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On the Interaction of Elementary Particles
II.

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§ Quantization of the U-field †.

U-function (Ũ) は次なる wave equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = -4\pi g \tilde{\Psi} \frac{\tau_1 \mp i\tau_2}{2} \Psi \quad (4)$$

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} \tilde{U} = -4\pi g \tilde{\Psi} \frac{\tau_1 + i\tau_2}{2} \Psi \quad (5)$$

$U^{\dagger}U + \tilde{U}\tilde{U}^{\dagger}$

$$L = -\frac{1}{\cancel{4\pi}}\left\{\frac{\partial \tilde{U}}{\partial x}\frac{\partial U}{\partial x} \mp \frac{\partial \tilde{U}}{\partial y}\frac{\partial U}{\partial y} \mp \frac{\partial \tilde{U}}{\partial z}\frac{\partial U}{\partial z} \mp \frac{1}{c^2}\frac{\partial \tilde{U}}{\partial t}\frac{\partial U}{\partial t} + \lambda^2 \tilde{U} U\right\}$$
$$+ \cancel{4\pi} g \left\{\tilde{U} \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \Psi + U \tilde{\Psi} \frac{\tau_1 + i\tau_2}{2} \Psi\right\}$$

$$\bar{L} = \int L \, dv$$

これ Lagrangian から derive される.
UとŨとは互に complex conjugate であるから これを real なものであらはし.

$$U = \frac{1}{\sqrt{2}}(V + iW)$$
$$\tilde{U} = \frac{1}{\sqrt{2}}(V - iW)$$

とすると.

$$L = -\frac{1}{8\pi}\left\{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 - \frac{1}{c^2}\left(\frac{\partial V}{\partial t}\right)^2 + \lambda^2 V^2 + \left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2 + \left(\frac{\partial W}{\partial z}\right)^2 - \frac{1}{c^2}\left(\frac{\partial W}{\partial t}\right)^2 + \lambda^2 W^2\right\}$$
$$+ \frac{g}{\sqrt{2}}\left(V \tilde{\Psi}\tau_1\Psi - W\tilde{\Psi}\tau_2\Psi\right)\}$$

† W. Pauli u. V. Weisskopf, Helvetica Physica. 7, 709, 1934.

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$$\cancel{\left(\frac{1}{4\pi c^2}\right)} Q^2 + R^2$$

$$= \left(\frac{1}{4\pi c^2}\right)^2 \cancel{2 U^\dagger U} = \cancel{}$$

$$2 \cdot \frac{\partial \tilde{U}}{\partial t} \frac{\partial U}{\partial t} = 2 \dot{U}^\dagger \dot{U}^\dagger$$

$$\frac{\partial L}{\partial\left(\frac{\partial V}{\partial t}\right)} = \frac{1}{4\pi c^2} \frac{\partial V}{\partial t} = Q$$

$$\frac{\partial L}{\partial\left(\frac{\partial W}{\partial t}\right)} = \frac{1}{4\pi c^2} \frac{\partial W}{\partial t} = R$$

$$-(\mathrm{grad}(V+iW))^\dagger \, (\mathrm{grad}(V-iW))^2 = 2\, \mathrm{grad}\, \tilde{U}\, \mathrm{grad}\, U$$

$$L = \frac{1}{2}\Big\{ 4\pi c^2 Q^2 - \frac{1}{4\pi}(\mathrm{grad}\, V)^2 - \frac{1}{4\pi}\lambda^2 V^2$$
$$+ 4\pi c^2 R^2 - \frac{1}{4\pi}(\mathrm{grad}\, W)^2 - \frac{\lambda^2}{4\pi} W^2 \Big\}$$
$$+ \frac{g}{\sqrt{2}}\left(V \tilde{\Psi} \tau_1 \Psi - W \tilde{\Psi} \tau_2 \Psi\right)$$

$$H = \frac{1}{2}\Big\{ 4\pi c^2 Q^2 + \frac{1}{4\pi}(\mathrm{grad}\, V)^2 + \frac{\lambda^2 V^2}{4\pi}$$
$$+ 4\pi^2 c^2 R^2 + \frac{1}{4\pi}(\mathrm{grad} W)^2 + \frac{\lambda^2 W^2}{4\pi} \Big\}$$
$$- \frac{g}{\sqrt{2}}\left(V \tilde{\Psi} \tau_1 \Psi + W \tilde{\Psi} \tau_2 \Psi \right)$$

$$\cancel{[Q(x,t), V(} \quad \bar{H} = \int H \, dv$$

$$[V(x,t), Q(x',t)] = i\hbar\, \delta(x, x')$$

$$[W(x,t), R(x',t)] = i\hbar\, \delta(x, x')$$

任意の field quantity F に対する field equation は

$$i\hbar \frac{\partial F}{\partial t} = [F, \bar{H}]$$

によつて與へられる.

尚 Energy-momentum tensor は $\left(\begin{array}{l} x_1=x,\ x_2=y,\ x_3=z \\ x_4=ict \text{ とおくと} \end{array}\right)$

$$T_{\mu\nu} = -\frac{1}{4\pi}\left(\frac{\partial\tilde{U}}{\partial x_\mu}\frac{\partial U}{\partial x_\nu} + \frac{\partial\tilde{U}}{\partial x_\nu}\frac{\partial U}{\partial x_\mu}\right) - L\,\delta_{\mu\nu}$$

によつて與へられる.　　(尚ここ infolge が問題になる)

又. total momentum は

$$G_k = \frac{i}{c}\int T_{4k}\,dV = -\int \cdots$$

$$-i\hbar\frac{\partial F}{\partial x_k} = [F, G_k]$$

次に continuity の equation を出すには

(4)$\times\tilde{U}$ − (5)$\times U$ を作ると.　　(以下省略)

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(\tilde{U}\frac{\partial U}{\partial t} - U\frac{\partial\tilde{U}}{\partial t}\right) - \frac{\partial}{\partial x}\,\mathrm{grad}\left\{\tilde{U}\,\mathrm{grad}\,U - U\,\mathrm{grad}\,\tilde{U}\right\}$$
$$= -4\pi g\,\tilde{\Psi}\left(\tilde{U}\frac{\tau_1 - i\tau_2}{2} - U\frac{\tau_1 + i\tau_2}{2}\right)\Psi \quad (7)$$

一方. heavy particle に対する wave equation

$$\left\{i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{4}\left(\frac{1+\tau_3}{M_N} + \frac{1-\tau_3}{M_P}\right)\Delta - \frac{1+\tau_3}{2}M_N c^2 - \frac{1-\tau_3}{2}M_P c^2\right.$$
$$\left. - g\left(\tilde{U}\frac{\tau_1 - i\tau_2}{2} + U\frac{\tau_1 + i\tau_2}{2}\right)\right\}\Psi = 0 \quad (6)$$

$$\left(\frac{1-\tau_3}{2}\right)\left(\frac{\tau_1-i\tau_2}{2}\right)-\left(\frac{\tau_1-i\tau_2}{2}\right)\left(\frac{1-\tau_3}{2}\right)$$

$$=\frac{1}{4}\left(i\tau_2+\tau_1-i\tau_2+\tau_1\right)$$

$$(\tau_1-i\tau_2)$$

$$=\frac{1}{2}(\tau_1\ i\tau_2)$$

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然るに $\{-i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{4}(\qquad)\Delta - \cdots\}\Psi = 0$ (8')

故に、$\tilde\Psi\frac{(1-\tau_3)}{2}\times(6)$ − $(6')\times\frac{(1-\tau_3)}{2}\Psi$ を作ると、

$$i\hbar\frac{\partial}{\partial t}\left(\tilde\Psi\frac{(1-\tau_3)}{2}\Psi\right) + \frac{\hbar^2}{2M_p}\mathrm{grad}\left(\tilde\Psi\frac{(1-\tau_3)}{2}\mathrm{grad}\Psi - \mathrm{grad}\tilde\Psi\frac{1-\tau_3}{2}\Psi\right)$$

$$- g\left(\tilde U\tilde\Psi\frac{\tau_1-i\tau_2}{2}\Psi - U\tilde\Psi\frac{\tau_1+i\tau_2}{2}\Psi\right) = 0, \quad (8)$$

$i\tau_3\tau_2 = \tau_1$

$$(1-\tau_3)\frac{(\tau_1-i\tau_2)}{2} - \frac{(\tau_1-i\tau_2)}{2}(1-\tau_3)$$
$$= -i\tau_2+\tau_1$$

$\frac{(7)}{2\hbar}$ + (8) ~~×ℏ~~.

$$\frac{\partial}{\partial t}\left\{\frac{1}{4\pi c^2}\left(\tilde U\frac{\partial U}{\partial t} - U\frac{\partial\tilde U}{\partial t}\right) + i\hbar\left(\tilde\Psi\frac{(1-\tau_3)}{2}\Psi\right)\right\}$$

$$- \mathrm{div}\left\{\frac{1}{4\pi}(\tilde U\,\mathrm{grad}\,U - \mathrm{grad}\,\tilde U\cdot U)\right.$$
$$\left.- \frac{\hbar^2}{2M_p}\left(\tilde\Psi\frac{(1-\tau_3)}{2}\mathrm{grad}\,\Psi - \mathrm{grad}\,\tilde\Psi\frac{1-\tau_3}{2}\Psi\right)\right\} = 0$$

所で $e\tilde\Psi\frac{(1-\tau_3)}{2}\Psi$ は heavy particle の charge density

∴ $-\frac{ei}{2\hbar c^2}\left(\tilde U\frac{\partial U}{\partial t} - U\frac{\partial\tilde U}{\partial t}\right)$ が U-quantum の charge density となる.

$\frac{e}{\hbar c^2}\frac{g^2W}{\hbar} = \frac{e^2}{\hbar c}\frac{g^2}{\hbar c}\frac{W}{e}$

$= \frac{W}{e}\frac{1}{\lambda} = \frac{mc^2}{e^2}\frac{1}{\lambda}e$

$\frac{g^2W}{\hbar}$ $\quad \frac{g^2eW}{\hbar^3c^2} =$

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所で　U の complex conjugate は $\frac{1}{8\pi c^2}\frac{\partial \tilde{U}}{\partial t} = U^{\dagger}$

なるから、$\frac{1}{4\pi c^2}\left\{U \frac{\partial \tilde{U}'}{\partial t} - \frac{\partial \tilde{U}'}{\partial t} U\right\} = i\hbar\,\delta(r-r')$

$\therefore\ \frac{\partial \tilde{U}(r)}{\partial t} U(r) = 4\pi c^2 \hbar \cdot N_-(r)$

$\therefore\ \frac{-ei}{2\hbar c^2}\left(\tilde{U}\frac{\partial U}{\partial t} - U\frac{\partial \tilde{U}}{\partial t}\right)$

$= \mathrm{const}(\infty) - e N_-(r) + e N_+(r).$

Protonがあれば周囲に $\tilde{U}$ 場を来す。従って $N_+(r)$ の平均値は 0 でない。
又 Neutronがあれば U fieldが来す、$N_-(r)$ の平均値は 0 でない。

$$H = \frac{1}{2}\left\{\cancel{4\pi c^2}\ 8\pi\hbar^2 c^2 U^{\dagger}\tilde{U}^{\dagger} + \frac{1}{2\pi}\,\mathrm{grad}\,\tilde{U}\,\mathrm{grad}\,U + \frac{\lambda^2}{2\pi} U\tilde{U}\right\}$$
$$- g\left\{\tilde{U}\tilde{\Psi}\frac{\tau_1 - i\tau_2}{2}\Psi + U\tilde{\Psi}\frac{\tau_1 + i\tau_2}{2}\Psi\right\}$$

今 Proton or Neutron の charge が ~~per~~ U-field のために どれ丈変化するか 計算して見る. 即ち P→N の Übergangs-wahrscheinlichkeit のために Proton の charge は多少減少している. それ以外に $\tilde{U}$-field として外部に diffuse してゐる わけである.

所で $\tilde{U} = g\dfrac{e^{-\lambda r}}{r}$

$U^{*} =$

~~次に~~ Proton + U-field に対する Hamiltonian は

$$H = \int \frac{1}{2}\left\{ 8\pi \hbar^2 c^2 U^{\dagger} \tilde{U}^{\dagger} + \frac{1}{2\pi} \operatorname{grad}\tilde{U} \operatorname{grad} U + \frac{\lambda^2}{2\pi} \tilde{U} U \right\}$$
$$- g\left\{ \tilde{U}\, \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \Psi + U\, \tilde{\Psi} \frac{\tau_1 + i\tau_2}{2} \Psi \right\}$$
$$+ \tilde{\Psi} \frac{\mathbf{p}^2}{2M} \Psi$$

$$\bar{H} = \int H\, dv$$

U を expand すると.

$$U = \iiint U(k)\, e^{ikr}\, dk$$
$$\dot{U}(k) = \kappa \tilde{U}^{\dagger}(k).$$
$$\dot{\tilde{U}}(k) = \kappa U^{\dagger}(k).$$
$$\dot{U}^{\dagger}(k) = \quad U(k) + \cdots$$

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Energiestörung の zweite Näherung 迄計算すれば気付くのは Selbstenergie があらはれることであらう.