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朝永振一郎, The Scattering of Neutrons by Protons

Wave Equation (Heisenberg)

$$\left\{\Delta_1+\Delta_2+\frac{2M}{\hbar^2}\left[\frac{1}{2}J(r_{12})(\rho_1^\xi\rho_2^\xi+\rho_1^\eta\rho_2^\eta)+D+W\right]\right\}\times\psi(r_1\,\rho_1^\zeta,\ r_2\,\rho_2^\zeta)=0$$

$$(\rho_1^\zeta+\rho_2^\zeta)\,\psi(r_1\rho_1^\zeta,\ r_2\,\rho_2^\zeta)=0$$

$$\rho^\xi=\begin{pmatrix}0&1\\1&0\end{pmatrix}\qquad\rho^\eta=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\qquad\rho^\zeta=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

Wave Equation for relative motion

$$\Delta+\frac{2M^*}{\hbar^2}\left\{\frac{1}{2}J(r)(\rho_1^\xi\rho_2^\xi+\rho_1^\eta\rho_2^\eta)+E\right\}\psi(r,\rho_1^\zeta\rho_2^\zeta)=0$$

$$M^*=\frac{M}{2}\qquad E=W+D$$

(centre of mass being at rest)

$$\psi(r,\rho_1^\zeta\rho_2^\zeta)=\varphi_1(r)\{\alpha(\rho_1^\zeta)\beta(\rho_2^\zeta)+\alpha(\rho_2^\zeta)\beta(\rho_1^\zeta)\}$$

or $$=\varphi_2(r)\{\quad ״\quad -\quad ״\quad\}$$

$$\alpha(\rho)=\delta_{\rho,1}\qquad\beta(\rho)=\delta_{\rho,-1}$$

$$\left.\begin{aligned}&\left\{\Delta+\frac{2M^*}{\hbar^2}[J(r)+E]\right\}\varphi_1(r)=0\\&\left\{\Delta+\frac{2M^*}{\hbar^2}[-J(r)+E]\right\}\varphi_2(r)=0\end{aligned}\right\}$$

(1) Mass Defect. $J(r)=\dfrac{Ae^{-\lambda r}}{r}$

$\varphi(r)=\dfrac{2}{\sqrt{4\pi}}q^{\frac{3}{2}}e^{-qr}$ & assume (λ, q & energy

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$$-\frac{2M^*}{\hbar^2}\bar{H} = \int \varphi^*(r)\left\{\Delta + \frac{2M^*}{\hbar}J(r)\right\}\varphi(r)\,dV$$

が min になる様にとる。 すると $\bar{H}$ が mass defect の appr. value になり、之に実験を入れると、 A と λ の関係が出る。

$$\bar{H} = \frac{\hbar^2}{2M^*}q^2 - A\frac{4q^3}{(2q+\lambda)^2}$$

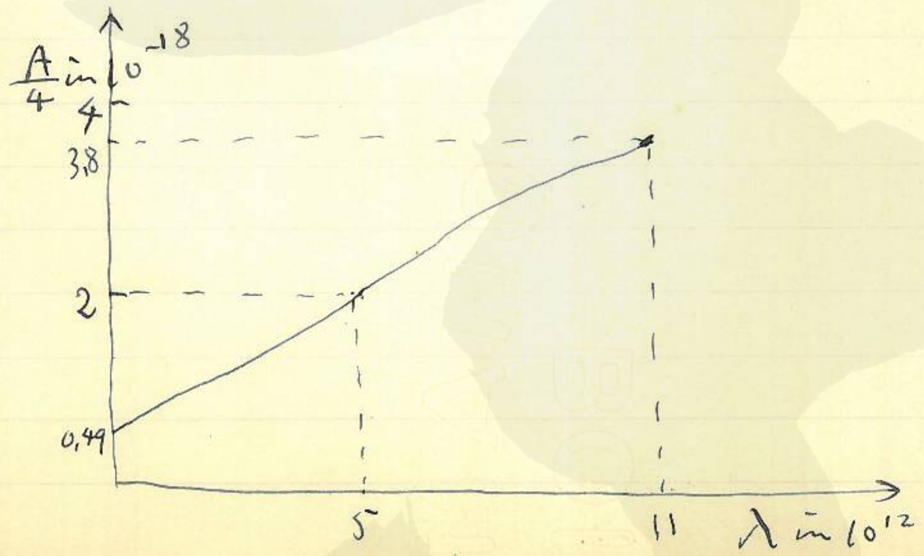
$$\frac{\partial \bar{H}}{\partial q} = 0.$$

$$\left.\begin{aligned}\lambda &= \frac{2q\,(Kq^2+\bar{H})}{Kq^2-3\bar{H}}\\ A &= \frac{4(Kq^2-\bar{H})^3}{(Kq^2-3\bar{H})^2}\end{aligned}\right\}\quad K = \frac{\hbar^2}{2M^*}$$

$H^2 = 2.01298$ $\qquad n_0^1 = 1.0067.$

$\bar{H} = -1.48\times10^{-6}$ erg.

この approx. を 実際 $\lambda = 8\times10^{12}$ の場合に numerical $\int$ をやって出した A と比較して見ると、 2.5 % の範囲で正しい。



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(2) Scattering

$$\left\{ \Delta + \frac{2M^*}{\hbar^2} J(r)(\rho \cdots) + k^2 \right\} \psi(r, \rho_1^3 \rho_2^3) = 0$$

$$k = \frac{M^* v}{2} \qquad v: \text{relative velocity} \qquad n \xrightarrow{v/2} \qquad \xleftarrow{v/2} p$$

$$\psi_+(r) = \psi(r, 1, -1) \underset{r \to \infty}{\longrightarrow} I + f_+(\theta) \cdot S$$

$$\psi_-(r) = \psi(r, -1, +1) \rightarrow f_-(\theta) \cdot S$$

$$I = e^{ikz} \qquad S = \frac{1}{r} e^{ikr}$$

$$\{\Delta + k^2\} \psi_+ + \frac{2M^*}{\hbar^2} J \psi_- = 0$$

$$\{\ \ '' \ \ \} \psi_- + \frac{2M^*}{\hbar^2} J \cdot \psi_+ = 0$$

$$\left. \begin{aligned} \varphi_1 &= \frac{\psi_+ + \psi_-}{2} \\ \varphi_2 &= \frac{\psi_+ - \psi_-}{2} \end{aligned} \right\} \qquad \begin{aligned} &\left\{ \Delta + \left[k^2 + \frac{2M^*}{\hbar^2} J \right] \right\} \varphi_1 = 0 \\ &\left\{ \Delta + \left[k^2 - \frac{2M^*}{\hbar^2} J \right] \right\} \varphi_2 = 0 \end{aligned}$$

$$\left. \begin{aligned} \varphi_1 &\rightarrow \tfrac{1}{2} I + F_1(\theta) S, \\ \varphi_2 &\rightarrow \tfrac{1}{2} I + F_2(\theta) S. \end{aligned} \right\} \qquad \begin{aligned} f_+(\theta) &= F_1 + F_2 \\ f_-(\theta) &= F_1 - F_2 \end{aligned}$$

spin parallel : $\left\{ \begin{aligned} \Psi_+^{anti}(r) &= \psi_+(r) - \psi_-(-r) \\ \Psi_-^{anti}(r) &= \psi_-(r) - \psi_+(-r) \end{aligned} \right.$

spin anti ∥el ; $\left\{ \begin{aligned} \Psi_+^{sym}(r) &= \psi_+(r) + \psi_-(-r) \\ \Psi_-^{sym}(r) &= \psi_-(r) + \psi_+(-r) \end{aligned} \right.$

$$\left. \begin{aligned} \Psi_+^{anti} &\sim e^{ikz} + \{ f_+(\theta) - f_-(\pi - \theta) \} \cdot S \\ \Psi_-^{anti} &\sim -e^{-ikz} + \{ -f_+(\pi - \theta) + f_-(\theta) \} \cdot S \end{aligned} \right\}$$

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$$\left.\begin{aligned}\Psi_+^{sym} &\sim e^{ikz} + \{f_+(\theta) + f_-(\pi-\theta)\}\cdot S\\ \Psi_-^{sym} &\sim e^{-ikz} + \{f_+(\pi-\theta) + f_-(\theta)\}\cdot S\end{aligned}\right\}$$

Prob. per unit solid angle of the neutron to be scattered in the direction in which the vector $r_1 - r_2$ after collision makes the angle θ to the z-direction; (Mott; Proc. 126, 259)

$$P(\theta) = \frac{1}{4}\{3\,|f_+(\theta) - f_-(\pi-\theta)|^2 + |f_+(\theta) + f_-(\pi-\theta)|^2\}$$
$$= \frac{1}{4}\{3\,|F(\theta) + G(\theta) - F(\pi-\theta) + G(\pi-\theta)|^2 + |F(\theta) + G(\theta) + F(\pi-\theta) - G(\pi-\theta)|^2\}$$

If the proton is initially at rest, the prob. .... scattered in the direction Θ is then

$$P'(\Theta) = \frac{1}{4}\{3\,|F(2\Theta) + \cdots\ |^2 + \cdots\ \}\cdot 4\cos\Theta$$

$$\theta = 2\Theta \qquad \sin\theta\, d\theta = 4\cos\Theta \sin\Theta\, d\Theta.$$

total cross section

$$Q = 2\pi\int_0^\pi P(\theta)\sin\theta\, d\theta = 2\pi\int_0^{\frac{\pi}{2}} P'(\Theta)\sin\Theta\, d\Theta.$$

mass absorption coef. $\frac{\mu}{\rho} = \frac{Q}{M}.$

(3) $F_j(\theta)$ の決定.

$$\varphi_j(r) = \sum_l P_l(\cos\theta)\,\Phi_{l_j}(r)/r$$

$$\frac{d^2\Phi_{l_j}(r)}{dr^2} + \left[k^2 - \frac{l(l+1)}{r} - (-1)^j \frac{2M^*}{\hbar^2} J(r)\right]\Phi_{l_j}(r) = 0$$

$$\Phi_{l_j}(r) \sim A_{l_j} \sin\left(kr - \frac{1}{2}l\pi + \delta_{l_j}\right)$$

$$F_j(\theta) = \frac{1}{4\pi i}\sum_l (2l+1)\left(e^{2i\delta_{l_j}} - 1\right)P_l(\cos\theta)$$

$$P(\theta) =$$

$$Q = \frac{\pi}{k^2}\sum_l (2l+1)\left\{[2-(-1)^l]\sin^2\delta_{l_1} + [2+(-1)^l]\sin^2\delta_{l_2}\right\}$$

$$J(r) = \frac{Ae^{-\lambda r}}{r}$$

$j=1$: attractive ($J>0$, $J<0$ なら repulsive)

$x = \frac{2k}{\lambda}$ $\quad \beta = \left(\frac{M^*}{\hbar^2}\frac{A}{\lambda}\right)^{1/2}$ (Morse,

$\rho = kr$.

$$\frac{d^2\Phi_{l_j}(\rho)}{d\rho^2} + \left(1 - (-1)^j \frac{4\beta^2}{x}\frac{e^{-2\frac{\rho}{x}}}{\rho} - \frac{l(l+1)}{\rho^2}\right)\Phi_{l_j}(\rho) = 0 \quad (11)$$

$$Q = \frac{\pi}{k^2}\{\underset{\text{attraction}}{\sin^2\delta_{01}} + 3\underset{\text{repulsion}}{\sin^2\delta_{02}}\} \quad J>0$$

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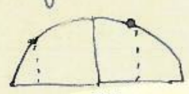
δ_{0j} の決定は(31)式の numeral integration が必要.
(Morse 参照)

(4) 実験と比較.　$V = 3.3 \times 10^9$　$k = 2.6 \times 10^{12}$

λ ~~$= 3.9 \times 10$~~	x	A	β^2	δ_1	δ_2	Q cm²	$\frac{\mu}{\rho}$
3.9×10^{12}	1.33	6.56×10^{-18}	1.286	1.81	0.587	0.87×10^{-24}	0.52
5.2	1.0	8.12	1.194	1.88	0.473	0.70	0.42
6.5	0.8	9.68	1.139	1.91	0.392	0.61	0.37

Curie Joliot, $\left(\frac{\mu}{\rho}\right)_H = 0.927$

(Bonner, $Q = 0.73 \times 10^{-24}$ (Phys. Rev. May 1, 1934))


$\frac{\pi}{2} = 1.5 \ldots$

$\therefore \lambda > 6.5 \times 10^{12}$.

$\lambda = 6.5 \times 10^{12}$,　$\beta^2 = 1.139$

v	eV	x	δ_1	δ_2	Q	$\frac{\mu}{\rho}$
2.06×10^9	2.2×10^6	0.5	0.264	0.98	0.98×10^{-24}	0.59
0.83	3.5×10^5	0.2	0.112	1.65	1.65	0.99
0.16	1.4×10^4	0.04	0.0227	1.85	1.85	1.12

Meitner, Be + Po $< 2 \times 10^6$ e.V.
$Q = 2 \times 10^{-24}$ cm²
で 2.8×10^5 e.V で sharp max.

slow neutron に対しては　Q は ρ が小さくて済む
∴ λ を大きくせねばならぬ..
Bonner, $Q = 2.53 \times 10^{-24}$　for $v = 1.3 \times 10^9$ cm/sec

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slow neutron の Q は ρ の小さな change に對して敏感にかはる.
$\rho^2 \approx 1$ とすると、上の値の3倍になる.($x \doteqdot 0.04$)
しかし、さうすると、$\lambda \to \infty$ にならなければ $H \to 0$
$\bar{E} \approx 0 \to \rho = 0.87$.

結局、slow neutron の absorption が Meitner の実験、
(B + Po, Li + Po) で非常に大きいことが説明出来ぬ。
しかし、例へば、recoil proton 自身の absorption のために、
neutron の absorp が実際より大きく見えるのかもしれぬ。

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朝永振一郎, Proton と Neutron が combine して radiation を出す prob.

variation of constant である.

$J > 0 \quad : \quad J(r) = \dfrac{Pe^{-\lambda r}}{1-e^{-\lambda r}}$

とおいて.

$\lambda = 10.0 \times 10^{12}$

$k_0 = 2.6 \times 10^{12}$ (neutron vel. 3.3×10^9 cm/sec)

$\cancel{k_{H^2} = 1.5 \times 10^{12}}$ (1

$Q_{H^2} = 0.69 \times 10^{-29}$ cm^2 (spin $\genfrac{}{}{0pt}{}{\text{anti parallel}}{\text{parallel}}$)

$\therefore \ Q_{H'^2} = \dfrac{Q_{H^2}}{4} = 3.1 \times 10^{-30}$ cm^2

$J < 0 \quad \ldots \quad Q_{H^2} = \ \ldots$ (spin ∥el)

$Q_{H'^2} = \dfrac{3}{4} Q_{H^2} =$

$$Q_{H_2} = \int \frac{1}{v_0^3} \frac{1}{\hbar^2} \frac{16\pi^2 e^2}{M^2 c^3} (\varepsilon^{(0)} - \varepsilon)^3 (\vartheta^2 Z)^2 [\bar{r}]^2.$$