$$p^m q^n: \quad \frac{(p^{m+1}-1)(q^{n+1}-1)-2p^m q^n}{(p-1)(q-1)}$$

$p^m = x;\ q^n = y$

$$= -\frac{xy(pq-2p-2q+2)+px+qy-1}{(p-1)(q-1)}$$

$p=2:$

$$= \frac{2xy-2x-qy+1}{q-1}$$

$$= \frac{2^{m+1}q^n - 2^{m+1} - q\cdot q^n + 1}{q-1}$$

$$= 2^{m+1}(1+q+\cdots+q^{n-1}) - q(1+\cdots+q^{n-1}) - 1$$

$$= (2^{m+1}-q)(1+q+\cdots q^{n-1}) - 1$$

$2^m(2^{m+1}-1)$ が完全数,
prime

(6, 28, 496, 8128, 33550336, ..)

$p \geq 3,$
$q > p.$

$$= -\frac{xy\{(p-2)(q-2)-2\}+px+qy-1}{(p-1)(q-1)} < 0$$

always.

$q = 3, 7, 31, 127, 8191,$

$m = 1, 2, 4, 6, 12,$

$N = 6, 28, 496, 8128, 33550336,$

$N = 2^m(2^{m+1}-1)$
$= 2^m q$

~~$2^{m+1}-1$ が prime ならば~~
~~$m+1$ が prime による~~

Fermat, Theorem
crystal p522.
207: 1882年
m は ずれたする
が、m=1
は別.

$p^m = x, \quad q^n = y, \quad r^k = z$

$$\frac{(p^{m+1}-1)(q^{n+1}-1)(r^{k+1}-1)}{(p-1)(q-1)(r-1)} - 2p^m q^n r^k$$

$$= \cancel{2} - \frac{(px-1)(qy-1)(rz-1) + 2xyz(p-1)(q-1)(r-1)}{(p-1)(q-1)(r-1)}$$

$$\cancel{z} - xyz\{(\overset{2}{2p}-1)(\overset{3}{q}-1)(\overset{5}{r}-1) - pqr\}$$

$p=2$:

$$= -\frac{2xyz(q-1)(r-1) - (2x-1)(qy-1)(rz-1)}{(q-1)(r-1)}$$

$$= \frac{2(q+r-1)xyz - (2qxy + 2rxz + qryz)}{(q-1)(r-1)}$$

$$+2x + rz + qy - 1$$

$q=3$:

$$= \frac{2(r+2)xyz - \cancel{2}(6xy + 2rxz + 3ryz)}{2(r-1)}$$

$$+2x + 3y + rz - 1$$

$r=5$:

$$= \frac{\cancel{2}14xyz - (6xy + 10x\overset{z}{\cancel{y}} + 15yz)}{8}$$

$$+2x + 3y + 5z - 1$$

$\cancel{y=2}$ $z=5$:

$$= \frac{70xy - (6xy + 50x + 75y)}{8} + 2x + 3y + 25 - 1$$

$$= \frac{64xy - 48x - 72y + 24}{8}$$

$$= 8xy - 6x - 9y + 3$$

$8xy - 6x - 9y + 3 = (x - \frac{9}{8})(8y - 6) - \frac{30}{8}$

$= 0$

$\cancel{y} - x(8y - 6) - (9y - \frac{54}{8}) - \frac{30}{8}$

$(1+3)3^{n-1}$

$8 \cdot 2^m 3^n - 6 \cdot 2^m - 9 \cdot 3^n + 3 = 0$

$3^n + 3^{n-1} = 1$

$2^{m+3} \cdot 3^n - 2^{m+1} \cdot 3 - 3^{n+2} + 3 = 0$

$3\{2^{m+3} 3^{n-1} - 2^{m+1} - 3^{n+1} + 1\} = 0.$

$m+1$

$2^{a+2} \cdot 3^b - 2^a - 3^{b+2} + 1 = 0.$

$3^b(2^{a+2} - 3^2) - (2^a \pm 1) = 0.$

$a = 4,\ m = 3$

$(64 - 9)\ \ 2^3 + 1 = 9\ \ -15$

$b =$

$a = 5$

$\boxed{2^m \cdot 3 \cdot 5 \text{ ナル形ノ完全数ナシ}}$

$\because\ 18 \cdot 2^m - 24 \neq 0$

ヲ満足スル m ナシ.

$\boxed{2^m \cdot 3^2 \cdot 5\ \ 〃\ \ 〃\ \ 〃}$

$\because\ 66 \cdot 2^m - 98 \neq 0$

$\boxed{2^m \cdot 3^3 \cdot 5\ \ 〃\ \ 〃\ \ 〃\ \ 〃}$

$6 \times 35 \cdot 2^m - 3 \times 80 \neq 0$

$27 \quad 9 \times 3 \quad 4 \times 2 \quad 8 \quad 81 \quad 6 \times 36 \quad 3 \quad 78$

一般に $2^m\cdot 3^n\cdot 5$ なる完全数なし。

$$\because\quad 2^m = \frac{9\cdot 3^n - 3}{8\cdot 3^n - 6}$$

$$= \frac{3(3^{n+1}-1)}{\cancel{3^{n-1}(8-}\ 6\cdot(4\cdot 3^{n-1}-1)}$$

$$2^{m+1} = \frac{3^{n+1}-1}{4(4\cdot 3^{n-1}-1)} \quad \text{nが大となれば,}\quad \xrightarrow[n\to\infty]{} \frac{3^{n+1}}{4\cdot 3^{n-1}} = \frac{9}{4}.$$

$n=1$, $\frac{8}{3}$

$n=2$ $\frac{26}{11}$

$\downarrow$

$n=\infty$ $\frac{9}{4}$

$$\frac{B+A}{C+A} - \frac{B}{C}$$

$$= \frac{(B+A)C - B(C+A)}{(C+A)\cdot C}$$

$$= \frac{A(C-B)}{(C+A)\cdot C}$$

$\frac{A}{A-D}$

$B+C$

$\frac{B}{C}$

21 問 = 完全数ハナイ.

次 = $2^m\cdot 3^n\cdot 5^k$ なる完全数なし

$$\frac{8}{8}\left(14xyz - (6xy+10xz+15yz) + 2x+3y+5z-1\right) = 0$$

$$x = \frac{15yz - 3y - 5z + 1}{14yz - 6y - 10z + 2}$$

$z=5$ $= \frac{\cancel{15y-3}\ 42y-24}{64y-48} = \cancel{\frac{21y-12}{32y-24}}$

$$= \frac{72y - \cancel{24}}{64y-48} = \frac{9y-3}{8y-6} = \frac{4}{3} \to \frac{9}{8}$$

$\frac{24}{72}$ $\to$ $\frac{21}{32}$

$z = 5^2$.

$$x = \frac{372y - 124}{344y - 248}$$

$$= \frac{93y - 31}{\cancel{344y} \; 86y - 62}$$

$$= \frac{31(3y-1)}{2(43y-31)}$$

$$= \frac{31 \times 8}{2 \times 98} \rightarrow \frac{93}{86}$$

$$\begin{array}{r} 25 \\ 15 \\ \hline 125 \\ 25 \\ \hline 375 \\ 3 \\ \hline 372 \end{array}$$

$14 \times 25 - 6$

124×3

$-10z - 2 \quad z = 5k.$

$$x = \frac{(15z-3)y - (5z-1)}{(14z-6)y - (10z-2)}$$

$$= \frac{1}{2}\,\frac{(5z-1)(3y-1)}{(7z-6)y - (5z-1)}$$

$7z - 3$

$y = 3$, $z = 5$ $\quad x = \frac{4}{3} \rightarrow$ $y = \infty$, $z = \infty$ $\quad x = \frac{15}{14}$

$$2xyz(p-1)(q-1)(r-1)$$
$$-(px-1)(qy-1)(rz-1)=0$$

$$x=\frac{(qy-1)(rz-1)}{2(p-1)(q-1)(r-1)yz-pqryz}$$

$$x\{2yz(p-1)(q-1)(r-1)-p(qy-1)(rz-1)\}$$

$$x=\frac{-(qy-1)(rz-1)}{2(p-1)(q-1)(r-1)yz-p(qy-1)(rz-1)}$$

$$\left.\begin{array}{l}p=2\\ q=3\end{array}\right\}\quad=\frac{-(3y-1)(rz-1)}{2\cdot 2(r-1)yz-2(3y-1)(rz-1)}$$

$$=\frac{(3y-1)(rz-1)}{2\{(r+2)yz-3y-rz+1\}}$$

$= 4(r-1)$

$-4(r-1)yz+6ryz$

$-6r$

$-2rz-6y+2$

$r\to\infty$
$y\to\infty$
$z\to\infty$

$$=\frac{3r}{2(r+2)}$$

$$=\boxed{\frac{3}{2}}$$

$6r+4$ $\quad 2(r+2)yz$

$42:$
$1+2+3+6$
$1+2+7+14+21$

$r\to\infty$

-2

$$=\frac{3yz}{2(y-1)z}=\frac{3y}{2(y-1)}$$

$$x=\frac{8(rz-1)}{2\{(r+2)3z-9-rz+1\}}$$

$2\cdot3\cdot r$

$$\frac{rz-1}{2(r+3)z-4}\qquad=4\frac{rz-1}{(2r+6)z-8}$$

$$3z=3\qquad=2\frac{rz-1}{(r+3)z-4}$$

$p=3$: $-\dfrac{xy(q-2)-x(y-3)-y(x-q)-1}{2(q-1)}$

$= -\dfrac{3qxy-\cancel{4}6xy-2qxy+2xy+3x+qy}{2\cdot(q-1)}$ $\quad C^{-1}$

$= -\dfrac{(q-4)xy+3x+qy-1}{2(q-1)}$

$q=5$ $= -\dfrac{}{2(q-1}$

$= -\dfrac{xy+3x+5y-1}{8}$

$3^m 5^n + 3\cdot 3^m + 5\cdot 5^n - 1 = 0$

$m=1$ $\quad 3\cdot 5^n + 9 + 5\cdot 5^n - 1 = 0$

$8\cdot 5^n + 10 = 0, \quad > 0$

$\dfrac{1+3+5}{6}$ $\quad 4($ $\quad (3q-8)$

$p=5$. $-\dfrac{xy(5q-\cancel{2}10-2q+2)+px+qy-1}{4(q-1)}$

一般ニ

$x = p^m q^n \cdots r^k \cdot s^l$

ニ於テ $y = p^m q^n \cdots r^k$ ガ 完全数又ハ過剰数ナレバ ハ

$x = p^m q^n \cdots r^k \cdot s^l = y \cdot s^l$

ハ又過剰数ナリ。

∵ y ノ約数ノ和ヲ Z トスレバ

$Z > y$. ~~実ニハ~~ y ノ約数ノ夫々

然ルニ x ノ約数 ハ ヲ含ンデ居ル

ニ s^l ヲカケタモノガ含マレテ居ル

∴ x ノ約数ノ和ハ $> s^l Z > s^l y > x$

中途結論. $2^m \cdot 3^n \cdots r^k$ 完全数

~~$p^m q^n$~~ (∵ $2^m 3^n$ ハ 2ノ過剰数)

ハ過剰数. $2^m \cdot 3^n \cdots r^k s^l$ モ然リ.

又 ~~数ニ~~ 従ッテ ∫ $2^m \cdot 3^n \cdots$

又 $2^m \cdot q^n$　$q \geq 5$. ニ於テ

$2^{m+1} > q$ ノ時 完全数又ハ過剰数,

∴ ~~2ノ時ハ~~ $2^m \cdot q^n \cdots r^k \cdot s^l$ ハ又過剰数,

for $2^{m+1} > q$.

~~$2^m \cdot q^n$~~ 例ヘバ $2^m q^n$ $q=5$　$m \geq 2$,　$20 \cdots\cdots$

$m = 2, 3, 4 \cdots$　$(2 \cdot 5)$ $20 \cdot 7^k \cdot 11^l \cdots\cdots$ ヲ(A)

$k, l = 0, 1, 2 \cdots$

$n = 1, 2, 3 \cdots$　$20 \cdot 7^k \cdot 11^l \cdots$

$2 \cdot 5^{n} \cdot r^{k} \ldots$

$$\frac{2(5+r-1)yz - (10y+2rz+5ryz)+2+5y+rz-1}{4(r-1)}$$

$$= \frac{(8-3r)yz - 10y - 2rz + 2 + 5y + rz - 1}{4(r-1)}$$

$+4+rz+5y-1$

$$= \frac{2(r+4)2yz - (20y+4rz+5ryz)}{4(r-1)}$$

$$= \frac{(16-r)yz - 15y - 3rz + 3}{4(r-1)}$$

$$= \frac{(16-r)yz - 15y - 3rz + 3}{4(r-1)}$$

$r=7$

$$= \frac{9yz - 15y - 21z + 3}{24}$$

$$= \frac{3yz - 5y - 7z + 1}{8} = 4$$

$y \geq 5$

$z \geq 7$

$$= \frac{3\times5\times7 - 5\times5 - 7\times7 + 1}{8} > 0$$

$70 - \quad 50 \times 16 \quad 1+2+5+7+10+14+35$

一般ニ アル数ガ過剰数ナレバ 其ノ倍数ハ
完全数又ハ
常ニ 過剰数ナリ.

∴ ~~一般ニ~~ $\cancel{x}$ $\cancel{x \geq 1}$

$2^m \cdot 3^n \cdot x$ $\quad m \neq 0,\ n \neq 0 \quad x > 1.$

ナル数ハ過剰数.

$2^m \cdot 5^n \cdot x$ $\quad m \geq 2,\ n \neq 0 \quad x \geq 1.$

モヨシ

一般ニ $2^m \cdot p q^n \cdot x$ $\quad 2^{m+1} > q \quad x > 1$

一般 $p^m \cdot q^n \cdot r^k \cdots$ ハ $p \cdot q \cdot r$ ガ過剰ノ数, 過剰ナリ
$m \neq 0,\ n \neq 0,\ k \neq 0$

pqr: $(p+1)(q+1)(r+1) - 2pqr$

$= -pqr + pq + qr + pr + p + q + r + 1.$

$p = 2 \quad q = 5 \quad r = 7.$
$-70 + 10 + 35 + 14 + 2 + 5 + 7 + 1 = 4 !!$

$p = 2 \quad q = 5 \quad r = 11$
$-110 + 10 + 55 + 22 + 2 + 5 + 11 + 1 = -4 !!$

$p = 3 \quad q = 5 \quad r = 7$
$-105 + 15 + 21 + 35 + 3 + 5 + 7 + 1$
$= -18$

pqr ナル形ノ完全数ハナイ。

$\because (p+1)(q+1)(r+1) \neq 2pqr,$

$\because$ $p=2$ トスレバ、~~p~~ $q=3$ デナケレバナラヌ。

従ッテ ~~$r+1$~~ $r+1 \neq r$ ナル故

コレモ成立セヌ。

又 $p \neq 2$ ナレバ、左辺ハ 8ノ倍数、然ルニ右辺ハ

2ノ倍数デハナイ。$\therefore$ 両辺ハ等シクナラヌ。

$p^m q r$ ナル形ノ

$$\therefore \frac{p^{m+1}-1}{p-1}(q+1)(r+1) \neq 2p^m q r.$$

$\therefore p=2$ トスレバ

$$(2^{m+1}-1)(q+1)(r+1) = 2^{m+1} q r$$

$q+1 = 2^{m+1}$ デハダメ。

$$2^{m+1}(q+r) - qr + 2^{m+1} - q - r = 2^{\cancel{m+1}} qr0$$

$$2^{m+1}(q+r+1) = (q+1)(r+1) - 1$$

偶 奇

$$p=3. \quad \frac{3^{m+1}-1}{2}(q+1)(r+1) \neq 2 \cdot 3^m \cdot q \cdot r$$

偶 偶 4ノ倍数 偶 奇 奇 奇 偶数

一般ニ p ハ奇数、$\therefore$ 上式ノ左辺ハ 4ノ倍数

右辺ハ 2以上ノ factor ヲフクマヌ。

又 $p^m q^n r^k$ ノ $\dfrac{p^{m+1}-1}{p-1} \cdot \dfrac{q^{n+1}-1}{q-1} \cdot \dfrac{r^{k+1}-1}{r-1} \neq 2p^m q^n r^k$

$p \neq 2$、m, n, k ノ少クモ二ツガ奇数ナラバ

完全数デナイ。

~~$\therefore$ 偶 其ノ一ツ外ヘバ m ヲ奇数トスレバ $p^{m+1}-1$ ハ $p+1$ デモ~~

~~タヾシ、右辺ハ一般ニ $p \neq 2$ ナル限リ $p+1$ デハナイコトハ~~
~~n, k ヲトッテモ同様~~

$\cancel{3^2 \cdot 5^2 \cdot 7^k}$

$p^2 \cdot q^2 \cdot r$:

$$\frac{p^{3}-1}{p-1} \cdot \frac{q^3-1}{q-1} \cdot \frac{r^2-1}{r-1}$$

$$= 2p^2 q^2 r$$

$$= (1+p+p^2)(1+q+q^2)(1+r) - 2p^2 q^2 r$$

p^2+p p^3+3p+9

$k >$

$$\left(1+\frac{1}{p}+\frac{1}{p^2}+\cdots\right)\left(1+\frac{1}{q}+\cdots\right)\left(1+\frac{1}{r}+\cdots\right)$$

$$-2 < \left(1+\frac{1}{p}+\frac{1}{p^2}+\cdots\right)^3-2$$

p, q, r ノ最小ナルモノヲ p トスレバ

$$< \left(\frac{1}{1-\frac{1}{p}}\right)^3 - 2 = \left(\frac{p}{p-1}\right)^3 - 2 < 0$$

for $\frac{p}{p-1} < \sqrt[3]{2} = 1.26\ldots$

$$\frac{0.10091}{3 \;)\; 0.30103}$$

$p=5$

∴ $p^m \cdot q^n \cdot r^k$ ニ於テ p, q, r ガ 5 又ハ夫以上ノ 不適当

$\frac{7}{6}$ $\frac{11}{10}$

$$\left(\frac{p}{p-1}\right)^3 < 2$$

$n \geq 4$

$$\frac{p}{p-1} \cdot \frac{q}{q-1} \cdot \frac{r}{r-1} < 2$$

$\cancel{\frac{4}{3}}$, $\frac{5}{4}$, $\frac{3}{2}$, $\frac{5}{4}$

for $p=3$, $q=5$, $r=17$

$p=3$, $q=7$, $r=11$

$$\frac{p}{p-1}\cdot\frac{q}{q-1}\cdots\cdots < 2,$$

$$\frac{1}{3}\cdot\frac{2\cdot4\cdot6\cdot8\cdots}{1\cdot3\cdot5\cdot7\cdots} = \pi$$

$$2\cdot\frac{\quad}{3}$$

$$\frac{3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8}$$

$$\frac{\quad}{3\cdot5\cdot7} > \frac{3\cdot5\cdot7\cdot11\cdots}{2\cdot4\cdot6\cdot10\cdots} > \frac{35}{16}$$

$$\int \sin^{2m+1}x\,\cancel{\cos x}\,dx = \frac{1}{m}\sin^{m+1}x\cdot\cos x$$

$$\int \sin^{2m+1}x\,dx = \int (1-\cos^2 x)^m \sin x\,dx$$

$$= \int \sin^{2m}x\,dx$$

$$= \sin^{2m+1}x$$

$$\frac{p}{p-1} \cdot \frac{q}{q-1} \cdot \frac{r}{r-1} \cdot \frac{s}{s-1}$$

$$\frac{3}{2} \cdot \frac{5}{4} \cdot \frac{7}{6} \cdot \frac{11}{10} \; \frac{13}{12} \leqq 2$$

720 × 4 = 2880; 2880 × 2 = 5760

35 × 13 = 105 + 35 = 455; 455 × 11 = 455 + 4550 = 5005

∴ $p^m q^n r^k s^l$ ニ於テ 之等ガモシ
5, 7, 11, 13 又ハ夫々ヨリ大ナル場合
不足数ナリ.

又 $\frac{3}{2} \cdot \cancel{\frac{8}{4}} \cdot \frac{11}{10} \; \cancel{\frac{13}{12}} \; \frac{17}{16}$

21 × 13 = 63 + 21 = 273; 273 × 11 = 273 + 2730 = 3003

1440

21 × 11 = 21 + 210 = 231

240 429

480

$\cancel{\frac{3}{2}} \; \frac{5}{4} \cdot \frac{7}{6} \cdot \frac{11}{10} \cdot \frac{13}{12} \cdot \frac{17}{16}$

120 × 2 = 240

16 × 144 = 24 + 38

231 × 17 = 1617 + 231 = 3927

39 × 39 / 429