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I. $N \to P + M_k^- \to P + M_{k'}^- \to N$

$$W_I = \sum_{k,k'} \sum \frac{H_{0k} H_{kk'} H_{k'0}}{(E_k + \varepsilon_k - Mc^2)(E_{k'} + \varepsilon_{k'} - Mc^2)}$$

$$= \frac{1}{3\pi} \left(\frac{f_2^2}{\hbar c}\right) \frac{e}{\kappa^2} (\sigma H) \int_0^{k_0} \frac{k^4\, dk}{(k^2 + \kappa^2)^2}$$

II. $N \to P + M_k^- \to \cancel{P} + M_{k'}^+ + \overset{0}{M}{}_k^- \to N$

III. $N \to N + M_k^+ + M_{k'}^- \to P + M_{k'}^- \to N$

$$W_{II} + W_{III} = W_I$$

IV. $N \xrightarrow[NM]{} P + M_k^- \xrightarrow[NME]{} N$

V. $N \xrightarrow[NME]{} P + M_k^- \xrightarrow[NM]{} N$

$\Big\}\ W_{IV} = W_V = 0$

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$$W_{P.S.} = W_{I} + W_{II} + W_{III} + W_{IV} + W_{V}$$
$$= -\mu H$$

Pseudoscalar $\quad \mu_{PS} = \frac{-2}{3\pi}\left(\frac{f_2^2}{\hbar c}\right)\frac{e\hbar}{\mu c}\,\sigma\, f(x)$

Vector $\quad \mu_{V} = 2\mu_{PS}$

$$f(x) = \int_0^x \frac{x^4\,dx}{(1+x^2)^2}$$

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校正送先 京都帝大
理学部 湯川秀樹

Short Note

Note on the Magnetic Moment of the Nucleon.

By Sizuo Ueno and Hironobu Kuze

In the present formalism of the meson theory, the anomalous magnetic moment of the nucleon is ascribed to the virtual existence of those ~~virtual~~ mesons around it, which are described by the vector field.[1] Recent trend of the theory indicates, however, the coexistence of mesons, which can be described by the pseudoscalar field and contribute to the nuclear force as much as the vector mesons. Thus we can expect that the pseudoscalar mesons are also responsible for the anomalous magnetic moment of the nucleon. But a closer investigation is neeeded in order to make this point clear, since the pseudoscalar mesons have no intrinsic magnetic moments in contrast to the vector mesons.[2]

Now we consider a neutron with the spin vector σ in the magnetic field **H.** The energy of the neutron changes on account of the interaction of the virtual mesons and protons in the intermediate states with the magnetic field.

There are four types of relevant processes:

$$\text{I.} \quad N \rightarrow P + M_k^- \xrightarrow{H} P + M_{k'}^- \rightarrow N$$

1) Fröhlich, Heitler and Kemmer, Proc. Roy. Soc. A 166(1938), 154; Yukawa, Sakata and Taketani, Proc. Phys.-Math. Soc. Japan 26(1938), 319.
2) Similar calculations were performed independently by Yamasaki and Uma (Proc. Phys.-Math. Soc. Japan, in press). The authors are indebted to Dr. Kobayasi for the communication of their results before publication.

II. $N \rightarrow P + M_k^- \rightarrow N + M_{k'}^+ + M_k^- \xrightarrow{H} N$

I~~V~~II. $N \xrightarrow{H} N + M_{k'}^+ + M_k^- \rightarrow P + M_k^- \rightarrow N$

IV. $N \xrightarrow{H} P + M_k^- \rightarrow N$ or $N \rightarrow P + M_k^- \xrightarrow{H} N$

where P, M_k^- and $M_{k'}^+$ denote a proton, a meson with the negative charge and the momentum $\hbar\mathbf{k}$ and a meson with the positive charge and the momentum $\hbar\mathbf{k}'$ respectively. The energy change due to the process I, for instance, is given by the well known formula

$$W_I = \sum_{k,k'} \sum \frac{H_{0k'} H_{k'k} H_{k0}}{(E_{k'} + \varepsilon_{k'} - Mc^2)(E_k + \varepsilon_k - Mc^2)} \qquad (1)$$

where H_{k0}, $H_{k'k}$ and $H_{0k'}$ denote the matrix elements of the interaction energy corresponding to the transitions between the initial state (0), the first intermediate state (k), the second intermediate state (k') and the final state (0), which is identical with the initial state, respectively. E_k, ε_k denote the energies of the proton and the meson, both having the momentum of magnitude $\hbar k$. The first summation $\sum_{k,k'}$ in (1), which should be performed over all possible values of the momenta $\hbar\mathbf{k}$, $\hbar\mathbf{k}'$ of the mesons in the intermediate states, can be transformed into the integration with respect to $\mathbf{k}$ and $\mathbf{k}'$. It reduces to the integral with respect to $\mathbf{k}$ alone, since the conservation of momentum requires $\mathbf{k} = \mathbf{k}'$. The second summation refers to possible orientation of the spin of the proton, the summation with respect to the spin of the meson being, of course, unnecessary in this case.

By using the expressions for the Hamiltonian in the pseudoscalar theory, which were obtained by Tanikawa and Yukawa[3], we arrive at a result ~~and by expanding the meson field~~

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3) Tanikawa and Yukawa, Proc. Phys.-Math. Soc. Japan 23(1941), 445.

$$W_{\mathrm{I}} = \frac{1}{3\pi}\left(\frac{f_2^2}{\hbar c}\right)\frac{e}{\varkappa^2}(\boldsymbol{\sigma}\boldsymbol{H})\int_0^{\infty}\frac{k^4 dk}{(k^2+\varkappa^2)^2} \qquad (2)$$

which has a similar form as in the case of the vector mesons, although only those magnetic moments ~~of themesons~~ which are caused by the orbital motion of the pseudoscalar mesons contribute to the energy change in our case. The upper limit of the integration in (2) should be replaced by a finite value k_0 instead of extending to infinity in order to avoid the well known divergence difficulty.

The energy changes $W_{\mathrm{II}}, W_{\mathrm{III}}$ due to the processes II and III can be calculat in a similar manner and we obtain the result

$$W_{\mathrm{II}} + W_{\mathrm{III}} = W_{\mathrm{I}}$$

On the other hand, the energy change W_{IV} due to the process IV originated by the simultaneous interaction of the meson with the nucleon and the electromagnetic field is found to be 0, so that the resultant energy change W takes the form.

$$W = \qquad = 2W_{\mathrm{I}} = -\mu H \qquad (3)$$

with

$$\mu = -\frac{2}{3\pi}\left(\frac{f_2^2}{\hbar c}\right)\frac{e\hbar}{\mu c}\boldsymbol{\sigma} f(x_0), \qquad (4)$$

where $x_0 = k_0/\varkappa$ and $f(x_0) = \int_0^{x_0}\frac{x^4 dx}{(1+x^2)^2}$

Thus μ is the additional magnetic moment of the neutron due to the virtual existence of pseudoscalar mesons.

This result can be compared with the corresponding moment

$$\mu' = -\frac{4}{3\pi}\left(\frac{g_2^2}{\hbar c}\right)\frac{e\hbar}{\mu c}\boldsymbol{\sigma} f(x_0) \qquad (5)$$

in the case of the vector mesons. If we take, for example, the constant ~~g_2~~ characterizing the dipole interaction between the nucleon and the pseudoscalar meson f_2 equal to that between the nucleon and the vector meson g_2, we find that the contribution of the pseudoscalar meson is just one half of that of the vector meson. Further discussions on the bearing of this conclusion to other problems of the meson theory ~~were~~ are made by Yamasaki and Uma4) on the basis of the single force hypothesis, which was put forward by Kobayasi.5)

In conclusion, the authors wishes to express their cordial thanks to Prof. Yukawa for his continual guidance and also to Dr. Sakata, Dr. Kobayasi and Mr. Tanikawa for valuable discussions. Their thanks are also due to the Japan Society for Promotion of Scientific Research for financial aid.

Institute of Theoretical Physics,
Kyoto Imperial University.

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4) Yamasaki and Uma, loc. cit.
5) Kobayasi, Proc. Phys.-Math. Soc. Japan <u>23</u>(1941), 891.

Errata

Sizuo Ueno and Hironobu Kuze: Note on the Magnetic Moment of the Nucleon.

(Proc. Phys.-Math. Soc. Japan 24(1942), 184)

The expression (4) on page 185 should be read

$$m = -\frac{2}{3\pi}\left(\frac{f_2^2}{\hbar c}\right)\frac{e\hbar}{\mu c} S f(x_0) \tag{4}$$

instead of

$$m = +\frac{2}{3\pi}\left(\frac{f_2^2}{\hbar \pi}\right)\frac{e\hbar}{\mu c} S f(x_0) \tag{4}$$

[参考 → F08 031]

Proc. Phys.-Math. Soc. Japan 24(1942)184-185,612.

SHORT NOTE.

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In the present formalism of the meson theory, the anomalous magnetic moment of the nucleon is ascribed to the virtual existence of those mesons around it, which are described by the vector field[(1)]. Recent trend of the theory indicates, however, the coexistence of mesons, which can be described by the pseudoscalar field and contribute to the nuclear force as much as the vector mesons. Thus we can expect that the pseudoscalar mesons are also responsible for the anomalous magnetic moment of the nucleon. But a closer investigation is needed in order to make this point clear, since the pseudoscalar mesons have no *intrinsic* magnetic moments in constrast to the vector mesons[(2)].

Now we consider a neutron with the spin vector $\boldsymbol{s}$ in the magnetic field $\boldsymbol{H}$. The energy of the neutron changes on account of the interaction of the virtual mesons and protons in the intermediate states with the magnetic field. There are four types of relevant processes:

I. $N \rightarrow P + M_k^- \underset{\boldsymbol{H}}{\rightarrow} P + M_{k'}^- \rightarrow N$

II. $N \rightarrow P + M_k^- \rightarrow N + M_{k'}^+ + M_k^- \underset{\boldsymbol{H}}{\rightarrow} N$

III. $N \underset{\boldsymbol{H}}{\rightarrow} N + M_{k'}^+ + M_k^- \rightarrow P + M_k^- \rightarrow N$

IV. $N \underset{\boldsymbol{H}}{\rightarrow} P + M_k^- \rightarrow N$ or $N \rightarrow P + M_k^- \underset{\boldsymbol{H}}{\rightarrow} N$

where P, M_k^- and $M_{k'}^+$ denote a proton, a meson with the negative charge and the momentum $\hbar k$ and a meson with the positive charge and the momentum $\hbar k'$ respectively. The energy change due to the process I, for instance, is given by the well known formula

$$W_{\rm I} = \sum_{k,k'} \sum \frac{H_{0k'} H_{k'k} H_{k0}}{(E_{k'} + \varepsilon_{k'} - Mc^2)(E_k + \varepsilon_k - Mc^2)} \qquad \cdots\cdots\cdots\cdots\cdots (1)$$

where H_{k0}, $H_{k'k}$ and $H_{0k'}$ denote the matrix elements of the interaction energy corresponding to the transitions between the initial state (0), the first intermediate state (k), the second intermediate state (k') and the final state (0), which is identical with the initial state, respectively. E_k, ε_k denote the energies of the proton and the meson, both having the momentum of magnitude $\hbar k$. The first summation $\sum_{k,k'}$ in (1), which should be performed over all possible values of the momentum $\hbar k$, $\hbar k'$ of the mesons in the intermediate states, can be transformed into the integration with respect to k and k'. It reduces to the integral with respect to k alone, since the conservation of momentum requires $k = k'$. The second summation refers to possible orientations of the spin of the proton, the summation with respect to the spin of the meson being, of course, unnecessary in this case.

By using the expressions for the Hamiltonian in the pseudoscalar theory, which were obtained by Tanikawa and Yukawa[(3)], we arrive at a result

$$W_{\rm I} = \frac{1}{3\pi}\left(\frac{f_2^2}{\hbar c}\right)\frac{e}{\kappa^2}(\boldsymbol{s}\boldsymbol{H})\int_0^\infty \frac{k^4 dk}{(k^2+\kappa^2)^2}, \qquad \cdots\cdots\cdots\cdots\cdots (2)$$

which has a similar form as in the case of vector mesons, although only those magnetic moments which are caused by the *orbital* motion of pseudoscalar mesons contribute to the energy change in our case. The upper limit of the integration in (2) should be replaced by a finite value k_0 instead of extending to infinity in order to avoid the well known divergence difficulty.

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(3) Tanikawa and Yukawa, Proc. Phys.-Math. Soc. Japan **23** (1941), 445.

The energy changes $W_{\rm II}$, $W_{\rm III}$ due to the processes II and III can be calculated in a similar manner and we obtain the result

$$W_{\rm II} + W_{\rm III} = W_{\rm I}.$$

On the other hand, the energy change $W_{\rm IV}$ due to the process IV originated by the simultaneous interaction of the meson with the nucleon and the electromagnetic field is found to be 0, so that the resultant energy change W takes the form

$$W = W_{\rm I} + W_{\rm II} + W_{\rm III} + W_{\rm IV} = 2W_{\rm I} = -m\boldsymbol{H} \qquad (3)$$

with

$$m = \frac{2}{3\pi}\left(\frac{f_2^2}{\hbar\pi}\right)\frac{e\hbar}{\mu c}\, s f(x_0), \qquad (4)$$

where $x_0 = k_0/\kappa$ and

$$f(x_0) = \int_0^{x_0} \frac{x^4 dx}{(1+x^2)^2}. \qquad (4)$$

This m is the additional magnetic moment of the neutron due to the virtual existence of pseudoscalar mesons.

This result can be compared with the corresponding moment

$$m' = -\frac{4}{3\pi}\left(\frac{g_2^2}{\hbar c}\right)\frac{e\hbar}{\mu c}\, s f(x_0) \qquad (5)$$

in the case of the vector mesons. If we take, for example, the constant characterizing the dipole interaction between the nucleon and the pseudoscalar meson f_2 equal to that between the nucleon and the vector meson g_2, we find that the contribution of the pseudoscalar meson is just one half of that of the vector meson. Further discussions on the bearing of this conclusion to other problems of the meson theory are made by Yamasaki and Uma[(4)] on the basis of the single force hypothesis, which was put forward by Kobayasi[(5)].

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Institute of Theoretical Physics,
Kyoto Imperial University.

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Errata. [PPMSJ 24(42), 612]

Sizuo Ueno and Hironobu Kuze: *Note on the Magnetic Moment of the Nucleon.*

(Proc. Phys.-Math. Soc. Japan, 24 (1942), 184.)

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instead of

$$m = +\frac{2}{3\pi}\left(\frac{f_2^2}{\hbar\pi}\right)\frac{e\hbar}{\mu c}\, s f(x_0). \qquad (4)$$

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