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Short Note

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Note on the Theory of Positrons

Recently attempts were made to eliminate the difficulty of infinite charge density in Dirac's theory of positrons. Heisenberg[1] introduced the ~~field~~ expression

$$R_s = \frac{1}{2}[\psi^\dagger(x'k')\psi(x''k'') - \psi(x''k'')\psi^\dagger(x'k')] \quad (1)$$

as the quantum electrodynamical analoggon to Dirac's symmetrical density matrix for the electron[2], where x', x'' denote the coordinates of any two points in four dimensional space and k', k'' both take the value 1, 2, 3 and 4.

Now we want to extend this ~~ass~~ formalism in the following manner. Every quantity which can be derived from the density matrix (by differentiation or integration with respect to the coordinates ~~or by other processes~~ etc.) should ~~have~~ take the symmetrical form similar to (1). For example, charge density ~~at the point x'~~ for the electron at the point x' should take the form

$$-\frac{e}{2}\sum_{k'=1}^{4}\{\psi^\dagger(x'k')\psi(x'k') - \psi(x'k')\psi^\dagger(x'k')\}. \quad (2)$$

Similarly the current density ~~takes~~ the form

$$-\frac{ec}{2}\sum_{k',l'=1}^{4}\{\psi^\dagger(x'k')\alpha_{k'l'}\psi(x'l') - \alpha_{k'l'}\psi(x'l')\psi^\dagger(x'k')\} \quad (3)$$

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These expressions are already free from the difficulty of infinite densities, ~~for, if all the~~, at least, in the case of no external field. For, if all the negative energy states are occupied and all the positive energy states are empty, the mean value of ~~(2)~~ the operator (12) is zero, ~~as the negat~~ which can be easyly understood from the symmetry of positive and negative states. If finite number of negative energy states are empty and finite number of positive energy states are occupied, ~~the~~ only these finite number of states will contribute to the density.

Next the symmetical Hamiltonian for the electron is

$$\overline{H}_e = \frac{c}{2}\iiint dx'dy'dz' \Big[\psi^{\dagger}(x'k') \cdot \{ \alpha_{k'l'}(p + \tfrac{e}{c}A) + \beta mc \} \psi(x'l') - \{ \alpha_{k'l'}(p + \tfrac{e}{c}A) + \beta mc \} \psi(x'l') \cdot \psi^{\dagger}(x'k') \Big], \quad (14)$$

where $p_x = -i\hbar \frac{\partial}{\partial x'}$ etc, so that the Hamiltonian for the system electron plus electromagnetic field becomes

$$\overline{H} = \overline{H}_e + \frac{1}{8\pi}\iiint (\mathbb{E}^2_{(x'y'z')} + \mathbb{H}^2_{(x'y'z')})\, dx'dy'dz'. \quad (15)$$

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From this Hamiltonian and the ordinary relations of commutation for the field quantities $\psi, \psi^{\dagger}, A, E$ and H, ~~the eq~~ field equations ~~for~~ are derived.
~~The only difference of the between the field equations~~ ~~thus~~ They differ from the ~~ordinary~~ equation of the ordinary theory only in that the charge and current densities take the form (2) and (3) respectively in our case.
~~all the ne~~ ~~In case, when all the negative~~ ~~From (4)~~ ~~we notice see that, if all the states of the electron are empty, the energy for the electron become negative infinity~~
We notice from the expression (4) that, if all the negative energy states are occupied and ~~all~~ the positive energy states are empty, the energy for the electron becomes negative infinity, which, however, is constant and is a sort of zero point energy.