Vector Meson Theory ニヨル Proton ノ Anomalous Magnetic Moment ノ計算。

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Vector Meson Theory ニヨル Proton ノ Anomalous Magnetic Moment ノ計算。

3次ノ攝動計算ヲ行フ。次ノ3ツノ過程ガ可能デアル。

(1). $P + (H) \xrightarrow[NM]{} N + M_k^+ + (H) \xrightarrow[ME]{} N + M_k^+ + (H) \xrightarrow[NM]{} P + (H)$

(2) $P + (H) \xrightarrow[NM]{} N + M_k^+ + (H) \xrightarrow[NM]{} P + M_k^- + M_k^+ + (H) \xrightarrow[ME]{} P + (H)$

(3) $P + (H) \xrightarrow[ME]{} P + M_k^+ + M_k^- + (H) \xrightarrow[NM]{} N + M_k^+ + (H) \xrightarrow[NM]{} P + (H)$

P : Proton; N : Neutron; M_k^+ : 運動量 $\vec{k}$ ヲ持ッタ正荷電ノ Meson; M_k^- : 運動量 $\vec{k}$ ヲ持ッタ負荷電ノ Meson; (H) : z-方向ニ強サ H ヲ持ッタ一様ノ磁場。 $\xrightarrow[ME]{}$, $\xrightarrow[NM]{}$ ハ夫々 Meson ト Electro-magnetic field, Heavy Particle ト Meson ノ interaction ニヨル Process ヲ示ス。

1. Interaction Energies ト Matrix Elements.

$$\begin{cases} H^{I}_{NM} = \tilde{u} \cdot \frac{f}{\kappa} \left[\pi (\vec{\sigma}, \operatorname{curl} \vec{\varphi}) + \pi^* (\vec{\sigma}, \operatorname{curl} \vec{\varphi})^* \right] \cdot u, \\ H^{II}_{NM} = \tilde{u} \cdot \frac{g}{\kappa} \left[\pi \operatorname{div} \vec{\psi} + \pi^* \operatorname{div} \vec{\psi}^* \right] \cdot u \\ H_{NM} = H^{I}_{NM} + H^{II}_{NM} \end{cases}$$

π^* : Proton ヲ Neutron ニ変ヘル operator,
$\vec{\varphi}$: Meson ノ wave function ノ transverse part.
$\vec{\psi}$: Meson ノ wave function ノ longitudinal part.
$\vec{\sigma}$: Pauli ノ spin vector (of the heavy particle).

u : heavy particle ノ wave function. (non-relativistic).

$$H_{ME}=\frac{ie}{4\pi\hbar c}\Big\{\big(\mathrm{curl}\,\vec{\phi},\,[\vec{A}\times(\vec{\phi}^*-\tfrac{1}{c\kappa}\cdot\dot{\vec{\psi}}^*)]\big)+\mathrm{div}\,\vec{\psi}\cdot\big(\vec{A},\,\vec{\psi}^*+\tfrac{1}{c\kappa}\dot{\vec{\phi}}^*\big)$$

$$+\frac{V}{c}(\vec{\phi}^*,\dot{\vec{\phi}})+\frac{V}{c}(\vec{\psi}^*,\dot{\vec{\psi}})-\text{compl. conj.}\Big\}$$

$$+\frac{e^2}{4\pi^2\hbar^2c^2}\Big\{|[\vec{A}\times\vec{\phi}]|^2+\frac{1}{c^2\kappa^2}|(\vec{A},\dot{\vec{\phi}})|^2+\frac{1}{c\kappa}(\vec{A}\cdot\vec{\phi}^*)(\vec{A}\cdot\dot{\vec{\psi}})$$

$$+\frac{1}{c\kappa}(\vec{A},\vec{\phi})(\vec{A},\dot{\vec{\psi}}^*)+\frac{1}{c\kappa}(\vec{A},\dot{\vec{\phi}}^*)(\vec{A},\vec{\psi})+\frac{1}{c\kappa}(\vec{A},\dot{\vec{\phi}})(\vec{A},\vec{\psi}^*)$$

$$+|(\vec{A},\vec{\psi})|^2+\frac{1}{c^2\kappa^2}|[\vec{A}\times\dot{\vec{\psi}}]|^2\Big\}.$$

今考ヘルノハ、z-方向ニ一様ナ磁場 H ガ存在スル場合ニ於ケル摂動エネルギーノ、H ニ比例スル項デアルカラ（$|\vec{A}|$ ガ H ニ比例スルコトヲ考ヘテ）、H_{ME} トシテハ

$$H_{ME}=\frac{ie}{4\pi\hbar c}\Big\{\big(\mathrm{curl}\,\vec{\phi},\,[\vec{A}\times(\vec{\phi}^*-\tfrac{1}{c\kappa}\dot{\vec{\psi}}^*)]\big)+\mathrm{div}\,\vec{\psi}\cdot\big(\vec{A},\,\vec{\psi}^*+\tfrac{1}{c\kappa}\dot{\vec{\phi}}^*\big)$$

$$-\text{compl. conj.}\Big\}$$

ヲトレバヨイ。更ニ上ノ H_{NM} ノ形カラ我々ノ計算ニ effect ヲ与ヘルノハ $\vec{\phi}$ ダケデアルカラ H_{ME} トシテハ更ニ簡単ニ

$$H_{ME}=\frac{ie}{4\pi\hbar c}\cdot\big(\mathrm{curl}\,\vec{\phi},\,[\vec{A}\times\vec{\phi}^*]\big)+\text{compl. conj.}\qquad(1)$$

$$\vec{A}=\left(-\frac{H}{2}y,\ \frac{H}{2}x,\ 0\right)$$

ヲ用ヒレバヨイ。又 H_{NM} トシテハ

$$H_{NM}=\tilde{u}\,\frac{f}{\lambda}\big[\Pi(\vec{\sigma},\mathrm{curl}\,\vec{\phi})+\Pi^*(\vec{\sigma},\mathrm{curl}\,\vec{\phi})^*\big]\qquad(2)$$

ヲ用ヒレバヨイ。

$\vec{\phi}, \vec{\phi}^*$ ヲ次ノ様ニ展開スル。

$$\begin{cases} \vec{\phi} = \frac{1}{\sqrt{L^3}} \sum_k i \cdot \sqrt{\frac{2\pi\hbar^2 c^2}{\varepsilon_k}} \cdot (a_k - b_k^*) \cdot \vec{j} \cdot e^{i(\vec{k}, \vec{r})} \\ \vec{\phi}^* = \frac{1}{\sqrt{L^3}} \cdot \sum_k -i \sqrt{\frac{2\pi\hbar^2 c^2}{\varepsilon_k}} \cdot (a_k^* - b_k) \cdot \vec{j} \cdot e^{-i(\vec{k}, \vec{r})} \end{cases} \quad (3)$$

$\varepsilon_k = \hbar c \sqrt{k^2 + \varkappa^2}$; $\varkappa = \frac{\mu c}{\hbar}$; μ: Meson ノ rest mass.

$\vec{j}$: $\vec{k}$ = 垂直ナ unit vector.

$$\begin{cases} [a_k, a_l^*] = [b_k, b_l^*] = \delta_{kl} \end{cases}$$

他ノ a's, b's ハ スベテ commute スル。

a_k^* : M_k^+ ヲ 1箇 emit スル operator.
b_k^* : M_k^- ヲ 〃 〃 〃 .
a_k : M_k^+ ヲ 1箇 absorb スル operator.
b_k : M_k^- ヲ 〃 〃 〃 .

Z-方向ノ spin 及ビ之ト反対方向ノ spin ヲ持ツ heavy particle ノ wave function ヲ夫々

$$\begin{cases} u^{(\alpha)} = \alpha e^{i(\vec{k}, \vec{r})} = \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix} \cdot e^{i(\vec{k}, \vec{r})} \\ u^{(\beta)} = \beta \cdot e^{i(\vec{k}, \vec{r})} = \begin{pmatrix} \beta_+ \\ \beta_- \end{pmatrix} \cdot e^{i(\vec{k}, \vec{r})} \end{cases} \quad (4)$$

トスレバ

$$\begin{cases} \alpha_+ = 1 \\ \alpha_- = 0 \end{cases} \qquad \begin{cases} \beta_+ = 0 \\ \beta_- = 1 \end{cases} \quad (4')$$

デアッテ

$$\alpha\tilde{\alpha}^* + \beta\tilde{\beta}^* = 1 \quad (\text{unit matrix}). \tag{5}$$

initial state 及ビ intermediate state ニ於ケル heavy particle ノ wave function ヲ夫々 $u^{(A)}$, $u^{(I)}$ デ示ス事ニスル。(3) ヲ (2) ニ入レテ

$$H^{NM} = \frac{f}{\kappa}\frac{1}{\sqrt{L^3}}\cdot\sum_k \sqrt{\frac{2\pi\hbar^2c^2}{\varepsilon_k}}\cdot \tilde{u}^{(I)}\cdot(a_k^* - b_k)(\vec{\sigma}, [\vec{j}\times\vec{k}])\cdot u^{(A)}\cdot \pi^* \cdot e^{-i(\vec{k},\vec{r})}$$

$$+\ \text{compl. conj.} \tag{6}$$

(3) ヲ (1) ニ入レテ

$$H^{ME} = \frac{\hbar e c}{2L^3}\cdot\sum_{k,k'}\frac{1}{\sqrt{\varepsilon_k\varepsilon_{k'}}}(a_k - b_k^*)(a_{k'}^* - b_{k'})([\vec{j}\times\vec{k}], [\vec{A}\times\vec{j}'])\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}$$

$$+\ \text{compl. conj.} \tag{7}$$

(6), (7) カラ 次ノ Matrix elements ヲ得ル。

$$\left\{\begin{aligned}
H^{NM}_{M_k^+\,\text{emission}} &= \frac{1}{\sqrt{L^3}}\cdot\frac{f}{\kappa}\cdot\sqrt{\frac{2\pi\hbar^2c^2}{\varepsilon_k}}\cdot\tilde{I}^*\cdot(\vec{\sigma},[\vec{j}\times\vec{k}])\cdot A = H_{k0}\\
H^{NM}_{M_k^+\,\text{absorption}} &= \frac{1}{\sqrt{L^3}}\cdot\frac{f}{\kappa}\sqrt{\frac{2\pi\hbar^2c^2}{\varepsilon_k}}\cdot\tilde{A}^*\cdot(\vec{\sigma},[\vec{j}\times\vec{k}])\cdot I = H_{0k}\\
H^{ME}_{M_k^+\to M_{k'}^+} &= \frac{\hbar e c}{2L^3}\cdot\frac{1}{\sqrt{\varepsilon_k\varepsilon_{k'}}}\int_V\{([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}']) + ([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\}\\
&\quad\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r} = H_{k'k}.\\
H^{NM}_{M_k^-\,\cancel{\text{absorption}}\,\text{emission}} &= -\frac{1}{\sqrt{L^3}}\cdot\frac{f}{\kappa}\sqrt{\frac{2\pi\hbar^2c^2}{\varepsilon_k}}\cdot\tilde{A}^*\cdot(\vec{\sigma},[\vec{j}\times\vec{k}])\,I. = H'_{k0}\\
H^{NM}_{M_k^-\,\text{absorption}} &= -\frac{1}{\sqrt{L^3}}\cdot\frac{f}{\kappa}\sqrt{\frac{2\pi\hbar^2c^2}{\varepsilon_k}}\cdot\tilde{I}^*(\vec{\sigma},[\vec{j}\times\vec{k}])\,A = H'_{0k}\\
H^{ME}_{M_k^+,M_{k'}^-\,\text{emission}} &= -\frac{\hbar e c}{2L^3}\cdot\frac{1}{\sqrt{\varepsilon_k\varepsilon_{k'}}}\int_V\{([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}']) + ([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\}\\
&\quad\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r} = H^e_{kk'}
\end{aligned}\right.$$

$$H^{ME}_{M^+_{k}M^-_{k'}\,absorption} = -\frac{\hbar e c}{2L^3}\cdot\frac{1}{\sqrt{\varepsilon_k\varepsilon_{k'}}}\int_V\left\{([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}'])+([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\right\}$$

$$\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r} = H^{a}_{kk'}. \qquad (8).$$

茲ニ　A, I ハ　夫々　α 或ハ β ニ等シイ。

2.　Process (1) ニヨル Perturbation Energy.

ハジメニ　Proton ガ静止シテヰルトスルト

$$W_I = \sum_{k,k'}\sum_{j,j'}\sum \frac{H_{k0}\cdot H_{kk'}\cdot H_{0k'}}{\varepsilon_k\cdot\varepsilon_{k'}}$$

ここニ　(7) ヲ入レルト

$$W_I = \frac{\pi f^2 e}{\hbar c\kappa^2\cdot L^6}\cdot\sum_{k,k'}\sum_{j,j'}\sum\frac{1}{(k^2+\kappa^2)(k'^2+\kappa^2)}\int_V\tilde{A}(\vec{\sigma},[\vec{j}'\times\vec{k}'])\,I\cdot\tilde{I}\,(\vec{\sigma},[\vec{j}\times\vec{k}])A$$

$$\cdot\left[([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}'])+([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\right]\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}.$$

茲ニ　$\sum$ ハ　スベテノ中間状態ニツイテノ和ヲ示シ、$\sum_{j,j'}$ ハ　$\vec{k},\vec{k}'$ ヲ一ツ定メタ時ノ　$\vec{j},\vec{j}'$ ノスベテノ方向ニツイテノ和ヲ示ス。ここニ ~~4~~ (5) ヲ用ヒ、且ハジメニ　Proton ガ z-方向ノ spin ヲ持ツテヰタトスルト

$$W_I = \frac{\pi f^2 e}{\hbar c\kappa^2 L^6}\cdot\sum_{k,k'}\sum_{j,j'}\frac{1}{(k^2+\kappa^2)(k'^2+\kappa^2)}\int_V\tilde{\alpha}\cdot(\vec{\sigma},[\vec{j}'\times\vec{k}'])(\vec{\sigma},[\vec{j}\times\vec{k}])\,\alpha$$

$$\cdot\left[([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}'])+([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\right]\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}.$$

$\sum_{j,j'}$ ヲ行ヒ、　$\alpha=\begin{pmatrix}1\\0\end{pmatrix}$,　$\tilde{\alpha}=(1\ 0)$ ヲ用ヒルト

$i\,\vec{\sigma}[(\vec{j}'\times\vec{k}')\times(\vec{j}\times\vec{k})]$

$$W_I = -\frac{\pi f^2 e}{\hbar c \varkappa^2 \cdot L^6}\sum_{k,k'}\frac{1}{(k^2+\varkappa^2)(k'^2+\varkappa^2)}\int_V \Big\{(\vec{k},\vec{k}')(\vec{A},\vec{k}+\vec{k}')$$

$$+ k^2(\vec{A},\vec{k}') + k'^2(\vec{A},\vec{k})\Big\}\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}$$

$$-\frac{i\pi f^2 e}{\hbar c\varkappa^2\cdot L^6}\sum_{k,k'}\frac{1}{(k^2+\varkappa^2)(k'^2+\varkappa^2)}\int_V\Big\{(\vec{A},\vec{k}+\vec{k}')[\vec{k}\times\vec{k}']_z$$

$$-(\vec{A},[\vec{k}\times\vec{k}'])(k_z+k_z')\Big\}\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}$$

$L\to\infty$ トスルト $\dfrac{1}{L^6}\sum_{k,k'} \longrightarrow \dfrac{1}{(2\pi)^6}\iint d\vec{k}\,d\vec{k}'$,

$\int_V d\vec{r} \longrightarrow \int_{全空間} d\vec{r}$ トナツテ

$$W_I = -\frac{\pi f^2 e}{64\pi^6\cdot\hbar c\varkappa^2}\iint\frac{1}{(k^2+\varkappa^2)(k'^2+\varkappa^2)}\int\Big\{(\vec{k}\vec{k}')(\vec{A}.\vec{k}+\vec{k}')+k^2(\vec{A}.\vec{k}')+k'^2(\vec{A}.\vec{k})\Big\}$$

$$\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}\,d\vec{k}\,d\vec{k}'$$

$$-\frac{i\pi f^2 e}{64\pi^6\hbar c\varkappa^2}\iint\frac{1}{(k^2+\varkappa^2)(k'^2+\varkappa^2)}\int\Big\{(\vec{A},\vec{k}+\vec{k}')[\vec{k}\times\vec{k}']_z$$

$$-(\vec{A},[\vec{k}\times\vec{k}'])(k_z+k_z')\Big\}e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}\,d\vec{k}\,d\vec{k}'.$$

1st. integral ハ $\vec{k}$- ヤ $\vec{k}'$ space ニ於ケル polar angle ニツイテノ積分ニヨツテ vanish スル。又、部分積分等ヲ用ヒテ計算スルト

$$\iint\frac{1}{(k^2+\varkappa^2)(k'^2+\varkappa^2)}\int\Big\{(\vec{A},\vec{k}+\vec{k}')[\vec{k}\times\vec{k}']_z-(\vec{A},[\vec{k}\times\vec{k}'])(k_z+k_z')\Big\}e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}\,d\vec{k}\,d\vec{k}'$$

$$=+4\pi^3 iH\iint\frac{(\vec{k}+\vec{k}',\vec{k})+(k_z+k_z')\cdot k_z}{(k^2+\varkappa^2)(k'^2+\varkappa^2)}\cdot\delta(\vec{k}-\vec{k}')\,d\vec{k}\,d\vec{k}'$$

$$=8\pi^3 iH\iint\frac{k^2+k_z^2}{(k^2+\varkappa^2)^2}\,d\vec{k}$$

$$=8\pi^3 iH\,\frac{16\pi}{3}\cdot\int\frac{k^4\,dk}{(k^2+\varkappa^2)^2}.$$

$$\therefore\quad W_{\mathrm{I}}(H) = -\frac{i f^2 e}{64\pi^5 \cdot \hbar c\kappa^2}\cdot\frac{128\pi^4 i H}{3}\int\frac{k^4\,dk}{(k^2+\kappa^2)^2}$$

$$= \underline{\underline{\frac{2f^2 eH}{3\pi\cdot\hbar c\kappa^2}\int\frac{k^4\,dk}{(k^2+\kappa^2)^2}.}}$$

3. Process (2) 及ビ(3) ニヨル Perturbation Energy.

W_{I} ノ時ト同様ニシテ

$$W_{\mathrm{II}} = \sum_{k,k'}\sum_{j,j'}\sum \frac{H^{a}_{kk'}\cdot H'_{k'0}\cdot H_{k0}}{\varepsilon_k\cdot(\varepsilon_k+\varepsilon_{k'})}$$

之ニ(7)ヲ用ヒヽト

$$W_{\mathrm{II}} = \frac{\pi f^2 e\hbar^3 c^3}{\kappa^2 L^6}\sum_{k,k'}\sum_{j,j'}\sum\frac{1}{\varepsilon_k^2\varepsilon_{k'}(\varepsilon_k+\varepsilon_{k'})}\int_V \tilde{A}(\vec{\sigma},[\vec{j}'\times\vec{k}'])\,I\cdot\tilde{I}(\vec{\sigma},[\vec{j}\times\vec{k}])A$$

$$\cdot\Big[([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}'])+([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\Big]\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}$$

同様ニシテ

$$W_{\mathrm{III}} = \frac{\pi f^2 e\hbar^3 c^3}{\kappa^2 L^6}\sum_{k,k'}\sum_{j,j'}\sum\frac{1}{(\varepsilon_k+\varepsilon_{k'})\cdot\varepsilon_k\varepsilon_{k'}^2}\int_V \tilde{A}(\vec{\sigma},[\vec{j}'\times\vec{k}'])\,I\cdot\tilde{I}(\vec{\sigma},[\vec{j}\times\vec{k}])A$$

$$\cdot\Big[([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}'])+([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\Big]\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}.$$

$$\frac{1}{\varepsilon_k^2\varepsilon_{k'}(\varepsilon_k+\varepsilon_{k'})}+\frac{1}{(\varepsilon_k+\varepsilon_{k'})\varepsilon_k\varepsilon_{k'}^2} = \frac{1}{\varepsilon_k^2\varepsilon_{k'}^2} = \frac{1}{\hbar^4 c^4(k^2+\kappa^2)(k'^2+\kappa^2)}$$

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$$W_{\mathrm{II}} + W_{\mathrm{III}}$$

$$= \frac{\pi f^2 e}{\hbar c\kappa^2 L^6}\sum_{k,k'}\sum_{j,j'}\sum\frac{1}{(k^2+\kappa^2)(k'^2+\kappa^2)}\int_V \tilde{A}(\vec{\sigma},[\vec{j}'\times\vec{k}'])\,I\cdot\tilde{I}(\vec{\sigma},[\vec{j}\times\vec{k}])A$$

$$\cdot\Big[([\vec{j}\times\vec{k}],[\vec{A}\times\vec{j}'])+([\vec{j}'\times\vec{k}'],[\vec{A}\times\vec{j}])\Big]\cdot e^{i(\vec{k}-\vec{k}',\vec{r})}\,d\vec{r}$$

$$= W_{\mathrm{I}}.$$

従ッテ　全攝動エネルギー　W(H) ハ

$$W(H) = W_{I} + W_{II} + W_{III}$$

$$= 2W_{I}.$$

$$= \frac{4f^2eH}{3\pi\cdot\hbar c\varkappa^2}\int\frac{k^4\,dk}{(k^2+\varkappa^2)^2}$$

(–)