Pseudo-scalar Meson Theory ニヨル Neutron ノ Anomalous Magnetic Moment ノ計算.

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2601. 11.

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$$W_I(H) = \sum \frac{H_{ok'}^{*} H_{k'k} H_{ko}}{(E_{k'} - E_o)(E_k - E_o)}$$

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1. Field equations.

$$\left.\begin{aligned} \left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)\Psi &= \kappa\Phi - 4\pi f_2 Q \\ -\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)\Phi &= \kappa\Psi + \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)\vec{P} - 4\pi f_1 R. \\ \kappa\vec{P} &= -\left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)\Psi + 4\pi f_2 P. \end{aligned}\right\} \quad (1)$$

$\kappa = \frac{\mu c}{\hbar}$; μ : Meson, rest mass.

Ψ : Pseudo-scalar ; $(\vec{P}, \Phi)$: Pseudo-four-vector.

$$R = \tilde{\psi}\Pi^*\rho_2\psi, \quad \vec{P} = \tilde{\psi}\Pi^*\vec{\sigma}\psi, \quad Q = \tilde{\psi}\Pi^*\rho_1\psi. \qquad (2)$$

$\psi, \tilde{\psi}$: wave function for the heavy particle.

Π^* : proton ヲ neutron ニ 變ヘル operator.

Commutation relations:

$$\left.\begin{aligned} [\Psi(\vec{x}), \Psi^\dagger(\vec{x}')] &= i\hbar\cdot\delta(\vec{x}-\vec{x}') \\ [\tilde{\Psi}(\vec{x}), \tilde{\Psi}^\dagger(\vec{x}')] &= i\hbar\cdot\delta(\vec{x}-\vec{x}') \end{aligned}\right\} \quad (3)$$

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$$\left.\begin{aligned} \Psi^\dagger &= \frac{\partial L}{\partial\left(\frac{\partial\Psi}{\partial t}\right)} = \frac{1}{4\pi\kappa c}\tilde{\Phi} \\ \tilde{\Psi}^\dagger &= \frac{\partial L}{\partial\left(\frac{\partial\tilde{\Psi}}{\partial t}\right)} = \frac{1}{4\pi\kappa c}\cdot\Phi \end{aligned}\right\} \quad (4)$$

L : Lagrangian for the system.

1.

<u>Hamiltonian for the total system</u>:

$$\overline{H} = \int \{ H_M + H_N + H_E + H' \} dv \qquad (5)$$

$$H' = H_{NM} + H_{ME} + H_{NE} + H_{NME} \qquad (5_1)$$

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$$H_{M} = 4\pi\kappa^2c^2\,\tilde{\Psi}^{\dagger}\Psi^{\dagger} + \frac{1}{4\pi\kappa^2}\,\mathrm{grad}\,\tilde{\Psi}\cdot\mathrm{grad}\,\Psi + \frac{1}{4\pi}\tilde{\Psi}\Psi, \qquad (5_2)$$

$$H_{NM} = -4\pi c f_2\cdot(Q\Psi^{\dagger} + \tilde{Q}\tilde{\Psi}^{\dagger}) - \frac{f_1}{\kappa}(R\tilde{\Psi} + \tilde{R}\Psi) \qquad (5_3)$$
$$- \frac{f_2}{\kappa^2}(\vec{P}\,\mathrm{grad}\,\tilde{\Psi} + \tilde{\vec{P}}\,\mathrm{grad}\,\Psi) + 4\pi\left(\frac{f_2}{\kappa}\right)^2\tilde{P}P.$$

$$H_{ME} = \frac{ie}{\hbar}A_0(\tilde{\Psi}^{\dagger}\tilde{\Psi} - \Psi^{\dagger}\Psi^{\dagger}) + \frac{ie}{4\pi\kappa^2\hbar c}(\vec{A}\tilde{\Psi}\,\mathrm{grad}\,\Psi - \vec{A}\Psi\,\mathrm{grad}\,\tilde{\Psi}) \qquad (5_4)$$

$$H_{NME} = \frac{if_2e}{\kappa^2\hbar c}(\vec{A}\Psi\tilde{\vec{P}} - \vec{A}\tilde{\Psi}\vec{P}), \qquad (5_5)$$

$\Psi, \Psi^{\dagger}$ ヲ plane waves デ expand スルト

$$\left.\begin{aligned} \Psi &= \frac{1}{\sqrt{L^3}}\sum_k -i\sqrt{\frac{2\pi\mu^2c^4}{\varepsilon_k}}(-a_k + b_k^*)\cdot\exp(i\vec{k}\vec{r}) \\ \Psi^{\dagger} &= \frac{1}{\sqrt{L^3}}\sum_k \sqrt{\frac{\hbar^2\varepsilon_k}{8\pi\mu^2c^4}}(a_k^* + b_k)\cdot\exp(-i\vec{k}\vec{r}) \end{aligned}\right\} \qquad (6)$$

(N, M, E ハ夫々 heavy particle, meson, electro-magnetic field ヲ意味スル.)

a's, b's ニ対スル commutation relations ハ

$$[a_k, a_l^*] = [b_k, b_l^*] = \delta_{kl} \qquad (7)$$

上ノ Ψ ヲ用ヒルト

$$H_M = \sum_k \varepsilon_k(a_k^*a_k + b_k^*b_k + 1),$$

Mesons ノ total charge $= \sum_k e(a_k^*a_k - b_k^*b_k)$.

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$$a_k^* a_k = N_k^+ : \quad (M_k^+, \text{數})$$
$$b_k^* b_k = N_k^- : \quad (N_k^-, \text{數}).$$

從ツテ

$$\left.\begin{array}{lll} a_k^* : & M_k^+ \text{ヲ} & \text{1箇増ス operator.} \\ a_k : & M_k^+ \text{ヲ} & \text{1箇減ジル operator.} \\ b_k^* : & M_k^- \text{ヲ} & \text{1箇増ス operator.} \\ b_k : & M_k^- \text{ヲ} & \text{1箇減ジル operator.} \end{array}\right\} \quad (8)$$

又上ニ於テ Lハ一辺ノ長サLノ立方体中ニ考フル system ガ存在スル 場合ニ從ッテ normalize シタ時ノ normalizing factor.

$$k_x = \frac{2\pi}{L} \cdot n_x, \quad k_y = \frac{2\pi}{L} \cdot n_y, \quad k_z = \frac{2\pi}{L} n_z. \quad (9)$$

$$n_x, n_y, n_z : \quad \text{integers.} \ (\geqq 0)$$

$$\varepsilon_k = \hbar c \sqrt{k^2 + \varkappa^2}. \quad (10)$$

(6)ヲ (5_3), (5_4), (5_5) ニ入レテ

$$H_{NM} = \frac{1}{\sqrt{L^3}} \sum_k \tilde{\psi}_b \left[-\frac{f_2}{\varkappa} \sqrt{2\pi\varepsilon_k} \cdot (a_k^* + b_k) \rho_1 + \frac{f_2 \hbar c}{\varkappa} \sqrt{\frac{2\pi}{\varepsilon_k}} (a_k^* - b_k)(\vec{\sigma}\vec{k}) \right.$$
$$\left. + i f_1 \hbar c \sqrt{\frac{2\pi}{\varepsilon_k}} (a_k^* - b_k) \rho_2 \right] \Pi^* \exp(-i\vec{k}\vec{r}) \cdot \psi_a + \text{compl. conj.} \quad (11_1)$$

$$H_{ME} = -\frac{e}{2L^3} \sum_{k,k'} \left[\sqrt{\frac{\varepsilon_k}{\varepsilon_{k'}}} (a_k + b_k^*)(-a_{k'}^* + b_{k'}) A_0 e^{i(\vec{k'}-\vec{k},\vec{r})} \right.$$
$$\left. + \frac{\hbar c}{\sqrt{\varepsilon_k \varepsilon_{k'}}} (+a_k^* + b_k)(-a_{k'}^* + b_{k'}^*)(\vec{A}, \vec{k'}) e^{i(\vec{k'}-\vec{k},\vec{r})} \right] + \text{compl. conj.}$$

$$H_{NME} = \frac{f_2 e}{\hbar c \varkappa^2} \cdot \frac{1}{\sqrt{L^3}} \sum_k \tilde{\psi}_b \sqrt{\frac{2\pi\mu^2 c^4}{\varepsilon_k}} (-a_k^* + b_k)(\vec{A}, \vec{\sigma}) \Pi^* e^{-i(\vec{k},\vec{r})} \psi_a \quad (11_2)$$
$$+ \text{compl. conj.} \quad (11_3)$$

$$(-a_{k}^* + b_{k}^*)(-a_{k'}^* + b_{k'})(\vec{A}, \vec{k'}) e^{-i(\vec{k'}-\vec{k})\vec{r}}$$

$k \to k'$, $k' \to k$

2.1. Process I.

$$\overset{a}{N} \xrightarrow[NM]{} \overset{b}{P + M_k^-} \xrightarrow[ME]{} \overset{b'}{P + M_{k'}^-} \xrightarrow[NM]{} \overset{a'}{N}.$$

H_{ME}^{I}

initial state, final state 及ビ 二ツノ intermediate states ニ於ケル wave function (heavy particle) ヲ夫々

$$\begin{cases} \psi_a = \dfrac{1}{\sqrt{L^3}} \cdot u^{(a)}, \\ \psi_{a'} = \dfrac{1}{\sqrt{L^3}} \cdot u^{(a)} e^{-i(\vec{k}-\vec{k}', \vec{r})} \\ \psi_b = \psi_{b'} = \dfrac{1}{\sqrt{L^3}} \cdot u^{(b)} e^{-i(\vec{k}, \vec{r})} \end{cases}$$

(u : Dirac's spinor)

各 Process ニ対スル Matrix element ヲ 夫々 $H_{0k}, H_{kk'}, H_{k0}$ トスルト

$$H_{0k} = \frac{1}{\sqrt{L^3}} \cdot \frac{1}{L^3} \int u^{(a)*} \left[-\frac{f_2}{\kappa}\sqrt{2\pi\varepsilon_k} \cdot \rho_1 - \frac{f_2 \hbar c}{\kappa}\sqrt{\frac{2\pi}{\varepsilon_k}} (\vec{\sigma}, \vec{k}) + i f_1 \hbar c \sqrt{\frac{2\pi}{\varepsilon_k}} \cdot \rho_2 \right] \cdot u^{(b)} \, d\vec{r}$$

i.e.

$$H_{0k} = \frac{1}{\sqrt{L^3}} u^{(a)*} K(k) u^{(b)} \qquad (12_1)$$

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$$\begin{cases} K(k) \equiv \alpha(k)\rho_1 + \beta(k)(\vec{\sigma}, \vec{k}) + \gamma(k)\rho_2 \\ \alpha(k) \equiv -\dfrac{f_2}{\kappa}\sqrt{2\pi\varepsilon_k}, \quad \beta(k) \equiv -\dfrac{f_2 \hbar c}{\kappa}\sqrt{\dfrac{2\pi}{\varepsilon_k}}, \quad \gamma(k) \equiv i f_1 \hbar c \sqrt{\dfrac{2\pi}{\varepsilon_k}} \end{cases} \qquad (12_2)$$

$K(k)(1+\rho_3)K^*(k') = 2\{\alpha(k)\alpha(k') + i\alpha(k)\gamma(k') - i\gamma(k)\alpha(k') - \gamma(k)\gamma(k') + \beta(k)\beta(k')((\vec{k}\vec{k}') + i[\vec{k}\,\vec{k}'])\}$

同様ニシテ

$$\left.\begin{aligned} H_{k0} &= \frac{1}{\sqrt{L^3}} u^{(b)*} K^*(k) u^{(a)} \\ K^*(k) &\equiv \alpha(k)\rho_1 + \beta(k)(\vec{\sigma}, \vec{k}) - \gamma(k)\rho_2 \end{aligned}\right\} \qquad (12_3)$$

$(-\vec{k}+\vec{k}')$

$$H_{kk'} = -\frac{\hbar e c}{2L^3}\cdot\frac{1}{\sqrt{\varepsilon_k\varepsilon_{k'}}}\int_v (\vec{A}(\vec{r}),\ \vec{k}+\vec{k}')\, e^{i(\vec{k}'-\vec{k},\vec{r})}\, d\vec{r} \qquad (12_4)$$

但シ $A_0 = 0,\quad \vec{A} = \left(-\frac{H}{2}y,\ \frac{H}{2}x,\ 0\right) = \frac{1}{2}[\mathbf{H}\,\mathbf{r}]$

故ニ Process I ニ ヨル Perturbation energy $W_I(H)$ ハ

$$W_I(H) = \sum_{k,k'}\sum \frac{H_{ok}\,H_{kk'}\,H_{k'o}}{(E_k+\varepsilon_k-Mc^2)(E_{k'}+\varepsilon_{k'}-Mc^2)}$$

$E_k = \pm\sqrt{\hbar^2c^2k^2+M^2c^4}$; M : heavy particle ノ rest mass.

之ニ 上ノ H_{ok}, $H_{kk'}$, $H_{k'o}$ ヲ 入レルト

$$W_I(H) = -\frac{\hbar e c}{2L^6}\sum_{k,k'}\sum \frac{u^{(a)*}K(k)u^{(b)}\cdot\int_v(\vec{A},\vec{k}+\vec{k}')e^{i(\vec{k}'-\vec{k},\vec{r})}d\vec{r}\cdot u^{(b)*}K^*(k')u^{(a)}}{\sqrt{\varepsilon_k\varepsilon_{k'}}\cdot(E_k+\varepsilon_k-Mc^2)(E_{k'}+\varepsilon_{k'}-Mc^2)}$$

($\sum$: heavy particle ノスベテノ中間状態ニツイテノ和.)

$$= -\frac{\hbar e c}{2L^6}\sum_{k,k'}\sum \frac{u^{(a)*}K(k)[E_k-\varepsilon_k+Mc^2]u^{(b)}\cdot u^{(b)*}[E_{k'}-\varepsilon_{k'}+Mc^2]K^*(k')\cdot u^{(a)}}{\sqrt{\varepsilon_k\varepsilon_{k'}}\cdot[E_k^2-(\varepsilon_k-Mc^2)^2][E_{k'}^2-(\varepsilon_{k'}-Mc^2)^2]}$$
$$\times\int_v(\vec{A},\vec{k}+\vec{k}')e^{i(\vec{k}'-\vec{k},\vec{r})}d\vec{r}$$

$$= -\frac{\hbar e c}{2L^6}\sum_{k,k'}\sum \frac{u^{(a)*}K(k)[H(k)-\varepsilon_k+Mc^2]u^{(b)}u^{(b)*}[H(k')-\varepsilon_{k'}+Mc^2]u^{(a)}}{\sqrt{\varepsilon_k\varepsilon_{k'}}\cdot[E_k^2-(\varepsilon_k-Mc^2)^2][E_{k'}^2-(\varepsilon_{k'}-Mc^2)^2]}$$
$$\times\int_v(\vec{A},\vec{k}+\vec{k}')e^{i(\vec{k}'-\vec{k},\vec{r})}d\vec{r}$$

negative energy state ニハ $E_k-\varepsilon_k+Mc^2$ ...

$$H(k) = \hbar c\,\rho_1(\vec{\sigma},\vec{k}) + \rho_3 Mc^2$$

$Mc^2 \gg \hbar c k,\ \mu c^2$ トスルト

$H(k)-\varepsilon_k+Mc^2 \cong (1+\rho_3)Mc^2.\qquad H(k')-\varepsilon_{k'}+Mc^2 \cong (1+\rho_3)Mc^2.$

$E_k^2-(\varepsilon_k-Mc^2)^2 \cong 2Mc^2\cdot\varepsilon_k.\qquad E_{k'}^2-(\varepsilon_{k'}-Mc^2)^2 \cong 2Mc^2\cdot\varepsilon_{k'}$

5.

$$\therefore\quad W_I(H) \cong -\frac{\hbar e c}{8L^6}\sum_{k,k'}\sum \frac{u^{(a)*}K(k)(1+\rho_3)u^{(b)}u^{(b)*}(1+\rho_3)K^*(k')u^{(a)}}{\varepsilon_k\varepsilon_{k'}\cdot\sqrt{\varepsilon_k\varepsilon_{k'}}}$$

$$\times\int_V(\vec{A},\vec{k}+\vec{k}')\,e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}$$

$$= -\frac{\hbar e c}{4L^6}\sum_{k,k'}\frac{u^{(a)*}K(k)(1+\rho_3)K^*(k')u^{(a)}}{\varepsilon_k\varepsilon_{k'}\cdot\sqrt{\varepsilon_k\varepsilon_{k'}}}\int_V(\vec{A},\vec{k}+\vec{k}')\,e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}.$$

$$u^{(a)} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad u^{(a)*} = (1\;0\;0\;0)\qquad \text{トスート}$$

$$2u^{(a)*}K(k)K^*(k')u^{(a)}$$

$$u^{(a)*}K(k)(1+\rho_3)K^*(k')u^{(a)} = \frac{4\pi f_2^2\hbar^2c^2}{\kappa^2\sqrt{\varepsilon_k\varepsilon_{k'}}}\cdot\{(\vec{k}\vec{k}') + i[\vec{k}\times\vec{k}']_z\}$$

$$\therefore\quad W_I(H) = -\frac{\pi f_2^2\hbar^3c^3e}{\kappa^2L^6}\sum_{k,k'}\frac{(\vec{k}\vec{k}')+i[\vec{k}\times\vec{k}']_z}{\varepsilon_k^2\varepsilon_{k'}^2}\int_V(\vec{A},\vec{k}+\vec{k}')\,e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}$$

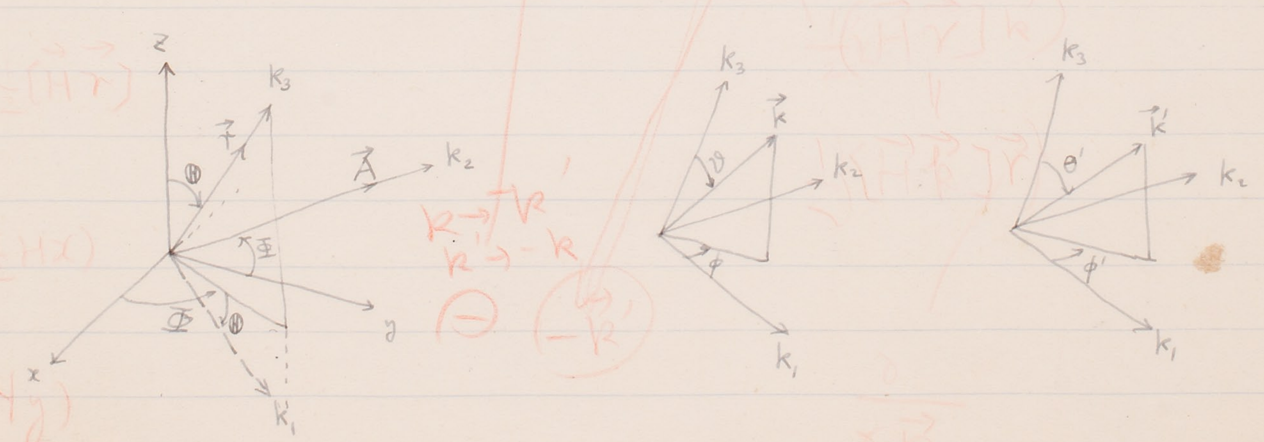
$$= -\frac{2\pi f_2^2e}{\hbar c\kappa^2L^6}\sum_{k,k'}\frac{(\vec{k}\vec{k}')}{(k^2+\kappa^2)(k'^2+\kappa^2)}\int_V(\vec{A},\vec{k})\cos(\vec{k}'-\vec{k},\vec{r})\,d\vec{r}$$

$$+\frac{2\pi f_2^2e}{\hbar c\kappa^2L^6}\sum_{k,k'}\frac{[\vec{k}\times\vec{k}']_z}{(k^2+\kappa^2)(k'^2+\kappa^2)}\cdot\int_V(\vec{A},\vec{k})\sin(\vec{k}'-\vec{k},\vec{r})\,d\vec{r}$$

$L\to\infty$ トシテ $\vec{k},\vec{k}'$ ニツイテノ積分ニ直セバ

$$= -\frac{f_2^2e}{(2\pi)^5\cdot\hbar c\kappa^2}\iiint\frac{(\vec{k},\vec{k}')\cdot(\vec{A},\vec{k})\cos(\vec{k}'-\vec{k},\vec{r})}{(k^2+\kappa^2)(k'^2+\kappa^2)}\,d\vec{r}\,d\vec{k}\,d\vec{k}'$$

$$+\frac{f_2^2e}{(2\pi)^5\cdot\hbar c\kappa^2}\iiint\frac{[\vec{k}\times\vec{k}']_z\,(\vec{A},\vec{k})\cdot\sin(\vec{k}'-\vec{k},\vec{r})}{(k^2+\kappa^2)(k'^2+\kappa^2)}\,d\vec{r}\,d\vec{k}\,d\vec{k}'$$



6.

$(\vec{A}\vec{k}) = \frac{1}{2}[\vec{H}\vec{r}]\vec{k} = \frac{1}{2}\vec{H}[\vec{r}\vec{k}]$

$H = H_z$ ニ ~~トレバ~~ $\rightarrow \frac{1}{2}H(xk_y - yk_x)$

fig. ノ様ニ $\vec{k}$-space, $\vec{k}'$-space, 各座標軸ヲトル

$$\left\{\begin{aligned}
(\vec{k}\vec{k}') &= kk'(\sin\vartheta\cos\phi\sin\vartheta'\cos\phi' + \sin\vartheta\sin\phi\sin\vartheta'\sin\phi' + \cos\vartheta\cos\vartheta') \\
(\vec{A},\vec{k}) &= |\vec{A}|\,k\sin\vartheta\sin\phi = \frac{H}{2}kr\sin\Theta\sin\vartheta\sin\phi, \\
[\vec{k}\times\vec{k}']_z &= kk'\{\cos\Theta\cdot\sin\vartheta\sin\vartheta'(\cos\phi\sin\phi' - \sin\phi\cos\phi') \\
&\qquad + \sin\Theta(\cos\vartheta\sin\vartheta'\sin\phi' - \sin\vartheta\cos\vartheta'\sin\phi)\} \\
(\vec{k}'-\vec{k},\vec{r}) &= k'r\cos\vartheta' - kr\cos\vartheta.
\end{aligned}\right.$$

$$\int_0^{2\pi}\sin\phi\,d\phi = \int_0^{2\pi}\cos\phi\,d\phi = 0, \quad \int_0^{2\pi}\sin^2\phi\,d\phi = \pi$$

ヲ用ヒテ ϕ, ϕ' ニツイテ積分ヲ行フト, 結局

$$W_I(H) = -\frac{f_2^2 eH}{32\pi^3\hbar c\kappa^2}\iiint \frac{k^4 k'^3\cdot\sin(k'r\cos\vartheta' - kr\cos\vartheta)\sin^3\vartheta\sin\vartheta'\cos\vartheta'}{(k^2+\kappa^2)(k'^2+\kappa^2)}$$

$$\cdot r^3\sin^3\Theta\,dr\,d\Theta\,d\Phi\cdot dk\,dk'\,d\vartheta\,d\vartheta'$$

Θ, Φ ニツイテ積分スルト

$$= -\frac{f_2^2 eH}{12\pi^2\hbar c\kappa^2}\iiint \frac{k^4 k'^3\cdot\sin(k'r\cos\vartheta' - kr\cos\vartheta)\cdot\sin^3\vartheta\sin\vartheta'\cos\vartheta'}{(k^2+\kappa^2)(k'^2+\kappa^2)}$$

$$\cdot r^3\,dr\,dk\,dk'\,d\vartheta\,d\vartheta'$$

$$\int_0^\pi \sin(k'r\cos\vartheta' - kr\cos\vartheta)\cdot\sin\vartheta'\cos\vartheta'\,d\vartheta'$$

$$= \cos(kr\cos\vartheta)\int_0^\pi \sin(k'r\cos\vartheta')\sin\vartheta'\cos\vartheta'\,d\vartheta' - \sin(kr\cos\vartheta)\int_0^\pi\cos(k'r\cos\vartheta')\sin\vartheta'\cos\vartheta'\,d\vartheta'$$

$$= \cos(kr\cos\vartheta)\int_{-1}^{+1}\sin(k'rz)\cdot z\,dz - \sin(kr\cos\vartheta)\int_{-1}^{+1}\underbrace{\cos(k'rz)\cdot z}_{\text{odd fn}}\,dz.$$

$$= \cos(kr\cos\vartheta)\cdot\frac{2}{k'r}\left(\frac{\sin k'r}{k'r} - \cos k'r\right)$$

$$\int_0^{\pi} \cos(kr\cos\vartheta)\cdot\sin^3\vartheta\, d\vartheta = \int_{-1}^{+1} (1-z^2)\cdot\cos(krz)\, dz$$

$$= \frac{4}{k^2r^2}\left(\frac{\sin kr}{kr} - \cos kr\right)$$

$$\therefore\quad W_I(H) = -\frac{2f_2^2 eH}{3\pi^2\hbar c\kappa^2}\iiint_{0\,0\,0}^{\infty\infty\infty} \frac{k^2 k'^2}{(k^2+\kappa^2)(k'^2+\kappa^2)}\left(\frac{\sin kr}{kr} - \cos kr\right)\left(\frac{\sin k'r}{k'r} - \cos k'r\right)\cdot dr\, dk'\, dk$$

トコロガ

$$\int_0^{\infty}\left(\frac{\sin kr}{kr} - \cos kr\right)\left(\frac{\sin k'r}{k'r} - \cos k'r\right) dr$$

$$= \frac{1}{kk'}\int_0^{\infty}\frac{\sin kr\cdot\sin k'r}{r^2}\, dr - \frac{1}{k'}\int_0^{\infty}\frac{\sin k'r\cdot\cos kr}{r}\, dr - \frac{1}{k}\int_0^{\infty}\frac{\sin kr\cdot\cos k'r}{r}\, dr$$

$$+ \int_0^{\infty}\cos kr\cdot\cos k'r\cdot dr$$

(1st. int. = partial integration ヲナスト integrated part ハ 両端デ消失シ、残リノ積分ハ 第二、第三ノ積分ト互ニCancel スル。)

$$= \int_0^{\infty}\cos kr\cdot\cos k'r\cdot dr.$$

$$= \frac{1}{2}\int_0^{\infty}\left[\cos(k+k')r + \cos(k-k')r\right] dr$$

$$= \frac{1}{2}\lim_{r\to\infty}\left[\frac{\sin(k+k')r}{k+k'} + \frac{\sin(k-k')r}{k-k'}\right]$$

$$= \frac{\pi}{2}\left[\delta(k+k') + \delta(k-k')\right]$$

$k, k' > 0$ デアルカラ $\delta(k+k') = 0.$

$$= \frac{\pi}{2}\cdot\delta(k-k').$$

$$\therefore\quad W_I(H) = -\frac{f_2^2 eH}{3\pi\hbar c\kappa^2}\iint \frac{k^2\cdot k'^2}{(k^2+\kappa^2)(k'^2+\kappa^2)}\cdot\delta(k-k')\cdot dk\, dk'$$

$(\sigma\mathbf{k})(\sigma\mathbf{k}') = (\mathbf{k}\,\mathbf{k}') + i\sigma[\mathbf{k}\,\mathbf{k}']$

$(\sigma_x k_x)(\sigma_y k_y') \to i\sigma_z k_x k_y'$

$$W_I = -\frac{\pi f_2^2 e}{\hbar c \varkappa^2 L^6} \sum_{kk'} \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} \int_V (\vec{A},\, \vec{k}+\vec{k}')\, e^{i(\vec{k}'-\vec{k})\vec{r}}\, d\vec{r}$$ ノ計算

$\left\{\frac{L^3}{(2\pi)^3}\right\}^2$ ヲカケテ $\sum_{k,k'}$ ヲ積分ニ直スト

$$W_I = -\frac{\pi f_2^2 e}{\hbar c \varkappa^2 L^6} \frac{L^6}{(2\pi)^6} \iint \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} d\vec{k}\, d\vec{k}' \int_V (\vec{A}, \vec{k}+\vec{k}')\, e^{i(\vec{k}'-\vec{k},\vec{r})}\, d\vec{r}.$$

コレハ $\vec{k}$ ト $\vec{k}'$ ニツイテ 對稱ナルから

$$= -\frac{f_2^2 e}{32\hbar c\varkappa^2\pi^5} \iint \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} d\vec{k}\, d\vec{k}' \int_V (\vec{A},\vec{k})\, e^{i(\vec{k}'-\vec{k},\vec{r})}\, d\vec{r}.$$

vector potential $\vec{A}$ ノ分値ハ $\left(-\frac{H}{2}y,\ \frac{H}{2}x,\ 0\right)$
コレヲ入レテ

$$W_I = -\frac{H f_2^2 e}{64\hbar c\varkappa^2\pi^5} \iiint \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} (-k_x y + k_y x)\, e^{i(\vec{k}'-\vec{k},\vec{r})}\, d\vec{r}.$$

x分値ノミヲ計算スル.

$$I_x = -\iint \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} k_x \int_{-\infty}^{+\infty} e^{i(k_x'-k_x)x}\, dx \int_{-\infty}^{+\infty} y\, e^{i(k_y'-k_y)y}\, dy \int_{-\infty}^{+\infty} e^{i(k_z'-k_z)z}\, dz\, d\vec{k}\, d\vec{k}'$$

$$= -(2\pi)^2 \iint \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} k_x\, \delta(k_x'-k_x)\,\delta(k_z'-k_z)\, \frac{1}{i}\frac{d}{dk_y'} \int_{-\infty}^{+\infty} e^{i(k_y'-k_y)y}\, dy.$$

dk_y' デ部分積分シテ

$$= -\frac{(2\pi)^3}{i} \int_{-\infty}^{+\infty}\!\!\cdots\!\!\int \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)} k_x\, dk_x\, dk_y\, dk_z\, dk_x'\, dk_y'\, \delta(\vec{k}'-\vec{k}) \Bigg|_{k_y'=-\infty}^{k_y'=+\infty}$$

$$+ \frac{(2\pi)^3}{i} \int_{-\infty}^{+\infty}\!\!\cdots\!\!\int \frac{d}{dk_y'} \frac{(k_x k_x' + k_y k_y' + k_z k_z') + i(k_x k_y' - k_y k_x')}{(k^2+\varkappa^2)(k_x'^2 + k_y'^2 + k_z'^2 + \varkappa^2)} k_x\, \delta(\vec{k}'-\vec{k})\, dk_x \cdots dk_x'\, dk_y'\, dk_z'$$

第一項ハ $k_y' = \pm\infty$ ヲ入レレバ integrand ガ 0 トナリ消エル.

$$= \frac{(2\pi)^3}{i} \iint \frac{(k_y + ik_x)(k'^2+\varkappa^2) - 2k_y'\{(\vec{k}\vec{k}') + i(k_x k_y' - k_y k_x')\}}{(k^2+\varkappa^2)(k_x'^2 + k_y'^2 + k_z'^2 + \varkappa^2)^2} k_x\, \delta(\vec{k}'-\vec{k})\, d\vec{k}\, d\vec{k}'$$

従ッテ　$l_x = \frac{(2\pi)^3}{i} \int \frac{-k_x k_y(\kappa^2 - k^2) + i k_x^2 (k^2+\kappa^2)}{(k^2+\kappa^2)^3} d\vec{k}.$

同様ニシテ y 分値ヲ計算スル。

$$l_y = \iint \frac{(\vec{k},\vec{k}') + i[\vec{k}\times\vec{k}']_z}{(k^2+\kappa^2)(k'^2+\kappa^2)} k_y \int_{-\infty}^{+\infty} x e^{i(k_x'-k_x)x} dx \int_{-\infty}^{+\infty} e^{i(k_y'-k_y)y} dy \int_{-\infty}^{+\infty} e^{i(k_z'-k_z)z} dz\, d\vec{k}\, d\vec{k}'$$

$$= \frac{(2\pi)^3}{i} \iint \frac{d}{dk_x'} \frac{(\vec{k}\cdot\vec{k}') + i[\vec{k}\times\vec{k}']}{(k^2+\kappa^2)(k'^2+\kappa^2)} k_y \cdot \delta(\vec{k}'-\vec{k})\, d\vec{k}\, d\vec{k}'.$$

$$= \frac{(2\pi)^3}{i} \int \frac{-k_y k_x(\kappa^2-k^2) + i k_y^2 (k^2+\kappa^2)}{(k^2+\kappa^2)^3} d\vec{k}.$$

コレヲ加ヘ合ハセテ

$$W_I = -\frac{f_2^2 e H}{64 \hbar c \kappa^2 \pi^5} (l_x + l_y)$$

$$= -\frac{8 f_2^2 e H \pi^3}{64 \hbar c \kappa^2 \pi^5 i} \int \frac{i(k_x^2+k_y^2)}{(k^2+\kappa^2)^2} d\vec{k}.$$

$$= -\frac{f_2^2 e H}{8 \hbar c \kappa^2 \pi^2} \int \frac{(k_x^2+k_y^2)}{(k^2+\kappa^2)^2} d\vec{k}.$$

$$\int \frac{k_x^2+k_y^2}{(k^2+\kappa^2)^2} d\vec{k} = \int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{k^2 \sin^2\theta}{(k^2+\kappa^2)^2} k^2 dk \sin\theta\, d\theta\, d\phi$$

$$= 2\pi \int_0^\infty \int_0^\pi \frac{k^4}{(k^2+\kappa^2)^2} \sin^3\theta\, d\theta$$

$$= \frac{8\pi}{3} \int_0^\infty \frac{k^4}{(k^2+\kappa^2)^2}$$

$$\therefore\ W_I = -\frac{f_2^2 e H}{3 \hbar c \kappa^2 \pi} \int_0^\infty \frac{k^4\, dk}{(k^2+\kappa^2)^2}.$$

$W_I = \mu_n (\sigma H)$　$\frac{e\hbar}{\mu c}$　$H \cdot \frac{e\hbar}{\mu c} \cdot \frac{\hbar^2}{\mu c}$　$\frac{f_2^2}{3\pi}$

$\frac{k_0}{\kappa} = x_0$　$\frac{1}{\kappa} \int_0^{x_0} \frac{k^4 dk}{(k^2+\kappa^2)^2} = \int_0^{x_0} \frac{x^4 dx}{(1+x^2)^2}$　$\frac{1}{3\pi}$　$\hbar$　$\left(\frac{f_2^2}{\hbar c}\right)$

$\frac{k}{\kappa} = x$

$$W_{\mathrm{I}} = -\frac{f_2^2 eH}{3\pi\hbar c\varkappa^2}\int\frac{k^4\,dk}{(k^2+\varkappa^2)^2}$$

22. Process II 及び III.

$$\mathrm{II}:\quad N \xrightarrow[NM]{} P + M_k^- \xrightarrow[NM]{} P + M_{k'}^+ + M_k^- \xrightarrow[ME]{} N.$$

$$\mathrm{III}:\quad N+(H) \xrightarrow[ME]{} N + M_k^+ + M_{k'}^- \xrightarrow[NM]{} P + M_{k'}^- \xrightarrow[NM]{} N.$$

上ト同様ニシテ計算スレバヨイ。

$$W_{\mathrm{II}}(H) = \sum_{k,k'}\sum\frac{H^{\mathrm{II}}_{ok}\,H^{\mathrm{II}}_{k'o}\cdot H^{\mathrm{II}}_{kk'}}{(E_k+\varepsilon_k-Mc^2)(E_{k''}+\varepsilon_k+\varepsilon_{k'}-Mc^2)}$$

$$(\vec{k}'' = \vec{k}-\vec{k}').$$

$$\left\{\begin{aligned} H^{\mathrm{II}}_{ok} &= \frac{1}{\sqrt{L^3}}\cdot u^{(a)*}\kappa(k)\,u^{(b)} \\ H^{\mathrm{II}}_{k'o} &= \frac{1}{\sqrt{L^3}}\cdot u^{(b)*}\cdot\kappa_1(k)\,u^{(a)} \\ H^{\mathrm{II}}_{kk'} &= \frac{\hbar ec}{2L^3}\cdot\frac{1}{\sqrt{\varepsilon_k\varepsilon_{k'}}}\int_V(\vec{A},\vec{k}+\vec{k}')\,e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}\end{aligned}\right.$$

$$\kappa_1(k) \equiv \alpha(k)\rho_1 - \beta(k)(\vec{\sigma},\vec{k}) + \gamma(k)\rho_2.$$

21. ノ時ト同じ近似デ計算スルト.

$$W_{\mathrm{II}}(H) = -\frac{\pi f_2^2\hbar^3c^3e}{\varkappa^2L^6}\sum_{k,k'}\frac{(\vec{k}\vec{k}')+i[\vec{k}\times\vec{k}']_z}{\varepsilon_k^2\varepsilon_{k'}(\varepsilon_k+\varepsilon_{k'})}\int_V(\vec{A},\vec{k}+\vec{k}')\,e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}$$

同様ニシテ

$$W_{\mathrm{III}}(H) = -\frac{\pi f_2^2\hbar^3c^3e}{\varkappa^2L^6}\sum_{k,k'}\frac{(\vec{k}\vec{k}')+i[\vec{k}\times\vec{k}']_z}{\varepsilon_k\cdot\varepsilon_{k'}^2(\varepsilon_k+\varepsilon_{k'})}\int_V(\vec{A},\vec{k}+\vec{k}')\cdot e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}$$

$$\frac{1}{\varepsilon_k^2\cdot\varepsilon_{k'}(\varepsilon_k+\varepsilon_{k'})} + \frac{1}{\varepsilon_k\varepsilon_{k'}^2(\varepsilon_k+\varepsilon_{k'})} = \frac{1}{\varepsilon_k^2\varepsilon_{k'}^2}$$

$$\therefore\ W_{\mathrm{II}}(H) + W_{\mathrm{III}}(H) = -\frac{\pi f_2^2\hbar^3c^3e}{\varkappa^2L^6}\sum_{k,k'}\frac{(\vec{k},\vec{k}')+i[\vec{k}\times\vec{k}']_z}{\varepsilon_k^2\varepsilon_{k'}^2}\int_V(\vec{A},\vec{k}+\vec{k}')\,e^{i(\vec{k}'-\vec{k},\vec{r})}\,d\vec{r}$$

$$= W_{\mathrm{I}}(H)$$

2₃. Process IV.

$$N \xrightarrow[NM]{} P + M_k^- \xrightarrow[NMH]{} N.$$

initial state (= final state), intermediate state ニ於ケル heavy particle ノ wave function ヲ夫々

$$\begin{cases} \psi_a = \psi_{a'} = \dfrac{1}{\sqrt{L^3}} \cdot u^{(a)} \\ \psi_b = \dfrac{1}{\sqrt{L^3}}\, u^{(b)} e^{-i(\vec{k},\vec{r})} \end{cases}$$

トスル。夫々ノ process ニ対スル Matrix element ヲ H_{0k}, H_{k0} トスレバ

$$\underline{H_{0k} = \frac{1}{\sqrt{L^3}} u^{(a)*} K(k)\, u^{(b)}} \qquad (13_1)$$

$$\underline{H_{k0} = \frac{1}{L^3\sqrt{L^3}} \cdot \frac{f_2 e}{\hbar c \kappa^2} \cdot \sqrt{\frac{2\pi \mu^2 c^4}{\varepsilon_k}} \int_V u^{(b)*} (\vec{A},\vec{\sigma})\, u^{(a)}\, d\vec{r}} \qquad (13_2)$$

故ニ Process IV ニヨル Perturbation energy $W_{IV}(H)$ ハ

$$W_{IV}(H) = \sum_{k}\sum \frac{H_{0k} H_{k0}}{E_k + \varepsilon_k - Mc^2}$$

$$= \frac{f_2 e \cdot \sqrt{2\pi\mu^2 c^4}}{\hbar c \kappa^2 \cdot L^6} \cdot \sum_k \sum \int_V \frac{u^{(a)*} K(k)\, u^{(b)} \cdot u^{(b)*} (\vec{A},\vec{\sigma})\, u^{(a)}}{\sqrt{\varepsilon_k} \cdot [E_k + \varepsilon_k - Mc^2]}\, d\vec{r}$$

$Mc^2 \gg \hbar c k,\ \hbar c \kappa$ トスレバ

$$\simeq \frac{f_2 e \sqrt{2\pi\mu^2 c^4}}{2\hbar c \kappa^2 \cdot L^6} \cdot \sum_k \int_V \frac{u^{(a)*} K(k) (1+\rho_3) (\vec{A}\vec{\sigma})\, u^{(a)}}{\varepsilon_k \cdot \sqrt{\varepsilon_k}}\, d\vec{r}$$

$$u^{(a)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \qquad u^{(a)*} = (1\ 0\ 0\ 0) \quad \text{トスレバ}$$

$$u^{(a)*} K(k)(1+\rho_3)(\vec{A}\vec{\sigma})\, u^{(a)} = -\frac{2 f_2 \hbar c}{\kappa} \cdot \sqrt{\frac{2\pi}{\varepsilon_k}} \cdot \left[(\vec{k}\vec{A}) + i [\vec{k} \times \vec{A}]_z \right].$$

$$\therefore\quad W_{IV}(H) = -\frac{f_2 e\sqrt{2\pi\mu^2c^4}}{2\hbar c\kappa^2\cdot L^6}\cdot\frac{2f_2\hbar c\cdot\sqrt{2\pi}}{\kappa}\sum_k\int_V\frac{(\vec{A},\vec{k})+i[\vec{k}\times\vec{A}]_z}{\varepsilon_k^2}\,d\vec{r}$$

$$= -\frac{2\pi f_2^2\cdot\hbar e c}{\kappa^2 L^6}\cdot\sum_k\int_V\frac{(\vec{A},\vec{k})+i[\vec{k}\times\vec{A}]_z}{\hbar^2c^2(k^2+\kappa^2)}\,d\vec{r}$$

$$= -\frac{2\pi\cdot f_2^2 e}{\hbar c\kappa^2\cdot L^6}\cdot\sum_k\int_V\frac{(\vec{A},\vec{k})+i[\vec{k}\times\vec{A}]_z}{k^2+\kappa^2}\,d\vec{r}$$

$L\rightarrow\infty$ トシテ 積分ニ直スト $\left(\frac{1}{L^3}\int_V d\vec{r} \longrightarrow \int d\vec{r}\right)$.

$$= -\frac{f_2^2 e}{4\pi^2\hbar c\kappa^2}\iint\frac{(\vec{A},\vec{k})+i[\vec{k}\times\vec{A}]_z}{k^2+\kappa^2}\,d\vec{r}\,d\vec{k}$$

之ハ $\vec{k}$-space ノ polar angle ニツイテノ 積分ニヨッテ vanish スル。故ニ

$$W_{IV}(H) = 0.$$

24. Process. V.

$$N \xrightarrow[NME]{} P + M_k^- \xrightarrow[NM.]{} N$$

コノ時ノ Matrix elements ハ

$$\begin{cases} H_{0k} = \dfrac{1}{L^3\sqrt{L^3}}\cdot\dfrac{f_2 e}{\hbar c\kappa^2}\cdot\sqrt{\dfrac{2\pi\mu^2c^4}{\varepsilon_k}}\displaystyle\int_V u^{(a)*}(\vec{A},\vec{\sigma})\,u^{(b)}\,d\vec{r}\ . \\ H_{k0} = \dfrac{1}{\sqrt{L^3}}\cdot u^{(b)*}K^*(k)\,u^{(a)} \end{cases}$$

$$\therefore\quad W_V(H) = \frac{f_2 e\cdot\sqrt{2\pi\mu^2c^4}}{\hbar c\kappa^2\cdot L^6}\sum_k\sum\int_V\frac{u^{(a)*}(\vec{A},\vec{\sigma})\,u^{(b)}\cdot u^{(b)*}K^*(k)\,u^{(a)}}{\sqrt{\varepsilon_k}\cdot[E_k+\varepsilon_k-Mc^2]}\,d\vec{r}$$

$$\cong \frac{f_2 e\cdot\sqrt{2\pi\mu^2c^4}}{2\hbar c\kappa^2\cdot L^6}\sum_k\int_V\frac{u^{(a)*}(\vec{A},\vec{\sigma})(1+\rho_3)K^*(k)\,u^{(a)}}{\varepsilon_k\cdot\sqrt{\varepsilon_k}}\,d\vec{r}$$

$$u^{(a)*}(\vec{A},\vec{\sigma})(1+\rho_3)K^*(k)\,u^{(a)} = -\frac{2f_2\hbar c}{\kappa}\cdot\sqrt{\frac{2\pi}{\varepsilon_k}}\cdot\left[(\vec{k},\vec{A}) - i[\vec{k}\times\vec{A}]_z\right]$$

故ニ $W_{IV}(H)$ ノ時ト同様ニシテ $W_V(H) = 0.$

従ツテ 攝動エネルギー ハ

$$W(H) = W_{I} + W_{II} + W_{III} + W_{IV} + W_{V} = 2W_{I}$$

$$= -\frac{2f_2^2 eH}{3\pi\hbar c\varkappa^2}\cdot\int\frac{k^4\,dk}{(k^2+\varkappa^2)^2}.$$

$\mu \uparrow \uparrow H$ $\uparrow \sigma$

(+) ↓ (−)

但シ 上ニ於テ. W_{I}, W_{II}, W_{III} ヲ求メル時ニ $\psi_{a'} + \psi_{a}$ デアツタカラ. コノ $W(H)$ ハ heavy particle ノ self-energy ト見做ス事ガデキナイガ. heavy particle ノ recoil ヲ neglect シテ計算シテ得ル結果ガ丁度上ノ $W(H)$ ニナツテヰル。recoil ヲ考ニ入レルトキニハ 上ノ樣ニ heavy particle ガ free デアルトシテハ ~~[illegible]~~ ソノ magnetic moment ハワカラナイ。

$$W(H) = -\mu(\text{neutron})\,H$$

$$\mu(\text{neutron}) = \frac{2f_2^2}{3\pi\hbar c\varkappa^2}\int\frac{k^4\,dk}{(k^2+\varkappa^2)^2}$$

12.