1) Capture of Neutrons

Bethe, Bulletin. Feb. 1935

Neutron m, Nucleus M Product Nucleus M'

$$\vec{p} + \vec{P} = \vec{P}'$$

$$\varepsilon + E = E'$$

$$m^2 = M^2 + M'^2 - 2(EE' - \vec{P}\vec{P}')$$

$$= (M' - M)^2 - 2(EE' - MM' - \vec{P}\vec{P}')$$

$$= (M' - M)^2 - 2\left\{\sqrt{M^2+P^2}\sqrt{M'^2+P'^2} - MM' - \vec{P}\vec{P}'\right\}$$

$$\leq (M' - M)^2 - 2\left\{\frac{M'P^2}{2M} + \frac{MP'^2}{2M'} - \vec{P}\vec{P}'\right\}$$

$$= (M' - M)^2 - \left(\sqrt{\frac{M'}{M}}\vec{P} \mp \sqrt{\frac{M}{M'}}\vec{P}'\right)^2$$

$$m \leq M' - M$$

$P = 0$; $m^2 \leq (M' - M)^2 - \frac{M}{M'}P'^2$

$$\vec{p} = \vec{P}'$$

$$\varepsilon + M = E' + \gamma$$

$$m^2 = (E' + \gamma - M)^2 - P'^2$$

$$= M'^2 + M^2 - 2E'M + 2\gamma(E' - M)$$

$$= (M' - M)^2 - 2\{(E' - M')M - \gamma(E' - M)\}$$

$$\gamma > \frac{E' - M}{(E' - M')M} = \frac{M' + \frac{P^2}{2M'} - M}{\frac{P^2}{2M'}M} = \frac{2M'^2 + P^2 - 2M'M}{P^2 M}$$

$$= \frac{2M'(M' - M) + P^2}{P^2 M}$$

$$(\varepsilon + M - \gamma)^2 - p^2 = M'^2$$

$$\gamma = -\sqrt{M'^2 + p^2} + \varepsilon + M$$

$$\cancel{= \varepsilon + M} = M + m + \frac{p^2}{2m} - M' - \frac{p^2}{2M'}$$

$$\cong D + \frac{p^2}{2m}$$

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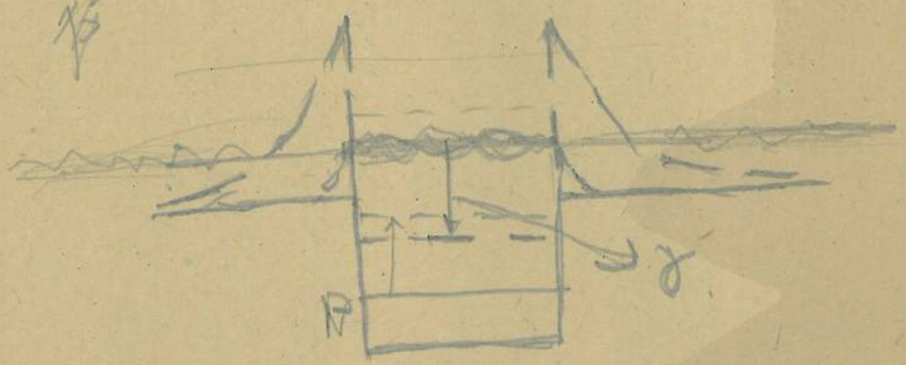
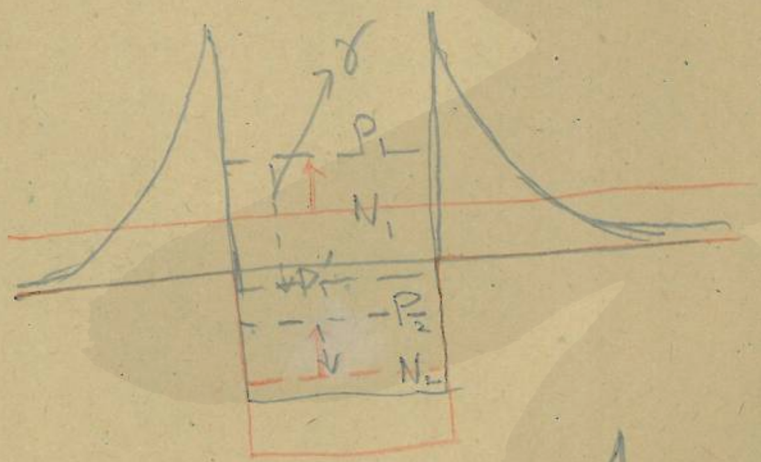
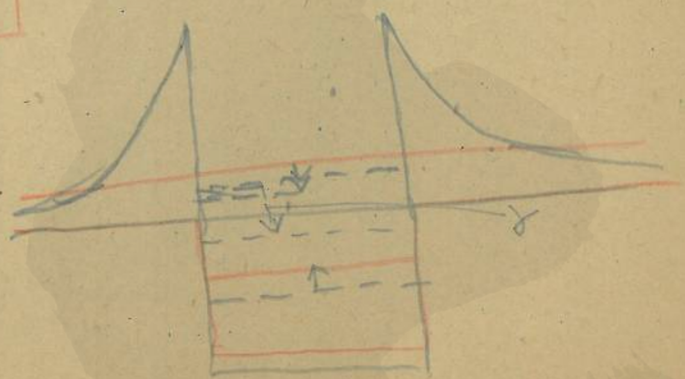






$$\left.\begin{aligned} \mu + k_0 &= E + k_1 \\ k_0 &= P\cos\chi + k_1\cos\theta \end{aligned}\right\} \quad E^2 = P^2 + \mu^2$$

$$P\sin\chi = k_1 \sin\theta.$$

$$P^2 = (k_0 - k_1\cos\theta)^2 + k_1^2\sin^2\theta$$

$$= (\mu + k_0 - k_1)^2 - \mu^2$$

$$-2k_0k_1\cos\theta = \underline{2\mu k_0} \mp 2\mu k_1 - 2k_0k_1$$

$k_0 \gg \mu$

$$\left.\begin{aligned} k_1 &= \frac{k_0}{1 + (1-\cos\theta)\frac{k_0}{\mu}} \cong \mu \\ E &= \mu + k_0 - \frac{k_0}{1 + (1-\cos\theta)\frac{k_0}{\mu}} \cong k_0 \\ \tan\chi &= \frac{k_1 \sin\theta}{k_0 - k_1\cos\theta}, \cong \frac{\mu}{k_0} \end{aligned}\right\}$$

$$\chi \cong O\left(\frac{\mu}{k_0}\right)$$

$$P^2 = k_0^2 - 2k_0k_1\cos\theta + k_1^2$$

$$= k_0^2 - k_1\left\{\frac{k_0}{1-(1-\cos\theta)\frac{k_0}{\mu}} - 2k_0\cos\theta\right\}$$

$$= k_0^2 - k_0\left\{\frac{k_0 - 2k_1\cos\theta + 2\frac{k_0^2}{\mu}\cos\theta - 2\frac{k_0^2}{\mu}\cos^2\theta}{\left(1-(1-\cos\theta)\frac{k_0}{\mu}\right)^2}\right.$$

$$= k_0^2 \frac{\left(1-(1-\cos\theta)\frac{k_0}{\mu}\right)^2 + \cos\theta \ldots - 2\frac{k_0}{\mu}\cos\theta}{\left(1-(1-\cos\theta)\frac{k_0}{\mu}\right)^2}$$

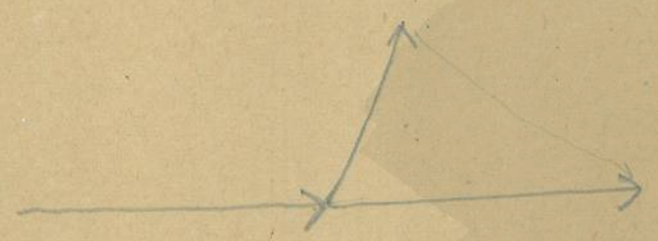
$$+ 2\frac{k_0}{\mu}\cos^2\theta$$

$$-2\cdot(1-\cos\theta)\frac{k_0}{\mu} + (1-\cos\theta)^2\frac{k_1^2}{\mu^2}$$

$$2\cos\theta - 2\frac{k_0}{\mu}\sin^2\theta + (1-\cos\theta)^2\frac{k_0^2}{\mu^2}$$

$k_0 = O(k_0) + O(\mu)$

$k_0 \cdot \frac{\mu}{k_0} = \mu$



$$P = E - O\left(\frac{\mu^2}{k_0}\right)$$

$$(E + k_1 - \mu)^2 = (P\cos\chi + k_1\cos\theta)^2$$