INSTITUTE OF THEORETICAL PHYSICS
KYOTO IMPERIAL UNIVERSITY.

No. June 4 1941

On the Interaction of Scalar (or Pseudoscalar) Meson with the Electromagnetic Field

I

Vector meson と Electromagnetic field との interaction には種々の強い singularity が現われる。
Scalar (or pseudoscalar) の場合にどうであるかしらべてみると、

$$\left\{\left(p + \frac{e}{c}A\right)^2 - \left(\frac{W}{c} + \frac{e}{c}V\right)^2 + m^2c^2\right\}\psi = 0$$

or

$$\cancel{-\hbar^2\frac{\partial}{\partial s}}$$

$$\left\{\left(-i\hbar\,\mathrm{grad} + \frac{e}{c}A\right)^2 - \left(\frac{i\hbar}{c}\frac{\partial}{\partial t} + \frac{e}{c}V\right)^2 + m^2c^2\right\}\psi = 0$$

$$-eV = U$$

$A = 0$ と置けば

$$\Delta\psi + \frac{4\pi^2}{h^2c^2}\left\{(W-U)^2 - \underbrace{m^2c^4}_{W_0^2}\right\}\psi = 0$$

Coulomb field の場合には

$$U = -\frac{Ze^2}{r}$$

と書ける。 (Sommerfeld, Atombau, Ergänzungsband. Kap. I. §9.

$$\psi = R P_l^m(\cos\theta)e^{im\phi}$$

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \left(A + \frac{2B}{r} + \frac{C}{r^2}\right)R = 0$$

$$A = \frac{4\pi^2}{h^2c^2}(W^2 - W_0^2), \quad B = \frac{4\pi^2}{h^2c^2} 2e^2 W$$

$$C = -l(l+1) + \frac{4\pi^2 Z^2 e^4}{h^2c^2}.$$

$$A = -\frac{1}{r_0^2}, \quad \rho = \frac{r}{r_0}, \quad R = e^{-\rho/2} v$$

$$v'' + \left(\frac{2}{\rho} - 1\right)v' + \left[\left(\frac{B}{\sqrt{A}} - 1\right)\frac{1}{\rho} + \frac{C}{\rho^2}\right]v = 0$$

$$C = -l(l+1) + \alpha^2 Z^2$$

$$v = \rho^\gamma \sum a_\nu \rho^\nu$$

$$l = 0: \quad \gamma = \sqrt{\tfrac{1}{4} - \alpha^2 Z^2} - \frac{1}{2}$$

$$= -\alpha^2 Z^2 + \cdots \quad < 0.$$

$$\frac{W}{W_0} = \left\{1 + \frac{\alpha^2 Z^2}{\left[n_r + \sqrt{(l+\frac{1}{2})^2 - \alpha^2 Z^2} + \frac{1}{2}\right]^2}\right\}^{-1/2}$$

この時の S-state の wave function は $r=0$ で conv. でないけれども $\frac{1}{r}$ より弱いので差支ない。