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Short Note

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Note ~~On~~ the Magnetic Moment of the Nucleon

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with charge + e ~~± e or e~~ and

In the present theory of the nuclear structure, the anomalous magnetic moments of the ~~nucl~~ neutron and the proton are ascribed to the ~~vir~~ those of the mesons, which ~~exist surround the nucleons~~ exist virtually around the nucleons. In fact the mesons, which ~~are~~ can be described by the vector field, ~~have~~ the with spin 1 intrinsic magnetic moment, in the direction of spin ~~+ or~~ , ~~so that~~ . of the magnitude of the order of $\frac{e\hbar}{2\mu c}$ so as to give rise to the ~~anomalous~~ additional magnetic moment ~~to the~~ in the case of the proton. Similarly the mesons with charge $-e$ and spin 1 ~~are~~ considered to be responsible in the case of the neutron. Now recent ~~de~~ modification trend of the meson theory indicates the possibility ~~to~~ of the existence of ~~the those~~ the mesons ~~wh~~ with spin 0, which can be described by the pseudoscalar field. Since the pseudoscalar mesons ~~with spin~~ have no intrinsic magnetic moment, it is not very clear whether they also contribute to the

~~made~~ performed a calculation in the case of the pseudoscalar meson
~~magnetic moment of the nucleon.)~~ ~~Thus it seems~~
~~worth while to calculate~~ ~~We calculated~~
and a closer investigation seems to be necessitated.
We thus ~~calculated~~ the required ~~terms~~ on the
line very much the same as in the case of the
vector mesons[1]. We want ~~to~~ give a brief account
of the result in the following.[2]

~~Consider~~ A neutron (N) ~~in a homogeneous~~ magnetic
field H with the spin described by a vector ③
changes its energy in the magnetic field H due upon
~~virtual existence~~ interaction of the by an amount W of the mesons as well as the protons
to the ~~full~~ process in the intermediate state with successive any two of

I. $N \rightarrow P + M_k^- \rightarrow P + M_{k'}^- \rightarrow N$,

where P and M_k^- ~~denote~~ mean a proton and a meson
with the ~~a~~ negative charge and the momentum $\hbar k$. If we
denote the matrix elements of the interaction energy
~~referring to relating~~ relating to the transitions between the initial state (0), the
first intermediate state (k) the second (k')
and the final state (0) ~~again denoted by~~ which is the
same as the initial state

Fröhlich,

1) Heitler and Kemmer, Proc. Roy. Soc. A 166 (1938), 154.
Yukawa, Sakata, ~~Kobayasi~~ and Taketani, Proc. Phys.
Math. Soc. Japan 20 (1938), 319.

2) Similar calculations are also made independently
by Kobayasi, Yamasaki and ~~Ma~~ Uma. The
authors are indebted to Dr. Kobayasi for the
communication of their results before publication.

respectively by H_{0k}, $H_{kk'}$, H_{k0}, ~~the~~ we obtain

$$W_{I} = \sum\sum_{k,k'} \frac{H_{0k}H_{kk'}H_{k0}}{(E_k+\varepsilon_k-Mc^2)(E_{k'}+\varepsilon_{k'}-Mc^2)}, \quad (1)$$

If we denote the energies of the initial state (0), the ~~in~~termediate states (k, k') by Mc^2, E_k, E where E_k, ε_k denote the energy of the proton and the meson both having the momentum $\hbar k$, respectively. The ~~second~~ first summation $\sum_{k,k'}$ in (1) ~~means~~ ~~refers to the~~ is performed over all possible values of the momentum ~~a~~ $\hbar k$, $\hbar k'$ of the meson in the intermediate state and the second summation $\sum$ refers to the spin of the proton. ~~in the~~ In the case of the pseudoscalar meson, ~~the we~~ there remain no need for extra degree of freedom in contrast to ~~that~~ vector mesons.

~~As~~ By using the expressions for the Hamiltonian in the pseudoscalar theory, which were ~~are~~ ~~der~~ ~~in the~~ by Tanikawa and Yukawa[1], and by expanding the meson field into plane waves, we can calculat~~age~~ the

[1] Tanikawa and Yukawa,

Vector

$$\mu_a = \mu_a' + \mu_a'' \qquad \varepsilon_k = \hbar c\sqrt{\varkappa^2+k^2}$$

$$\mu_a' = \frac{ie}{4\pi\hbar c}\frac{g^2}{\varkappa^2}\pi^*\pi\frac{1}{(2\pi)^6}\int\frac{1}{\varepsilon_k\varepsilon_{k'}}(\vec\sigma\,\mathrm{curl}\,\phi_{k'}^*)[\vec r\times\mathrm{curl}[\phi_k\times\phi_k^*]](\vec\sigma\,\mathrm{curl}\,\phi_k)\,d\vec k\,d\vec k'\,d\vec r$$

$$= \frac{ie}{4\hbar c}\frac{g^2}{\varkappa^2}\pi^*\pi\frac{1}{\pi^2}\int\frac{1}{k^2+\varkappa^2}\vec k([\vec\sigma\times\vec\sigma]\vec k)\,d\vec k$$

$$\mu_a'' = \quad '' \quad \frac{1}{2\pi^2}\int\frac{1}{k^2+\varkappa^2}(\vec k([\vec\sigma\times\vec\sigma]\vec k) - k^2[\vec\sigma\times\vec\sigma])\,d\vec k$$

Pseudo-vector

$$\mu_a^s = \frac{ie}{4\pi\hbar c}\frac{f^2}{\varkappa^2}\pi_1^*\pi\frac{1}{(2\pi)^6}\int\frac{1}{\varepsilon_k\varepsilon_{k'}}(\vec\sigma\,\mathrm{grad}\,\phi^*)[\vec r\times(\phi^*\mathrm{grad}\,\phi' - \phi'\mathrm{grad}\,\phi^*)](\vec\sigma\,\mathrm{grad}\,\phi')\,d\vec k\,d\vec k'\,d\vec r$$

$$= \frac{ie}{4\hbar c}\frac{f^2}{\varkappa^2}\frac{1}{\cancel{(2\pi)^2}_{\,2\pi^2}}\int\frac{1}{(k^2+\varkappa^2)}(\vec k([\sigma\times\sigma]\vec k) - k^2[\sigma\times\sigma])\,d\vec k$$

$$\mu_a' = \mu_a'' = \mu_a^s = \frac{e}{\hbar c}\frac{(g^2 \text{ or } f^2)}{\varkappa^2}\frac{2}{3\pi}\vec\sigma\int_{a\varkappa}^{\infty}\frac{k^4}{(k^2+\varkappa^2)^2}\,dk$$

$$\vec\mu_0 \equiv \vec\sigma\frac{e\hbar}{2Mc}$$

$$= \vec\mu_0\frac{(g^2 \text{ or } f^2)}{\hbar c}\frac{M}{\mu}\frac{4}{3\varkappa\pi}\int\frac{k^4}{(k^2+\varkappa^2)^2}\,dk = \vec\mu$$

Vector $-$ $2\vec\mu$!!

P. S $-$ $\vec\mu$.