

Bohr's model

Neutron's mass

Rigid sphere with radius $a \ll \lambda$ cross section is $4\pi a^2$

\therefore nucleus, proton, electron etc

UV cross section is $4\pi a^2$

electron's mass is small

Neutron's interaction energy 10^{12} volt

W.W.E. 10^6 volt electron's Neutron's

cross section is Born's approx. $4\pi \left(\frac{2M}{\hbar^2} V \frac{a^3}{3}\right)^2$

$$M = \frac{M_e M_n}{M_e + M_n} \quad V = 10^6 \text{ volt} \quad a = 2 \cdot 10^{-13} \text{ cm} \quad \approx (10^{-17} \text{ cm})^2$$

10^{11} cm Wegstrecke in Jonenpaar

Kern's stop

$$4\pi \left(\frac{2MV}{\hbar^2} \frac{r_0^3}{3}\right)^2 \approx \pi r_0^2$$

$$M = M_K M_N / (M_K + M_N) \approx M_N$$

$$r_0 \approx 10^{-12} \text{ cm} \quad \rightarrow V \approx \frac{3}{4} \frac{\hbar^2}{M r_0^2} \approx 3 \cdot 10^5 \text{ volt}$$

$$\left\{ \Delta + \frac{2M}{\hbar^2} (E - V) \right\} \psi = 0$$

$$\psi = e^{ikz} + \psi'$$

$$\int \frac{e^{-ikr}}{r} \cdot \frac{2M}{\hbar^2} V \cdot e^{ikz} \cdot r^2 dr d\theta \sin\theta d\phi$$

$$\frac{e^{ikr} - e^{-ikr}}{ik}$$

$$= 2\pi \frac{2MV}{\hbar^2} \int_0^a e^{-ikr(1-\cos\theta)} r dr \sin\theta d\theta$$

$$e^{-ikr} \left(\frac{e^{ikr} - e^{-ikr}}{ik} \right) r dr$$

$$\approx 2\pi \cdot \frac{2MV}{\hbar^2}$$

$$H_0 \psi = E_0 \psi$$

$$H_0 = \frac{\hbar^2 k^2}{2m}$$

$$i\hbar \frac{\partial \psi}{\partial t} = (H_0 + V) \psi$$

$$\psi = \sum_n C_n \exp \left[\frac{i}{\hbar} (k_n x - \omega_n t) \right]$$

$$i\hbar \dot{C}_n = \left(\frac{\hbar^2 k_n^2}{2m} - \omega_n \right) C_n e^{\frac{i}{\hbar} (\omega_n - \omega_0) t} + V_{n0} C_n e^{\frac{i}{\hbar} (\omega_n - \omega_0) t}$$

$$i\hbar \dot{C}_n(t) = \left(\frac{\hbar^2 k_n^2}{2m} - \omega_n \right) C_n e^{\frac{i}{\hbar} (\omega_n - \omega_0) t} + V_{n0} \frac{e^{\frac{i}{\hbar} (\omega_n - \omega_0) t} - 1}{\frac{i}{\hbar} (\omega_n - \omega_0)} C_n(0)$$

$$i\hbar \int \dot{C}_n(t) d\omega_n = V_{n0} \int \frac{e^{\frac{i}{\hbar} (\omega_n - \omega_0) t} - 1}{\frac{i}{\hbar} (\omega_n - \omega_0)} d\omega_n$$

$$\hbar^2 |C_n(t)|^2 = |V_{n0}|^2 \frac{2(1 - \cos \frac{\omega_n - \omega_0}{\hbar} t)}{(\omega_n - \omega_0)^2}$$

$$d|C_n(t)|^2 = dt \cdot |V_{n0}|^2 \frac{2 \sin \frac{\omega_n - \omega_0}{\hbar} t}{\omega_n - \omega_0}$$

$$= dt |V_{n0}|^2 \frac{2\pi}{\hbar} \delta(\omega_n - \omega_0)$$

$$\int_{-\infty}^{\infty} \frac{\sin \frac{\omega_n - \omega_0}{\hbar} t}{\omega_n - \omega_0} d(\omega_n - \omega_0) = \pi$$

$$\omega_n = \frac{\hbar^2 k_n^2}{2m}$$

$$dk \int \delta(\omega_n - \omega_0) d\omega_n$$

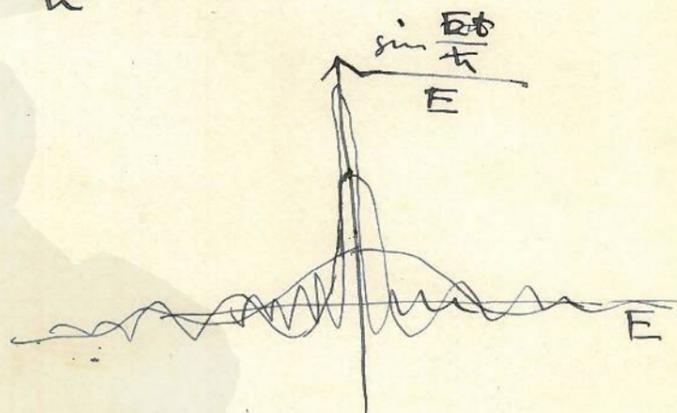
$$= \int \frac{\hbar k dk}{m}$$

$$\delta(\omega_n - \omega_0) = \frac{m}{\hbar^2 k} \delta(k_n - k_0)$$

$$4\pi \frac{\hbar^2 k^2}{\hbar^3} dk$$

$$\left(\frac{\hbar k}{\hbar} \right)^2 dk$$

$$|V_{n0}|^2 \frac{2\pi}{\hbar} \cdot \frac{4\pi m k_0}{\hbar^3} dk \delta(k_n - k_0) \cdot \frac{4\pi a^3}{3}$$



$$\sum |C_n(t)|^2 = \sum |V_{n0}|^2 \frac{2(1 - \cos \frac{\omega_n - \omega_0}{\hbar} t)}{(\omega_n - \omega_0)^2}$$

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$$\iiint V(\mathbf{r}') \frac{1}{r} \delta(\mathbf{r}-\mathbf{r}') d^3r' = \iiint V(\mathbf{r}) \delta(\mathbf{r}-\mathbf{r}) d^3r = V$$

$$\langle \mathbf{k} | V | \mathbf{k}' \rangle = \frac{1}{h^3} \int e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{r}} V(\mathbf{r}) d^3r$$

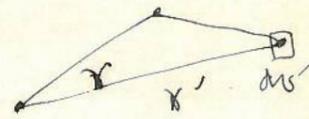
$$= \frac{1}{h^3} \int e^{i\mathbf{k}' \cdot \mathbf{r}} V(\mathbf{r}) d^3r$$

$$= \frac{2\pi}{h^3} \int_0^a (e^{ik'r} - e^{-ik'r}) \frac{1}{ik'r} V r^2 dr$$

$$= \frac{2\pi V}{h^3 ik'} \int_0^a (e^{ik'r} - e^{-ik'r}) r dr$$

$$A_V = \frac{[A, r]}{r^3}$$

$$(\Delta + k^2) \psi_1 = \frac{\alpha_x y - \alpha_y x}{r^3} e^{ikz}$$



$$\psi_1 = \iiint \frac{e^{ik(r'-r)+z'}}{|\mathbf{r}'-\mathbf{r}| r^3} \alpha_x y' - \alpha_y x' d^3r'$$

$$\alpha_x y' - \alpha_y x' =$$

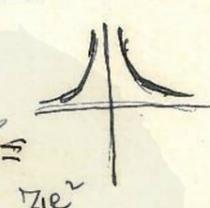
$$= \frac{e^{ik(r'-r)+z'}}{|\mathbf{r}'-\mathbf{r}|} (\alpha_x \frac{y'}{r'} - \alpha_y \frac{x'}{r'}) \sin\theta' \sin\theta'' \sin\theta' d\theta' d\theta'' d\phi'$$

$$\frac{m}{2\pi\hbar^2} \frac{V_{mean}}{r}$$

$$\hbar v_0 = e_1 e_2$$

$$m v_0 = \frac{e_1 e_2}{r}$$

$$\hbar v_0 = \frac{e_1 e_2}{\alpha \frac{\hbar}{mc}}$$



$$\frac{ze^2}{r} \quad \frac{e^2}{\hbar c}$$