt{1-\beta^2}}h\nu'$$

$$\cancel{h\nu'=\frac{h\nu\frac{mc^2\beta}{\sqrt{1-\beta^2}}\cos\varphi}{h\nu\cos\theta+\frac{mc^2\beta}{\sqrt{1-\beta^2}}(\cos\varphi\cos\theta-\sin\varphi\sin\theta)}}$$

$\theta=\frac{\pi}{2}$

$$=$$

$$\{h\nu+E-h\nu\cos\theta-E\cdot\beta(\cos\varphi\cos\theta-\sin\varphi\sin\theta)\}h\nu'$$
$$=h\nu E(1-\beta\cos\varphi)$$

$\theta=\frac{\pi}{2}$:
$$h\nu'=\frac{h\nu\cdot E(1-\beta\cos\varphi)}{h\nu+E(1+\beta\sin\varphi)}=\frac{E(1-\beta\cos\varphi)}{1+\frac{E}{h\nu}(1+\beta\sin\varphi)}$$

$$\lambda'\sim hc\cdot\frac{1+E/h\nu\,(1+\beta\sin\varphi)}{E(1-\beta\cos\varphi)}>\frac{hc}{E}$$

$$\cancel{h\nu+mc}$$

$$\mathbf{P}+\mathbf{p}=\mathbf{P}'+\mathbf{p}' \qquad \frac{E\varepsilon}{c^2}-\mathbf{P}\mathbf{p}=\frac{E'\varepsilon'}{c^2}-\mathbf{P}'\mathbf{p}'$$
$$E+\varepsilon=E'+\varepsilon'$$

$$\frac{1}{c^2}(E+\varepsilon-E')^2-(\mathbf{P}+\mathbf{p}-\mathbf{P}')^2=m^2c^2$$

$$2M^2c^2\cancel{+m^2c^2}-2\frac{(E+\varepsilon)E'}{c^2}+2\frac{E\varepsilon}{c^2}+2(\mathbf{P}+\mathbf{p})\mathbf{P}'-2\mathbf{P}\mathbf{p}$$
$$=0$$

E'

Furry and Oppenheimer (Phys. Rev. 45, 245, 1934)

$$Hx - xH = \left(\frac{\hbar c}{i}\right)\alpha_x$$

$$H\alpha_x + \alpha_x H = 2cp_x$$

$$H\alpha_x - \alpha_x H = 2H(\alpha_x - cH^{-1}p_x) = 2H\eta$$

$$H(\alpha_x - cH^{-1}p_x) - (\alpha_x - cH^{-1}p_x)H =$$

$$= (H\eta - \eta H) = 2H\eta \qquad \therefore H(\alpha_x - \eta) - (\alpha_x - \eta)H = 0$$

$$H(H\eta - \eta H) - (H\eta - \eta H)H = 2H \cdot 2H\eta$$

$$\xi = x + \left(\frac{\hbar c}{2i}\right)H^{-1}\eta$$

$$H\xi - \xi H = \left(\frac{\hbar c}{2i}\right)(\alpha_x - \eta)$$

~~$H(\alpha_x - \eta) - (\alpha$~~

$$H(H\xi - \xi H) - (H\xi - \xi H)H = 0.$$

$$H'(H'|\xi|H'') - (H'|\xi|H'')H'' = 0.$$

$(H'|\xi|H'') = 0$ unless $H' = H''$.

$(H'|H\xi - \xi H|H'') = 0$ unless $H' = H''$.

$H'(H'|\xi|H'') - (H'|\xi|H'')H'' = 0$ unless $H' = H''$.

$(H' - H'')(H'|\xi|H'') = 0$ unless $H' = H''$.

$(H'|\xi|H'') = 0$ 〃

$$\xi = g(x) = x - \left(\frac{\hbar c}{2i}\right)H^{-1}(\alpha_x - cH^{-1}p_x)$$

$$= x - \left(\frac{\hbar c}{2i}\right)(\alpha_x H_0^{-1} - cp_x H_0^{-2})$$

$$g(x) f_i(x) = g \cdot f_i(x).$$

$$f_i(x) = \int \exp(+ixp_x/\hbar) \cdot b_i(p_x)\, dp_x.$$

$$g(x) f_i(x) = \int \left\{ i\hbar \frac{db_i}{dp_x} - \left(\frac{\hbar c}{2i}\right) [\alpha_x^{ij} H_0^{-1} + c\, p_x H_0^{-2}] b_j \right\}$$

$$\times \exp(ixp_x/\hbar)\, dp_x = \int g\, b_i \exp(ixp_x/\hbar)\, dp_x$$

$$b_i = \exp(-igp_x/\hbar) \cdot a_i$$

$$\alpha_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}$$

$$\left\{ \begin{aligned} \frac{da_1}{dp_x} &= -\frac{c}{2}\left(H_0^{-1} a_4 + c\, p_x H_0^{-2} a_1\right) \\ \frac{da_4}{dp_x} &= -\frac{c}{2}\left(H_0^{-1} a_1 + c\, p_x H_0^{-2} a_4\right) \end{aligned} \right.$$

$$\frac{da_2}{dp_x} =$$

$$\frac{da_3}{dp_x} =$$

$$c_1 = a_1 + a_4 \qquad \frac{dc_1}{dp_x} = -\frac{c}{2}\left(H_0^{-1} + c\, p_x H_0^{-2}\right) c_1$$

$$c_1 = \exp \cdot \int -\frac{c}{2}\left(H_0^{-1} + c\, p_x H_0^{-2}\right) dp_x$$

$$\frac{d}{dp_x}\left(H_0^{-1} p_x\right) = H_0^{-1} - H_0^{-2} H_0^{-1} c^2 p_x^2$$

$$= m^2 c^4,$$

p. 260.

$$|V_{rel}|^2 = \frac{|\int \tilde{\psi}_n V \psi_n|^2}{\int\int \psi_p \psi_p \int \psi_n \psi_n}$$

ψ_p ;

$$\psi_1 = -\frac{Ap_3 + B(p_1 - ip_2)}{mc + W/c} S, \quad \psi_2 = -\frac{A(p_1+ip_2) - Bp_3}{mc + W/c} S$$

$$\psi_3 = AS \qquad \psi_4 = BS$$

ψ_n :

$$\psi_1 = A'S' \qquad \psi_2 = B'S'$$

$$\psi_3 = -\frac{A'p_3' + B'(p_1' - ip_2')}{mc - W'/c}, \quad \psi_4 = -\frac{A'(p_1'+ip_2') - Bp_3'}{mc - W'/c}$$

$$\frac{(A^2 \cancel{p_3 p^2} + B^2)p^2}{\cancel{(mc+\frac{W}{c})^2}} \left\{ \frac{p^2}{(mc+\frac{W}{c})^2} + 1 \right\} = (A^2+B^2)\frac{2\frac{W}{c}(mc+\frac{W}{c})}{(mc+\frac{W}{c})^2} = (A^2+B^2)\frac{2\frac{W}{c}}{mc+\frac{W}{c}}$$

$$p^2 + m^2c^2 = \frac{W^2}{c^2}$$

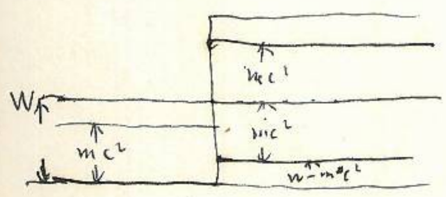
$$2mW + \frac{W^2}{c^2}$$

$$2\frac{W}{c}(mc + \frac{W}{c})$$

$$\int \frac{k^2 dk}{(1+k^2)^{+\frac{n}{2}}} = \frac{k^3 \, dk}{3(1+k^2)^{+\frac{n}{2}}} \Bigg| - \int \frac{k^2 \, dk}{3(1+k^2)^{+\frac{n}{2}}}$$

$$+ \int \frac{\cancel{\frac{n}{2}}\, n k^4 \, dk}{3(1+k^2)^{+\frac{n}{2}+1}}$$

$$+ \int \frac{n k^2 (1+k^2)}{3(1+k^2)^{+\frac{n}{2}+1}} - \int \frac{n k^2}{3(\quad)^{\frac{n}{2}+1}}$$



$+ \quad n$

$$\Psi^{(L)}_{1,0}(r_1)\cancel{\cdots} = \cancel{(N+1)^{-\frac{1}{2}}\cdots}$$

$$= \cancel{C^{(L)}_{r_1}\,\Psi^{(0)}_{0,0}(\cdots r_{i-1}, r_{i+1}\cdots)}$$

$$+ \sum_{p'} C^{(L)}_{p'}\,\Psi^{(0)}_{1,1}(r;\,p')$$

$$\sum_r C^{(L)}_r\,\tilde{u}_r(x) = u_L(x).$$

$$\sum_r \Psi^{(L);0)}_{N+1,N}(r,p)\,u_r(x) = u_L(x)\,\Psi^{0}_{N,N}(\cdots)$$

$$V_{rp} = \int V\cdot \exp$$

$w<0$ $W>0$ $\cancel{\Psi_3 = A}$

小 $\Psi_1 = -\dfrac{A p_3 + B(p_1 - i p_2)}{mc + W/c}S$, $\Psi_2 = -\dfrac{A(p_1 + i p_2) - B p_3}{mc + W/c}S$

大 $\Psi_3 = A S$, $\Psi_4 = B S$

大 小 $\Psi_1 = A'S'$ $\Psi_2 = B'S'$

小 大 $\Psi_3 = -\dfrac{A' p_3' + B'(p_1' - i p_2')}{mc - W'/c}$, $\Psi_4 = -\dfrac{A'(p_1' + i p_2') - B p_3'}{mc - W'/c}$

	Partikel 1	Partikel 2	Strahlungsfeld
mass:	m_1	m_2	
Anfangszustand:	$\mathfrak{p}_1$	$\mathfrak{p}_2$	$\mathfrak{k}_1 \; \mathfrak{k}_2 \; n_1, n_2, \ldots$
Endzustand:	$\mathfrak{p}_1'$	$\mathfrak{p}_2'$	$\mathfrak{k}' \; n_1', n_2' \ldots$
Zwischenzustand:	$\mathfrak{p}_1''$	$\mathfrak{p}_2''$	$\mathfrak{k}'' \; n_1'', n_2'' \ldots$

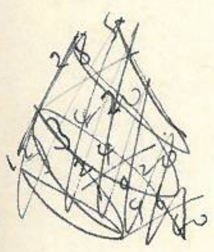
Direkte Übergang

$$\mathfrak{p}_1 + \mathfrak{p}_2 + \sum_i n_i \mathfrak{k}_i = \mathfrak{p}_1' + \mathfrak{p}_2' + \sum_{ji} n_i \mathfrak{k}_{ji}$$

$$\frac{m_1 \cancel{c}}{\sqrt{1-\beta_1^2}} + \frac{m_2 \cancel{c}}{\sqrt{1-\beta_2^2}} + \sum_i n_i \gamma_i = \frac{m_1 c}{\sqrt{1-\beta_1'^2}} + \frac{m_2 c}{\sqrt{1-\beta_2'^2}} + \sum_i n_i \gamma_i$$

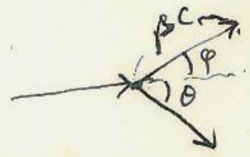
$$\cancel{n_i = n_i' = 0}$$

$$\frac{m_1 \beta_1}{\sqrt{1-\beta_1^2}} + \frac{m_2 \beta_2}{\sqrt{1-\beta_2^2}} + \sum_i n_i \gamma_i = \frac{m_1 \beta_1'}{\sqrt{1-\beta_1'^2}} + \frac{m_2 \beta_2'}{\sqrt{1-\beta_2'^2}} + \sum_i n_i \gamma_i'$$



$$M_1 \beta_1 + M_2 \beta_2 = M_1' \beta_1' + M_2' \beta_2'$$

$$M_1 + M_2 = M_1' + M_2'$$



$$h\nu = h\nu' \cos\theta + \frac{mc^2 \beta}{\sqrt{1-\beta^2}} \cos\varphi$$

$$h\nu = h\nu' + \frac{mc^2}{\sqrt{1-\beta^2}} - mc^2$$

$$h\nu' \sin\theta = \frac{mc^2 \beta}{\sqrt{1-\beta^2}} \sin\varphi$$

$$(h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2 = \frac{(mc^2\beta)^2}{1-\beta^2}$$

$$h^2\nu^2 - 2h^2\nu\nu'\cos\theta + h^2\nu'^2 = \frac{(mc^2\beta)^2}{1-\beta^2}$$

$$h^2\nu^2 - 2h^2\nu\nu' + h^2\nu'^2 + (mc^2)^2 + 2mc^2(h\nu - h\nu') = \frac{(mc^2)^2}{1-\beta^2}$$

$$2mc^2(h\nu - h\nu') = 2h^2\nu\nu'(1-\cos\theta)$$

$$h\nu'(mc^2 + h\nu(1-\cos\theta)) = mc^2 h\nu.$$

$$h\nu' = \frac{h\nu}{1 + \frac{2h\nu}{mc^2}\sin^2\frac{\theta}{2}}$$

p.248. $(x, i/g, \sigma+) = 2^{-\frac{1}{2}} \int_{-\infty}^{\infty} dp \exp[i(x-g)p/\hbar]$

$$\times [(m^2c^2+p^2)^{\frac{1}{2}} \{mc^2 + p^2(m^2c^2+p^2)^{\frac{1}{2}}\}]^{-\frac{1}{2}} \quad a_i^\sigma \qquad (1.6)$$

$$\left.\begin{aligned} a_1^1 &= -p \\ a_3^1 &= mc + (m^2c^2+p^2)^{\frac{1}{2}} \\ a_2^1 &= a_4^1 = 0 \end{aligned}\right\} \qquad \left.\begin{aligned} a_2^2 &= p \\ a_4^2 &= mc + (m^2c^2+p^2)^{\frac{1}{2}} \\ a_1^2 &= a_3^2 = 0, \end{aligned}\right\}$$

$$g(x) f_i(x) = \cancel{x f_i(x)} - \left(\frac{\hbar c}{2i}\right)^2 [\cancel{\alpha_x^{ij} H_0^{-1} f_j} + c p_x H_0^{-2} f_i] \quad j^i j f_j \quad \alpha$$

$$f_i(x) = \int \exp(i x p_x/\hbar)\, b_i(p_x)\, dp_x.$$

$$x f_i(x) = (-i\hbar) \int \frac{d \exp(\cdot\cdot)}{dp_x} b_i(p_x)\, dp_x$$

$$= \int \exp(\ \ ) \cdot i\hbar \frac{db_i}{dp_x} dp_x$$

$$\frac{db_i}{dp_x} + \frac{c}{2} [\cancel{\alpha_x^{ij} b_j H} \ [\alpha_x H_0^{-1} + c p_x H_0^{-2}]^{ij} b_j = g b_i \quad \left(\frac{i}{\hbar}\right)$$

$$\frac{db_i}{dp_x} + \frac{c}{2} \frac{(\alpha_x H_0)^{ij} \cancel{+ c p_x} b_j + c p_x b_i}{H_0^2} = \left(\frac{i}{\hbar}\right) g b_i$$

$$b_i = \exp \frac{-i g p_x}{\hbar} \cdot a_i$$

$$\frac{db_i}{dp_x} + \frac{c}{2} \frac{(\alpha_x H_0)^{ij} b_j + c p_x b_i}{H_0^2} = 0.$$

$\alpha_x = \rho_1 \sigma_x$ etc

$\rho = \rho_3$

$\alpha_x \alpha_y = i \sigma_z$ etc

$\alpha_x \rho = -i \rho_2 \sigma_x$

$$H_0 = -\rho mc^2 - c(\alpha p)$$

$$\frac{db_i}{dp_x} + \frac{ic^2}{2} \cancel{\sigma_z p_y \ p_y \sigma_z (\sigma_y p_z - \sigma_z p_y)} \frac{(\sigma_z p_y - \sigma_y p_z + \rho_2 \sigma_x mc)^{ij}}{H_0^2}$$

$$\rho_2 \sigma_x = \begin{pmatrix} & & & -i \\ & & -i & \\ & i & & \\ i & & & \end{pmatrix}$$

$$\frac{db_1}{dp_x} + \frac{1}{2} \frac{mc\, b_4}{m^2c^2+p_x^2} = 0, \quad \frac{db_2}{dp_x} + \frac{1}{2} \frac{mc\, b_3}{m^2c^2+p_x^2} = 0$$

$$\frac{db_4}{dp_x} - \frac{1}{2} \frac{mc\, b_1}{m^2c^2+p_x^2} = 0, \quad \frac{db_3}{dp_x} - \frac{1}{2} \frac{mc\, b_2}{m^2c^2+p_x^2} = 0.$$

$\cancel{\frac{db_1}{dp_x}} =$ $\quad \cancel{b_4 = (m^2c^2+p_x^2)^{\frac{1}{4}}}$

$$\cancel{\frac{db_1}{dp_x} = -\frac{1}{2}\frac{mc}{(m^2c^2+p_x^2)^{\frac{5}{4}}}}$$

$$\frac{d}{dp_x}\left\{(m^2c^2+p_x^2)\frac{db_1}{dp_x}\right\} + \frac{m^2c^2}{4}\frac{b_1}{(m^2c^2+p_x^2)^{\frac{3}{2}}} = 0.$$

$$\overline{x^2} - \overline{x}^2 =$$

$$\overline{x} = \frac{\int x \; \cancel{(i\hbar\frac{d}{dp})\exp[i(x-g)p/\hbar]} \; |(x, i/g, \sigma+)|^2 dx}{\int |(/\ )|^2 dx}$$

$$\cancel{\approx}$$

$$x$$

$$\cancel{\times} \approx \int x \exp\{i x (p-p')/\hbar\}\, dx$$

$$\overline{x} = \sum x W(x)$$

$$\cdots = \cancel{x \cdot (p-p')}\ \frac{x}{(p-p')} \qquad (x-\overline{x})^2 = \cancel{(x-\sum x W(x))^2}$$

$$= x^2 - 2x\cdot\overline{x} + (\overline{x})^2$$

$$(\overline{x}-\overline{x})^2 = \overline{x^2} - \overline{x}^2 > 0 \qquad \overline{(x-\overline{x})^2} = \sum x^2 W(x) - \overline{x}^2$$

$$(m^2c^2+p^2)^{\frac{1}{2}}\{mc+(m^2c^2+p^2)^{\frac{1}{2}}\}^{-1}\{p^2+(mc+(m^2c^2+p^2)^{\frac{1}{2}})^2\}$$

$$= (\quad)(\quad)^{-1}\{2(p^2+m^2c^2) + 2mc(m^2c^2+p^2)^{\frac{1}{2}}\}$$

$$= 2.$$

Ω^{Γ}

$\Omega_{rs}\,\omega_{rs} + \Omega_{\rho s}\,\omega_{\rho s} + \Omega_{r\sigma}\,\omega_{r\sigma} + \Omega_{\rho\sigma}\,\omega_{\rho\sigma}$

$a_r^{\dagger} a_s$

$\sum \Omega_{rr} + \sum \Omega_{\rho\rho}$ $\qquad \left(\begin{smallmatrix}(\ ) & \\ & (\ )\end{smallmatrix}\right)$

$\mathrm{spur}(A\Omega A^{-1}) = \mathrm{spur}\,\Omega$

$\mathrm{spur}\,\Omega^{\Gamma} = a_{\mu\nu}\,(A\Omega^{\nu}A^{-1})$
spur

$$V_{r\rho} = \int V \exp(kr)\, dk_x\, d\, dV.$$

$$V = C + \mathrm{grad}\, V \cdot \mathbf{r}$$
$$= C + \mathbf{E}\cdot\mathbf{r}$$

$$\int x \cdot \exp k_x x \cdot dx = x \cdot \frac{\exp \cdot k_x x}{k_x}\Big|$$

$$\frac{E_x x\, dy\, dz}{k} + \cdots$$

$\frac{e^2\hbar^2}{m^2c^2}$ $\qquad \frac{E\cdot d\bar{V}}{k}.$ $\qquad \boxed{\frac{E^2}{k^2}}$

$\frac{k\hbar}{mc}$

$$V_{r\rho} = \int V \exp(kr)\, dv \cdot \frac{k\hbar}{mc}$$

$$= eE \int r \exp(kr)\, dv \cdot \frac{k\hbar}{mc}$$

$$= \frac{e\hbar E_n}{mc} \cdot \frac{\left(\frac{\hbar}{mc}\right)^3 k}{k}$$

$$\frac{e}{mc} = \frac{e\hbar}{mc} E_n \qquad dx = \frac{\hbar}{mc} d\xi$$

$d\xi$

$$\int \frac{|\nabla V_{r\rho}|^2 dv}{E_r - E_\rho} = \sum \frac{\left(\frac{e\hbar}{mc} E_n\right)^2}{c\left\{\sqrt{\left(m^2c^2 + k^2\hbar^2\right)} + \sqrt{m\cdot}\right.}$$

$$= \sum \frac{\left(\frac{e\hbar}{mc} E_n\right)^2}{mc^2 \left(1 + \frac{k_1^2\hbar^2}{2m^2c^2} + 1 + \frac{k_2^2\hbar^2}{2m^2c^2}\right)}$$

$$= E_e \frac{e^2\hbar}{c} \frac{1}{\cancel{\frac{\hbar^2 k(k_1-k_2)}{2m^2c^2}}\ 1 + k^2}$$

$$= E_e \frac{e^2}{\hbar c} \frac{1}{1 + k^2} \qquad V_{r\rho}$$

$$V_{r\rho} = \int V \exp(2mc\,kr)\, dv \cdot k = eE \int r \exp(2mc\,kr)\, dv \cdot k$$

$$= e\sum E_n \cdot \frac{\left(\frac{\hbar}{mc}\right)}{2mc}$$

$$\sum \frac{|V_{r\rho}|^2}{E_r - E_\rho} = 4\pi \int k^2 dk \cdot \cancel{eE_n} \frac{\left(\frac{\hbar}{mc}\right)}{2} \quad e^2$$

$$V_{rp} = 4\pi \int V \exp(ip r/\hbar) r \, dr \cdot \frac{\hbar}{mc}$$

$$= 4\pi \cdot \frac{p}{mc} \int r^3 dr \exp(ipr)$$

$$= 4\pi \qquad V_{rp} = eE \cdot \int x \exp(ipx/\hbar)\, dx \cdot \frac{\hbar}{mc}$$

$$= eE \cdot \frac{\hbar}{p} \cdot \frac{p}{mc} \cdot \cancel{\frac{\hbar}{mc}}$$

$$\frac{e^2 E^2 \cdot \left(\frac{\hbar}{mc}\right)^2}{E^2 \cdot \left(\frac{\hbar}{mc}\right)^3} \int \frac{k^2 \, dk}{mc^2 (1 + k^2)}$$

$$\frac{\hbar}{mc}$$

$$k = \frac{p}{2mc}$$

$$\int u'(x) v(x) dx = u(x) v(0) - \int u v'$$

$$= \frac{\lambda_0}{R} \iint \frac{U}{} dk\, dk'$$

$$\frac{m^2 c^3}{\hbar e}$$

$$= \frac{\lambda_0}{R} \iint \frac{e}{}$$

$$\frac{Ze^2}{r^2} < m^2 c^3/\hbar e \qquad r^2 > \frac{Ze^2}{\hbar c} \cdot \frac{\hbar^2}{m^2 c^2}$$

$$r > \frac{mc^2}{} \quad \frac{Z \hbar^{\frac{1}{2}} e^{\frac{1}{2}}}{e^{\frac{1}{2}}} \frac{e^2}{mc^2} \qquad \lambda_0 = \frac{h}{mc} = \cancel{2.4 \times 10^{-10}}$$

$$\iiint \frac{|V_{rp}|^2}{E_r - E_p} \qquad Z \qquad \frac{\hbar}{mc}$$

$$\cancel{e_1 e_2} \qquad \frac{ee_1}{r_1} + \frac{ee_2}{r_2}$$

$$\int \boxed{} \frac{e_1}{r_1} e^{i(k-k')(r_1 - r_{10})/\lambda_0}$$

$$\times \int \boxed{} \frac{e_2}{r_2} e^{i(\cdots)(r_2 - r_{20})/\lambda_0} \qquad (1+k^2)$$

$$\frac{k k' - k k'}{k k' (k+k') |k-k'|^4} \qquad \cancel{1 - \cos\theta} \qquad \frac{(1-\cos\theta) k^2 dk}{\cancel{1+k-k}\; k^4}$$

$\iiint dk \quad e^{i \mathbf{K} R/\lambda_0}$

$\mathbf{k} + \mathbf{k}' = \mathbf{K}$

$\mathbf{k} - \mathbf{k}' = \boldsymbol{\kappa}$

$\mathbf{K} + \boldsymbol{\kappa}$

$$\frac{\sqrt{}\sqrt{}}{(1+k^2)^{\frac{3}{2}}(1+k'^2)^{\frac{3}{2}} \cdot \kappa^4}$$

$\left(\cancel{2+k^2+k'^2}\right)$

$(\sqrt{} + \sqrt{})$

$\frac{1}{4}(\mathbf{K}+\boldsymbol{\kappa})(\mathbf{K}-\boldsymbol{\kappa}) \qquad \kappa^4$

$= \cancel{\mathbf{K}^2} \frac{1}{4}(K^2 - \kappa^2)$

$\left\{1 + \frac{1}{4}(K^2+\kappa^2-2\mathbf{K}\boldsymbol{\kappa})\right\}\left\{1+\frac{1}{4}(K^2+\kappa^2+2\mathbf{K}\boldsymbol{\kappa})\right\}$

$=$

κ

$\mathbf{K}\cdot R/\lambda_0 = p$

$\mathbf{K}'\cdot R/\lambda_0 = p'$