©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室
Yukawa Hall Archival Library
Research Institute for Fundamental Physics
Kyoto University, Kyoto 606, Japan

On the Theory of Electrons
and Positrons

Nov. 27, 1934.

By H. Yukawa

F 02150

1) V. Fock, C. R. Leningrad (N.S.) 1933, S. 267-271. Nr 6.

2) W. H. Furry and J. R. Oppenheimer, Phys. Rev. 45, 245, 1934)

3) R. Peierls, Proc. Roy. Soc. 146,

4) P. A. M. Dirac, Proc. Camb. Phil. Soc. 30, 150, 1934

5) V. Weisskopf, ZS. f. Phys. 89, 27, 1934

---

6) M. Delbrück, ZS. f. Phys. 84, 144, 1933

7) O. Halpern, Phys. Rev. 44, 885, 1934

8) Heisenberg, ZS. f. Phys. 90, 209, 1934

9) W. Pauli und V. Weisskopf, Helvetica Physica 709, 1934

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

charge density

$$\rho = -\frac{e}{2}\sum_k \{\psi_k^\dagger \psi_k - \psi_k \psi_k^\dagger\}$$

current density

$$I = -\frac{ec}{2}\sum_{kk'} \{\psi_k^\dagger \alpha_{kk'} \psi_{k'} - \psi_{k'} \alpha_{kk'} \psi_k^\dagger\}$$

field equations

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) V = -4\pi\rho$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A = -\frac{4\pi I}{c}$$

$$\operatorname{div} A + \frac{1}{c}\frac{\partial V}{\partial t} = 0.$$

wave equations $\quad i\hbar\dot{\psi} = H_m \psi = c$

$$\left\{\frac{W}{c} + \frac{e}{c}A_0 + \alpha\left(p + \frac{e}{c}A\right) + \beta mc\right\}\psi = 0$$

$$\left\{\frac{W}{c} - \frac{e}{c}A_0 + \tilde{\alpha}\left(p - \frac{e}{c}A\right) - \tilde{\beta} mc\right\}\psi^\dagger = 0 \quad -i\hbar\dot{\psi}^\dagger = \tilde{H}_m \psi^\dagger$$

Vertauschungsrelationen

$$\psi_k^\dagger(x)\psi_{k'}(x') + \psi_{k'}(x')\psi_k^\dagger(x) = \delta_{kk'}\delta(x, x')$$

$$\psi_k^\dagger(x)\psi_{k'}^\dagger(x') + \psi_{k'}^\dagger(x')\psi_k^\dagger(x) = 0 \quad \text{etc}$$

Invariance

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\overline{H} = \iiint dv \left\{ \frac{1}{2} (\psi^\dagger \ldots \right.$$

$$\left. + \frac{1}{8\pi} (E^2 + H^2) \right\}$$

$$i\hbar \dot{\psi} = \psi \overline{H} - \overline{H} \psi$$

$$= H^{(m)}_{mn} \psi \qquad + W_m$$

$$i\hbar \dot{\psi}^\dagger = - \tilde{H}^{(m)}_{mn} \psi \quad - W_m$$

Selbstenergie ~~eines~~ Electrons

$$\psi = \sum_m a_m u_m$$

$$\psi^\dagger = \sum_n a_n^\dagger \tilde{u}_n$$

$$\overline{H} = \frac{1}{2} \sum_m (a_m^\dagger a_m - a_m a_m^\dagger) E_m - \frac{e}{2} \sum_{m,n} (a_m^\dagger a_n - a_n a_m^\dagger) A_{mn}$$

$$N_m \qquad N_m$$

$$- a_p^\dagger a_q \cancel{a_r^\dagger} a_s (\delta_{rs} - a_r^\dagger a_s)$$

$$\shortparallel$$

$$\frac{1}{4} \sum_{pqrs} \{ a_p^\dagger a_q a_r^\dagger a_s A_{pqrs} - a_p^\dagger a_q a_s a_r^\dagger A_{pqrs}$$

$$- a_q a_p^\dagger a_r^\dagger a_s A_{pqrs} + a_q a_p^\dagger a_s a_r^\dagger A_{pqrs} \}$$

$$(\delta_{pq} - a_p^\dagger a_q)(\delta_{rs} - a_r^\dagger a_s)$$

$$= \frac{1}{4} \sum_{pqrs} a_p^\dagger a_q a_r^\dagger a_s A_{pqrs} - \frac{1}{4} \sum_{p,q} a_p^\dagger a_q \sum_r (A_{pqrr} + \cancel{a_r^\dagger a_s} A_{rrpq})$$

$$+ \frac{1}{4} \sum_{r,s} A_{rr,ss} + \frac{1}{4} \sum_{p,q} a_p^\dagger a_q \sum_r (A_{pq,rr} + A_{rr,pq}) + \cancel{\sum a_r^\dagger a_s}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$H_0 \Psi_0 = E_0 \Psi_0$$

$$(H_0 - E_0)$$

$$\Psi_1 = (H_0 - E_0)^{-1} (E_1 - V) \Psi_0$$

$$V \Psi_0$$

$$H_0 \Psi_1 = H_0 (H_0 - E_0)^{-1} (E_1 - V) \Psi_0$$

$$\cancel{E_1 \Psi_0 = H_0 \Psi_1}$$

$$\int (\tilde{\Psi}_0 + \tilde{\Psi}_1 + \cancel{\tilde{\Psi}_2})(H + V)(\Psi_0 + \Psi_1 + \cancel{\Psi_2})\, dv$$

$$= \quad E_0 + \int \tilde{\Psi}_0 V \Psi_0 \, dv$$

$$+ \int$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$H = H_0 + V$$

$$\psi = \psi_0 + \psi_1 + \psi_2 + \cdots \qquad = (E_0 + E_1 + E_2 \cdots)(\cdots\cdots)$$

$$\mathcal{E} = E_0 + E_1 + E_2 +$$

$$H_0 \psi_0 = E_0 \psi_0$$

$$\tilde{\psi}_0 \qquad (H_0 - E_0)\psi_1 = (E_1 - V)\psi_0$$

$$\tilde{\psi}_0 \qquad (H_0 - E_0)\psi_2 = (E_1 - V)\psi_1 + E_2 \psi_0$$

$$\int \tilde{\psi}_0 V \psi_0 = E_1$$

$$\int \tilde{\psi}_0 (V - E_1)\psi_1 = E_2$$

$$\psi_1 = \sum_k c_k \psi^{(k)} \qquad V\psi_0 = \sum_k V_{k0} \psi^{(k)}$$

$$\cancel{\tilde{\psi}_0 V = \sum_k \tilde{\psi}^{(k)} V_{0k}}$$

$$(H_0 - E^{(k)})\psi^{(k)} = 0$$

$$\sum_k c_k (E^{(k)} - E_0)\psi^{(k)} = -\sum_{k \neq 0} V_{k0} \psi^{(k)}$$

$$c_k = -\frac{V_{k0}}{E^{(k)} - E_0}$$

$$E_2 = -\int \tilde{\psi}_0 (V - E_1) \sum_{k \neq 0} \frac{V_{k0}}{E^{(k)} - E_0} \psi^{(k)} = -\sum_k \frac{V_{0k} V_{k0}}{E^{(k)} - E_0}$$

$$= -\sum_k \frac{|V_{k0}|^2}{E^{(k)} - E_0} \qquad Q = \begin{pmatrix} \frac{1}{E^{(k)} - E_0} & 0 \\ & \frac{1}{E^{(k)} - E_0} \\ & & \ddots \end{pmatrix}$$

$$(VQV)_{00}$$

$$\frac{1}{8\pi}\int(\mathbf{E}^2+\mathbf{H}^2)\,dv \qquad -H_x\frac{\partial A_z}{\partial y}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$-\frac{1}{8\pi}\int\left\{\mathbf{E}\cdot\left(\mathrm{grad}\,V+\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right)+\mathbf{H}\,\mathrm{curl}\,\mathbf{A}\right\}dv$$

$$=\frac{1}{2}\int\rho\cdot V\mp\frac{1}{8\pi c}\int\mathbf{E}\frac{\partial\mathbf{A}}{\partial t}\pm\frac{1}{2}\int\left(\frac{\mathbf{I}}{c}+\frac{1}{4\pi c}\frac{\partial\mathbf{E}}{\partial t}\right)\mathbf{A}\,dv$$

$$=\frac{1}{2}\int\rho V\,dv-\frac{1}{2c}\int\mathbf{I}\mathbf{A}\,dv$$

$$\pm\frac{1}{8\pi c}\int\left(\mathbf{E}\frac{\partial\mathbf{A}}{\partial t}+\mathbf{A}\frac{\partial\mathbf{E}}{\partial t}\right)dv$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\overline{H} = \frac{1}{2}\sum_m (2N_m - 1) \ldots$$

$$+ \sum a_m^+ a_n (A_{mn}^\mu b_\mu + B_{mn}^\mu b_\mu^+)$$

$$E_0 = -\frac{1}{2}\sum W_{m+} + \frac{1}{2}\sum W_{m-} + W_0 + \cancel{\tfrac{1}{2} W_{0-}}$$

$$+ \frac{1}{2}\sum_\mu h\nu_\mu + \sum a_m^+ a_n (A_{mn}^\mu b$$

$a_{m+}^+ a_{m-}$

$$V \rightarrow V'$$

V_{kj}

$$V'_{jk} = \frac{1}{E_j - E_k} V_{jk}$$

$$V(H_0^{-1} V - V H_0^{-1})$$

$$+ \sum a_m^+ a_n (\ldots) + \sum_\mu A_{0m}^\mu B_{m0}^\mu \Big/ E_m - E_0 + h\nu_\mu$$

$$- \sum \frac{\quad}{E_m - E_0 + h\nu_\mu}$$

$$H = H_0 + \lambda V$$

$$H_0^{mn} = H_m \delta_{mn}$$

$$H^{mn} = H_0^{mn} + \lambda V^{mn}$$

$$= H_m \delta_{mn} + \lambda V^{mn}$$

$$U = U_0 + \lambda U_1 + \cdots$$

$$U =$$