Theory of Internal Conversion by Pair Formation

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Hulme (Proc. Roy. Soc. 138, 643, 1932) の計算結果によると internal conversion coefficient の実験値は $\frac{h\nu}{mc^2}=2$ の附近に於て計算値と大きな deviation を示してゐる。尚 $\frac{h\nu}{mc^2}=4$ の附近にも小さな deviation の maximum がある。

Taylor and Mott (ibid. 665, 1932) は ~~之を~~ dipole の ~~外~~ 外に quadripole をも考へることによつて説明せんとした。しかしいづれにしても int. conv. coef は $\frac{h\nu}{mc^2}$ が増すと共に monotone に減ることになり、実験 ~~curve~~ と一致しない。

この大きな方の deviation は electron pair formation の prob をあらはすことは殆んど疑ひない。

小さい方の dev. は ~~恐らく~~、Perrin (　　　) の electron の field による pair form. によるとすれば説明は可能らしい。

最近 Nedelsky, Oppenheimer (Phys. Rev 44, 949, 1933) は $\frac{2\pi Ze^2}{hc} \ll 1$ なる仮定の下に nuclear field を neglect して、Pair Formation による internal conversion coef を計算した。しかし之は Radioactive Element に對しては当てはまらぬ。

1. Calculation of Perturbation Matrix Elements

Dipole Radiation を考へる場合には Perturbation の Potential

$$A_z = \frac{\dot{p}(t-\frac{r}{c})}{cr}$$

$$A_0 = \left(\frac{\dot{p}(t-\frac{r}{c})}{cr} + \frac{p(t-\frac{r}{c})}{r^2}\right)\frac{z}{r}$$

従って Pert. Matrix は initial state 及び final state の energy total momentum j の q.n. ~~に~~ depend する

$$(w', j'u'|V|w'', j''u'')$$

$$= (w'|-eA_0|w'')\,\delta_{j'j''} + (w', j'u'|-e\overset{(\rho_1\sigma_z)}{A_z}|w''j''u'')$$

$$= \left(w'\left|\frac{e\dot{p}(t-\frac{r}{c})}{cr}\right|w''\right)\delta_{j'j''}\delta_{u'u''} + \cancel{\left(w'\left|\frac{e\dot{p}(t-\frac{r}{c})}{cr} + \frac{p(t-\frac{r}{c})}{r^2}\right|w''\right) \times (j'u'|\frac{z}{r}|j''u'')}$$

但し．
$$W = -V - c(\varepsilon p_r + i\varepsilon\rho_3\frac{j\hbar}{r} + \rho_3 mc)$$

$$\varepsilon = \frac{\rho_1(\sigma r)}{r} \qquad \varepsilon^2 = 1$$

$(\sigma r)\;(m_z + \frac{\hbar}{2}\sigma_z) - (\;)(\sigma r)$
$= i\hbar(\sigma_x y - \sigma_y x) + i\hbar(\sigma_x y - \sigma_y x)$
$x m_z - m_z x = $
$(-i\hbar)(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x})$

$$j\hbar = \rho_3\{(\sigma m) + \hbar\}$$
$$(r p) = r p_r + i\hbar,$$
$$(\sigma m) = \rho_3 j\hbar - \hbar,$$

$$(\sigma r)(\sigma p) = r p_r + i\rho_3 j\hbar.$$

or $W = -V - c(\rho_2 p_r - \rho_1 j\hbar/r + \rho_3 mc)$

$$u = m_z + \frac{\hbar}{2}\sigma_z$$

$\cancel{(j'u'|\frac{z}{r}|j''u'')}$

$-i\sigma_x = \sigma_z\sigma_y$

$$j\hbar z - z j\hbar = (-i\hbar)\rho_3(\sigma_x y - \sigma_y x) = \hbar\rho_3(\sigma_z\sigma_y y + \sigma_z\sigma_x x) = \hbar\rho_3\left(\sigma_z(\sigma r) - z\right)$$

$$j\sigma_z + \sigma_z j = 2\rho_3(m_z + \sigma_z\hbar)$$

$$j\hbar\sigma_z - \sigma_z j\hbar = 2i\rho_3(\sigma_x m_y - \sigma_y m_x)$$

$$j\hbar\rho_1 + \rho_1 j\hbar = 0.$$

$$j\hbar\rho_1\sigma_z z - \rho_1\sigma_z z j\hbar = (j\hbar\rho_1 + \rho_1 j\hbar)\sigma_z z - \rho_1(j\hbar\sigma_z + \sigma_z j\hbar)z + \rho_1\sigma_z(j\hbar z - z j\hbar)$$
$$= -2\rho_1\rho_3(m_z + \sigma_z\hbar)z + (-i\hbar)\rho_1\rho_3(i\sigma_y y + i\sigma_x x)$$
$$= 2i\rho_2(m_z + \tfrac{1}{2}\sigma_z\hbar)z + i\rho_2\hbar(\sigma r) - 2i\rho_2\hbar(\sigma_z z)$$

$j\rho_1\sigma_z z - \rho_1\sigma_z z j = 2i\rho_2(u_z + \frac{1}{2}\sigma_z\hbar)z$
$+ i\rho_2\hbar(\sigma x) - 2i\rho_2\hbar(\sigma_z z)$

$[j, (j\rho_1\sigma_z z - \rho_1\sigma_z z j)]$

$= j$

$j(j\sigma_z + \sigma_z j) + (j\sigma_z + \sigma_z j)j = j\cdot 2\rho_3(u + \frac{\hbar}{2}\sigma_z) + 2\rho_3(u + \frac{\hbar}{2}\sigma_z)j$
$+ \rho_3\hbar(j\sigma_3 + \sigma_3 j)$

$j\hbar\sigma_3 z + \sigma_3 z j\hbar = (j\hbar\sigma_3 + \sigma_3 j\hbar)z - \sigma_3(j\hbar z - z j\hbar)$

$= 2\rho_3(m_z + \sigma_z\hbar)z + \rho_3\sigma_z i\hbar(\sigma_x y - \sigma_y x)$

$= 2\rho_3(m_z + \sigma_z\hbar)z - \rho_3\hbar(\sigma_y y + \sigma_x x)$

$= 2\rho_3(m_z + \frac{\hbar}{2}\sigma_z)z - \rho_3\hbar(\sigma x) + 2\rho_3\cdot\sigma_z z$

$j(j\sigma_z + \sigma_z j) + (j\sigma_z + \sigma_z j)j = j\rho_3\sigma_z\hbar + \rho_3\sigma_z\hbar j$

$= \hbar\rho_3(j\sigma_z + \sigma_z j)$

$\rho_3\ (j' + j'')^2(j'|\sigma_z|j'') = \hbar(j'|\rho_3|j'') - j'$

$(j' + j'' + 1)(j' - j'' - 1)$
$= (j' - j'')^2 - 1$
$= j'^2 \pm 2$

$(\sigma, m)j\hbar\sigma_z - \sigma_z j\hbar = 2i(\sigma_x m_y - \sigma_y m_x)$

$j\hbar(j\hbar\sigma_3 - \sigma_3 j\hbar) - (\quad)j\hbar$

$j(jA \pm Aj) \pm (jA \pm Aj)j$
$= A.$

$= 2\hbar i\{j(\sigma_x m_y - \sigma_y m_x) - (\sigma_x m_y - \sigma_y m_x)j\}$

$= 2\hbar i\{(j\sigma_x - \sigma_x j)m_y + \sigma_x(jm_y - m_y j)$
$\cancel{-\sigma_y(\quad)} - (j\sigma_y - \sigma_y j)m_x + \sigma_y(jm_x - m_x j)\}$

$= 2\hbar i\{2(\sigma_y m_z - \sigma_z m_y)m_y + m_\cdot$

$\cancel{\alpha t} \quad \cancel{\sigma_2}$

$\cancel{\sigma_z m_z \frac{\hbar}{}}$

$(m_z + \frac{\hbar}{2}\sigma_z)^2 = m_z^2 + \hbar m_z \sigma_z + \frac{\hbar^2}{4}$

$\sum (m_z + \frac{\hbar}{2}\sigma_z)^2 = m^2 + \hbar(\sigma m) + \frac{3\hbar^2}{4}$

$$(m_z + \frac{\hbar}{2}\sigma_z)\psi_u(\varphi) = u\hbar\psi_u(\varphi)$$

$m_z^2 = \hbar^2\{u^2 + \frac{1}{4} - u\sigma_z\}$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(-i\hbar\frac{\partial}{\partial\varphi} \pm \frac{\hbar}{2})\psi_u(\varphi) = u\hbar\psi_u(\varphi)$$

$$\psi_u(\varphi) = e^{i(u \mp \frac{1}{2})\varphi}$$

$\psi_u(\varphi, \sigma) = e^{i(u - \frac{\sigma}{2})\varphi} c(\varphi)$

$$= (c_1 e^{i(u \mp \frac{1}{2})\varphi}, \; c_2 e^{i(u + \frac{1}{2})\varphi})$$

$$\rho_3(\sigma m + \hbar)\psi_{j,u}(\theta,\varphi) = j\hbar\psi_{j,u}(\theta,\varphi)$$

$$\psi_{j,u} = \{\rho_3(\sigma m) - j'\hbar\}\chi_{j,u}$$

$$\{m^2 - j^2\hbar^2 + \rho_3 j\hbar^2\}\chi_{ju} = 0$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\chi_{ju}}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2\chi_{ju}}{\partial\varphi^2} + j(j - \rho)\chi_{ju} = 0$$

$$-(u - \frac{\sigma}{2})^2 = -(u^2 + \frac{1}{4} - u\sigma)$$

$$\chi_{ju}(\theta, \varphi, \sigma, \rho) = c(r, \sigma, \rho) P^{u - \frac{\sigma}{2}}_{|j - \frac{1}{2}\rho| - \frac{1}{2}}(\theta, \varphi)$$

$c_1 + c_2$

$\cancel{\psi_{j,u} = c(r, \sigma, \varphi\rho)}$

$$P_l^m = (l-m)! \sin^m\theta \left(\frac{d}{d\cos\theta}\right)^{l+m} \frac{(-\sin^2\theta)^l}{2^l l!} e^{im\varphi}$$

$c_1 \; c_2$

$$\cancel{(i\hbar)^{-1}}(\sigma m) = (i\hbar)\begin{pmatrix} \frac{\partial}{\partial\varphi} & -ie^{-i\varphi}\frac{\partial}{\partial\theta} - e^{-i\varphi}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\varphi} \\ ie^{i\varphi}\frac{\partial}{\partial\theta} - e^{i\varphi}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\varphi} & -\frac{\partial}{\partial\varphi} \end{pmatrix} = \cancel{(i\hbar)^{-1}}\begin{pmatrix} m_z & m_x - im_y \\ m_x + im_y & -m_z \end{pmatrix}$$

$$\cancel{(-i\hbar)^{-1}(\sigma m) P_l^{u-\frac{1}{2}}} \; -ie^{-i\varphi}\frac{\partial}{\partial\theta} - e^{-i\varphi}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\varphi} P_l^m = (l-m)! \sin^{m-1}\theta \left(\frac{d}{d\cos\theta}\right)^{l+m} \frac{(-\sin^2\theta)^l}{2^l l!} e^{i(m-1)\varphi}$$

$$\frac{\partial}{\partial\theta} = -\sin\theta\frac{\partial}{\partial\cos\theta}$$

$= \sigma(u - \frac{\sigma}{2})$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\chi_{ju}}{\partial\theta}) - \frac{1}{\sin^2\theta}(u - \frac{\sigma}{2})^2\chi_{ju} + j(j-\rho)\chi_{ju} = 0 \quad (u + \frac{\sigma}{2})$$

$$\chi_{ju}(\theta, \sigma, \rho) = c(r, \sigma, \rho) P^{|u - \frac{\sigma}{2}|}_{|j - \frac{1}{2}\rho| - \frac{1}{2}}(\theta) e^{i(u - \frac{\sigma}{2})\varphi}$$

$$\psi_{ju} = c'(r, \sigma, \rho) P^{u - \frac{\sigma}{2}}_{|j - \frac{1}{2}\rho| - \frac{1}{2}}(\theta, \varphi)$$

$$\Sigma = \rho_1 \frac{(\sigma r)}{r} = \rho_1 \{\sigma_x \sin\theta \cos\varphi + \sigma_y \sin\theta \sin\varphi + \sigma_z \cos\theta\}$$

$$= \rho_1 \begin{Bmatrix} \cos\theta & e^{-i\varphi}\sin\theta \\ e^{i\varphi}\sin\theta & -\cos\theta \end{Bmatrix}$$

$$r = \sqrt{x^2+y^2+z^2} \qquad \frac{\frac{x}{z\sqrt{\ }}}{1+\frac{x^2+y^2}{z^2}} = \frac{r^2\cos\varphi\sin\theta\,\cancel{\cos\theta}}{r^2\cdot r\sin\theta}$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$$

$$\varphi = \tan^{-1}\frac{y}{x} \qquad \frac{-\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{-y}{x^2+y^2} = \frac{1}{r\sin\theta}\sin\varphi$$

$$W = -V - c(\rho_1(\sigma p) + \rho_3 mc)$$

$$p_x = \frac{\partial r}{\partial x}p_r + \frac{\partial\theta}{\partial x}p_\theta + \frac{\partial\varphi}{\partial x}p_\varphi = \cos\varphi\sin\theta\, p_r + \frac{\cos\varphi\cos\theta}{r}p_\theta - \frac{\sin\varphi}{r}p_\varphi$$

etc.

$$\begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial\theta}{\partial x} & \frac{\partial\varphi}{\partial x} \\ \frac{\partial r}{\partial y} & \cdot & \cdot \\ \frac{\partial r}{\partial z} & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cos\varphi\sin\theta & \frac{\cos\varphi\cos\theta}{r} & -\frac{\sin\varphi}{r\sin\theta} \\ \sin\varphi\sin\theta & \frac{\sin\varphi\cos\theta}{r} & \frac{\cos\varphi}{r\sin\theta} \\ \cos\theta & \cancel{\frac{\cos\varphi}{r\sin\theta}}\ -\frac{\sin\theta}{r} & 0 \end{pmatrix}$$

$$(\sigma p) = \sigma_r p_r + \frac{1}{r}(\sigma_\theta p_\theta + \sigma_\varphi p_\varphi)$$

$$\sigma_r = \sigma_x\cos\varphi\sin\theta + \sigma_y\sin\varphi\sin\theta + \sigma_z\cos\theta.$$

$$\sigma_\theta = \sigma_x\cos\varphi\cos\theta + \sigma_y\sin\varphi\cos\theta - \sigma_z\sin\theta.$$

$$\sigma_\varphi = -\frac{\sigma_x\sin\varphi}{\cancel{\sin\theta}} + \frac{\sigma_y\cos\varphi}{\cancel{\sin\theta}}.$$

$$\sigma_r\sigma_\theta = i\sigma_\varphi \quad \cancel{etc} = -\sigma_\theta\sigma_r \quad etc \qquad \sigma_r^2 = 1\ etc.$$

$$\cancel{p_r\sigma_r - \sigma_r p_r = 0}$$

$$\sigma_r p_r - p_r\sigma_r = 0 \qquad \sigma_\theta p_r - p_r\sigma_\theta = 0 \qquad \sigma_\varphi p_r - p_r\sigma_\varphi = 0$$

$$\sigma_r p_\theta - p_\theta\sigma_r = i\hbar\sigma_\theta \qquad \sigma_\theta p_\theta - p_\theta\sigma_\theta = i\hbar\sigma_r \qquad \sigma_\varphi p_\theta - p_\theta\sigma_\varphi = 0$$

$$\sigma_r p_\varphi - p_\varphi\sigma_r = i\hbar\sigma_\varphi\sin\theta \qquad \sigma_\theta p_\varphi - p_\varphi\sigma_\theta = i\hbar\sigma_\varphi\cos\theta$$

$$\sigma_\varphi p_\varphi - p_\varphi\sigma_\varphi = -i\hbar(\sigma_r\sin\theta + \sigma_\theta\cos\theta)$$

$$\cancel{(rp) = rp_r + i}$$

$$\frac{(\sigma r)}{r} = \sigma_r \qquad \cancel{[\sigma r] = \sigma_y\cos\theta - \sigma_z\sin\varphi\sin\theta =}$$

$$\cancel{[rp] = \quad p_y}$$

$$\cancel{(\sigma m) =}\ .$$

$$[rp]_x = \sin\varphi\,\cancel{\sin\varphi}(\cos\theta\, p_r - \frac{\sin\theta}{r}p_\theta) - \cos\theta(\sin\varphi\sin\theta\, p_r + \frac{\sin\varphi\cos\theta}{r}p_\theta + \frac{\cos\varphi}{r\sin\theta}p_\varphi)$$

$$= -\frac{\sin\varphi}{r}p_\theta - \frac{\cos\varphi\cos\theta}{r\sin\theta}p_\varphi$$

$$[rp]_y = \cos\theta\left(\cos\varphi\sin\theta\, p_r + \frac{\cos\varphi\cos\theta}{r}p_\theta - \frac{\sin\varphi}{r\sin\theta}p_\varphi\right) - \cos\varphi\sin\theta\left(\cos\theta\, p_r - \frac{\sin\theta}{r}p_\theta\right)$$

$$= \frac{\cos\varphi}{r}p_\theta - \frac{\sin\varphi\cos\theta}{r\sin\theta}p_\varphi$$

$$[rp]_z = \cos\varphi\sin\theta\left(r\sin\varphi\sin\theta\, p_r + \frac{\sin\varphi\cos\theta}{r}p_\theta + \frac{\cos\varphi}{r\sin\theta}p_\varphi\right)$$
$$-\sin\varphi\sin\theta\left(r\cos\varphi\sin\theta\, p_r + \frac{\cos\varphi\cos\theta}{r}p_\theta - \frac{\sin\varphi}{r\sin\theta}p_\varphi\right)$$
$$= p_\varphi .$$

$$(\sigma m) = \left(-\frac{\sin\varphi}{r}\sigma_x + \frac{\cos\varphi}{r}\sigma_y\right)p_\theta$$
$$-\left(\frac{\cos\varphi\cos\theta}{\sin\theta\, r}\sigma_x + \frac{\sin\varphi\cos\theta}{\sin\theta\, r}\sigma_y \pm \sigma_z\right)p_\varphi$$
$$= \frac{\sigma_\varphi}{r}p_\theta - \frac{\sigma_\theta}{\sin\theta}p_\varphi .$$

$$(\sigma p) = \sigma_r\left(p_r + \frac{i}{r}\left(\sigma_\varphi p_\theta - \frac{\sigma_\theta}{\sin\theta}p_\varphi\right)\right) = \sigma_r\left\{p_r + \frac{i}{r}(\sigma m)\right\}$$

$$m p_r - p_r m = 0 .$$
$$(\sigma m)(\sigma r) + (\sigma r)(\sigma m) = -2\hbar(\sigma r)$$
$$\{(\sigma m)+\hbar\}(\sigma r) + (\sigma r)\{(\sigma m)+\hbar\} = 0 .$$

∴ $\rho_3(\sigma m+\hbar) = j\hbar$ は $\rho_1\sigma_r$ と commute する.

$$W = -V - c\rho_1\sigma_r\left\{p_r + \frac{i}{r}(\rho_3 j - 1)\hbar\right\} - \rho_3 mc^2$$

今 $r, \theta, \varphi, \sigma_r, \rho_3$ を diagonal にする representation を考へると

$$W,\ j,\quad u = \hbar^{-1}\left(m_z + \frac{\hbar}{2}\sigma_z\right)$$

は互に commute するから その simultaneous eigenstate を求める.

$$\hbar^{-1}\left(m_z + \frac{\hbar}{2}\sigma_z\right)\cdot\hbar^{-1}\left(m_z - \frac{\hbar}{2}\sigma_z\right) = \hbar^{-2}m_z^2 - \frac{1}{4}.$$

$$u\psi_u = \left\{-i\sin\theta\frac{\partial}{\partial\varphi} + \frac{1}{2}(\sigma_r\cos\theta - \sigma_\theta\sin\theta)\right\}\psi_u$$
$$\psi_u(\theta,\varphi,\sigma_r) = \left\{-i\sin\theta\frac{\partial}{\partial\varphi} - \frac{1}{2}(\sigma_r\cos\theta - \sigma_\theta\sin\theta)\right\}\chi_u$$
$$= -\sin^2\theta\left(\frac{\partial^2}{\partial\varphi^2} - \frac{1}{4}\right)\chi_u =$$

$$\psi(r,\theta,\varphi,\sigma_r,\rho_3) = \psi_0 + \sigma_r\psi_1$$

$$p_r\psi(r,\theta,\varphi,\sigma_r,\rho_3) = -i\hbar\frac{\partial\psi}{\partial r}$$
$$p_\theta\psi = -i\hbar\frac{\partial\psi}{\partial r} - i\hbar\sigma_\theta\psi_1$$
$$p_\varphi\psi = -i\hbar\frac{\partial\psi}{\partial\varphi} - i\hbar\sigma_\varphi\sin\theta\,\psi_1$$

$$\left\{ i\frac{\partial}{\partial\varphi} - \frac{\sigma_r}{2}\cos\theta + u \right\}\psi = 0.$$

$$\psi = e^{i(u - \frac{\sigma_r}{2}\cos\theta)\varphi}\chi. \quad \chi \text{ ; indep of } \varphi.$$

$$\left\{ i\left(\sigma_\varphi \frac{\partial}{\partial\theta} - \frac{\sigma_\theta}{\sin\theta}\frac{\partial}{\partial\varphi}\right) + \rho_3 j \right\} e^{i(u-\frac{\sigma_r}{2}\cos\theta)\varphi}\chi = 0$$

$$e^{i(u-\frac{\sigma_r}{2}\cos\theta)} = \sum_n \frac{(u-\frac{\sigma_r}{2}\cos\theta)^n}{n!}$$

$$\frac{\partial}{\partial\theta} e^{i(u-\frac{\sigma_r}{2}\cos\theta)\varphi} = e^{i(u-\frac{\sigma_r}{2}\cos\theta)\varphi}\frac{\partial}{\partial\theta} + i\frac{\sigma_r}{2}\sin\theta\varphi e^{\cdots}$$

$$i\, e^{i(u+\frac{\sigma_r}{2}\cos\theta)\varphi}\left\{\left(\sigma_\varphi \frac{\partial}{\partial\theta} + \frac{\sigma_\theta}{2}\cos\theta\varphi\right) - \frac{\sigma_\theta}{\sin\theta}\left(\frac{\sigma_r}{2}\cos\theta + u\right)\right\}\chi$$

$$+ \rho_3 j\, e^{i(u-\frac{\sigma_r}{2}\cos\theta)\varphi}\chi = 0,$$

$$e^{i(u+\frac{\sigma_r}{2}\cos\theta)}$$

$$\left\{ i\frac{\partial}{\partial\theta} + \frac{\sigma_r}{2}\cos\theta\varphi - \frac{\cdots}{2}\frac{\cos\theta}{\sin\theta} - \frac{\sigma_r}{2}\frac{u}{\sin\theta} \right.$$

$$\left. + \rho_3 j\, e^{-i\sigma_r\cos\theta\varphi} \right\}\chi = 0.$$

$$\chi = e^{i\left\{\frac{\sigma_r}{2}\left(\cos\theta - \frac{u}{\sin\theta}\right)\varphi - \frac{1}{2}\frac{\cos\theta}{\sin\theta} + \rho_3 j e^{-i\sigma_r\cos\theta\varphi}\right\}} v$$

$$\left\{ -V - c\rho_1\sigma_r\left[p_r + \frac{i}{r}(\rho_3 j - 1)\hbar\right] - \rho_3 mc^2 \right\}\psi$$

$$=$$

$\rho_3\{(\sigma m)+\hbar\}\psi = \rho_3\{\sigma_\varphi p_\theta - \sigma_\theta p_\varphi + \hbar\}\psi$

$= \rho_3\ \sigma_r(-i\hbar)\left[\{\sigma_\varphi \frac{\partial}{\partial\theta} - \sigma_\theta \frac{\partial}{\partial\varphi}\} + i \}\psi + i\sigma_r\ \psi_1\right]$

$\sigma_\theta\ \psi = (\psi_0 - \sigma_r\psi_1)$

$\sigma_\varphi\ \psi = i\sigma_r(\psi_0 - \sigma_r\psi_1)$

然るに

$p_\theta\ \sigma_r - \sigma_r\ p_\theta = -i\hbar\sigma_\theta,$

$\therefore$

$p_r\psi(r,\theta,\varphi,\sigma_r,\rho_3) = -i\hbar\frac{\partial\psi}{\partial r}$

$p_\theta\ \psi = -i\hbar\left(\frac{\partial}{\partial\theta} - \frac{i}{2}\sigma_\varphi\right)\psi$

$p_\varphi\ \psi = -i\hbar\left(\frac{\partial}{\partial\varphi} + \frac{i}{2}\sin\theta\,\sigma_\theta\right)\psi.$

$j\hbar\psi = \rho_3\{(\sigma m)+\hbar\}\psi$

$= \rho_3\ (-i\hbar)\left(\sigma_\varphi\frac{\partial}{\partial\theta} - \frac{\sigma_\theta}{\sin\theta}\frac{\partial}{\partial\varphi}\right\} \cancel{\pm\frac{i}{2}(1-\sin\theta)})\psi$

or

$\left\{i\left(\sigma_\varphi\frac{\partial}{\partial\theta} - \sigma_\theta\frac{\partial}{\partial\varphi}\right) - \cancel{\frac{i}{2}(1-\sin\theta)} + \rho_3 j\right\}\psi = 0$

$\left[(-i\hbar)\left\{\frac{\partial}{\partial\varphi} + \frac{i}{2}\sin\theta\,\sigma_\theta\right\} + \frac{\hbar}{2}(\sigma_r\cos\theta - \sigma_\theta\sin\theta)\right]\psi = \ldots$

$\left\{i\frac{\partial}{\partial\varphi} \pm \frac{\sigma_r}{2}\cos\theta + \cancel{\frac{\sigma_\theta}{2}\sin\theta(1-\sin\theta)} + u\right\}\psi = 0.$

$\psi = e^{\cancel{\frac{i\sigma_r\sin\theta\cos\theta}{2} - \frac{\sigma_\theta}{2}}\ i\left(\frac{\sigma_r}{2}\frac{\cos\theta}{\sin\theta} - \frac{\sigma_\theta}{2}(1-\sin\theta) - \frac{u}{\sin\theta}\right)\varphi}\chi.$

$\cancel{\left\{i\sigma_\varphi\frac{\partial}{\partial\theta} + \sigma_\theta\left(\frac{\sigma_r}{2}\frac{\cos\theta}{\sin\theta} - \frac{\sigma_\theta}{2}(1-\sin\theta) - \frac{u}{\sin\theta}\right)\right.}$

$\left.-\frac{1}{2}(1-\sin\theta) + \rho_3 j\right\}\psi e^{\cdots}\ \chi$

$=$

$$\left\{ i \frac{\partial}{\partial \varphi} - \frac{\sigma_r}{2} \cos\theta + u \right\} \psi = 0$$

$$\psi = e^{i\left(u - \frac{\sigma_r}{2}\cos\theta\right)\varphi} \chi. \qquad \chi : \text{indep of } \varphi.$$

$$\left\{ i \left(\sigma_\varphi \frac{\partial}{\partial \theta} - \frac{\sigma_\theta}{\sin\theta} \frac{\partial}{\partial \varphi} \right) + \cancel{\sin\theta}\ 1 + \rho_3 j \right\} e^{i\left(u + \frac{\sigma_r}{2}\cos\theta\right)\varphi} \chi = 0.$$

$$\left\{ i \frac{\partial}{\partial \theta} - \frac{\sigma_r}{\sin\theta} \frac{\partial}{\partial \varphi} + \sigma_\varphi \cancel{\sin\theta} + \rho_3 \sigma_\varphi j \right\} e^{i\left(u + \frac{\sigma_r}{2}\cos\theta\right)\varphi} \chi = 0$$

$$\left(-i\hbar \frac{\partial}{\partial \varphi} \cancel{\hbar} + \frac{\hbar}{2}\sigma_z \right) \psi = u\hbar\psi$$

$$\left(i \frac{\partial}{\partial \varphi} + u - \frac{\sigma_z}{2} \right) \psi = 0.$$

$$\psi = e^{i\left(u - \frac{\sigma_z}{2}\right)\varphi} \chi.$$

$$\frac{1}{8}$$

$$\rho_3 \left\{ \left(\sigma_\varphi p_\theta - \frac{\sigma_\theta}{\sin\theta} p_\varphi \right) + \hbar \right\} \psi = j\hbar\psi$$

$$\sigma_\varphi = \sigma_x \sin\varphi + \sigma_y \cos\varphi$$

$$\sigma_\theta = \sigma_x \cos\varphi \cos\theta + \sigma_y \sin\varphi \cos\theta - \sigma_z \sin\theta$$

$$\sigma_r = \sigma_x \cos\varphi \sin\theta + \sigma_y \sin\varphi \sin\theta + \sigma_z \cos\theta$$

$$\left[\hbar \left\{ p_\theta + \frac{i\sigma_r}{\sin\theta} p_\varphi \right\} + \sigma_\varphi - \rho_3 j \sigma_\varphi \right] \psi = 0.$$

$$\left\{ i \frac{\partial}{\partial \theta} - \frac{\sigma_r}{\sin\theta} \frac{\partial}{\partial \varphi} + \sigma_\varphi + \rho_3 \sigma_\varphi j \right\} \psi = 0$$

$$\left(m_z + \frac{\hbar}{2}\sigma_z\right)\psi = u\hbar\psi. \qquad m_z = -i\hbar\frac{\partial}{\partial\varphi}$$

$$\psi = e^{i(u-\frac{\sigma_z}{2})\varphi}\chi = \cancel{e^{iu}\cos}\sum_n \frac{i^n(u-\frac{\sigma_z}{2})^n\varphi^n}{n!} = e^{iu\varphi}\left\{\cos\frac{\varphi}{2} - i\sigma_z\sin\frac{\varphi}{2}\right\}\chi$$

$$\rho_3(\sigma m + \hbar)\psi = j\hbar\psi.$$

$$\psi = \{\rho_3(\sigma m) - j\hbar\}v$$

$$\{m^2 - j^2\hbar^2 + \rho_3 j\hbar^2\}v = 0.$$

$$v = P^{u-\frac{\sigma_z}{2}}_{|j-\frac{\rho_3}{2}|-\frac{1}{2}}(\theta)\cdot w$$

$$m_x = -\frac{\sin\varphi}{r}p_\theta - \frac{\cos\varphi\cos\theta}{r\sin\theta}p_\varphi$$

$$m_y = \frac{\cos\varphi}{r}p_\theta - \frac{\sin\varphi\cos\theta}{r\sin\theta}p_\varphi$$

$$m_z = p_\varphi$$

$$\left[(\sigma_x m_x),\ e^{iu\varphi}\left\{\cos\frac{\varphi}{2} - i\sigma_z\sin\frac{\varphi}{2}\right\}\right]$$

$$= \sigma$$

$$\psi = e^{i(u-\frac{\sigma_z}{2})\varphi}\cdot P^{u-\frac{\sigma_z}{2}}_{|j-\frac{\rho_3}{2}|-\frac{1}{2}}(\theta)\cdot w$$

$$w = \cancel{w_1 + \rho_3 w_2}\left\{\frac{(1+\sigma_z)}{2}\sqrt{\frac{l+m-1/2}{2l-1}}\,\dot{\chi} - \frac{(1-\sigma_z)}{2}\sqrt{\frac{l-m-1/2}{2l-1}}\,\chi\right\}\frac{(1+\rho_3)}{2}if(r)$$

$$+\left\{\frac{(1+\sigma_z)}{2}\sqrt{\frac{l-m+\frac{1}{2}}{2l+1}} + \frac{(1-\sigma_z)}{2}\sqrt{\frac{l+m+\frac{1}{2}}{2l+1}}\right\}\frac{(1-\rho_3)}{2}g(r)$$

W

$$f(r) = \sqrt{1-\varepsilon}\cdot(\varphi_1 - \varphi_2)$$

$$g(r) = \sqrt{1+\varepsilon}\cdot(\varphi_1 + \varphi_2)$$