5. August 2025
YITP, Kyoto

Exactly solvable space-time channels as tools for average-computation benchmarking

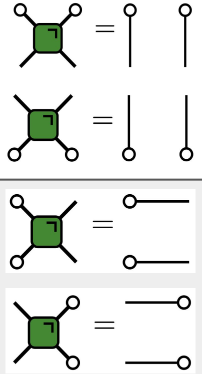
arXiv: 2507.18708

Trace preservation

Unitality

Left unitality

Right unitality



Pavel Kos

MAX-PLANCK-INSTITUT
FÜR QUANTENOPTIK

Non-equilibrium dynamics

What happens to a simple product quantum state under quantum evolution?

$|000\ldots\rangle$ under evolution under some Hamiltonian or quantum computer?

What are the **universal features**?
Universality classes of non-equilibrium matter?

I investigate correlations, entanglement and information spreading, local operator spreading, indicators of chaos and localization…

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If you don't care: contracting tensor network states

Outline

- Part I: Quantum dynamics
  - Setting: quantum **circuit** dynamics
  - Solvable models: **dual-unitary** quantum circuits
  - Solvable models: space-time quantum channels
- Part II: Intermezzo
  - Dual-isometric PEPS
- Part III: Exactly solvable space-time channels as tools for average-computation benchmarking of quantum devices
- Conclusions

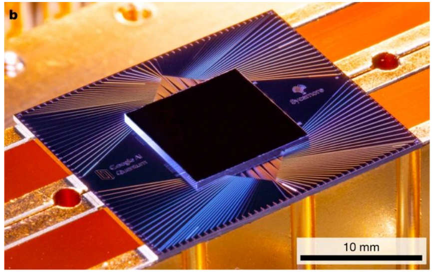
Part 1: Quantum dynamics

Introduction

- Behaviour of “generic”/chaotic quantum many-body systems?
- **Difficult** problem
- There are “statistically” solvable examples:
  - RMT, random unitary circuits, SYK
  - **Dual-unitary(DU)** models (even for clean systems)
  - Space-time quantum channels
  - ???

wikipedia.org

- Relevant for:
  - Non-equilibrium statistical mechanics, condensed matter
  - Quantum information and computation
  - High energy theory and quantum gravity
  - ...



Arute et al. Nature **574**, 505-510 (2019)

Setting

Simple setting with basic ingredients:

- Quantum mechanics
- Discrete space and time
- Locality

Setting

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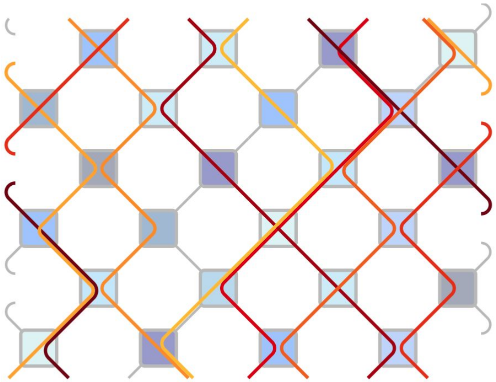
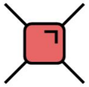
Quantum circuits

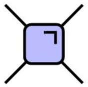


One can add conservation laws, gauge degrees of freedom [not in this talk]

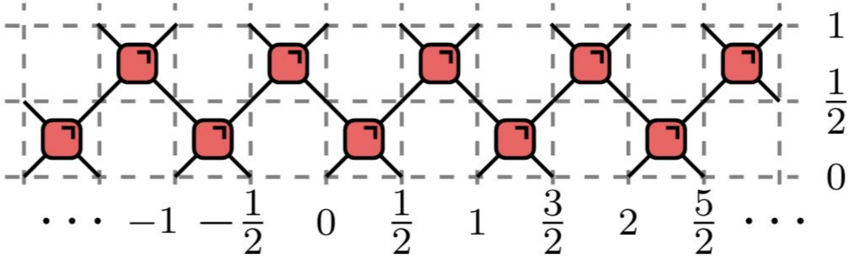
Setting: a Quantum Circuit

1+1D circuit made of two-qudit gates

$U =$ 

$U^\dagger =$ 

A Floquet dynamics, with

$\mathbb{U} =$ 

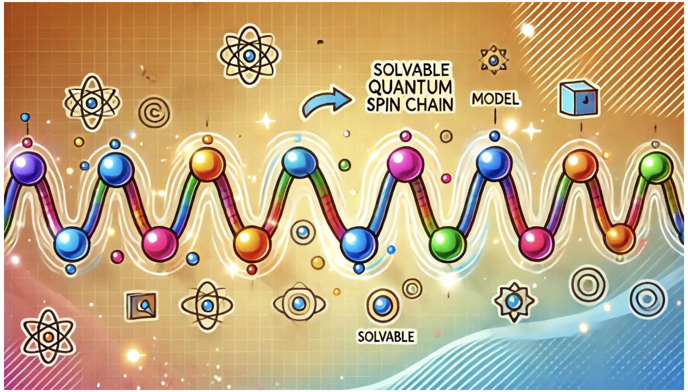
$$\mathbb{U}^e = U^{\otimes L} \qquad \mathbb{U} = \mathbb{U}^o \mathbb{U}^e = \mathbb{T}_{2L} U^{\otimes L} \mathbb{T}_{2L}^\dagger U^{\otimes L}$$

Solvability

Need an extra property for analytical treatment

Could average: random unitary circuits

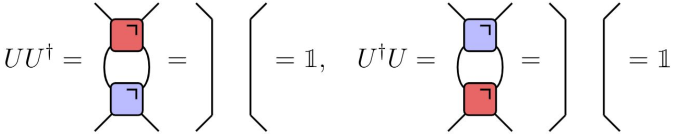
Instead:



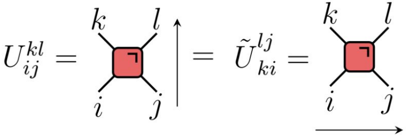
Dual-unitary (DU) Quantum Circuits

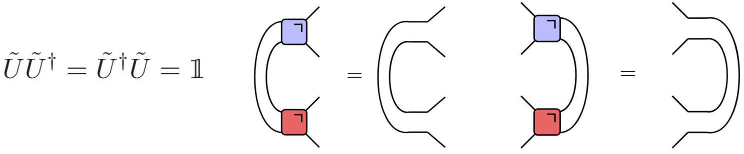
B Bertini, PK, T Prosen
PRL **123**, 210601 (2019)

- Unitarity:



- Dual-unitarity:





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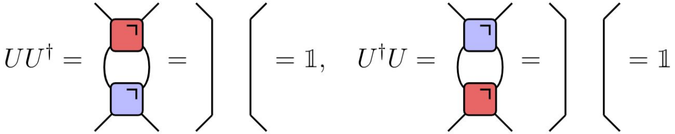
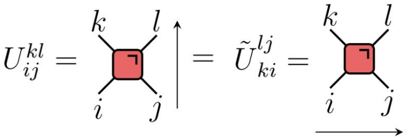
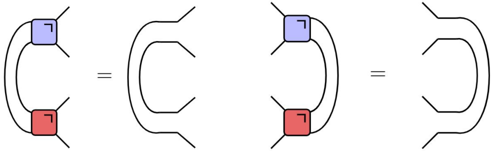


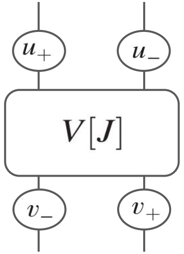
- Dual-unitarity:



$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger\tilde{U} = \mathbb{1}$



Generic dual-U(4) gate:

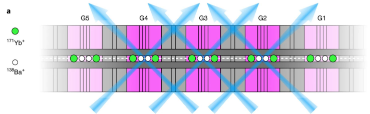


$$U = e^{i\phi}(u_+ \otimes u_-)V[J](v_- \otimes v_+) \quad u_\pm, v_\pm \in \mathrm{SU}(2)$$

$$V[J] = \exp\left[-i\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right)\right]$$

Dual-unitarity fixes 2 out of 16 real parameters w.r.t. generic U(4)

Contain universal gate set; free, integrable and **chaotic** models



Realised in experiments E. Chertkov et al. Nat. Phys. 18, 1074–1079 (2022), X. Mi et al. Science 374,1479-1483(2021)

Some overview of the results

I will show how this leads to solvable **correlation functions** on a simple example

Exact solutions also for: entanglement spreading, quantum chaos, operator spreading, deep thermalization, Hayden-Preskill recovery protocol, …

L Piroli, B Bertini, JI Cirac, T Prosen PRB 2020,
B Bertini, P Kos, T Prosen CMP 2021,
B Bertini, P Kos, T Prosen SciPost Phys. 2020
MA Rampp, R Moessner, PW Claeys PRL 2023
MA Rampp, PW Claeys Quantum 2024, …

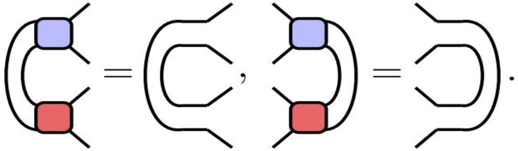
Generalizations of dual unitarity: perturbation theory, more general conditions, 2+1D…

PK, B Bertini, T Prosen PRX 2021
C Jonay, V Khemani, M Ippoliti, PRR 2021.
PK, G Styliaris Quantum 2023
XH Yu, Z Wang, PK Quantum 2024
MA Rampp, PW Claeys Quantum 2024
XH Yu, JI Cirac, PK*, G Styliaris* PRL 2024
RM Milbradt et al PRL 2023
Etc.

Introduction

There are surprising exact results for **dual-unitary(DU)** models

B. Bertini, PK, T. Prosen, PRL **123**, 210601 (2019)



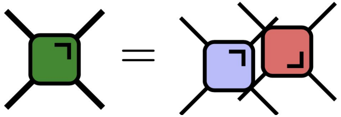
Computing correlations only requires space unitality

PK, B Bertini, T Prosen, PRX **11**, 011022 (2021)

We can demand only this **weaker condition** in open quantum dynamics

Folded picture - vectorization

$$|m\rangle\langle n| \xmapsto{\text{vec}} |m\rangle \otimes |n\rangle$$



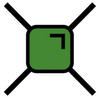
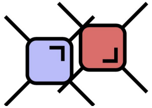
$$= U \otimes U^*$$

Circuits for open systems: local quantum channels

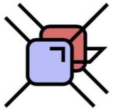
Kraus form $\rho(t+1) = \sum_k F_k \rho(t) F_k^\dagger$ $\qquad \sum_k F_k(\cdot)F_k^\dagger \overset{\text{vec}}{\longmapsto} \sum_k F_k \otimes F_k^*$

Local gates in vectorized form:

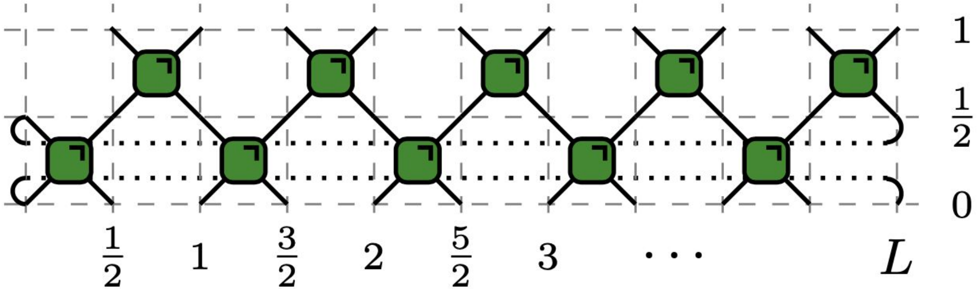
- Unitary

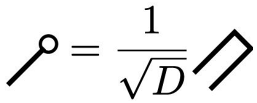
 $=$  $= U \otimes U^*$

- Generalization

$q =$  $=$  $= \sum_k E_k \otimes E_k^*$

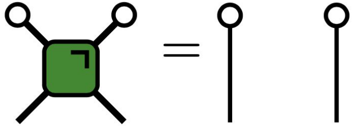
- 1 time step

$\mathbb{Q} =$ 



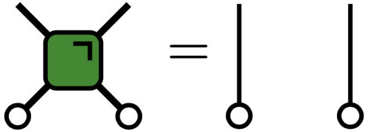
Unitality conditions

Trace preservation



$$\sum_k E_k^\dagger E_k = \mathbb{1}$$

Unitality



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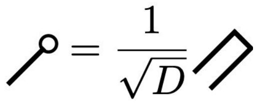
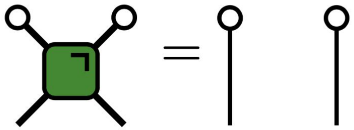


Trace preservation



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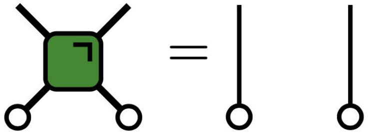
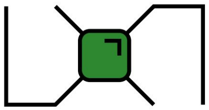
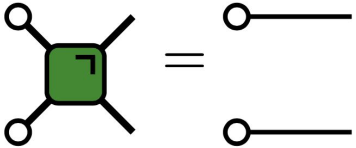
Unitality



$$\sum_k E_k E_k^\dagger = \mathbb{1}$$

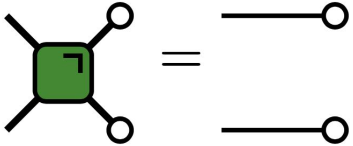
Left unitality

$\tilde{q} =$ 



$$\sum_k (\tilde{E}_k)^\dagger \tilde{E}_k = \mathbb{1}$$

Right unitality



$$\sum_k \tilde{E}_k (\tilde{E}_k)^\dagger = \mathbb{1}$$

weaker condition than du, need only 3

PK* and G. Styliaris*, Quantum 7, 1020 (2023)

Dimensions of different families

Type	Conditions	dimension	co-dimension
CPTP	i	$d^8 - d^4$	0
unital	i,ii	$(d^4-1)^2$	d^4-1
2-way	i,iii	$d^8 - 2d^4 + d^2$	$d^4 - d^2$
3-way	i-iii	$(d^4-1)^2 - (d^2-1)^2$	$2d^4 - 2d^2$
4-way	i-iv	$(d^4-1)^2 - 2(d^2-1)^2$	$3d^4 - 4d^2 + 1$

Many more parameters than in DU

3-way is enough to compute correlation functions!

Dual unitaries and 4-way unital quantum channels

$$U(J) = (W_1 \otimes W_2) \exp(iJs_3 \otimes s_3) S(V_1 \otimes V_2), \qquad (21)$$

P. W. Claeys and A. Lamacraft, PRL **126**, 100603 (2021)

Proposition III.2. *Any 4-way unital quantum channel can be decomposed as an affine combination of dual-unitary gates, i.e.,*

$$\begin{aligned} &(i)\text{-}(iv), \quad \Longrightarrow \\ &\qquad \exists \quad \lambda_k \in \mathbb{R},\ U_k \ \textit{dual unitary:} \\ &\qquad q = \sum_k \lambda_k U_k \otimes U_k^*, \quad \sum_k \lambda_k = 1\,. \qquad (22) \end{aligned}$$

In particular, dual-unitary gates of the form (21) *suffice.*

Unbiased average around DU gates → 4-way channels

One possible way to get 4 way channels

$$\mathcal{E}_p = \int d\lambda\, p(\lambda) U_\lambda(\cdot) U_\lambda^\dagger$$

$$U_\lambda = W_1 \otimes W_2 \exp\left(i \sum_{j=1}^{3} \theta_j \sigma_j \otimes \sigma_j \right) V_1 \otimes V_2$$

Symmetric average over around DU leads to 4 way. More precisely we need

$$p(\lambda) = p_{WV}(\lambda_W, \lambda_V) p_\theta(\theta_1, \theta_2, \theta_3)$$

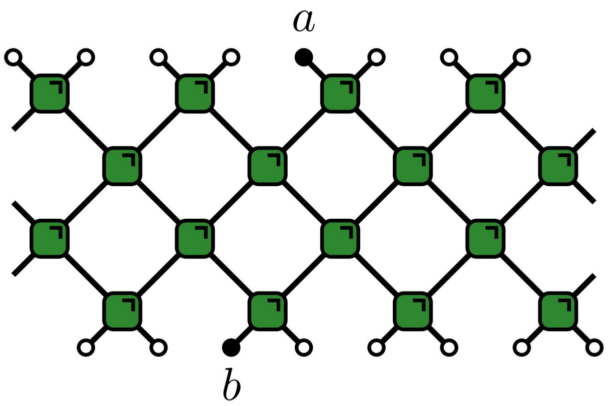
$$\begin{aligned} & p_\theta(\pi/4 + \delta_1, \pi/4 + \delta_2, \theta_3) \\ = & p_\theta(\pi/4 - \delta_1, \pi/4 + \delta_2, \theta_3) \\ = & p_\theta(\pi/4 + \delta_1, \pi/4 - \delta_2, \theta_3)\,, \end{aligned}$$

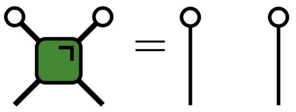
“Symmetric noise” even in the **absence** of DU results in solvability!

Relevant for experiments with DU:
E. Chertkov et al., Nat. Phys. 18, 1074-1079 (2022)
X. Mi et al., Science 374, 1479 (2021)

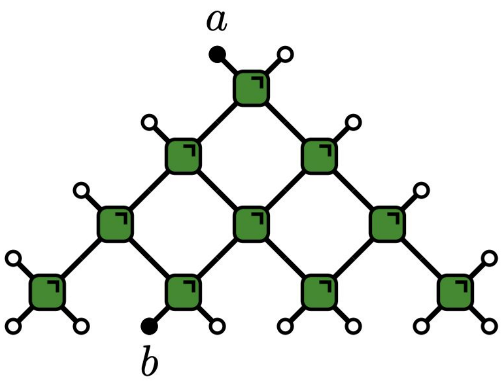
Spatio-temporal correlation functions

$$\langle a_i(t) b_j \rangle =$$





$$\langle a_i(t) b_j \rangle =$$



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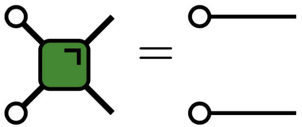
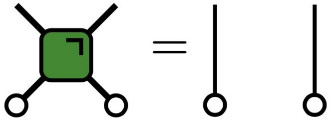
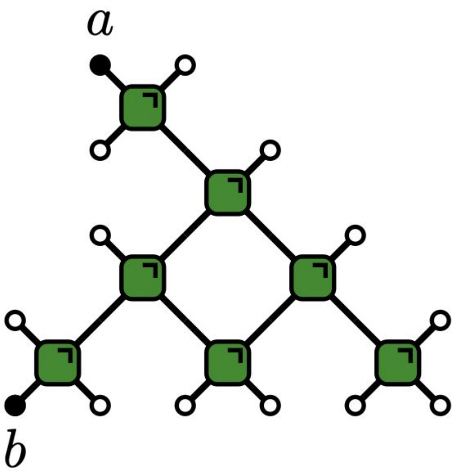




$\langle a_i(t) b_j \rangle =$ 

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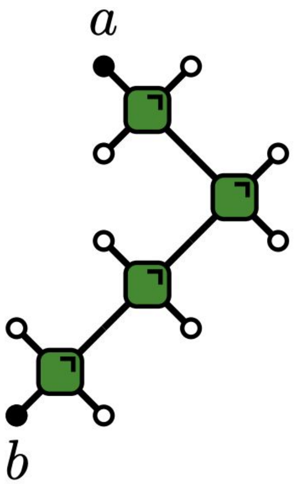
$\langle a_i(t) b_j \rangle =$ 

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Spatial correlation functions after a quench

Analogous with DU:
L. Piroli, B. Bertini, J. I. Cirac, and T. Prosen, PRB **101**, 094304 (2020)

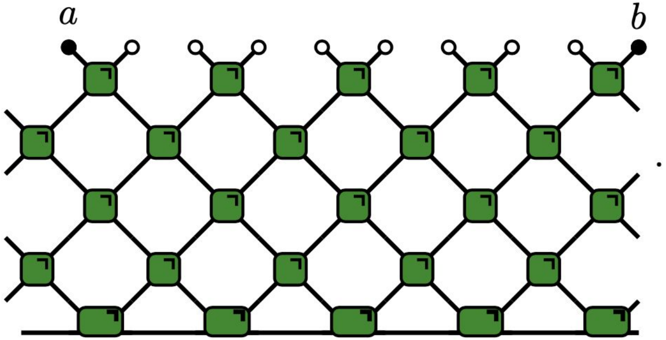
Start from matrix product density operator (MPDO) in a vectorised form

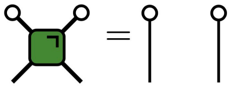
$$|\rho(0)\rangle = \sum_{s_{1/2},\ldots,s_L} \mathrm{Tr}\left[T^{(s_{1/2},s_1)} \ldots T^{(s_{L-1/2},s_L)}\right] |s_{1/2} \ldots s_L\rangle$$

$$= \frac{1}{d^L}$$  . (46)

Spatial correlation functions after a quench

$C_{ij}(t) =$





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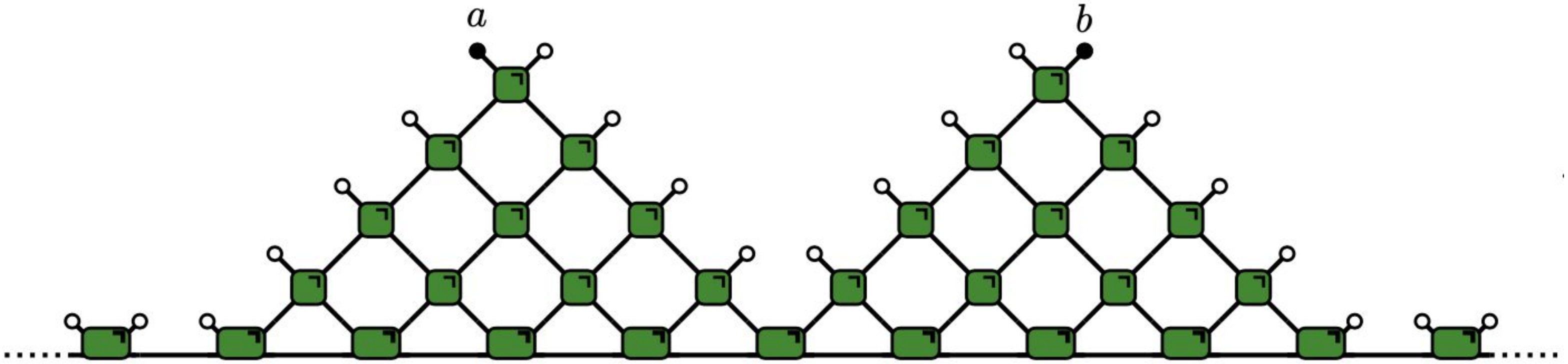

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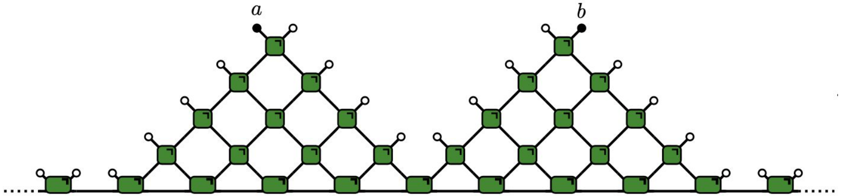
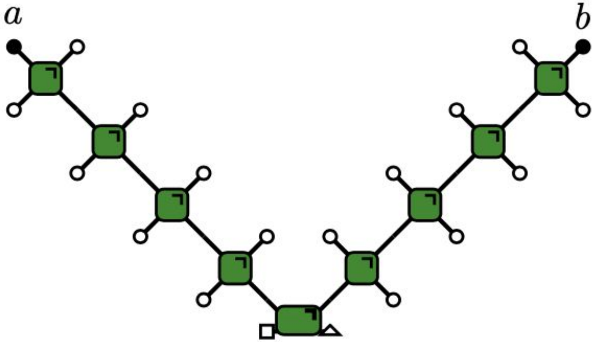
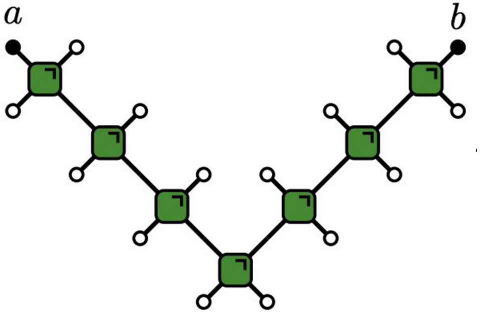
$C_{ij}(t) =$ 

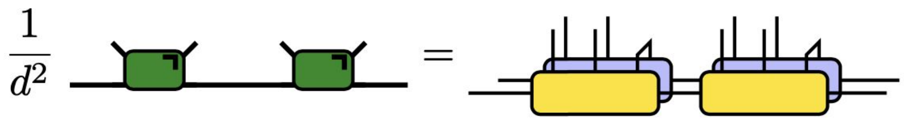
$E(0) =$  has unique leading fixed point $|\triangle\rangle\langle\square|$

Solvable MPDO 

$C_{ij}(t) =$ 

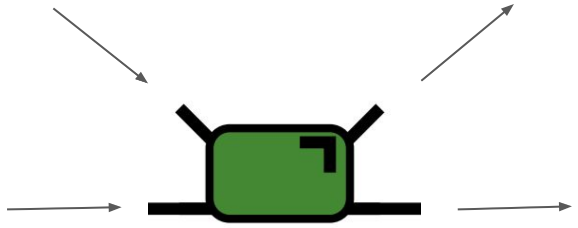
$C_{ij}(t) =$ 

Classification of solvable MPDOs with a local purification (LP)



Solvable MPDOs with LP are one to one with quantum channels

Analogous with DU:
L. Piroli, B. Bertini, J. I. Cirac, and T. Prosen, PRB **101**, 094304 (2020)



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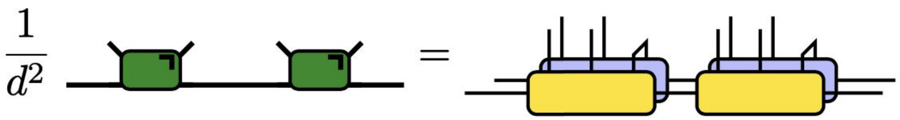

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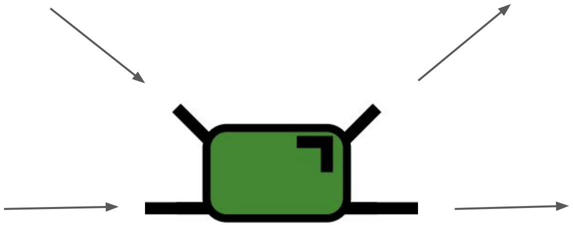


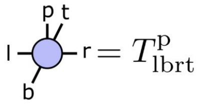
Bell pairs $|\cup \cdots \cup\rangle$

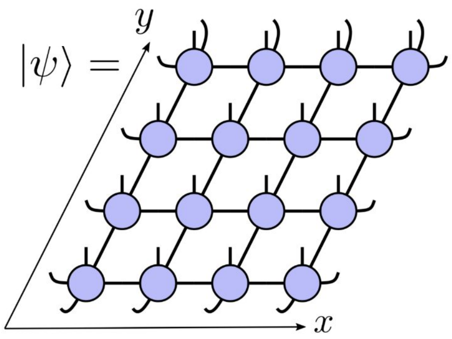
$\Phi(\sigma) = \alpha\sigma + (1-\alpha)\mathbb{1}$ Interpolates between $|\bigcirc \cdots \bigcirc\rangle$ and Bell pairs $|\cup \cdots \cup\rangle$

Outline

- Part I: Quantum dynamics
  - Setting: quantum **circuit** dynamics
  - Solvable models: **dual-unitary** quantum circuits
  - Solvable models: space-time quantum channels
- Part II: Intermezzo
  - Dual-isometric PEPS
- Part III: Exactly solvable space-time channels as tools for average-computation benchmarking of quantum devices
- Conclusions

TN 2D and beyond: projected entangled pair states (PEPS)

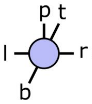




F. Verstraete and J. I. Cirac (2004), arXiv:cond-mat/0407066

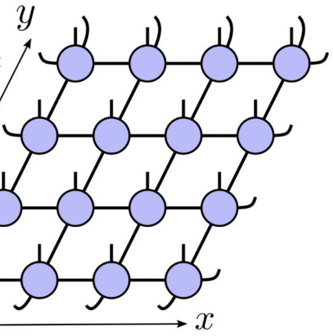
Capture area law entangled states
Good ansatz for ground states in 2D

2D and beyond: projected entangled pair states (PEPS)



$= T^{\mathrm{p}}_{\mathrm{lbrt}}$

$|\psi\rangle =$



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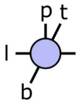
A lot of **success**, BUT:
Exact contraction of PEPS is #P-**hard** even for **typical** case (computing $\langle\psi|\hat{O}|\psi\rangle$)

N. Schuch, M. M. Wolf, F. Verstraete, and J. I. Cirac, PRL 98, 140506 (2007),
J. Haferkamp, D. Hangleiter, J. Eisert, and M. Gluza, PRR 2, 013010 (2020),

2D and beyond: projected entangled pair states (PEPS)



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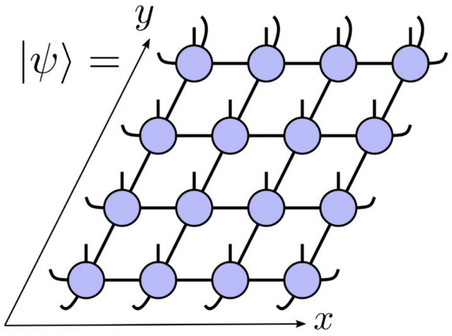

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J. Haferkamp, D. Hangleiter, J. Eisert, and M. Gluza, PRR 2, 013010 (2020),

Approximate contraction is also costly
Sometimes uncontrolled errors

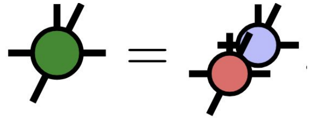
J. Haferkamp, D. Hangleiter, J. Eisert, and M. Gluza, PRR 2, 013010 (2020),
Romain Vasseur et al. PRB 100, 134203 (2019)
Sofía González-García et al. PRB 109, 235102 (2024),
Jielun Chen et al. arXiv:2404.19023
Jiaqing Jian et al. arXiv:2410.05414

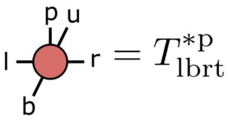
Hard or easy? Complexity phase transitions?

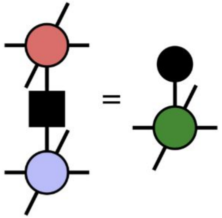
Vectorization

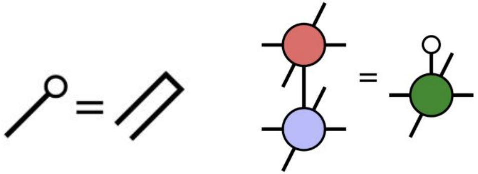
$$\sum_{ij} O_{ij} \, |i\rangle\langle j| \mapsto \sum_{ij} O_{ij} \, \langle i| \, \langle j|$$

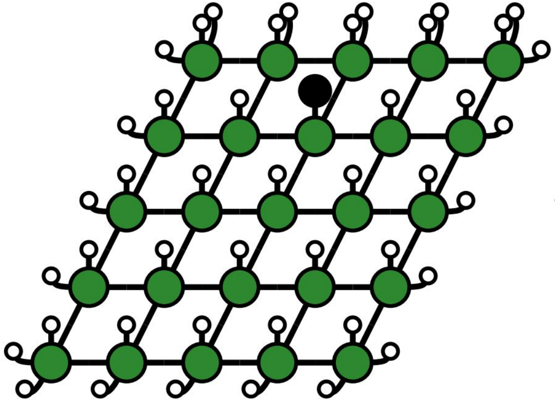
$$\langle\psi|\hat{O}|\psi\rangle = \langle O|(|\psi^*\rangle \otimes |\psi\rangle)$$

$T =$ 







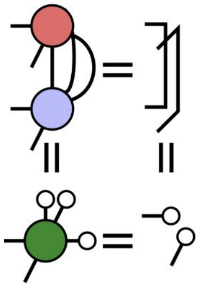
$\langle\psi|O|\psi\rangle =$ 

Isometric PEPS (iso-PEPS)

Extends the canonical form of 1D MPS to higher dimensions

M. P. Zaletel and F. Pollmann
PRL 124, 037201 (2020)

R. Haghshenas et al., PRB 100, 054404 (2019)



$$\sum_{\mathrm{r,t,p}} T^{\mathrm{p}}_{\mathrm{l_1 b_1 r t}} T^{*\mathrm{p}}_{\mathrm{l_2 b_2 r t}} = \delta_{\mathrm{l_1, l_2}} \delta_{\mathrm{b_1, b_2}}$$

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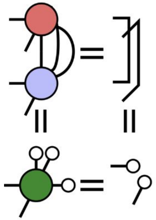

$$\sum_{\mathrm{r,t,p}} T^{\mathrm{p}}_{\mathrm{l_1 b_1 rt}} T^{*\mathrm{p}}_{\mathrm{l_2 b_2 rt}} = \delta_{\mathrm{l_1,l_2}}\delta_{\mathrm{b_1,b_2}}$$

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Extends the canonical form of 1D MPS to higher dimensions

Normalization; observables at orthogonal hypersurface can be computed
But moving hypersurface is still uncontrolled
Computing local observables is BQP-complete

D. Malz and R. Trivedi, arXiv:2402.07975

Preparation of all iso-PEPS is possible through sequential unitary circuits
Isometric condition defines a time axis

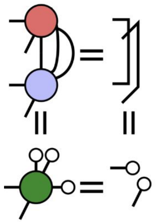
$$U^{\mathrm{prt}}_{\mathrm{lb}|0\rangle} = T^{\mathrm{p}}_{\mathrm{lbrt}}$$

Z.-Y. Wei, D. Malz, and J. I. Cirac,
PRL 128, 010607 (2022).

Can capture states with complex correlations, such as **topological** models and the associated phase transitions

Y.-J. Liu, K. Shtengel, and F. Pollmann, arXiv:2312.05079

Isometric PEPS (iso-PEPS)



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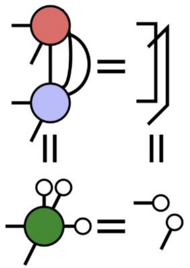
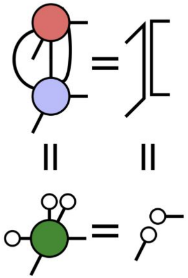
Y.-J. Liu, K. Shtengel, and F. Pollmann, arXiv:2312.05079

Dual-isometric PEPS (DI-PEPS)

XH Yu, JI Cirac, PK*, G Styliaris*,
PRL 133, 190401 (2024)



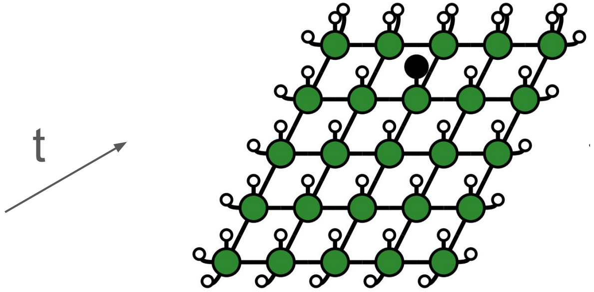
Additionally we introduce:



$$\sum_{\mathrm{l,t,p}} T^{\mathrm{p}}_{\mathrm{lb_1r_1t}} T^{*\mathrm{p}}_{\mathrm{lb_2r_2t}} = \delta_{\mathrm{r_1,r_2}} \delta_{\mathrm{b_1,b_2}}$$

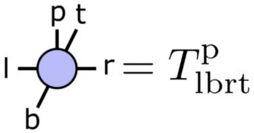
Correspondence with quantum channels

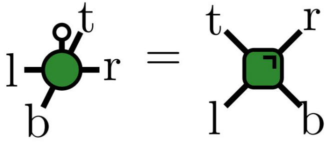
Calculation of expectation values in 2D PEPS ↔ 1+1D circuit of CP maps in virtual space

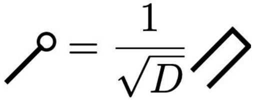


Kraus operators

$$E^{\mathrm{p}}_{\mathrm{tr,lb}} = T^{\mathrm{p}}_{\mathrm{lbrt}}$$







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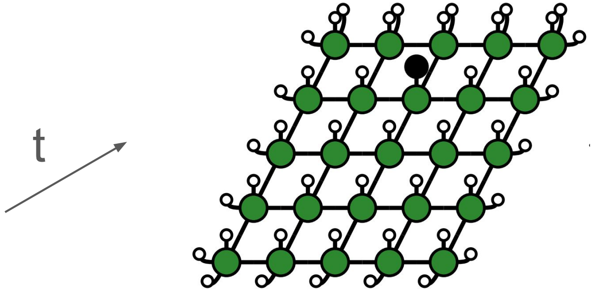

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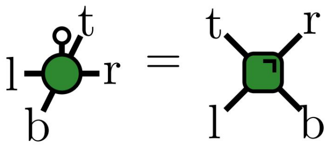


Isometric condition = trace preservation [hard problem in general]

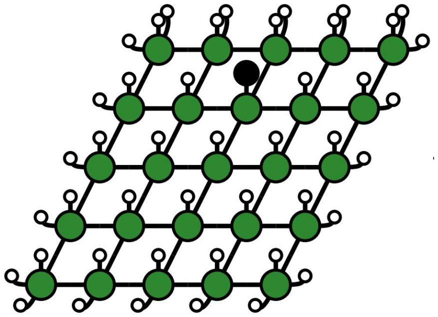
DI-PEPS = also space trace preservation

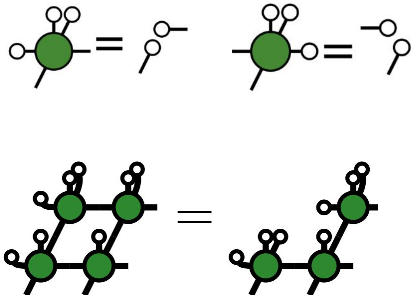
= space channels (2 unitality) [solvable*]

P Kos*, G Styliaris*
Quantum 7, 1020 (2023)

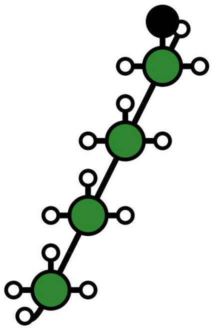
Observables

$$\langle\psi|O|\psi\rangle =$$





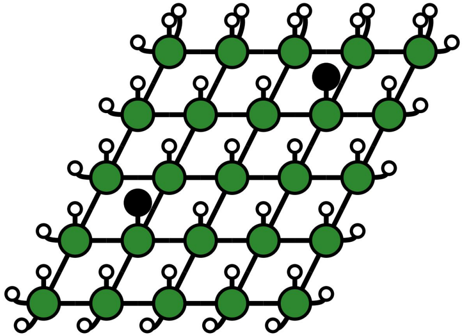
$$\langle\psi|O|\psi\rangle =$$

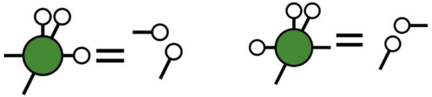


Local observables can be always computed efficiently!

Two-point correlations

$$\langle\psi|O_1O_2|\psi\rangle =$$





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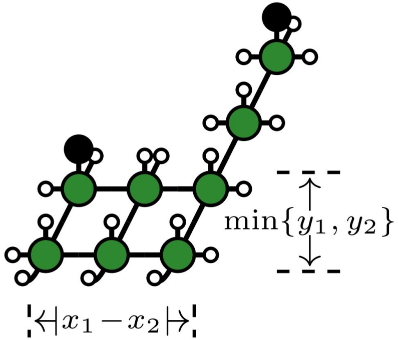
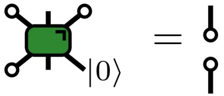


Some examples of DI-PEPS

Can be prepared with sequential quantum circuit satisfying



$$\sum_{\mathrm{ptl}} U^{\mathrm{pr_1t}}_{\mathrm{lb_1}|0\rangle} (U^{\mathrm{pr_2t}}_{\mathrm{lb_2}|0\rangle})^* = \delta_{\mathrm{b_1,b_2}}\delta_{\mathrm{r_1,r_2}}$$

Permutation-phase gates

$$U = P_{231} D$$

$P_{231} =$ 

D is diagonal gate, also arbitrary single qudit gates

Controlled-dual unitaries

$$U^{\mathrm{prt}}_{\mathrm{lba}} = \sum_i |i\rangle_{\mathrm{p}} \langle i|_{\mathrm{a}} V^i_{\mathrm{tr,lb}}$$

also arbitrary single qudit gates

Examples of DI-PEPS

Toric code

Contains 2D String-Net Liquids

Extend results by Tomohiro Soejima et. al. PRB 2020

Explicit family connecting topological and trivial phases

Etc.

Parameter counting and computational complexity

DI-PEPS is rich!

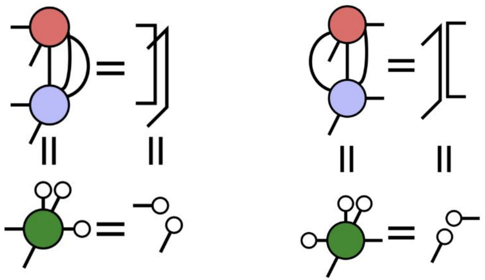
Real parameters of normal DI-PEPS $2(d-1)\chi^4$; normal PEPS $2d\chi^4 - 4\chi^2 + 2$

Computational complexity of the DI-PEPS:

With post-selection, it can encode universal quantum computation

⇨ Sampling of DI-PEPS is classically hard to simulate

Conclusions of part two



Introduced DI-PEPS

Rich family where local observables can be computed

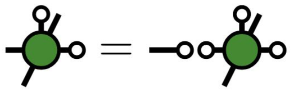
Can generalize to PEPS on other lattices or higher dimensions:

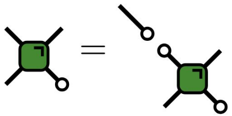
n isometric conditions → up to (n-1) correlations are solvable

Ansatz for numerics?

Equivalence of PEPS and 1+1D dynamics of CP maps:

Can use other solvable PEPS (e.g. SGS) to get solvable dynamics

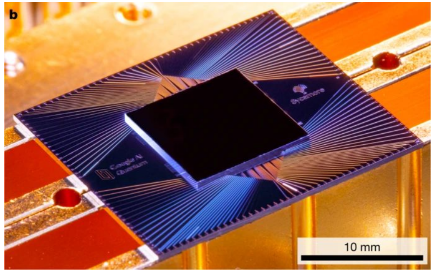




M. C. Bañuls et al. Sequentially generated states for the study of 2D systems, Phys. Rev. A 77, 052306 (2008).

Outline

- Part I: Quantum dynamics
  - Setting: quantum **circuit** dynamics
  - Solvable models: **dual-unitary** quantum circuits
  - Solvable models: space-time quantum channels
- Part II: Intermezzo
  - Dual-isometric PEPS
- Part III: Space-time channels as tools for average-computation **benchmarking** of quantum devices arXiv: 2507.18708
- Conclusions



Arute et al. Nature **574**, 505-510 (2019)

How would you use a quantum computer?

To study many body dynamics far from equilibrium!

How can we check if a quantum computer works?

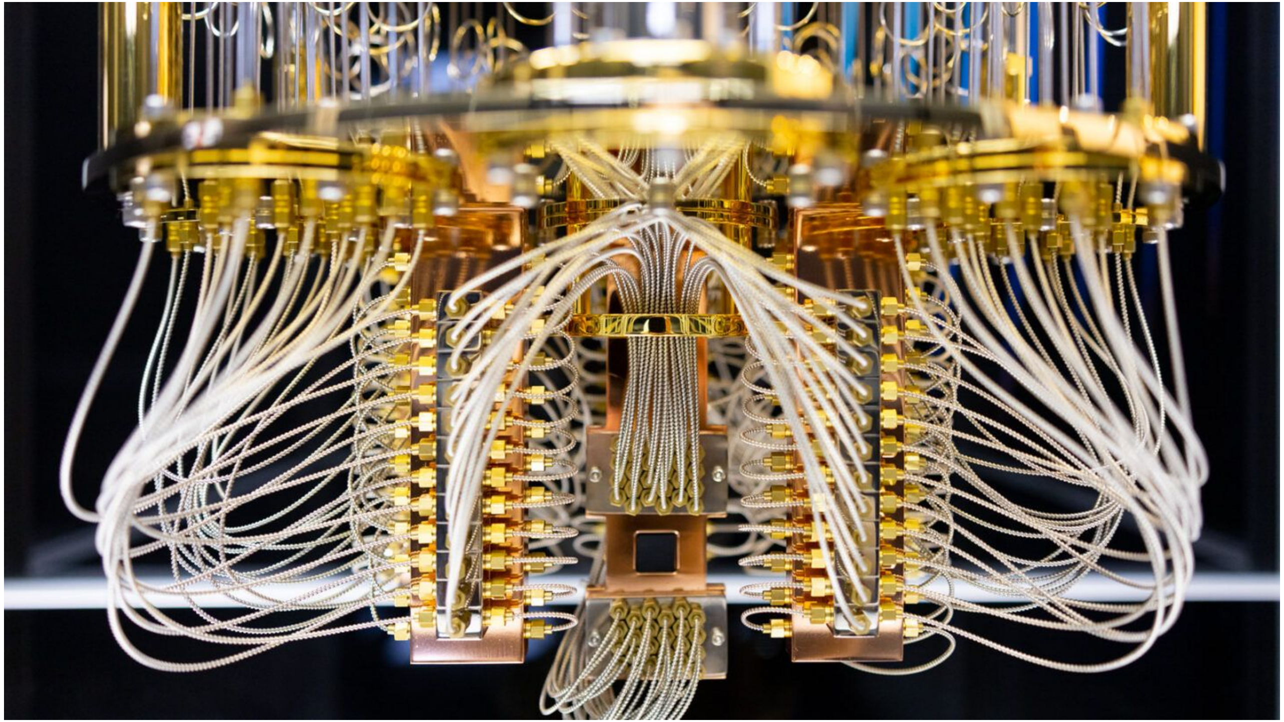


Image from: https://www.snexplores.org/article/heres-why-scientists-want-a-good-quantum-computer

ArXiv: 2507.18708, in Collaboration With

Flavio Baccari

UNIVERSITÀ
DEGLI STUDI
DI PADOVA

Georgios Styliaris

MAX-PLANCK-INSTITUT
FÜR QUANTENOPTIK

The Challenge of Benchmarking Quantum Devices

- Approaching the quantum advantage regime, it becomes increasingly difficult to benchmark
- Testing single gates is not enough
- Want to test something very close to target computation
- Previous approaches: shorter circuits, discarding hard gates and keeping only Cliffords or matchgates

S. Bravyi and D. Gosset, PRL 116, 250501 (2016).
R. S. Bennink et al., PRA 95, 062337 (2017).
J. Carrasco, M. Langer, A. Neven, and B. Kraus, PRR 6, L032074 (2024).

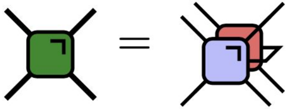
Average-Computation Benchmarking

- We develop benchmarking which does not alert the architecture, by **randomizing** the computation

$$U_i \longrightarrow \{U_{i,\alpha}\}_\alpha$$

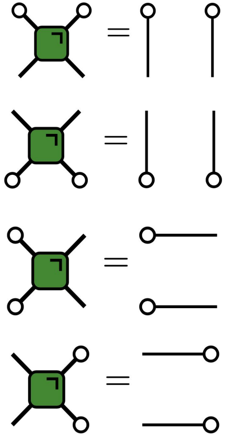
- Single instances are hard, but average becomes classically easy
- Keep (some) correlations of the original computation
- Concrete example: make the average be **space-time channel**
- Then we can test the correlation functions

4-way space-time channels



$$q = \ldots = \ldots = \sum_k E_k \otimes E_k^*$$

Make the average gate satisfy the conditions:



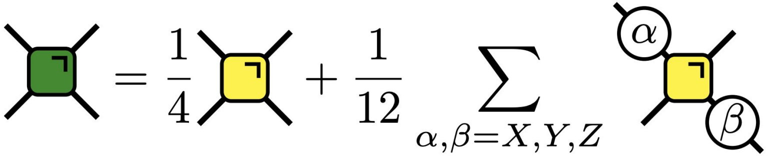
Qubit decomposition of the gate

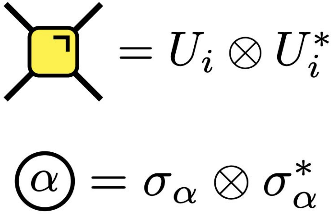
$$U_i = (W_A \otimes W_B) \exp\left(i \sum_{\alpha = X,Y,Z} \theta_\alpha^{(i)} \sigma_\alpha \otimes \sigma_\alpha \right) (V_A \otimes V_B)$$

$$\vec{\theta}^{(i,\pm\pm)} = \left(\tfrac{\pi}{4} \pm \delta_x, \tfrac{\pi}{4} \pm \delta_y, \theta_z\right)$$

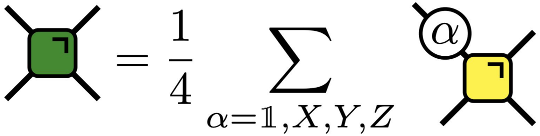
Equal mixture of $\{U_i^{(++)}, U_i^{(+-)}, U_i^{(-+)}, U_i^{(--)}\}$

Other recipes





3-way



How to find recipes?

3 or 4 unitality which keeps as much of the initial correlations as possible

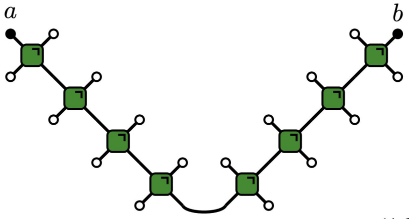
Easier if we have a parametrization

Educated guess

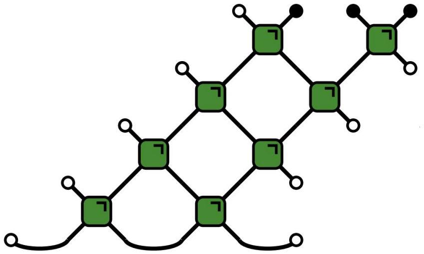
Search for a given rescaling supermap as a semidefinite program (SDP) with some properties

Computable examples

$\langle O^{(i,i+2T+1)} \rangle_{\text{avg}} =$



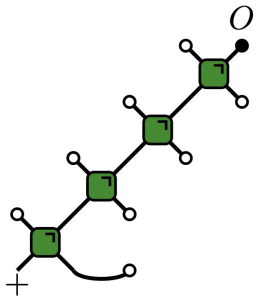
$\langle O^{(i,i+1,i+2)} \rangle_{\text{avg}} =$



$$|+\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$$

$$\rho(0) = |\Psi_0\rangle \langle \Psi_0| \,, \quad |\Psi_0\rangle = |+\Phi^+\Phi^+ \ldots\rangle$$

$\langle O^{(T)} \rangle_{\text{avg}} =$



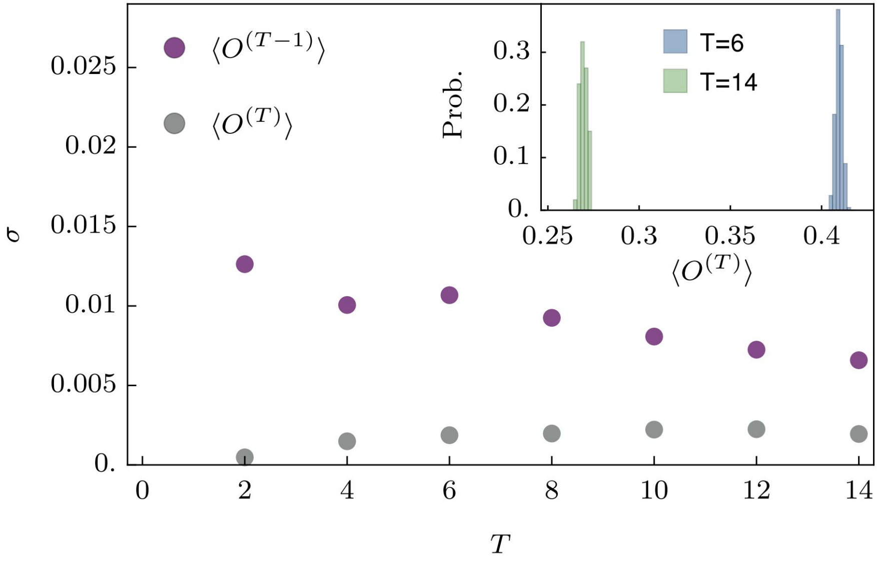
Considerations for practical implementations

Sampling complexity?

M samples for error

$$\epsilon \sim 1/\sqrt{M}$$

Variance stays small

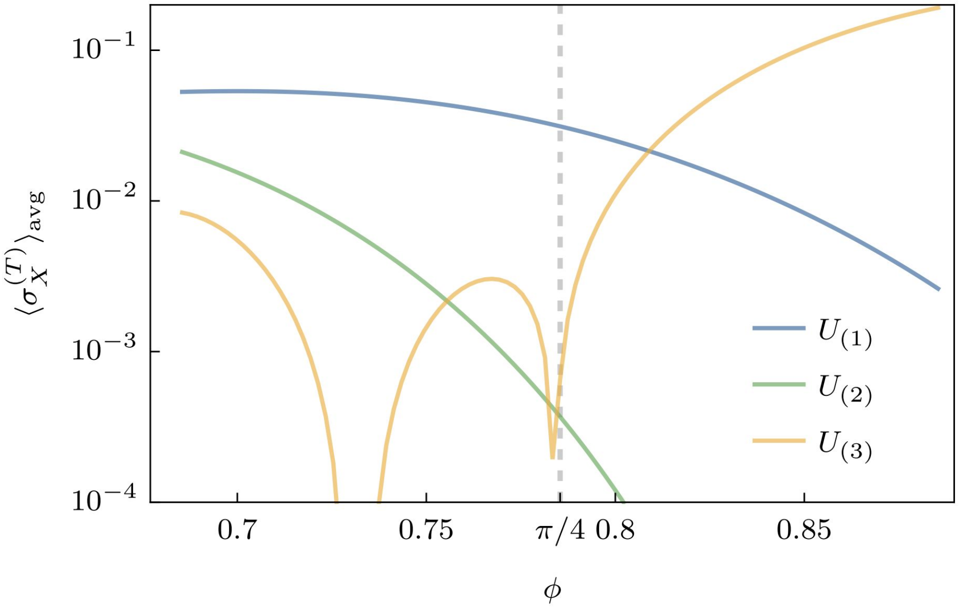


Benchmarking noise of T gate

$$T = \mathrm{diag}(1, e^{i\phi})$$

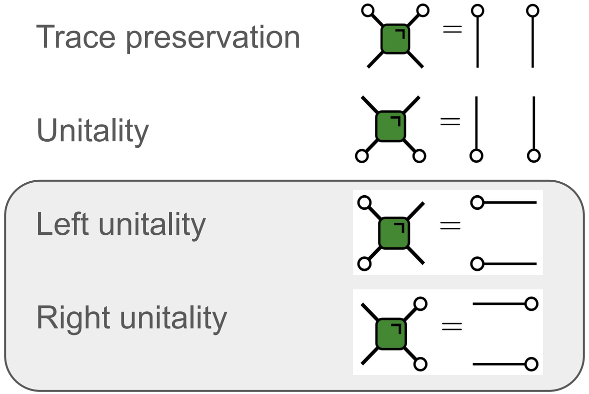
$$U_{(n)} = \mathrm{CNOT}_{12}(u_n \otimes u_n)\mathrm{CNOT}_{21}(u_n \otimes u_n)\mathrm{CNOT}_{12}(u_n \otimes u_n)$$

$$u_1 = HT^2HTH, u_2 = HT^2HTHTH, u_3 = HT^2HTHT$$



Conclusions

- **Exact solutions** in chaotic models are possible!
- We need something: **dual-unitarity** (or avg. over randomness) B. Bertini, PK, T. Prosen, PRL 123, 210601 (2019)
- Space-time quantum channels PK*, G Styliaris*, Quantum 7, 1020 (2023)



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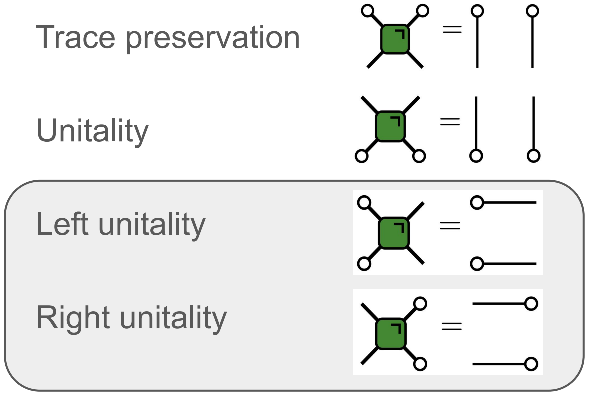
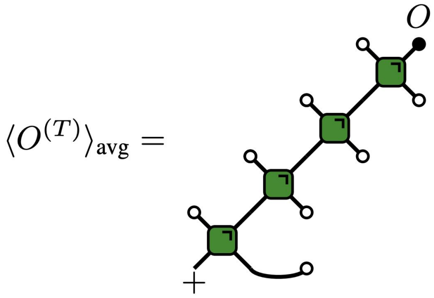
- We need something: **dual-unitarity** (or avg. over randomness) B. Bertini, PK, T. Prosen, PRL 123, 210601 (2019)
- Space-time quantum channels PK*, G Styliaris*, Quantum 7, 1020 (2023)
- Dual-isometric PEPS XH Yu, JI Cirac, PK*, G Styliaris*, PRL 133, 190401 (2024)
- Use these ideas to benchmark quantum computers F Baccar*, PK*, G Styliaris*, arXiv: 2507.18708
- Can be also used to **benchmark classical** approximate methods
- Beyond exponentially decaying signals?



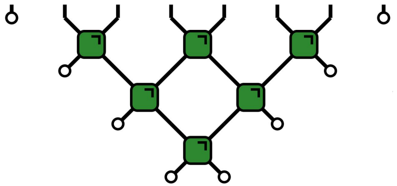


Thank You!

Pavel Kos, YITP 2025

Non-equilibrium steady states

Focusing on finite region A in TD limit, or boundary with completely depolarising channels $|\bigcirc\rangle\langle\bigcirc|$

3-way from solvable MPDO, t>|A|/2: exact unique NESS $|\rho(t)\rangle =$ 

Can they show rich physics?

Some random examples: Negativity was zero and mutual information was small, suggesting that the steady states are thermal-like area law states.