Quantum complexity and generalized area law in fully connected models

Tomotaka Kuwahara
RIKEN, RQC & CPR,
Analytical Quantum Complexity RIKEN Hakubi Research Team

7th August 2025
Recent Developments and Challenges in Tensor Networks:
Algorithms, Applications to science, and Rigorous theories

Main message

1: Area law holds without geometric structures

2: The Local Hamiltonian Problem (LHP) conjecture is (almost) true for fully connected models
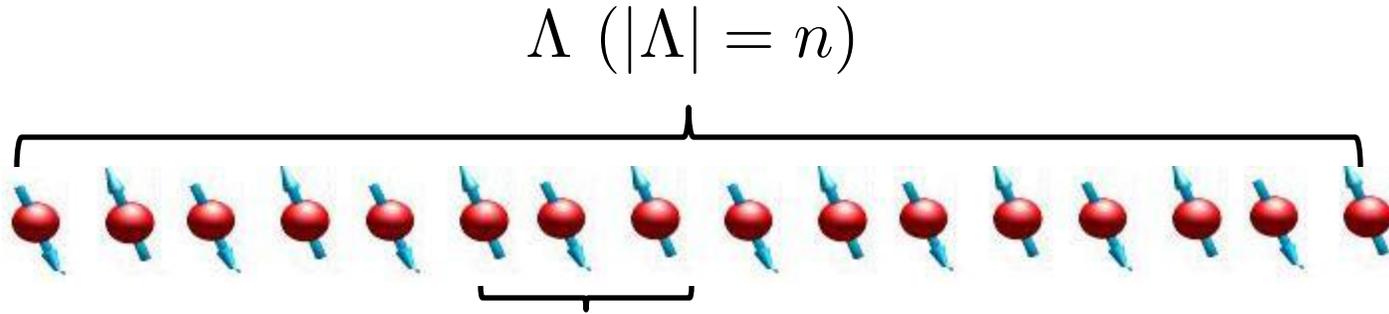
Introduction and Motivation

⭐ Area law conjecture (c.f. Itai's talk on 29$^\text{th}$ Jul.)

⭐ Generalized area law on infinite dimensional graph

Local Hamiltonian Problem (LHP)

- k-Local Hamiltonian involves at most k-body interactions

$$H = \sum_{Z \subset \Lambda : |Z| \leq k} h_Z$$

$\Lambda \ (|\Lambda| = n)$

$h_Z \ (|Z| = 3)$

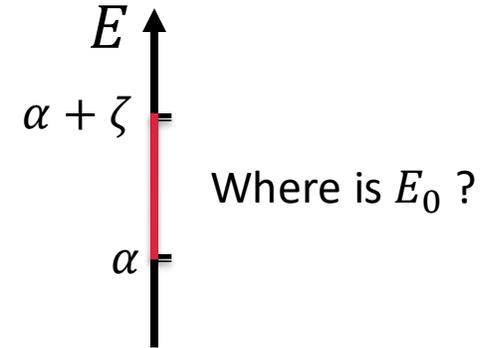- Can we solve the eigenproblem of H?

- n spin system: $2^n \times 2^n$ matrix

→ Usually difficult to solve even for very simple model!
(e.g., Haldane conjecture, PRL, 1983)

Problem

Given a local Hamiltonian H, decide if $E_0 \in [\alpha, \alpha + \zeta]$

E_0: minimum eigenvalue of H

Where is E_0 ?

Thm (Kitaev '99) The local Hamiltonian problem is QMA-complete for $\zeta = 1/\text{poly}(n)$

(analogue Cook-Levin thm)

QMA is the quantum analogue of NP, where the proof and the computation are quantum.

$$H = \sum_{Z \subset \Lambda : |Z| \leq k} h_Z$$

When the ground state is gapped, the complexity class of LHP is as easy as the classical cases:

⟶ The 1D case is in P

⟶ The 2D, 3D, ... cases are in NP

⭐ Gapped: $\Delta = E_1 - E_0 = \Omega(1)$

⭐ Gapped cases correspond to non-critical cases

⟶ determines the quantum phase of matter

E_0: minimum eigenvalue of H

E_1: the second minimum eigenvalue of H

When the ground state is gapped, the complexity class of LHP is as easy as the classical cases:

⟶ The 1D case is in P

⟶ The 2D, 3D, … cases are in NP

⭐ Classical simulation (computation) of a given ground state

⭐ In 1D this has been proved by Landau, Vazirani & Vidick

Z. Landau, U. Vazirani and T.Vidick, Nature Physics, 11, 566–569 (2015)

⭐ In higher D, the problem is wide open.
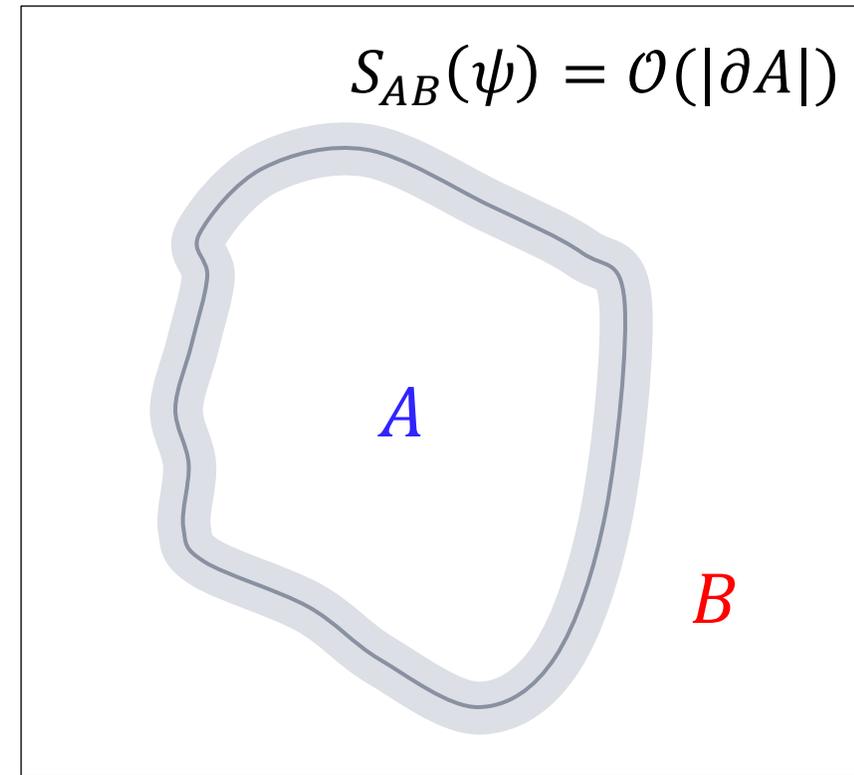
⟶ Entanglement of the ground state is the key

"How much entanglement between A and B?"

Less entanglement means easier to simulate.

$$|\psi\rangle = \sum_{s=1}^{2^n} \lambda_s |\psi_{A,s}\rangle \otimes |\psi_{B,s}\rangle$$

$$S_{AB}(\psi) = -\sum_{s=1}^{2^n} \lambda_s^2 \log(\lambda_s^2) \leq \text{const.} |\partial A|$$

Area law means the entanglement is proportional to the boundary of A

$$S_{AB}(\psi) = \mathcal{O}(|\partial A|)$$

A

B

(Area Law in 1D) $S_{AB}(\psi) \leq$ **const.**

Area Law Conjecture

> **When the ground state $|\Omega\rangle$ is gapped, the area law holds in arbitrary quantum many-body systems, $S_{AB}(\Omega) = \mathcal{O}(|\partial A|)$**

⭐ Gapped: $\Delta = E_1 - E_0 = \mathcal{O}(1)$

E_0: minimum eigenvalue of H

E_1: the second minimum eigenvalue of H

⭐ Proved only in 1D systems (short-range, long-range, bosonic interactions)

M. B. Hastings, J. Stat. Mech. (2007) P08024.
F. Brandao and M. Horodecki, Nat. Phys **9** (2013), 721
T. Kuwahara and K. Saito, Nat. Commun. **11,** 4478 (2020), Kim-Kuwahara, arXiv:2411.02157

⭐ In 2D-, even the proof of sub-volume law is not known

➡ Conditional proofs:
L. Masanes, Phys. Rev. A 80, 052104 (2009).
K. Van Acoleyen, M. Mariën, and F. Verstraete, Phys. Rev. Lett. 111, 170501 (2013).
A. Anshu, I. Arad, and D. Gosset, STOC 2022

Area Law Conjecture

When the ground state $|\Omega\rangle$ is gapped, the area law holds in arbitrary quantum many-body systems, $S_{AB}(\Omega) = \mathcal{O}(|\partial A|)$

⭐ Gapped: $\Delta = E_1 - E_0 = \mathcal{O}(1)$

E_0: mini___m

E_1: the sec___

In this talk, I assume non-degeneracy

Degenerate cases were treated in
Arad et al., Commun. Math. Phys. 356 (1), 65-105 (2017)
Arad et al., arXiv:2310.19028

⭐ Proved only in 1D systems (short-range, long-

M. B. Hastings, J. Stat. Mech. (2007) P08024.
F. Brandao and M. Horodecki, Nat. Phys **9** (2013), 721
T. Kuwahara and K. Saito, Nat. Commun. **11,** 4478 (2020), Kim-Kuwaha

⭐ In 2D-, even the proof of sub-volume law is not known

➡️ Conditional proofs:
L. Masanes, Phys. Rev. A 80, 052104 (2009).
K. Van Acoleyen, M. Mariën, and F. Verstraete, Phys. Rev. Lett. 111, 170501 (2013).
A. Anshu, I. Arad, and D. Gosset, STOC 2022
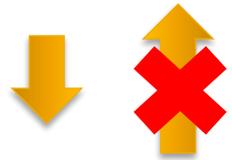
Why Area Law is important?

- When the ground state is efficiently described by <u>tensor network,</u>

  Efficient classical description of quantum state

- The ground state satisfies <u>the area law</u>

  Necessary condition !

- Breakdown of Area law

→ all the algorithms and the classifications based on tensor network break down!

Area law is a cornerstone in modern physics

Why Area Law is important?

- When the ground state is efficiently described by <u>tensor network,</u>

  Efficient classical description of quantum state

  Area law is not the sufficient condition

  Y. Ge, J. Eisert, New J. Phys. **18**, 083026 (2016)

- The ground state satisfies <u>the area law</u>

  Necessary condition **!**

- Breakdown of Area law
→ all the algorithms and the classifications based on tensor network break down!

  Area law is a cornerstone in modern physics

Generalized Area Law

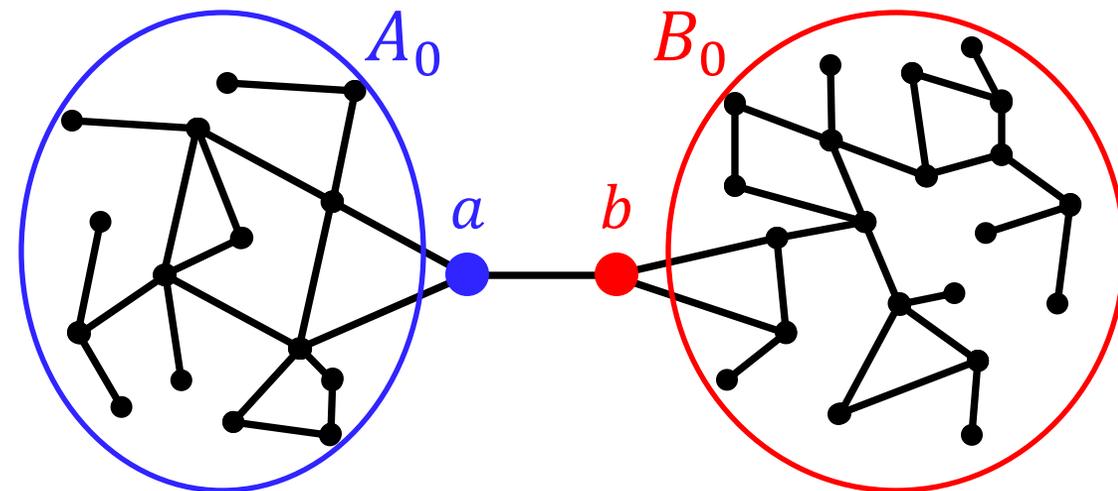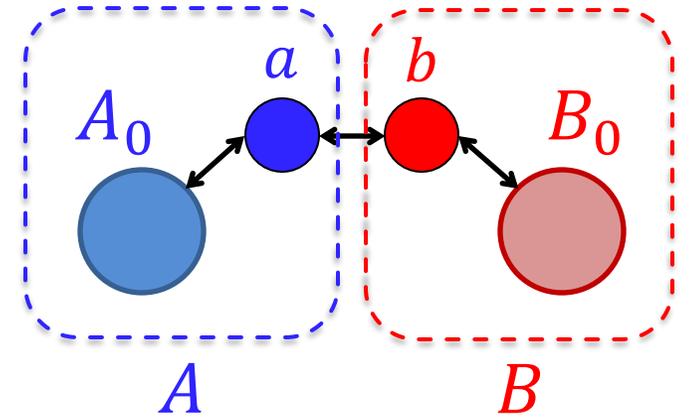- More general setup: 4-body systems ($A_0 a \leftrightarrow b B_0$)

$$S_{AB}(\Omega) \overset{?}{\propto} \log[\min(d_a, d_b)]$$

True for

Thermal area law M. Wolf et al., Phys. Rev. Lett. **100**, 070502

Dynamical area law (SIE thm.)

K. Van Acoleyen et al., Phys. Rev. Lett. **111**, 170501 (2013).

Generalized Area Law

- More general setup: 4-body systems ($A_0 a \leftrightarrow b B_0$)

$$S_{AB}(\Omega) \overset{?}{\propto} \log[\min(d_a, d_b)]$$

True for

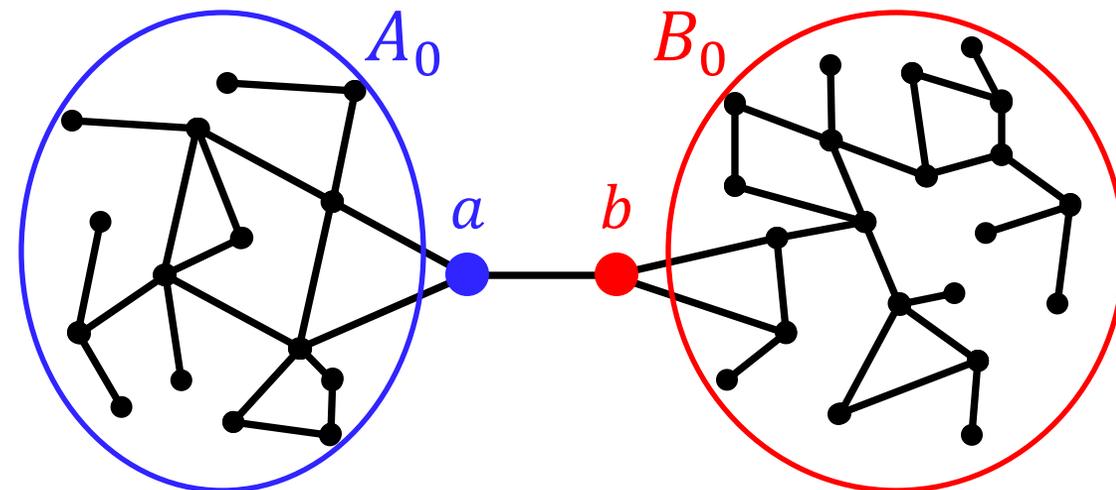Thermal area law   M. Wolf et al., Phys. Rev. Lett. **100**, 070502
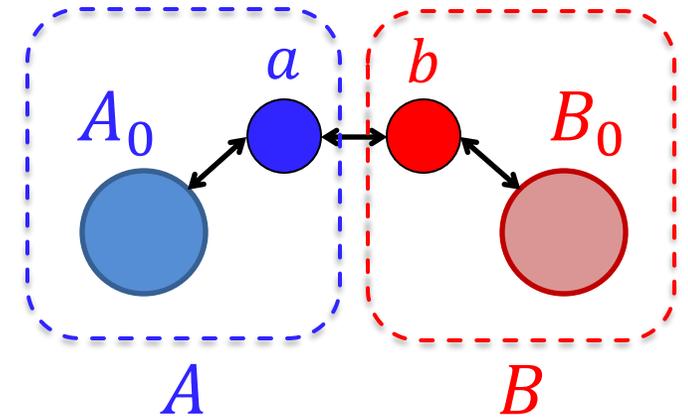Dynamical area law (SIE thm.)

K. Van Acoleyen et al., Phys. Rev. Lett. **111**, 170501 (2013).

Not True for

Ground-state area law

D. Aharanov, et al., FOCS (2014) pp. 246-255

Fully connected graph

- **Fully connected graph (all-to-all interactions)**

  (Sherrington-Kirkpatrick model, Lipkin-Meshkov-Glick model, Sachdev-Ye-Kitaev model, etc.)

  ➡ Quantum computing

  W. Lechner, P. Hauke, and P. Zoller, Science Advances **1**, e1500838 (2015).
  M. Vidal, J. J. Wesdorp, C. K. Andersen, PRX Quantum **6**, 010308 (2025)

  Quantum chemistry

  Y. Cao, et al., Chem. Rev. **119**, 10856 (2019).

- **Product state well approximates ground energy**

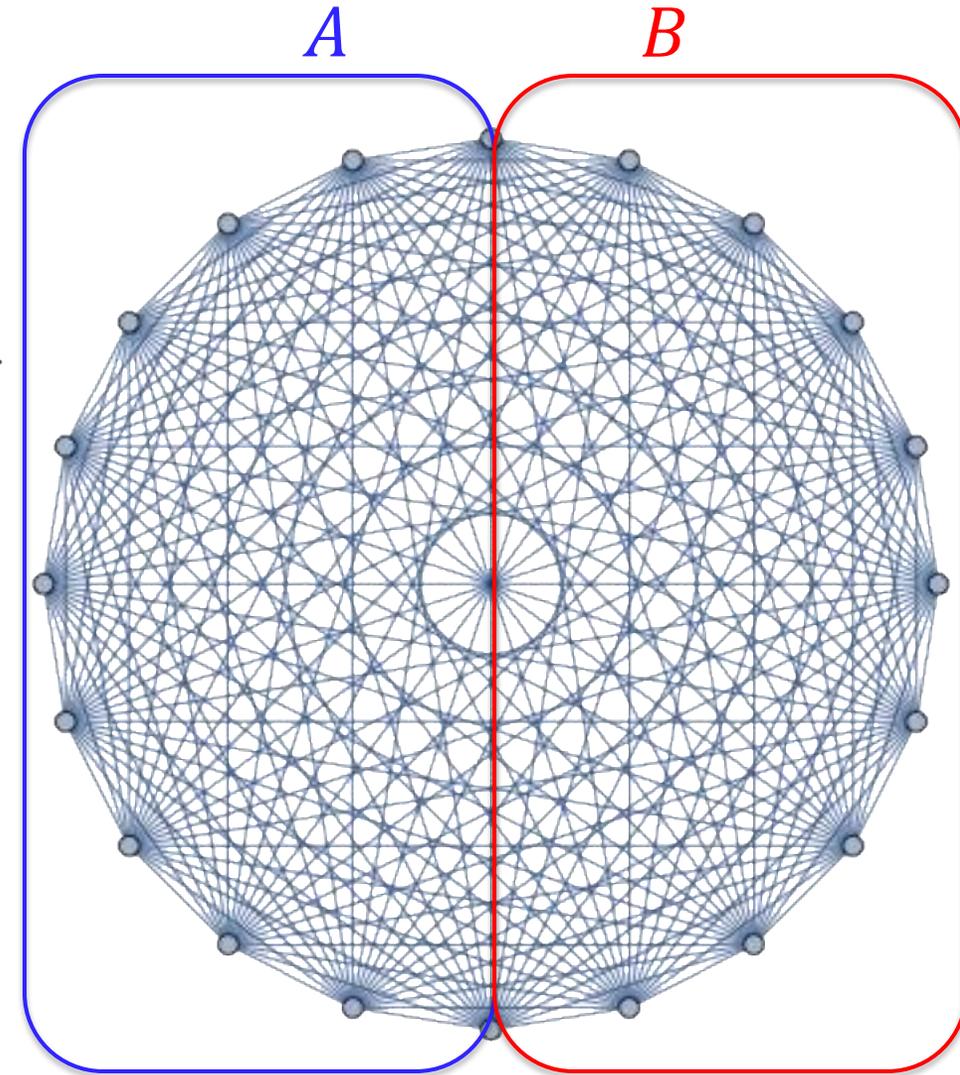 **(up to $\min[\sqrt{n/\Delta}, n^{2/3}]$ error)**

  Brandao-Harrow, STOC' 13
  Kuwahara et al., Quantum Sci. Technol. **2**, 015005 (2017).

- **Classical simulation**

  Bravyi et al., Nature Physics 18, 1367 (2022).

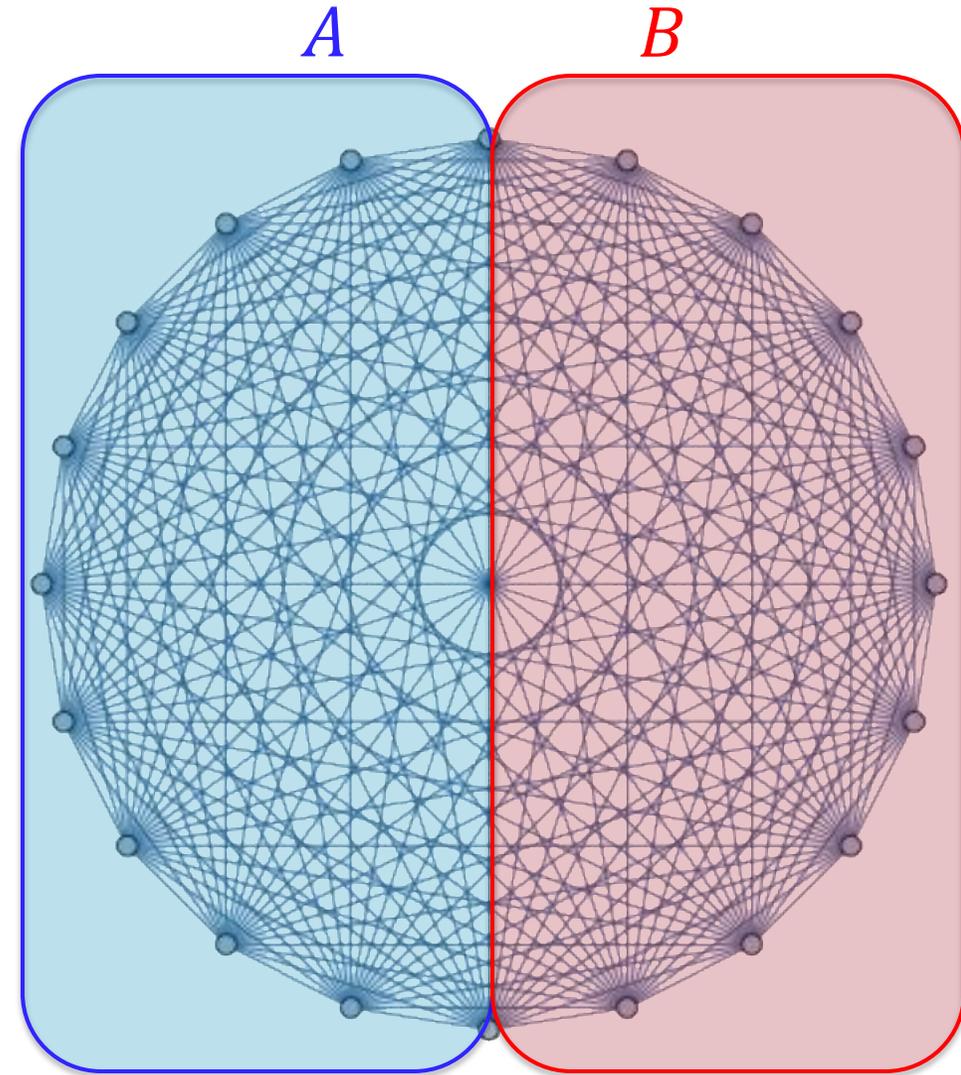 J Tindall et al., Nature Communications 13 (1), 7445 (2022)

A B

Generalized area law in fully connected graph

- No Boundary?

  All the qudits in A interact with all the qudits in B

A B

- **No Boundary?**

  All the qudits in A interact with all the qudits in B

- **Effectively mapping to constant size boundary**

➡ $H_{AB} = H_A + H_B + \boxed{h_{AB}}$

Small Schmidt rank

➡ Generalized area law may give $S_{AB}(\Omega) = \text{const.}$
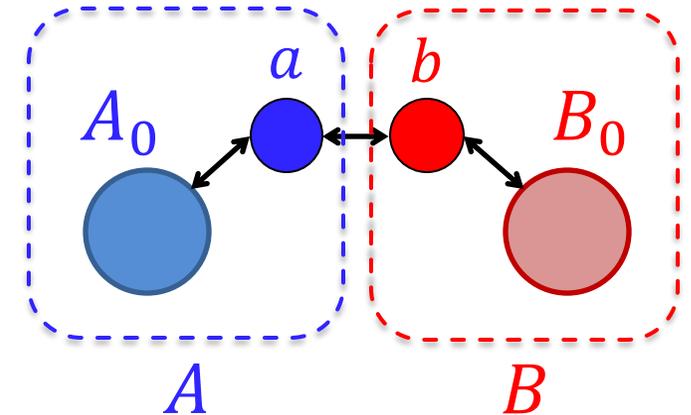
Purpose of this work

1: Proving "generalized area law" in the fully connected models
2: Efficient classical description of gapped ground states

Setup and Main results

⭐ Fully connected model with small Schmidt ranks

⭐ Generalized area law and Matrix-Product-State approximation

Set up: fully connected model

- Many-body Hamiltonian (n sites) Λ: total system

$$H = \sum_{i,i' \in \Lambda} h_{i,i'} + \sum_{i \in \Lambda} h_i \,,$$

➡ Generalized to arbitrary k-body interactions

- Subset-subset interaction: H_{AB}

$$H_{AB} = \sum_{i \in A} \sum_{i' \in B} h_{i,i'} \,,$$

- Ground state: $|\Omega\rangle$, spectral gap: Δ

- **Entanglement entropy:** $S_{AB}(\Omega) = -\mathrm{tr}[\rho_A \log(\rho_A)]$

$$\rho_A = \mathrm{tr}_B(|\Omega\rangle\langle\Omega|),$$
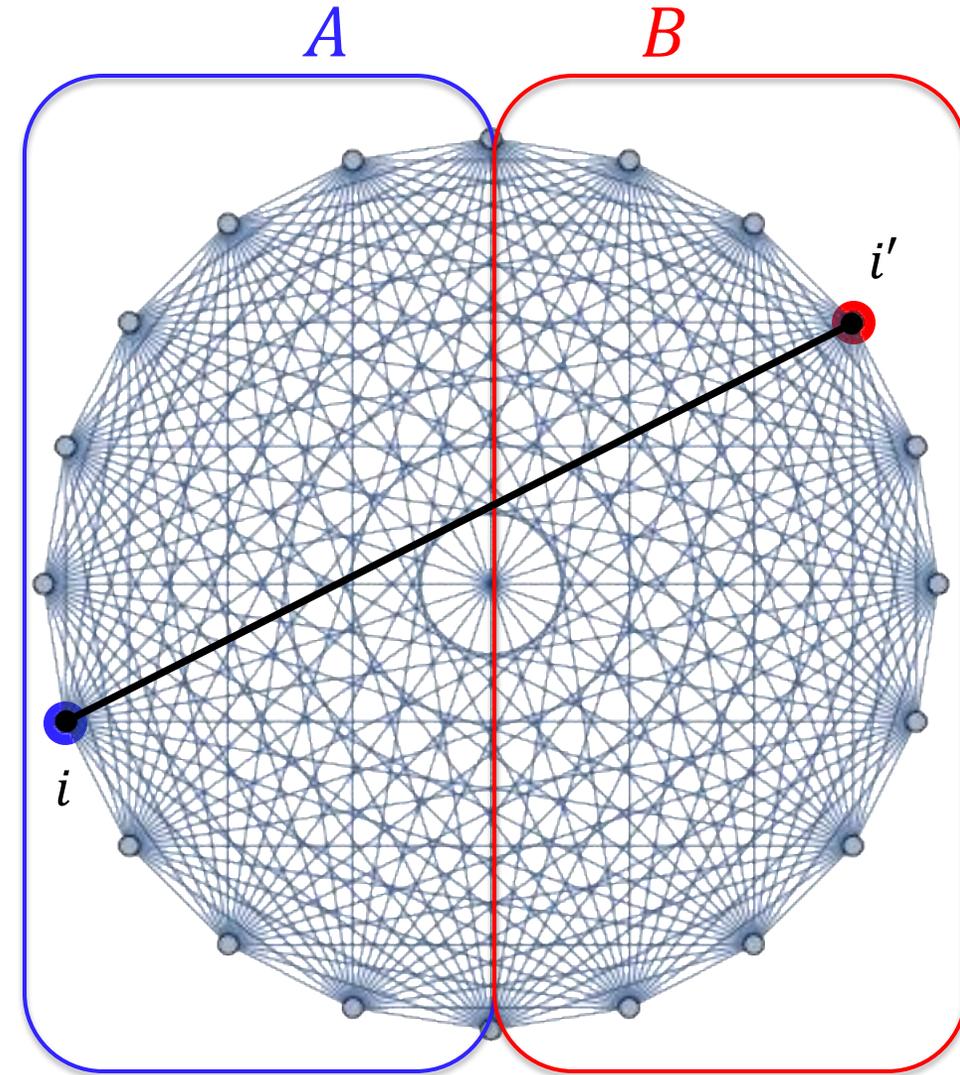
A B

Assumption: small Schmidt rank

$$H = \sum_{i,i' \in \Lambda} h_{i,i'} + \sum_{i \in \Lambda} h_i \,,$$

- Subset-subset interaction: H_{AB}

$$H_{AB} = \sum_{i \in A} \sum_{i' \in B} h_{i,i'} = \sum_{s=1}^{d_H} \frac{J_s}{n} H_{A,s} \otimes H_{B,s}, \qquad \sum_{s=1}^{d_H} |J_s| \leq \bar{g}_1$$

$$H_{A,s}, H_{B,s} : 1 \text{ local}$$

Assumption

- The coefficient $|J_s|$ should linearly decays

- E.g. LMG model, $H = \sum_{i,i' \in \Lambda} \left(\dfrac{J_x \sigma_{i,x} \otimes \sigma_{i',x} + J_y \sigma_{i,y} \otimes \sigma_{i',y}}{n} \right)$

$$H_{AB} = \frac{J_x}{n} \sum_{i \in A} \sigma_{i,x} \otimes \sum_{i' \in B} \sigma_{i',x} + \frac{J_y}{n} \sum_{i \in A} \sigma_{i,y} \otimes \sum_{i' \in B} \sigma_{i',y}$$

➡ $d_H = 2, \ \bar{g}_1 = J_x + J_y$

Assumption: small Schmidt rank

$$H = \sum_{i,i' \in \Lambda} h_{i,i'} + \sum_{i \in \Lambda} h_i,$$

- **Subset-subset interaction:** H_{AB}

$$H_{AB} = \sum_{i \in A} \sum_{i' \in B} h_{i,i'} = \sum_{s=1}^{d_H} \frac{J_s}{n} H_{A,s} \otimes H_{B,s}, \qquad \sum_{s=1}^{d_H} |J_s| \leq \bar{g}_1$$

$H_{A,s}, H_{B,s}$: 1 local

Assumption

A B
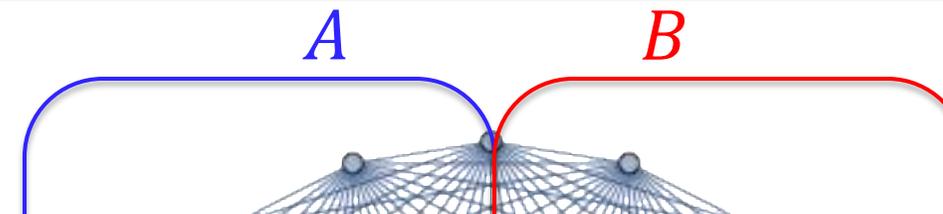
Note. SYK model

$$\sum_{i_1 < i_2 < i_3 < i_4} \frac{J_{i_1,i_2,i_3,i_4}}{n^{3/2}} c_{i_1} c_{i_2} c_{i_3} c_{i_4}$$

does not satisfy the condition.
But, the disorder-free case

$$\sum_{i_1 < i_2 < i_3 < i_4} \frac{J}{n^3} c_{i_1} c_{i_2} c_{i_3} c_{i_4}$$

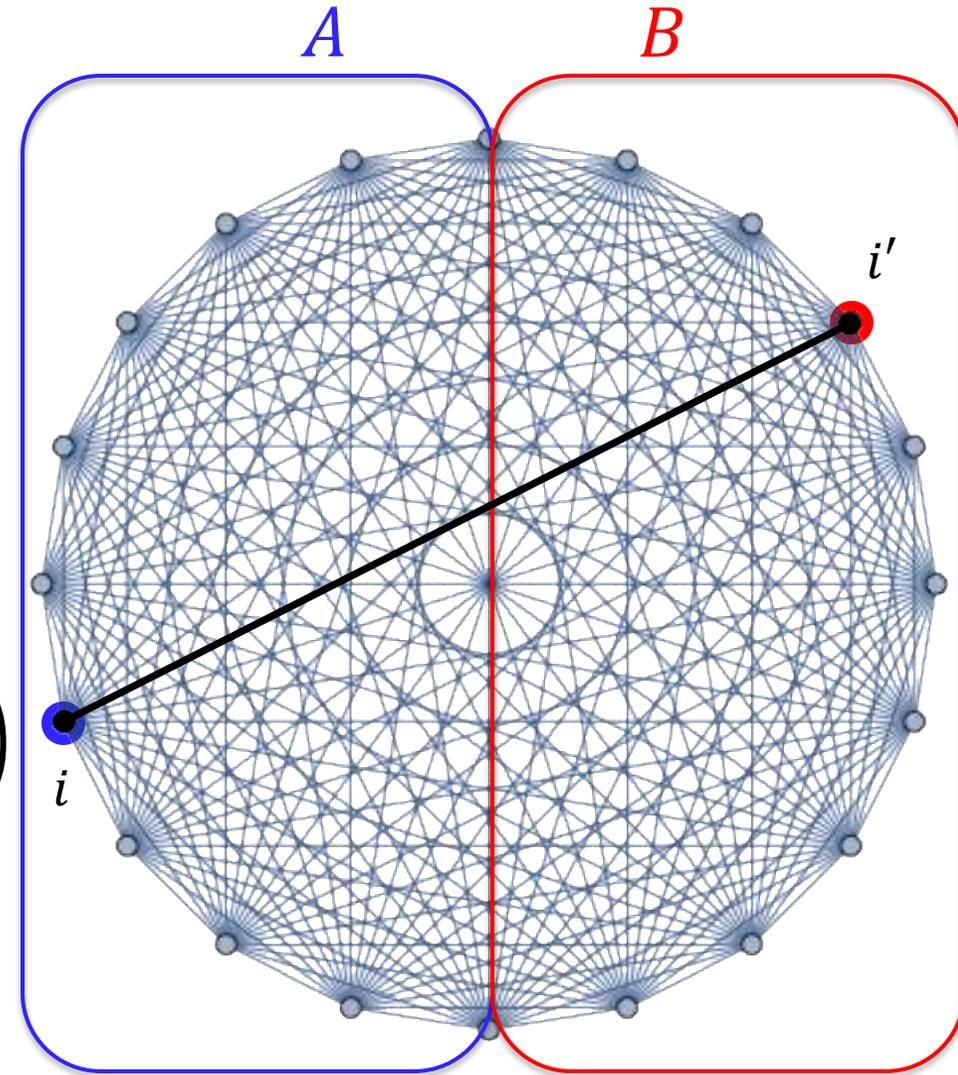satisfies the condition.

S. Ozaki, H. Katsura, PRR, **7**, 013092 (2025)

- The coefficient $|J_s|$ should linearly decays

- E.g. LMG model, $H = \sum_{i,i' \in \Lambda} \left(\frac{J_x \sigma_{i,x} \otimes \sigma_{i',x} + J_y \sigma_{i,y} \otimes \sigma_{i'}}{n} \right.$

$$H_{AB} = \frac{J_x}{n} \sum_{i \in A} \sigma_{i,x} \otimes \sum_{i' \in B} \sigma_{i',x} + \frac{J_y}{n} \sum_{i \in A} \sigma_{i,y} \otimes \sum_{i' \in B} \sigma_{i',y}$$

⟹ $d_H = 2, \ \bar{g}_1 = J_x + J_y$

Assumption: small Schmidt rank

$$H = \sum_{i,i' \in \Lambda} h_{i,i'} + \sum_{i \in \Lambda} h_i \, ,$$

- Subset-subset interaction: H_{AB}

A B

$$H_{AB} = \sum_{i \in A} \sum_{i' \in B} h_{i,i'} = \sum_{s=1}^{d_H} \frac{J_s}{n} H_{A,s} \otimes H_{B,s}, \qquad \sum_{s=1}^{d_H} |J_s| \le \bar{g}_1$$

$H_{A,s}, H_{B,s}$: 1 lo

Assumption

The condition can be removed
(Kim-Kuwahara, in preparation)

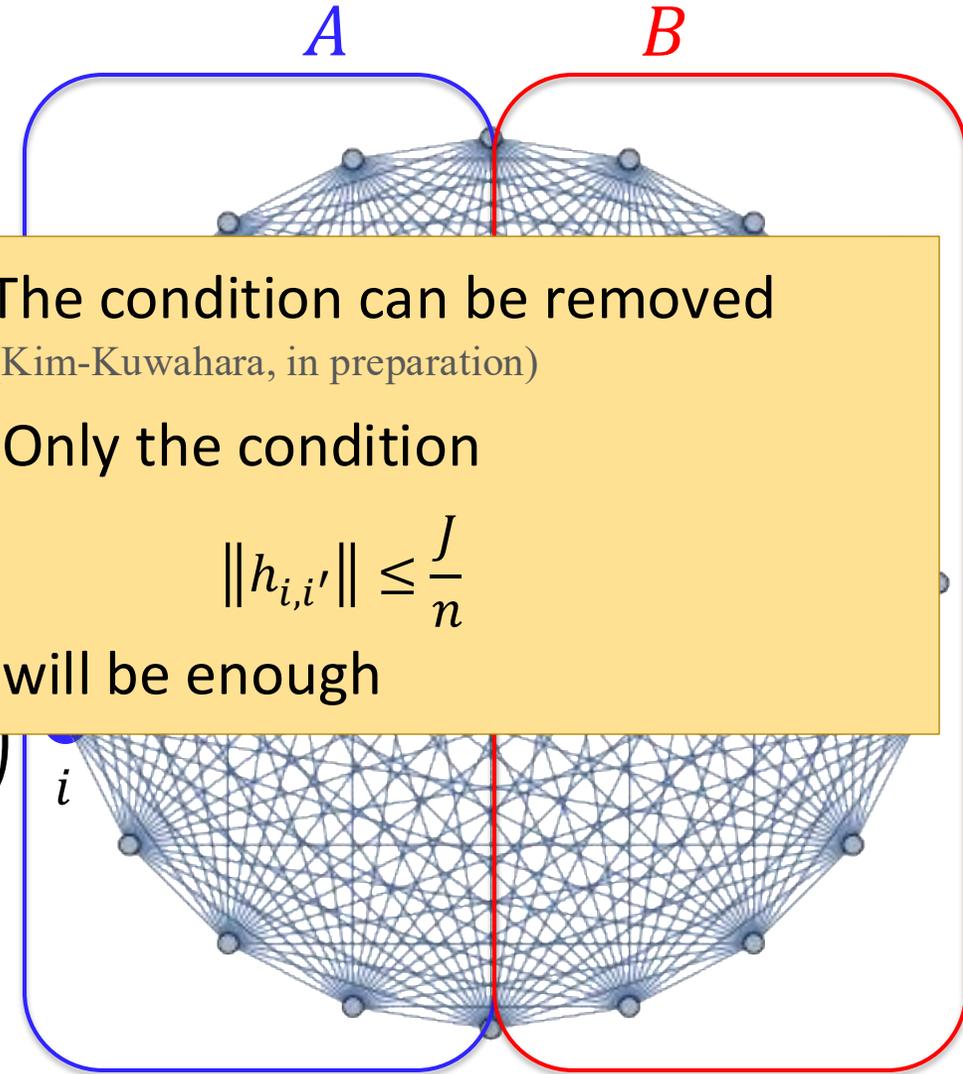Only the condition

$$\|h_{i,i'}\| \le \frac{J}{n}$$

will be enough

- The coefficient $|J_s|$ should linearly decays

- E.g. LMG model, $H = \sum_{i,i' \in \Lambda} \left(\dfrac{J_x \sigma_{i,x} \otimes \sigma_{i',x} + J_y \sigma_{i,y} \otimes \sigma_{i'} \cdots}{n} \right)_i$

$$H_{AB} = \frac{J_x}{n} \sum_{i \in A} \sigma_{i,x} \otimes \sum_{i' \in B} \sigma_{i',x} + \frac{J_y}{n} \sum_{i \in A} \sigma_{i,y} \otimes \sum_{i' \in B} \sigma_{i',y}$$

➡ $d_H = 2, \ \bar{g}_1 = J_x + J_y$
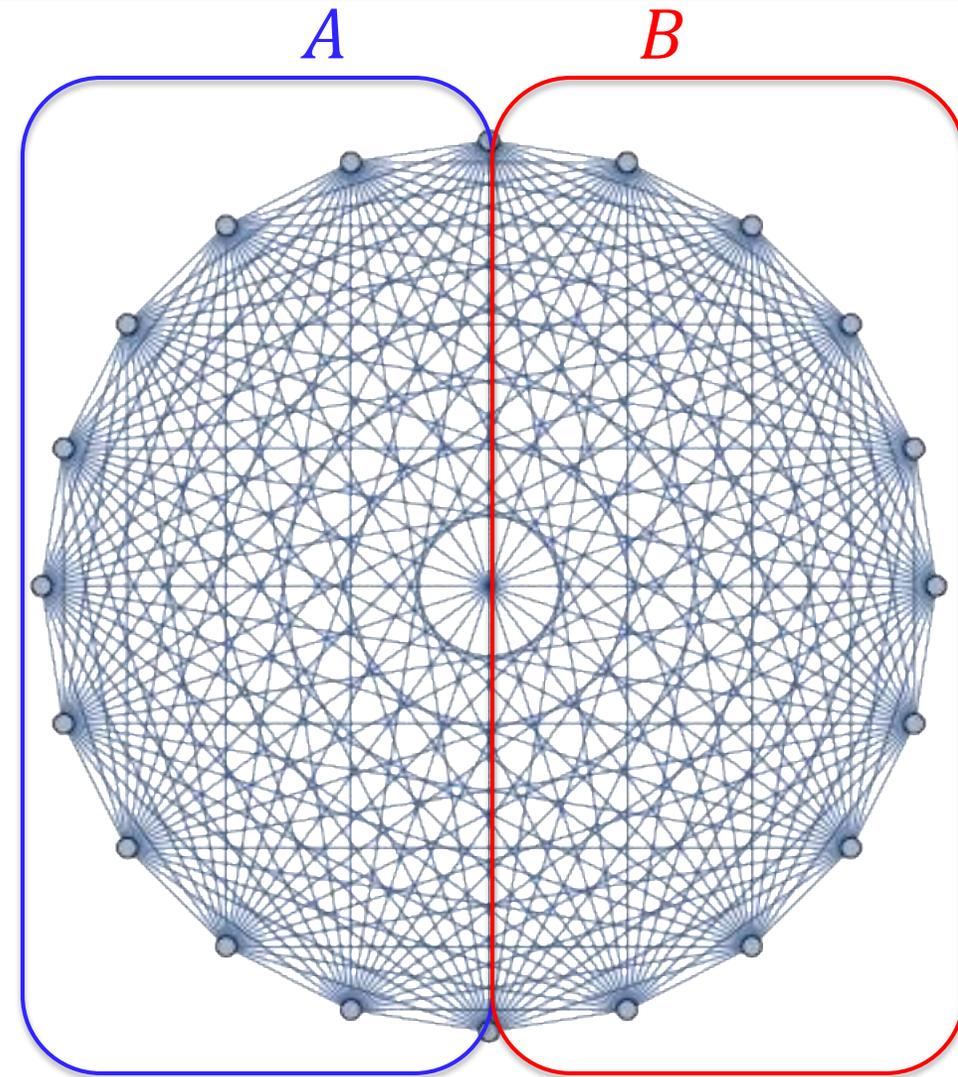
Main results: generalized area law

- **Generalized area law up to polylog factor**

$$S_{AB}(\Omega) \leq 2[\log(n)]^{\alpha_1 + \alpha_2 \log(f(\bar{g}_1, \Delta))} + \log(d) + 2$$

d: local Hilbert space dimension

$\alpha_1 = 230\log(10386ek^2), \quad \alpha_2 = 230, \quad f(\bar{g}_1, \Delta) := \max\{2, \log(324\bar{g}_1/\Delta)\}$

- **We obtain $S_{AB}(\Omega) = \text{polylog}(n)$ for $\Delta > \text{const}$.**

- Optimality?

  →Several numerical results indicate a constant upper bound

- **[Open Q.] What is the optimal upper bound?**

A B

Main results: Matrix Product State approximation

- **For 1D arranging, we get a MPS $|\Omega_D\rangle$ such that**

$$\|\text{tr}_{X^c}(|\Omega\rangle\langle\Omega|) - \text{tr}_{X^c}(|\Omega_D\rangle\langle\Omega_D|)\| \leq \delta|X| \quad \forall X \subseteq \Lambda$$

$$\text{by choosing } D = \exp\left[(\log n)^{c \, \text{loglog}(1/\delta)}\right]$$
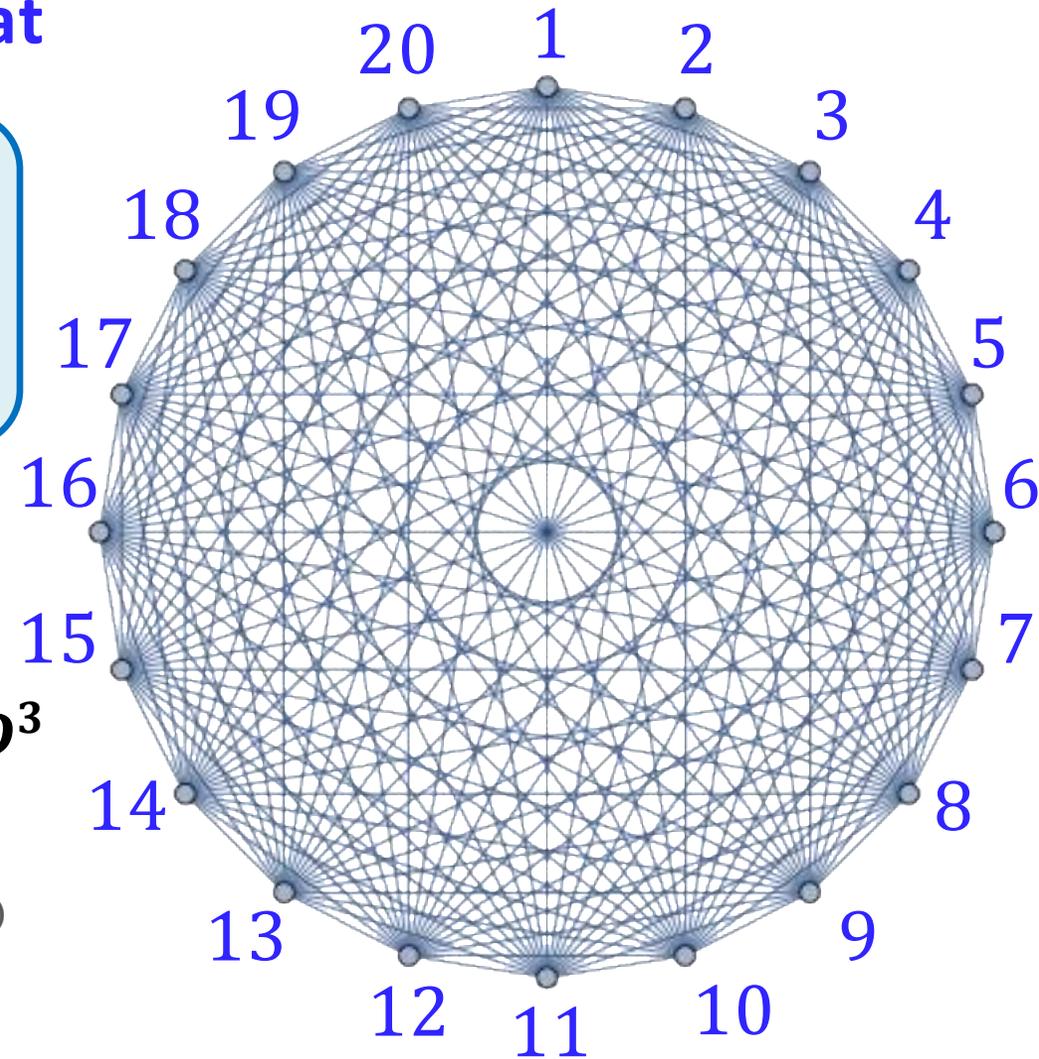
- **The bond dimension is quasi-polynomial:** $e^{\text{polylog}(n)}$

MPS is classically simulatable by computational time nD^3

→ Theoretical justification of numerical successes

J Tindall et al., Nature Communications 13 (1), 7445 (2022)

Main results: Matrix Product State approximation

- **For 1D arranging**

$$\|\mathrm{tr}_{X^c}(|\Omega\rangle\langle\Omega|) - \mathrm{tr}$$

by choosing $D =$

LHP conj.

When the ground state is gapped, the complexity class of LHP is as easy as the classical cases:
$\longrightarrow$ The 1D case is in P

$\longrightarrow$ The 2D, 3D, ... cases are in NP

LHP conj. on the dense graph is *almost* true.

- **Can we improve the result to polynomial form?**

- **The bond dimensi**

MPS is classically simulatable by computational time nD^3

$\rightarrow$ Theoretical justification of numerical successes

J Tindall et al., Nature Communications 13 (1), 7445 (2022)
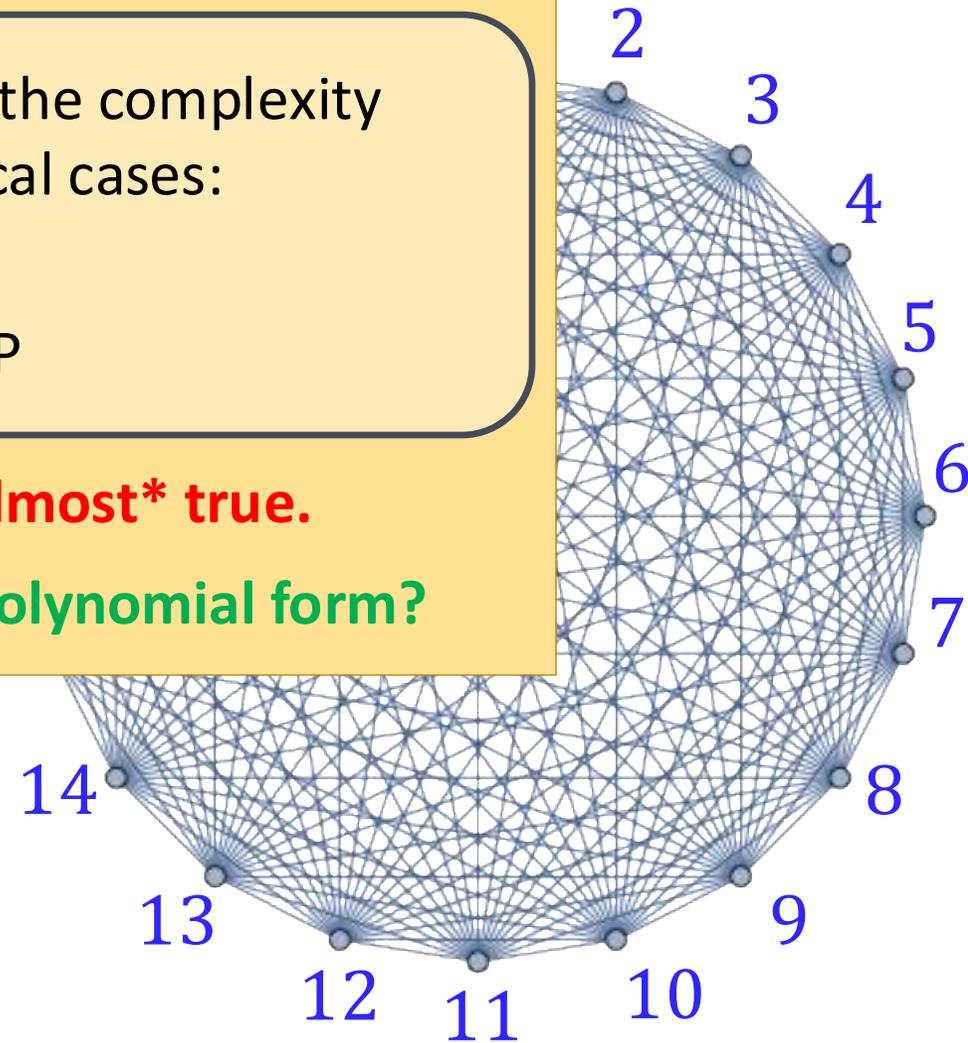
Main Technique

Mean-field-renormalization-group (MFRG) technique
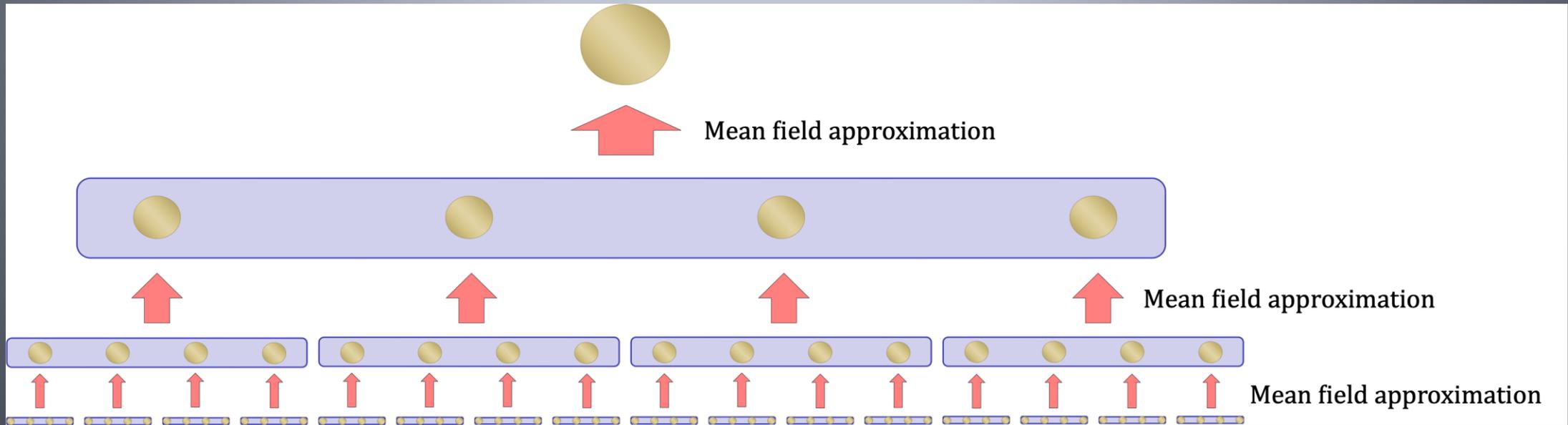
Step 1 of MFRG: mean-field approximation

- Product state approximation of the ground state $|\Omega\rangle$

[Schmidt decomposition at site i]

$$|\Omega\rangle = \sum_{s=0}^{d-1} \lambda_s \boxed{|s\rangle_i} \otimes |\phi_s\rangle_{\Lambda\backslash\{i\}} \qquad (\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{d-1})$$

i

- Mean-field states: $\{|0\rangle_i\}_{i\in\Lambda}$

- Computation of the mean-field states is usually **NP-hard**
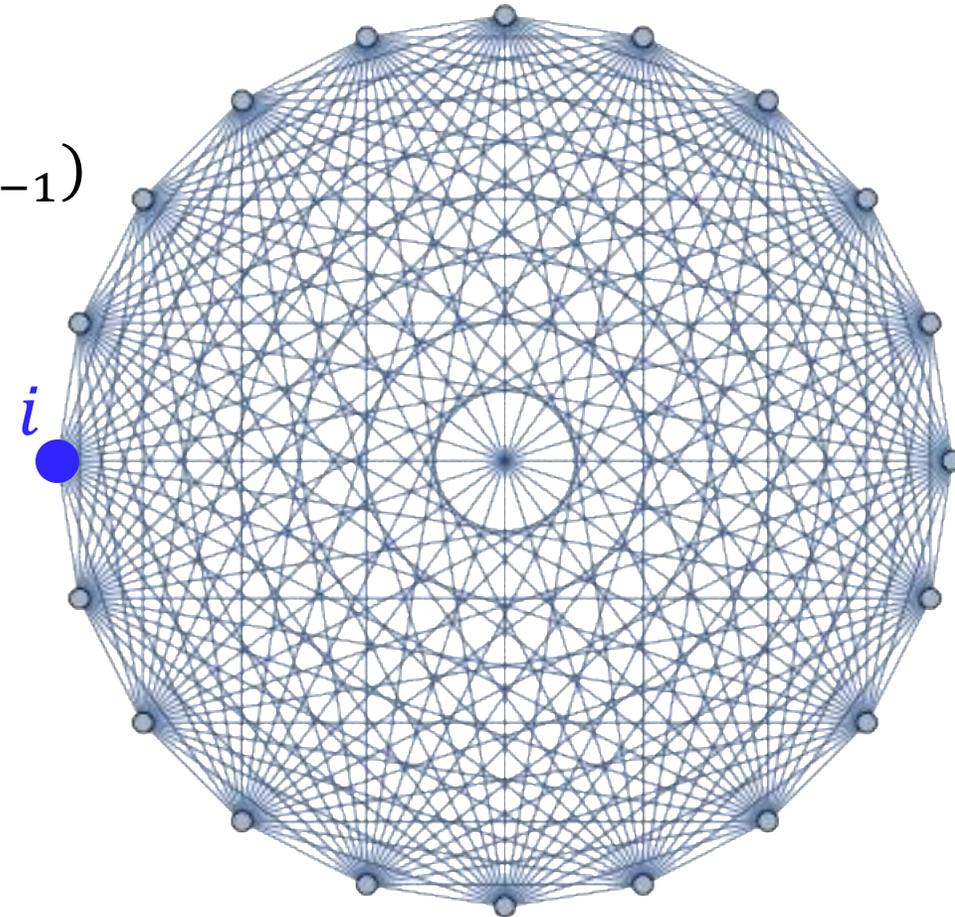
Step 2 of MFRG: truncating the block Hilbert space

- Decomposing the total system into blocks $\{L_s\}_s$

$$|L_s| = n^{1/8}$$

- Block size: $n^{1/8}$ No. of blocks: $n^{7/8}$
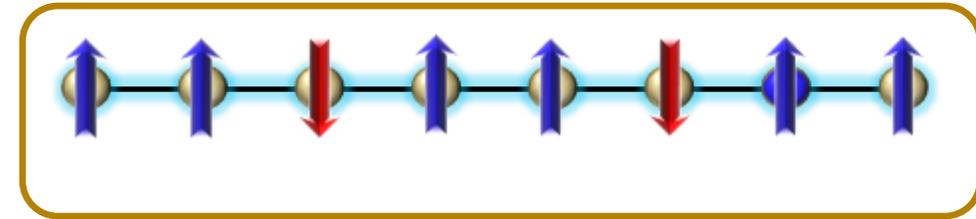
- The block's Hilbert space dimension $= e^{cn^{1/8}}$

Subexponential!

Step 2 of MFRG: truncating the block Hilbert space

- Decomposing the total system into blocks $\{L_s\}_s$

$$|L_s| = n^{1/8}$$

- Block size: $n^{1/8}$ No. of blocks: $n^{7/8}$

$|0\rangle$ $|0\rangle$ $|2\rangle$ $|0\rangle$ $|0\rangle$ $|5\rangle$ $|0\rangle$ $|0\rangle$
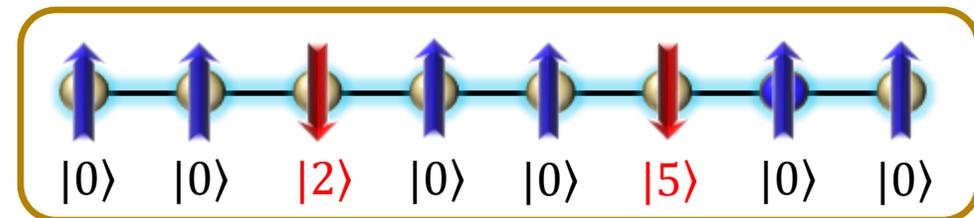
- The block's Hilbert space dimension = $e^{cn^{1/8}}$

[Renormalized bases]

Only z sites deviate from the mean-field states

Renormalized qudit

$\quad$ *Dimension* $\leq d^z n^{z/8} = \mathrm{poly}(n)$

Step 3 of MFRG: constructing renormalized Hamiltonian

- Renormalization projection: $\Pi^{(0)} = \bigotimes_s \Pi_{L_s, \leq z}$

$$|L_s| = n^{1/8}$$

$$|0\rangle \quad |0\rangle \quad |2\rangle \quad |0\rangle \quad |0\rangle \quad |5\rangle \quad |0\rangle \quad |0\rangle$$

- Renormalized Hamiltonian

$$H^{(1)} = \sum_{i,i' \in \Lambda} \Pi^{(0)} h_{i,i'} \Pi^{(0)} + \sum_{i \in \Lambda} \Pi^{(0)} h_i \Pi^{(0)} ,$$

$$\Pi_{L_s, \leq z} \ (z = 2)$$

Renormalized qudit

$$\Pi^{(0)} H_{L_s L_{s'}} \Pi^{(0)} = \sum_{i \in L_s} \sum_{i' \in L_{s'}} \Pi^{(0)} h_{i,i'} \Pi^{(0)}$$

Renormalized qudit-qudit interaction
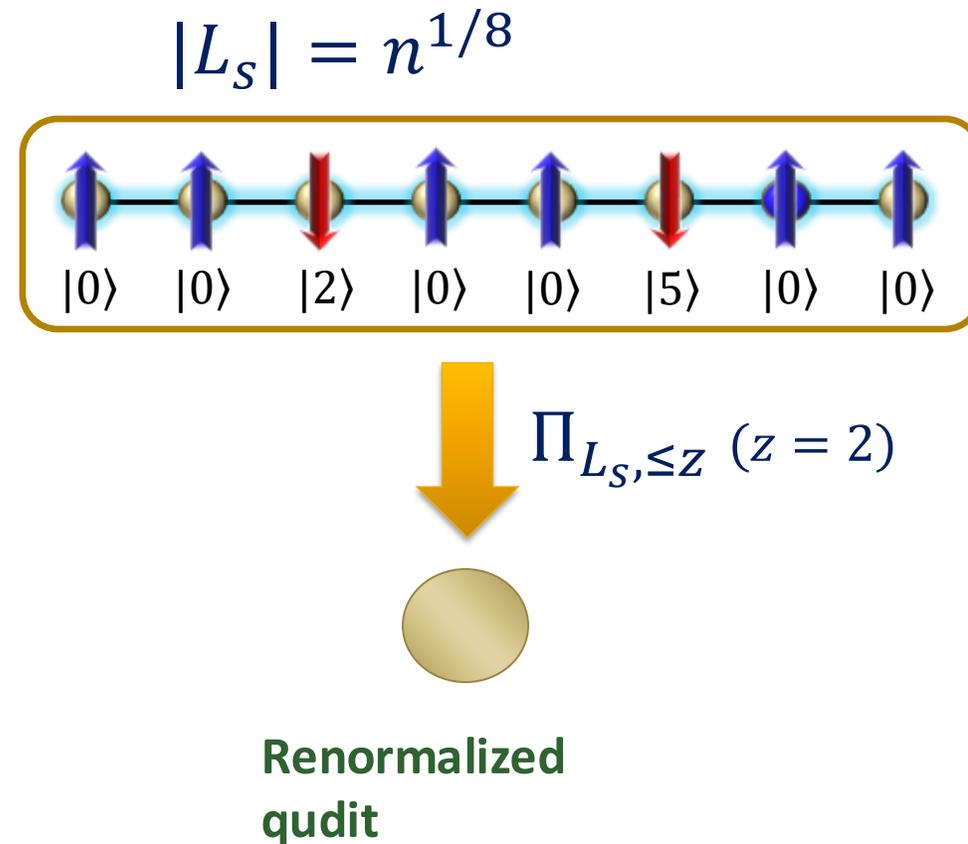
Step 4 of MFRG: iterating the processes

- Renormalized system size: $n \longrightarrow n^{7/8} \longrightarrow n^{49/64} \longrightarrow \cdots \longrightarrow n^{(7/8)^{s_0}}$ (after s_0 processes)

- Renormalized qudit's dimension:

$$d \longrightarrow d^z n^{z/8} \longrightarrow (d^z n^{z/8})^z n^{7z/64} \longrightarrow \cdots \longrightarrow (dn)^{\mathcal{O}(z^{s_0})}$$

➡ By choosing $s_0 \propto \log\log(n)$, system size = $\mathcal{O}(1)$

➡ By choosing $z = $ const., final dimension= $\boxed{(dn)^{\text{polylog}(n)}}$

Desired dimension!

Step 4 of MFRG: iterating the processes

- Renormalized system size: $n \longrightarrow n^{7/8} \longrightarrow n^{49/64} \longrightarrow \cdots \longrightarrow n^{(7/8)^{s_0}}$ (after s_0 processes)

- Renormalized qudit's dimension:

$$d \longrightarrow d^z n^{z/8} \longrightarrow ($$

➡ By choosing $s_0 \propto \mathrm{loglog}(n)$, system size

➡ By choosing $z = \mathrm{const.}$, final dimension= $(dn)^{\mathrm{polylog}(n)}$

The main technical challenge is error guarantee for the truncation z

Desired dimension!

Technical ingredients

Robustness of the ground state

- ## Variance-gap trade-off inequality

$$\boldsymbol{\Delta \cdot Var(A_L) \leq \gamma^2 \bar{g}_1 |L|}$$

A_L: k-local operator on L
γ: $\mathcal{O}(1)$ constant,
$\bar{g}_1$: local energy

Prop. 1 of arXiv:2411.02140

Robustness inequality

When the unitary u_i preserves the Schmidt bases,

$$\left|\langle\Omega|\left(u_i^\dagger H u_i - H\right)|\Omega\rangle\right| \leq \delta_{\mathbf{Rob}} \propto (n\Delta)^{-1/2}$$

$$|\langle s|u_i|s\rangle_i| = 1$$

$$|\Omega\rangle = \sum_{s=0}^{d-1} \lambda_s |s\rangle_i \otimes |\phi_s\rangle_{\Lambda\setminus\{i\}}$$

Robustness of the ground state

- Variance-gap trade-off i

$$\Delta \cdot \mathbf{Var}(A_L) \le \gamma^2$$

The ground state is locally disentangled in the sense

$$|\lambda_0|^2 \ge 1 - \frac{\delta_{Rob}}{2\Delta} \sim 1 - n^{-1/2}$$

Prop. 2 of arXiv:2411.02140

Robustness inequality

When the unitary u_i preserves the Schmidt bases,

$$\left|\langle \Omega|(u_i^\dagger H u_i - H)|\Omega\rangle\right| \le \delta_{\mathbf{Rob}} \propto (n\Delta)^{-1/2}$$

$$|\langle s|u_i|s\rangle_i| = 1$$

$$|\Omega\rangle = \sum_{s=0}^{d-1} \lambda_s |s\rangle_i \otimes |\phi_s\rangle_{\Lambda\setminus\{i\}}$$

Block Hilbert space truncation

- Small block L s.t. $|L| \ll n^{1/2}$, we have

$$\left| \langle \Omega | \left(1 - \prod_{i \in L} |0\rangle\langle 0|_i \right) |\Omega\rangle \right| \lesssim |L| \delta_{\text{Rob}} \propto \frac{|L|}{n^{1/2}} \approx 0$$

- Probability to observe deviations from the mean-field states?

➡️ **Obtaining probability distribution of 1-local operator $M_L = \sum_{i \in L} (1 - |0\rangle\langle 0|_i)$**

Ground state $|\Omega\rangle$ approximately minimize M_L, $\langle \Omega | M_L | \Omega \rangle \approx 0$

Block Hilbert space truncation

- Small block L s.t. $|L| \ll n^{1/2}$, we have

$$\left| \langle \Omega | \left(1 - \prod_{i \in L} |0\rangle\langle 0|_i \right) | \Omega \rangle \right| \lesssim |L| \delta_{\mathrm{Rob}} \propto \frac{|L|}{n^{1/2}} \approx 0$$

- Probability to observe deviations from the mean-field states?

→ **Obtaining probability distribution of 1-local operator $M_L = \sum_{i \in L}(1 - |0\rangle\langle 0|_i)$**

→ **Exponential concentration with a variance $\sqrt{|L|/\Delta}$**   Kuwahara, J. Stat. Mech. (2016) 053103

→ **Refining it to the strong decay as $\left(|L| \delta_{\mathrm{Rob}}^{1/2} \right)^z$ to observe eigenvalues of M_L larger than z**

Block Hilbert space truncation

- Small block L s.t. $|L| \ll n^{1/2}$, we have

$$\left| \langle \Omega | \left(1 - \prod_{i \in L} |0\rangle\langle 0|_i \right) | \Omega \rangle \right| \lesssim |L| \delta_{\text{Rob}} \propto \frac{|L|}{n^{1/2}} \approx 0$$

- Probability to observe deviations from the mean-field states?

→ **Obtaining probability distribution of 1-local operator $M_L = \sum_{i \in L} (1 - |0\rangle\langle 0|_i)$**

→ **Exponential** [Kuwahara, J. Stat. Mech. (2016) 053103]

More than z sites should deviate from the mean-field state

→ **Refining it to the strong decay as $\left(|L| \delta_{\text{Rob}}^{1/2} \right)^z$ to observe eigenvalues of M_L larger than z**

Block Hilbert space truncation

- Small block L s.t. $|L| \ll n^{1/2}$, we have

$$\left| \langle \Omega | \left(1 - \prod_{i \in L} |0\rangle\langle 0|_i \right) |\Omega\rangle \right| \lesssim |L| \delta_{\mathrm{Rob}} \propto \frac{|L|}{n^{1/2}} \approx 0$$

- Probability to observe deviations from the mean-field states?

Decay rate is given by $\left(|L|/n^{1/4} \right)^z$

➡ **Choice of $|L| = n^{1/8}$ makes** $\left(n^{-1/8} \right)^z$ ➡ $z = \mathcal{O}(1)$ **is enough to ensure** $1/\mathrm{poly}(n)$

➡ **Refining it to the strong decay as** $\left(|L| \delta_{\mathrm{Rob}}^{1/2} \right)^z$ **to observe eigenvalues of M_L larger than z**

Prop. 4 of arXiv:2411.02140

Renormalized interaction amplitude

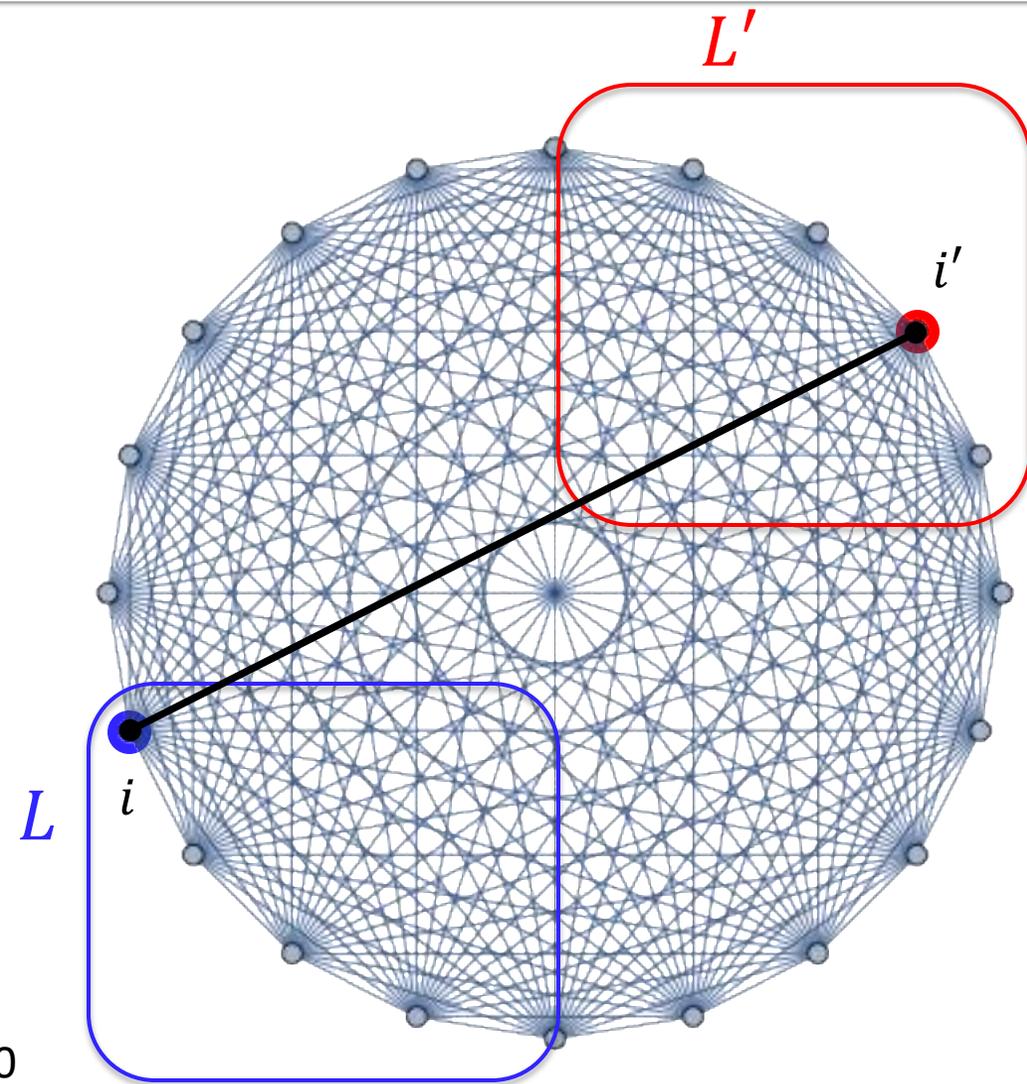- Block-block interaction $H_{L,L'}$

  ➡️ $\left\|\Pi^{(0)} H_{L,L'} \Pi^{(0)}\right\| \propto |L| \times |L'|/n$

- We need to suppress the amplification

  ➡️ Appropriate decomposition of

  $$H_{L,L'} = \Delta H_{L,L'} + \widetilde{H}_L + \widetilde{H}_{L'} + \text{const.}$$

  ➡️ $\left\|\Pi^{(0)} \Delta H_{L,L'} \Pi^{(0)}\right\| \propto z|L|/n$

  Prop. 5 of arXiv:2411.02140

L'

i'

L i

Renormalized interaction amplitude
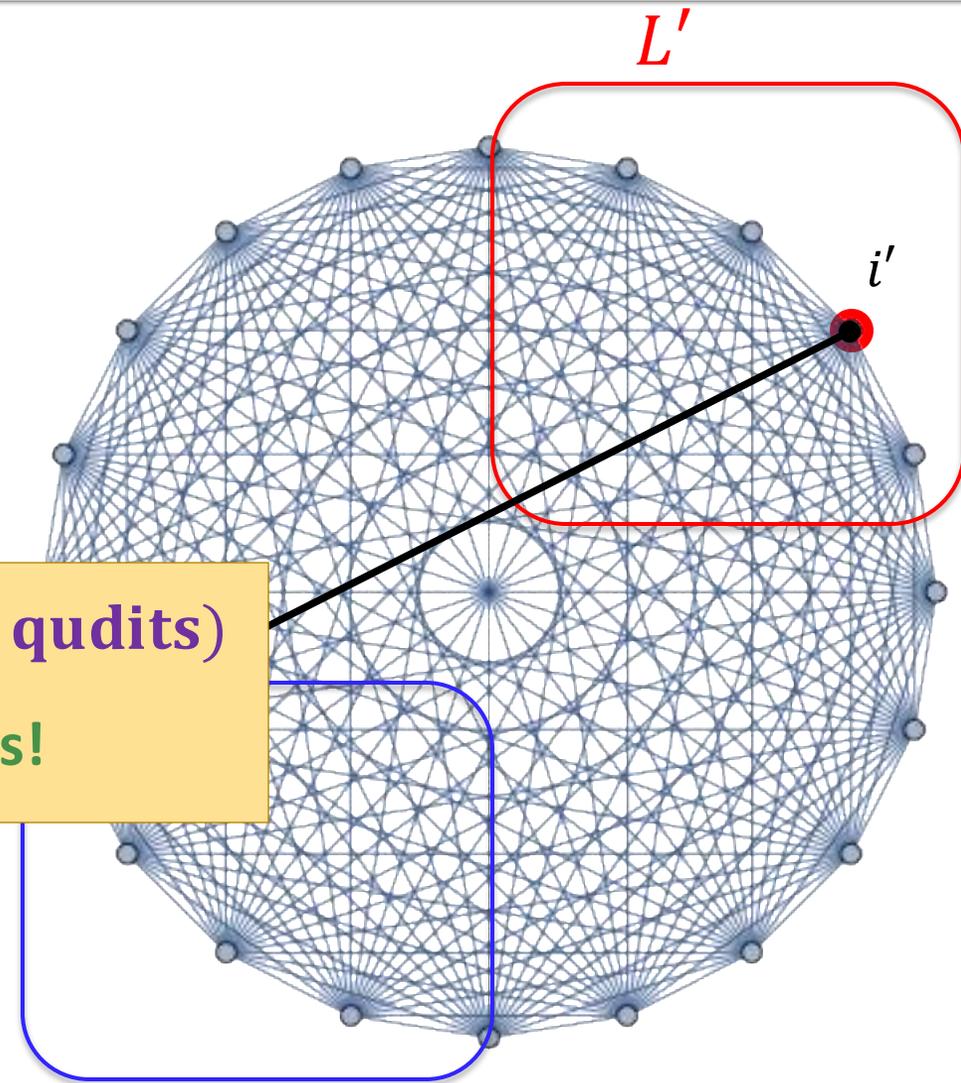
- Block-block interaction $H_{L,L'}$

  ➤ $\left\| \Pi^{(0)} H_{L,L'} \Pi^{(0)} \right\| \propto |L| \times |L'|/n$

- We need to suppress the amplification

Qudit-qudit interaction $\propto z/(\text{No. of renormalized qudits})$

➤ **The interaction amplification is at most z times!**

➤ $\left\| \Pi^{(0)} \Delta H_{L,L'} \Pi^{(0)} \right\| \propto z/(n/|L|)$
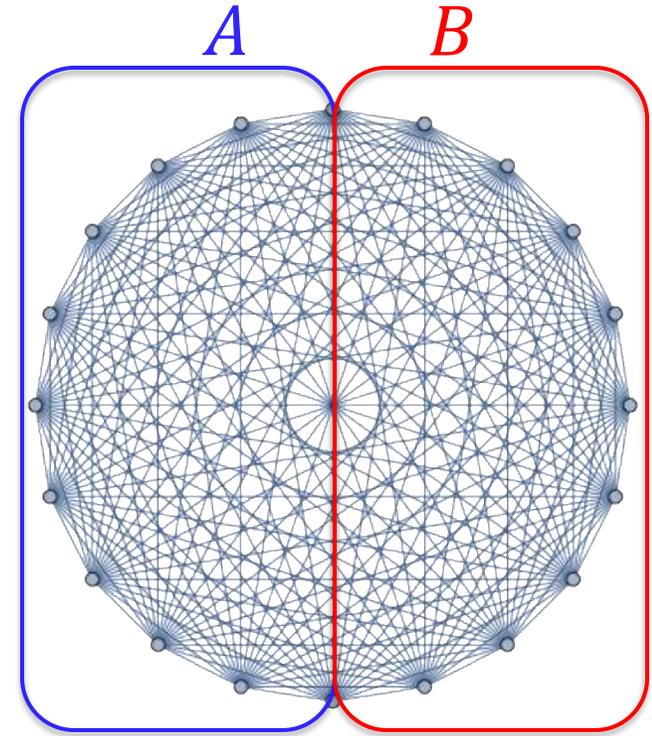
Summary and future problems

Summary

- Local Hamiltonian Problem on the fully connected graph

- Generalized area law and MPS approximation

$$\star \quad S_{AB}(\Omega) \leq 2[\log(n)]^{\alpha_1 + \alpha_2 \log(f(\bar{g}_1, \Delta))} + \log(d) + 2$$

$$\star \quad D = \exp\left[(\log n)^{c \, \log\log(1/\delta)}\right]$$

$$\text{for } \|\text{tr}_{X^c}(|\Omega\rangle\langle\Omega|) - \text{tr}_{X^c}(|\Omega_D\rangle\langle\Omega_D|)\| \leq \delta|X|$$

- The non-critical ground states have quasi-polynomial complexity

Open problems

(Q1) Can we remove the poly-logarithmic factor?

(Q2) Can we achieve polynomial complexity?

Thank you for listening!