$\rho(x'x'')$

Quantized,

$\psi^\dagger(x')\psi(x'') =$

number

$\nu_n \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ operator

$\boxed{a_m^\dagger a_n} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}_m \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_n$

$\nu_m = \prod \cancel{(1-2N_j)}(2N_m - 1)$

$S_1 \quad S_2 \quad S_3$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \begin{pmatrix} -1 & \\ & +1 \end{pmatrix}$

$S_1 + iS_2$

$\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}$ $\cancel{1+S}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\cancel{\prod S_m}$

$a_n = \frac{1}{2}(S_1 - iS_2)$

$a_n^\dagger = \frac{1}{2}(S_1 + iS_2)$

$a_m^\dagger a_n = \frac{1}{2}(S_1^{(m)} + iS_2^{(m)})$

$a_n = \prod_{m=1}^{n-1} S_3^{(m)} \cdot \frac{1}{2}(S_1^{(n)} - iS_2^{(n)})$ $\prod_{k=m}^{n-1} S_3^{(k)} \cdot \frac{1}{2}(S_1^{(n)} - iS_2^{(n)})$

$a_n^\dagger = \frac{1}{2}(S_1^{(n)} + iS_2^{(n)}) \prod_{m=1}^{n-1} S_3^{(m)}$

$$\rho_{mn} = a_n^{*} a_m^{\dagger} = \frac{1}{2}\left(S_1^{(n)} - iS_2^{(n)}\right) \times \prod_{k=m}^{n-1} S_3^{(k)} \cdot \frac{1}{2}\left(S_1^{(m)} + iS_2^{(m)}\right)$$

$$= -\frac{1}{2}\left(S_1^{(n)} - iS_2^{(n)}\right) \frac{1}{2}\left(S_1^{(m)} + iS_2^{(m)}\right) \times \prod_{k=m}^{n-1} S_3^{(k)}$$

$$= -\frac{1}{4}\left(S_1^{(m)} S_1^{(n)} + S_2^{(n)} S_2^{(m)}\right) \prod_{k=m}^{n-1} S_3^{(k)}$$

$$a_n a_n^{\dagger} = \frac{1}{2}\left(1 + S_3^{(n)}\right)$$

$$\rho_{nm} = a_m^{\dagger} a_n = \frac{1}{4}\left(S_1^{(m)} S_1^{(n)} + S_2^{(n)} S_2^{(m)}\right) \prod_{k=m}^{n-1} S_3^{(k)}$$

$$\rho^{(S)}_{nm} = \frac{1}{2}\left(a_m^{\dagger} a_n - a_n a_m^{\dagger}\right)$$

$$= \frac{1}{4}\left(S_1^{(m)} S_1^{(n)} + S_2^{(n)} S_2^{(m)}\right) \prod_{k=m}^{n-1} S_3^{(k)}$$

$$a_n^{\dagger} a_n = \frac{1}{2}\left(1 - S_3^{(n)}\right)$$

$$\rho^{(S)}_{nn} = \cancel{\frac{1}{4}\left(S_1^{(n)} \cdots \frac{1}{2}\right.} \ \frac{1}{2} S_3^{(n)}$$

$S_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $S_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $S_3 = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}$

$\cancel{\frac{1}{4}} S_3^{(1)} \quad \frac{S_1^{(1)} S_1^{(2)} + S_2^{(1)} S_2^{(2)}}{4} S_3^{(1)}$

$- \quad S_3^{(2)}$

$- \frac{S_1^{(1)} S_2^{(3)} + S_2^{(1)} S_2^{(3)}}{4} S_3^{(2)} S_3 \quad \frac{S_2^{(2)} S_2^{(3)} + S_2^{(2)} S_2^{(3)}}{4} S_3^{(2)} S_3$

$S_3^{(n')}$

$D(Q \cdot \rho^{(S)})$

$D(\rho^{(S)})$ = total number of electrons or total charge $(-e)$

$N \rho^{(S)} \neq 0$

$H_{11} S_3^{(1)} \neq H_{12} \frac{S_1^{(1)} S_1^{(2)} + S_2^{(1)} S_2^{(2)}}{4} S_3^{(1)}$

$- H_{13} \cdots - \cdots \neq 0.$

~~total number of~~ but finite number of
all the negative energy state is occupied and the finite number of positive energy state is occupied.

$$D(\rho^{(S)}) = \text{number of pos. energy state occupied} - \text{negative energy state unoccupied.}$$

(1) one electron problem (in the state 1.)

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} S_3^{(1)} (1, 0 \,;\, 0, 1 \quad)$$

$$(1, 0)$$

$$\begin{pmatrix} S_3^{(1)} & \frac{S_1^{(1)} S_1^{(2)} + S_2^{(1)} S_2^{(2)}}{4} S_3^{(1)} \\ -(\quad) & S_3^{(2)} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \quad -\tfrac{1}{2} \quad \tfrac{1}{2} \quad -\tfrac{1}{2} \quad \tfrac{1}{2} \quad 1 \quad 1 \quad -1 \quad -1$$

$$\begin{pmatrix} \frac{1}{2} & & & & & \\ & \frac{1}{2} & & & & \\ & & \frac{1}{2} & & & \\ & & & \ddots & & \\ & & & & \frac{1}{2} & \\ & & & & & \frac{1}{2} \\ & & & & & & \frac{1}{3} \end{pmatrix}$$

$a_n, a_m,$

$1, S_1^{(i)}, S_2^{(i)}, S_3^{(i)}$

など、その linear combination を element とする matrix, その multiplication は Quasi multiplication を採用.

$$\begin{pmatrix} 1+S_3 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$\frac{a_{mn}a_{nk} + a_{mn}a_{nk}}{2}$$

$\frac{1}{4}(1+1+\cdots)$

$\frac{1}{2}(1+S_3)$

$\frac{1}{2}(1-S_3)$

$$(a_1^\dagger a_2^\dagger \cdots) * (a_1\, a_2 \cdots\cdots\cdots)$$

$$= \left(\frac{a_1^\dagger a_1 + a_1 a_1^\dagger}{2}\right.$$

$a_m^\dagger a_n + a_n a_m^\dagger = \delta_{mn}$

$C_{mn} = a_m^\dagger a_n - \cancel{a_m a_n^\dagger}$

$C_{nm} = a_n^\dagger a_m$ $\quad \cancel{C_{mn} - C_{nm}}$

$C_{kl} C_{mn} - C_{mn} C_{kl} = a_m^\dagger a_n a_k^\dagger a_l$

$- a_k^\dagger a_l a_m^\dagger a_n = a_m^\dagger \delta_{kn} a_l + a_m^\dagger a_k^\dagger a_n$

$- \quad " \quad " \quad = a_m^\dagger a_l \delta_{kn} + a_k^\dagger a_n \delta_{ml}$

$C_{kl} C_{mn} - C_{mn} C_{kl} = \cancel{a_k^\dagger a_n} + \cancel{\delta_{kl}}$

$C_{kn} + C_{ml} \delta_{kn}$

$k = n$ $\quad C_{kl} C_{lk} - C_{lk} C_{kl} = C_{kk} + C_{ll}$

$k \neq l$

1 2 3 4

(241)

(412)

$C_{12} C_{34} \to 21, 43$

$C_{34} C_{12} \to 21, 43$

$C_{12} C_{24} \to (1432)$

(2431)

$C_{24} C_{12} \to (2134)$

(4132)