

New Frontiers in Tensor Networks

Computing and Compressing Quantum and Classical Fields

Hiroshi SHINAOKA
Saitama University



Collaborators from TU Wien, LMU Munich, Univ. Fribourg, Riken, CEA Grenoble....

New online seminar series

Asian–Pacific Condensed Matter Physics (CMP) seminars

About

The Asian-Pacific Condensed Matter Physics Seminars (AP-CMP) is a series of online seminars that brings together researchers from across the Asia-Pacific region and beyond. Our goal is to foster scientific exchange and collaboration in the field of condensed matter physics.

Mailing List

Please join [our public mailing list](#) to receive the meeting information. If you have a Google account, you can join the mailing list by clicking the “Join Group” button on the Google Groups page.

If you does not have a Google account, please submit your request to [a request form](#). Note that we will add corrected e-mail addresses regularly but not immediately. We strongly recommend you to join the mailing list using your Google account if any. If you do not receive an invitation within a few days, please check the trash folder and then contact us.

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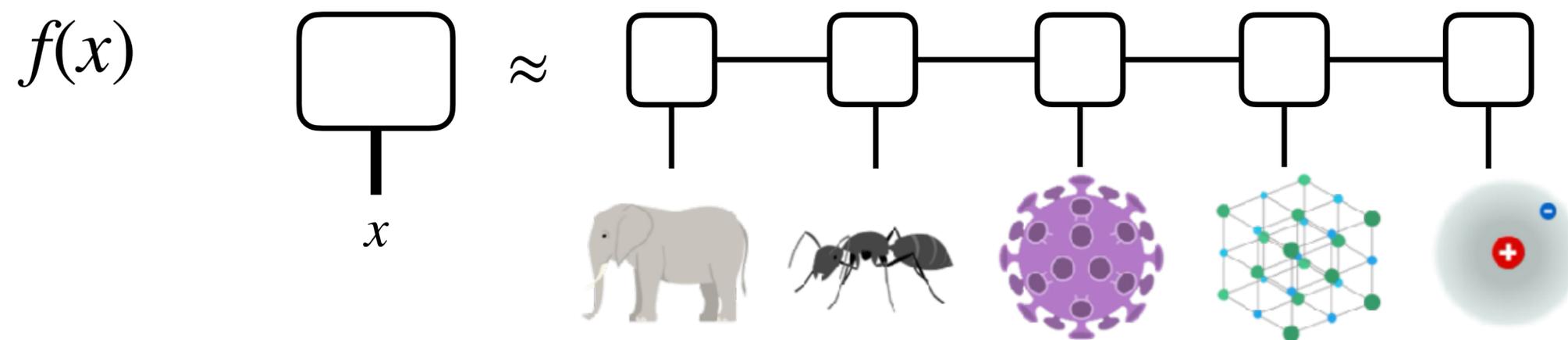
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- Tokuro Shimokawa (Okinawa Institute of Science and Technology Graduate University)
- Nayuta Takemori (The University of Osaka)
- Yuan Wan (Chinese Academy of Sciences)

Name	Affiliation	Title	Time	Chair
Yantao Wu	Institute of Physics, Chinese Academy of Sciences	Alternating and Gaussian fermionic Isometric Tensor Network States	July 22, 2025 (Tuesday) 11:00–12:00 JST	Lei Wang → Hiroshi Shinaoka
Po-Yao Chang		Diagnosing Many-Body Systems with Entanglement: Insights from Non-Unitary CFTs and TQFTs	August 12, 2025 (Tuesday) 11:00–12:00 JST	
Mingu Kang		TBA	August 26, 2025 (Tuesday) 11:00–12:00 JST	
Tsung-Han Lee		TBA	Sep 2, 2025 (Tuesday) 11:00–12:00 JST	
Tatsuya Miki	Saitama University	First-principles Calculations of Electron Chirality in Solids	October 14, 2025 (Tuesday) 11:00–12:00 JST	Hiroshi Shinaoka

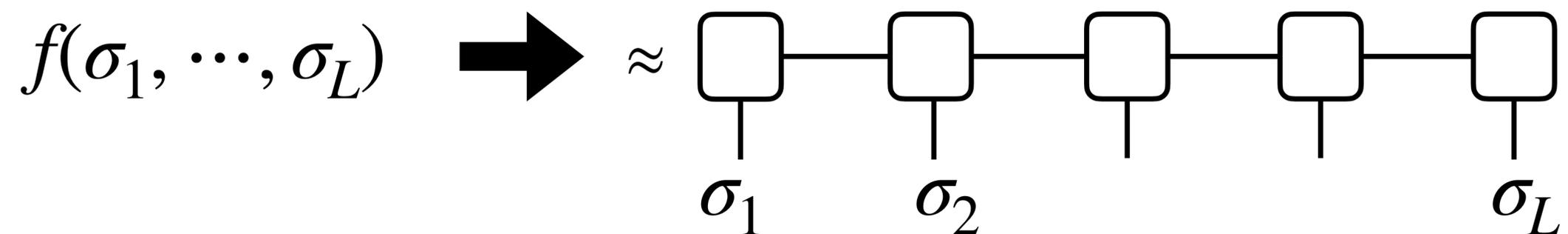
In a Nutshell

Quantics Tensor Train (QTT): exploiting scale separation



Convolution, Fourier transform...

Tensor Cross Interpolation (TCI): adaptively learn a TT from a function



Integration...

tensor4all community

tensor4all
<https://tensor4all.org/>



This website collects information from the tensor4all group which is working on tensor network methods based on tensor cross interpolation (TCI) and related tensor learning algorithms such as quantics/quantized tensor trains.

News

The House of Quantum Alps, the University of Saitama and the University of Munich LMU co-organize an International workshop on tensor cross interpolation and other algorithms for learning tensor networks.

- 📅 From October 6th to October 10th, 2025
- 📍 Y Spot Partners, Grenoble, France
- 📄 Registration is free but subject to approval (We only have only 50 available slots in total).
- 👤 Call for oral presentations and poster is open.

If you want to register, you need to create an account on scienceconf.org and register by following the steps on <https://tensornetworks.sciencesconf.org>.

Literature

A pedagogical introduction to tensor network methods, which includes an overview of the existing literature and also new algorithms, can be found in:

Yuriel Núñez Fernández, Marc K. Ritter, Matthieu Jeannin, Jheng-Wei Li, Thomas Kloss, Thibaud Louvet, Satoshi Terasaki, Olivier Parcollet, Jan von Delft, Hiroshi Shinaoka, and Xavier Waintal, "Learning tensor networks with tensor cross interpolation: new algorithms and libraries", [arXiv:2407.02454](https://arxiv.org/abs/2407.02454).

Please check the [reference](#) page for more information on TCI and quantics tensor trains.

- References
- Lecture notes
- C++/Python/Julia tutorials
- Monthly online meeting
- Onsite meeting (October 2025)



X. Waintal (CEA Grenoble)



J. von Delft (LMU Munich)

and H. Shinaoka (Saitama U.)

MACHINE LEARNING TENSOR NETWORK WORKSHOP

Topic of the workshop: Tensor network techniques are well-established numerical tools for solving seemingly exponentially hard problems in quantum many-body physics, with important successes in the simulation of model correlated electronic systems, quantum magnetisms and quantum computers. In this type of application, tensor networks are used to obtain compressed representations of quantum many-body wave functions, allowing for an exponential speed-up of otherwise impossible calculations.

Grenoble, October 2025

In the last few years, tensor network methods have spread far beyond applications in many-body physics, due to two key developments. First, it has been realized that many mathematical objects encountered in computational physics, such as the solutions of partial differential equations, are compressible too and admit low-rank tensor network representations. These enable manipulations of functions (integration, convolution, Fourier transform, ...) to be performed exponentially faster. Second, new exponentially fast learning algorithms are emerging for finding accurate tensor network representations of huge but compressible tensors – a key example being tensor cross interpolation (TCI). These algorithms – akin to machine learning – find compact representations of the entire object from a tiny training data set.

Together, these two developments provide a path for transferring much of the tensor network methodology developed for quantum many-body computations to entirely new classes of mathematical problems, such as high-dimensional integrals, partial differential equations, or function optimization. New applications, e.g. in computational fluid dynamics, plasma physics, optimized basis sets in chemistry, evaluation of Feynman diagrams in quantum field theory, to name but a few, are rapidly emerging.

This workshop will gather important players in this emerging field, from both the applied mathematics and theoretical physics communities, to discuss these exciting new applications and also activities within the tensor4all collaboration (<https://tensor4all.org/>). The schedule will provide a room for scientific discussions.

Fluid dynamics

Registration closed, next year
in Asia?

CONFIRMED INVITED SPEAKERS:

CHUNG Chia-Min, National Sun Yat-Sen University (Taiwan) ; **JAKSCH Dieter**, University of Hamburg (Germany) ; **KAO Ying-Jer**, National Taiwan University (Taiwan) ; **KAUCH Anna**, Vienna University of Technology (TU Wien, Austria) ; **KAYE Jason**, Flatiron Institute (USA) ; **KHOO Yuehaw**, University of Chicago (USA) ; **LADO Jose**, Aalto University (Finland) ; **LINDSAY Michael J.**, University of California, Berkeley (USA) ; **NUNEZ-FERNANDEZ Yurriel**, Néel Institute, CNRS (France) ; **OSELEDETS Ivan**, Skolkovo Institute of Science and Technology (Russia) ; **PARCOLLET Olivier**, Flatiron Institute (USA) ; **RITTER Marc**, Ludwig Maximilian University of Munich (Germany) ; **SAKURAI Rihito**, University of Tokyo (Japan) ; **SAVOSTYANOV Dmitry**, University of Essex (UK) ; **STOUDENMIRE Miles**, Flatiron Institute ; **WALLERBERGER Markus**, Vienna University of Technology (TU Wien, Austria).

Quantum physics

Applied Math

Outline

Part I

- Introduction
 - Quantics Tensor Train (QTT)
 - Tensor Cross Interpolation (TCI)
- Applications to quantum field theories
- New TCI algorithms & open-source libraries

Part II

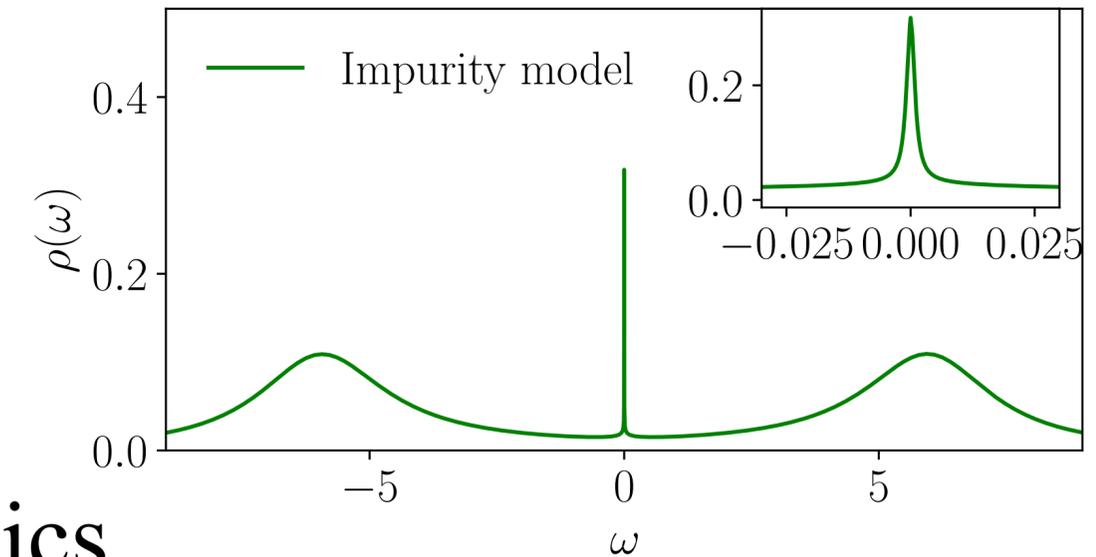
Short lecture on TCI

Coexisting scales in physics

- Classical example
 - **Turbulence**
from macroscopic to microscopic vortices
 - **Plasma**
ion and electron dynamics from nano- to milliseconds, microns to meters



- Quantum example
 - **Thermal equilibrium**
Kondo physics, bandwidth vs low T_c
 - **Nonequilibrium dynamics**
Fast electron relaxation and slow phonon dynamics



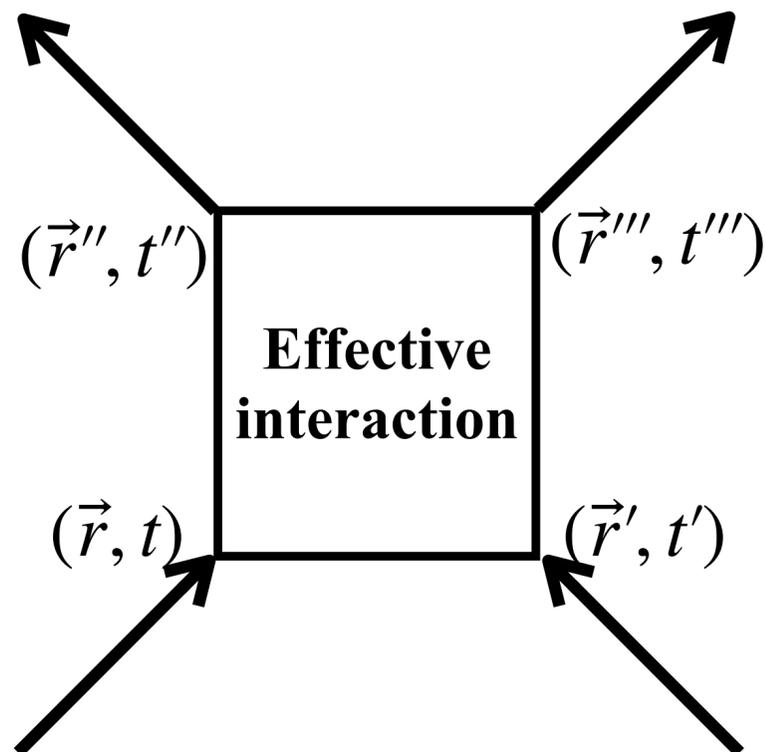
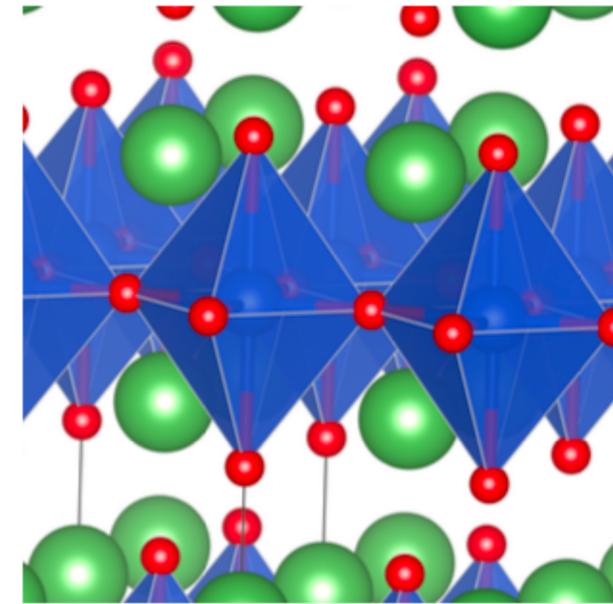
How to describe such a field and perform computation?

My own motivation

Novel theories

- Migdal-Eliashberg theory
- GW
- Dynamical mean-field theory
- Functional RG
- ...

Novel correlated physics

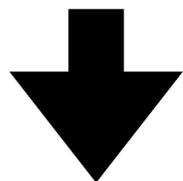
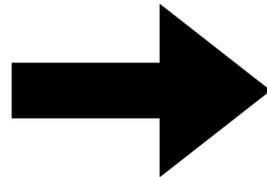
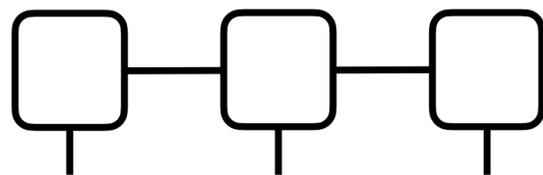


Needed: compression and computation!

Tensor networks

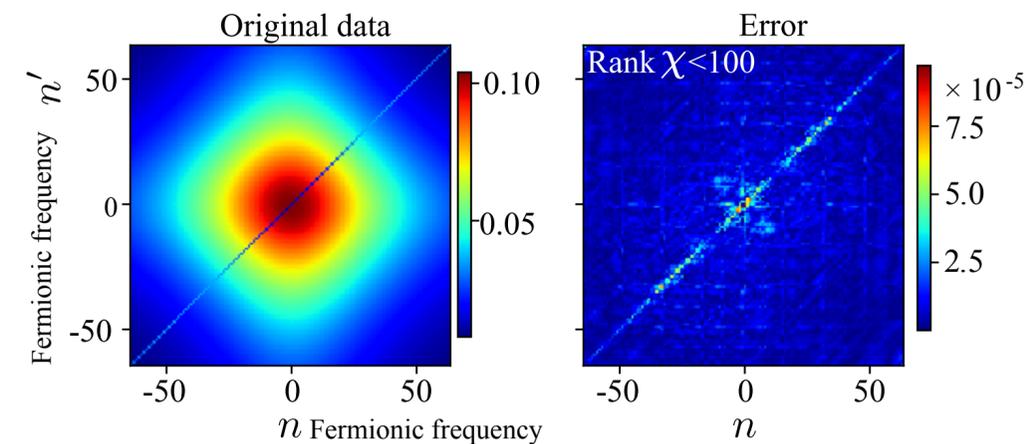
Conventional targets

Low-dimensional quantum states

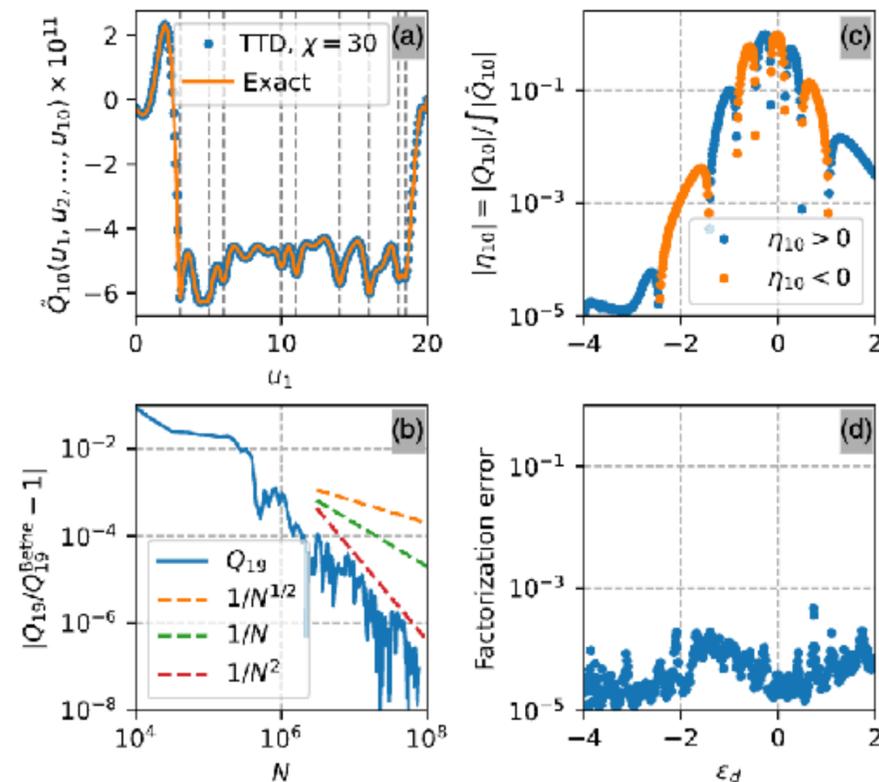


Quantum field theories

HS *et al.*, PRX **13**, 021015 (2023)



Y. Núñez Fernández *et al.*, PRX **12**, 041018 (2022)



Navier-Stokes equations

N. Gourianov *et al.*, Nat. Comput. Sci. **2**, 30 (2022)

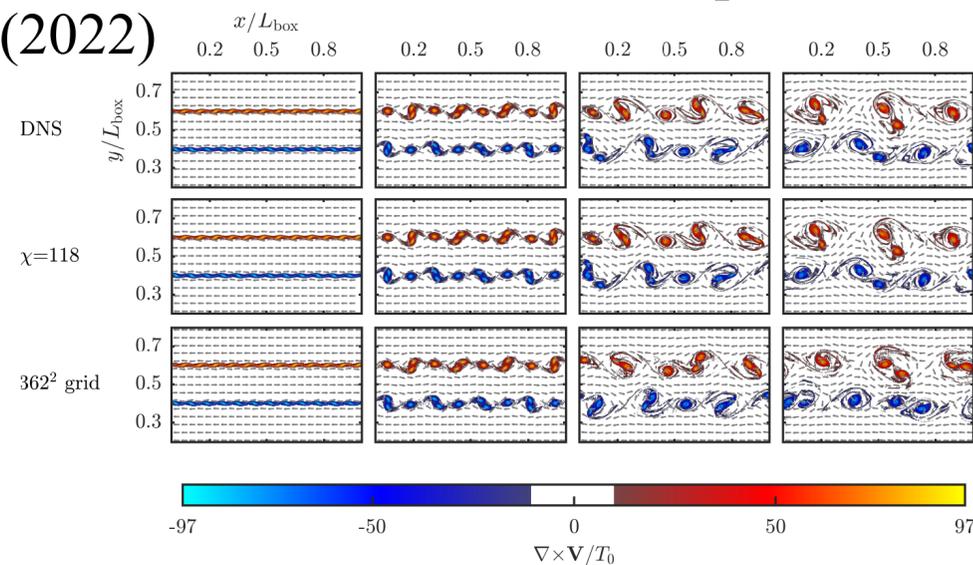


Image compression

José I. Latorre, arXiv:quant-ph/0510031v1

Machine learning

Zhao-Yu Han *et al.*, PRX **8**, 031012 (2018)

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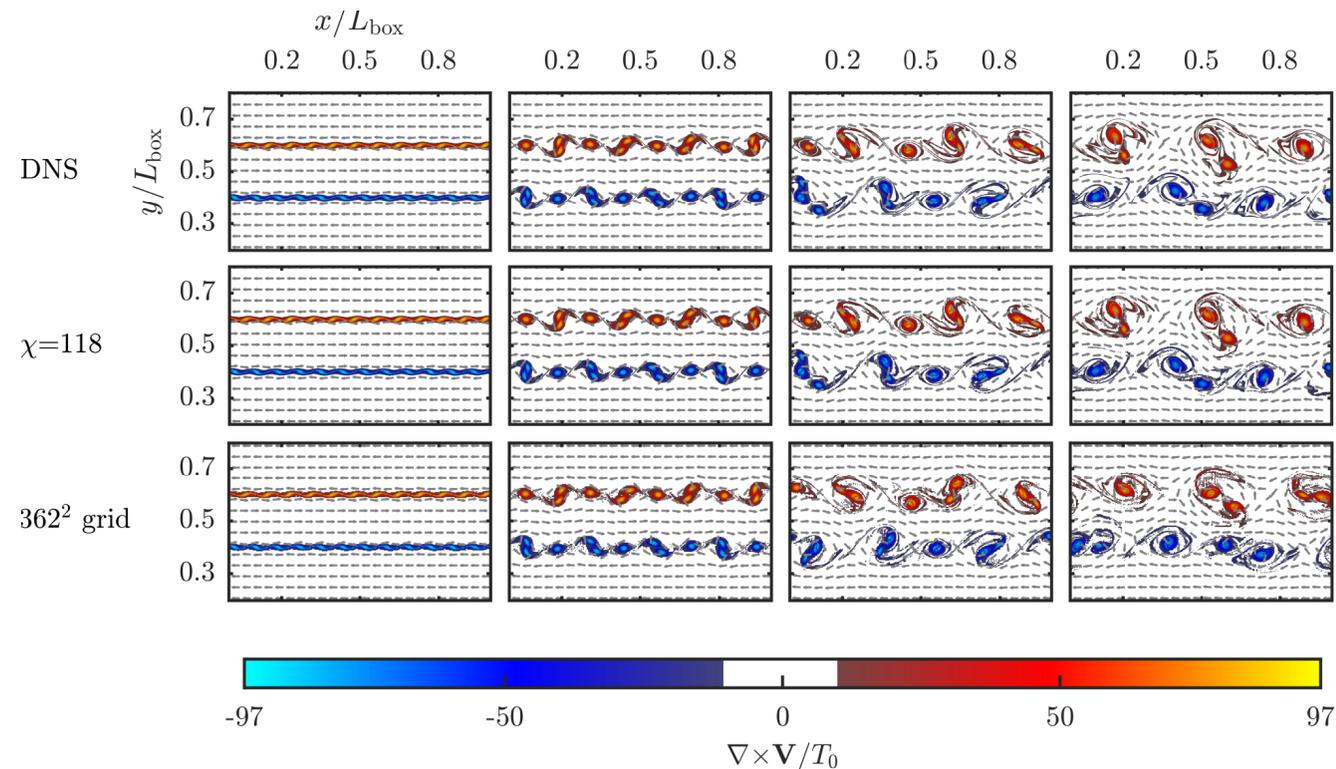
A golden rule for efficient computation: **Exploit structure!**

- Periodic functions \rightarrow Fourier basis
- Symmetry-invariant systems \rightarrow Group theory
- Low entanglement entropy \rightarrow Tensor networks
- **Separation of length scales \rightarrow Quantics representation**

Scale separation

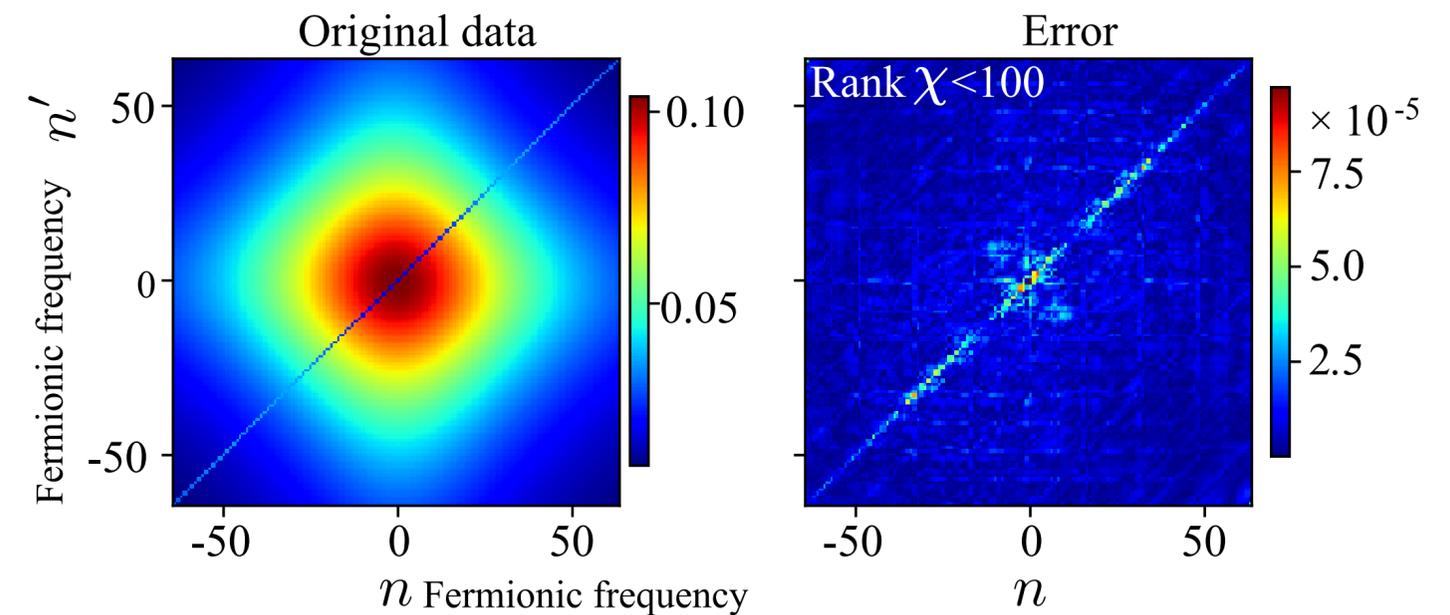
Solving Navier-Stokes equations for turbulent flows

N. Gourianov *et al.*, Nat. Comput. Sci. **2**, 30 (2022)



Quantum field theories

HS *et al.*, PRX **13**, 021015 (2023)



Vlasov-Poisson equations for collisionless plasmas

E. Ye and N. F. G. Loureiro, PRE **106**, 035208 (2022)

Tensorized orbitals for quantum chemistry

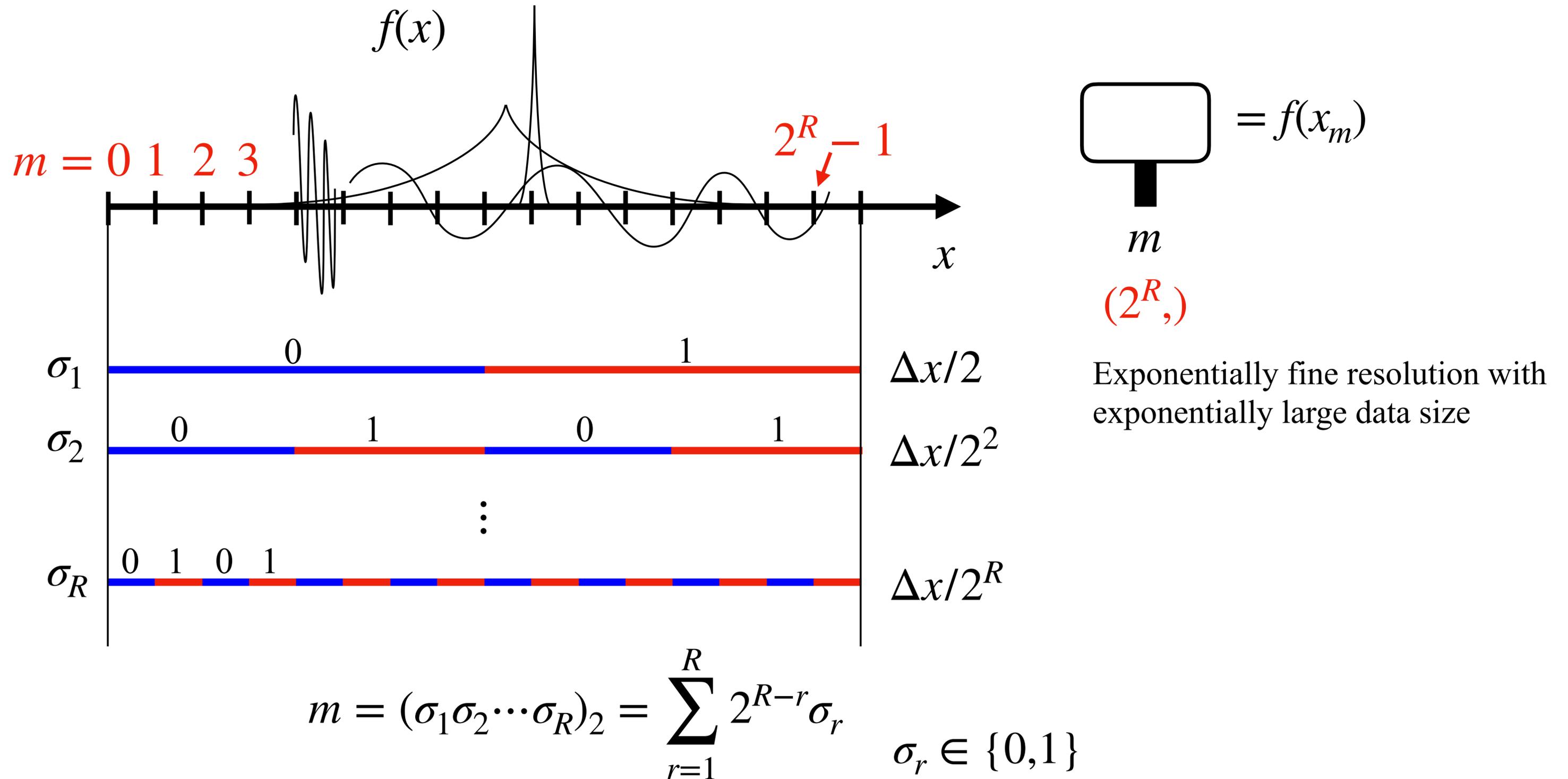
N. Jolly, Y. Núñez Fernández, X. Waintal, Phys. Rev. B **111**, 245115 (2025)

and more and more!

Quantics Tensor Train (QTT)

I. V. Oseledets, Doklady Math. **80**, 653 (2009)

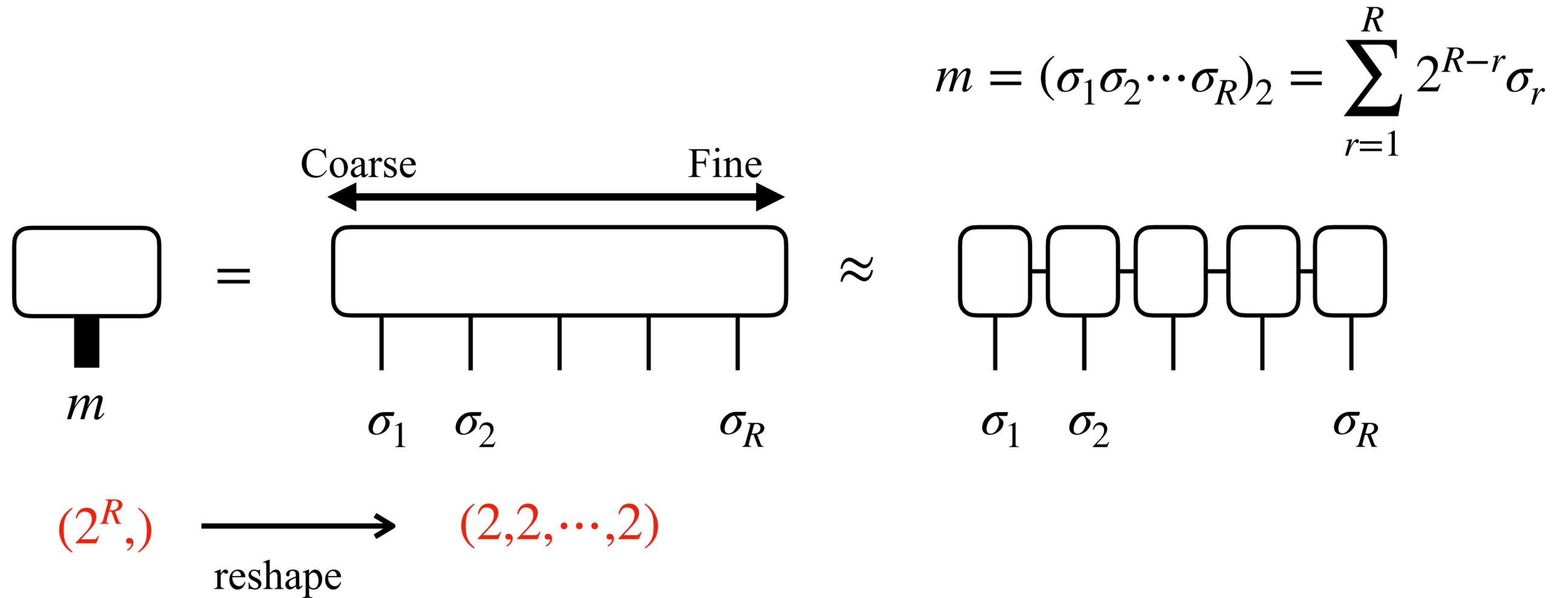
B. N. Khoromskij, Constr. Approx. **34**, 257 (2011)



Quantics Tensor Train (QTT)

I. V. Oseledets, Doklady Math. **80**, 653 (2009)

B. N. Khoromskij, Constr. Approx. **34**, 257 (2011)



Exponentially fine resolution, multivariate variables, computable!

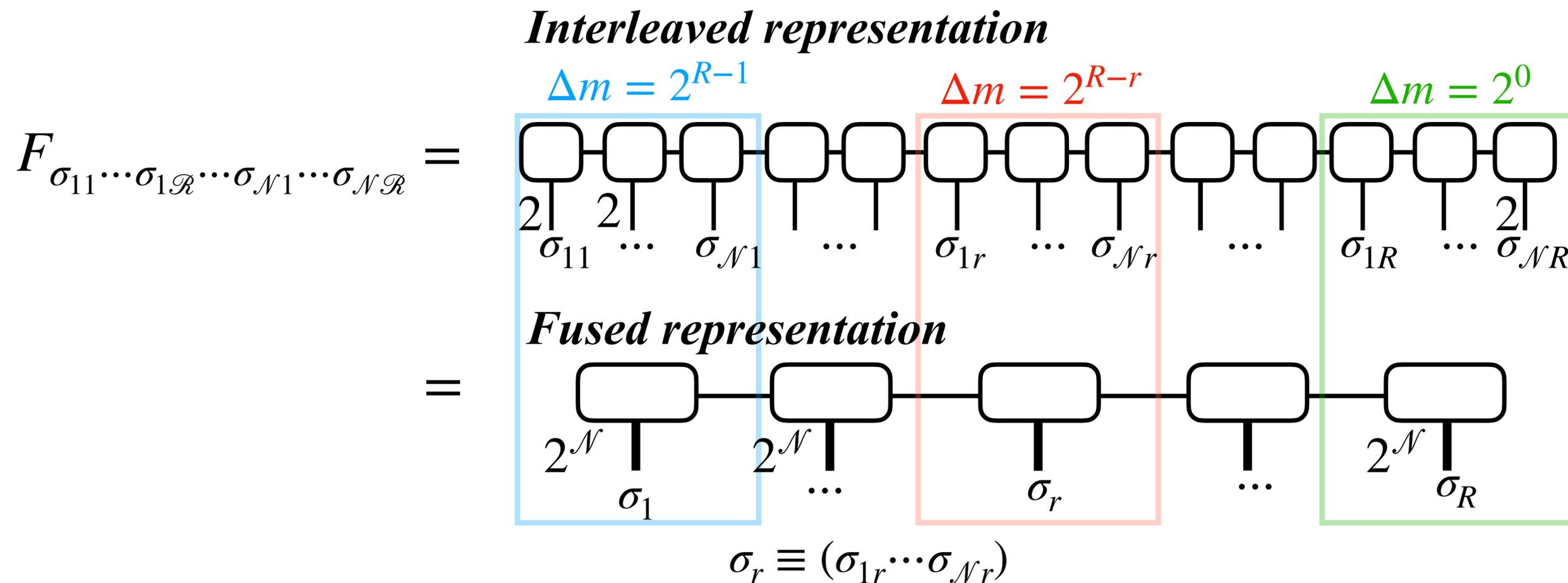
Multivariate function

I. V. Oseledets, Doklady Math. **80**, 653 (2009)

B. N. Khoromskij, Constr. Approx. **34**, 257 (2011)

\mathcal{N} -dimensional function $f(x_1, \dots, x_n, \dots, x_{\mathcal{N}})$

Binary coding for n -th variable $m_n = (\sigma_{n1} \cdots \sigma_{nR-1} \sigma_{nR})_2$



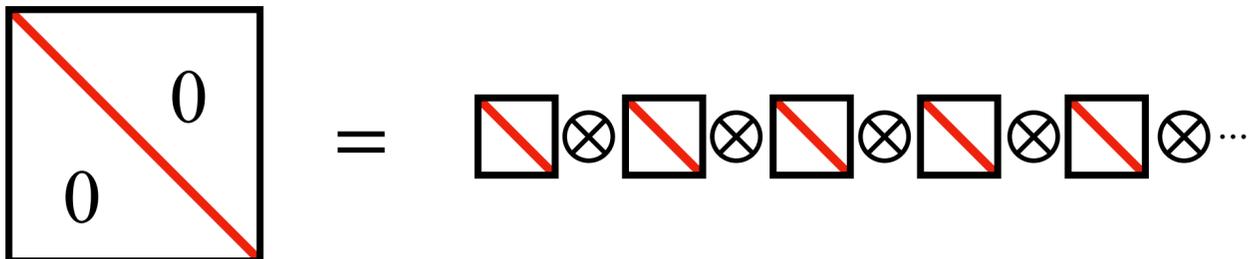
Degrees of freedom at the same length scale are usually highly entangled.

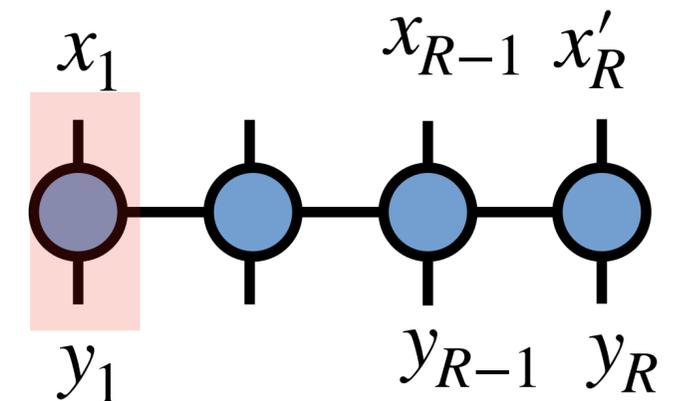
Compressible functions

Exponential function $f(x) = e^x = e^{x_1/2} e^{x_2/2^2} \dots e^{x_n/2^n} \dots$ $\chi = 1$
 $x = (0.x_1x_2\dots x_n\dots)_2 \in [0,1)$

The sum of N exponential functions can be represented as a QTT of rank at most N .
 \because Bond dimensions are added when MPSs are added.

Identity matrix $f(x, y) = \delta_{x,y} = \delta_{x_1,y_1} \delta_{x_2,y_2} \dots$ $\chi = 1$

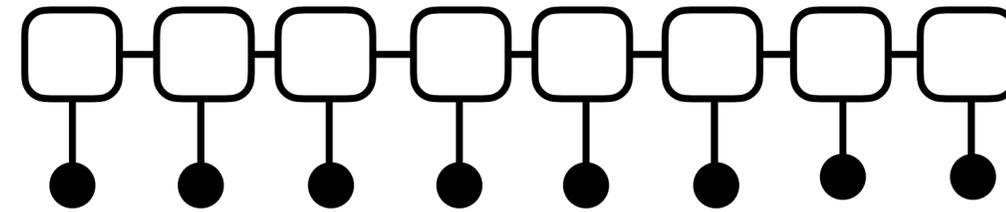
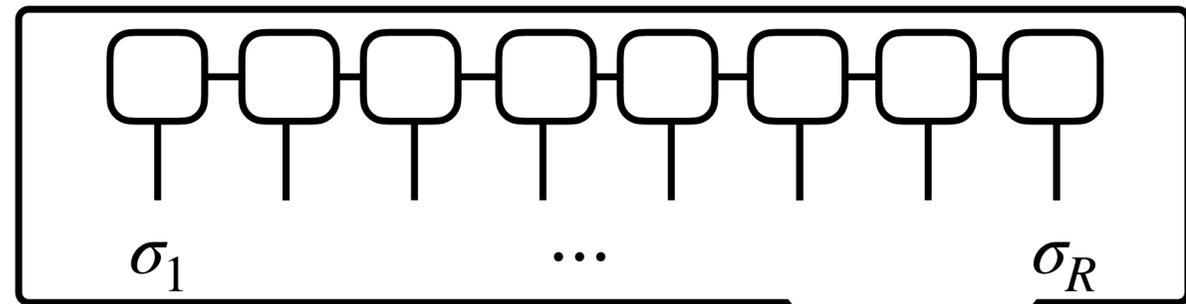




- Representation of Continuous Functions: <https://tensornetwork.org/functions/>
- B. N. Khoromskij, Tensor Numerical Methods in Scientific Computing, doi:10.1515/978311036591

by Miles Stoudenmire

Integration



↓ = (1,1)

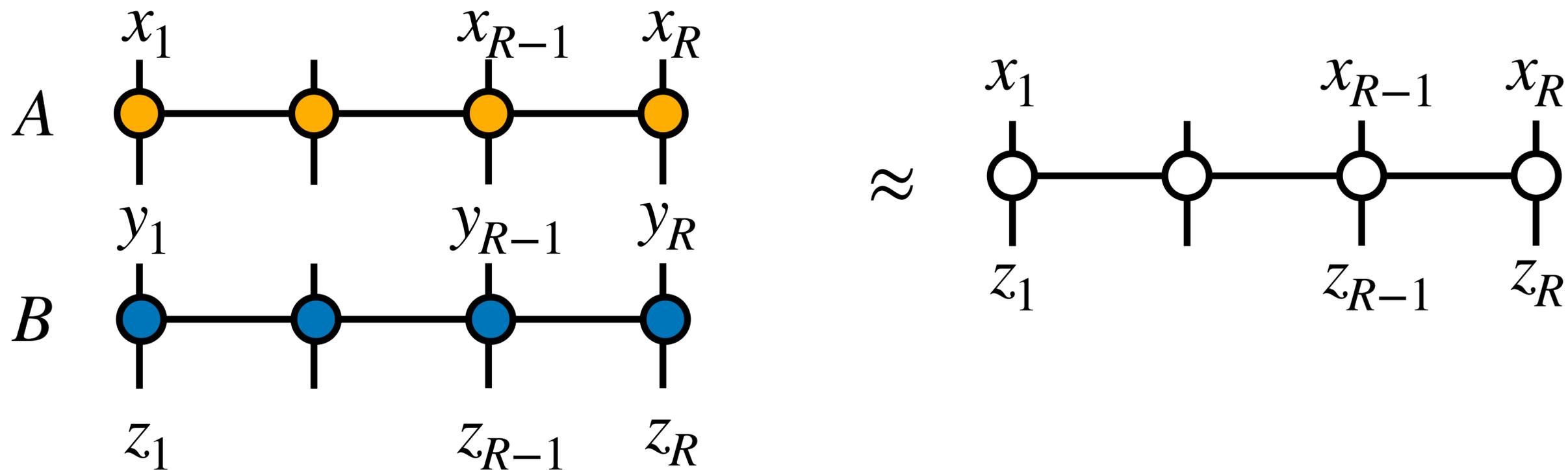
$$\int dx f(x) = 2^{-R} \sum_{\sigma_1, \dots, \sigma_R} f(\sigma_1, \dots, \sigma_R) + O(2^{-R})$$

Efficient integration with exponentially small discretization errors!

Convolution = MPO-MPO contraction!

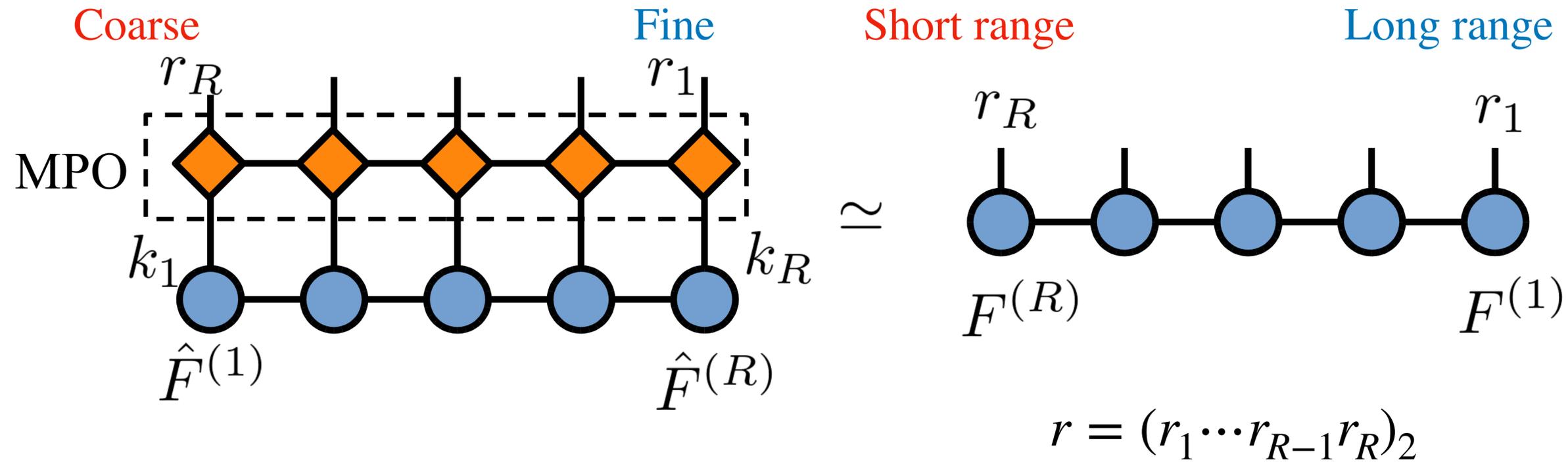
$$\int dy A(x, y) B(y, z) \approx C(x, z)$$

$$\sum_{y_1, \dots, y_R} A(x_1, y_1, \dots, x_R, y_R) B(y_1, z_1, \dots, y_R, z_R) \approx C(x_1, y_1, \dots, x_R, z_R)$$



Fourier Transform

$$F(r) = \int dk \hat{F}(k) e^{ikr}$$



Matrix product operator (MPO) is equivalent to the Quantum Fourier Transform quantum circuit.

K. J. Woolfe *et al.*, Quantum Inf. Comput. **17**, 1 (2017)

HS *et al.*, PRX **13**, 021015 (2023)

J. Chen, E.M. Stoudenmire, S. R. White, PRX Quantum **4**, 040318 (2023)

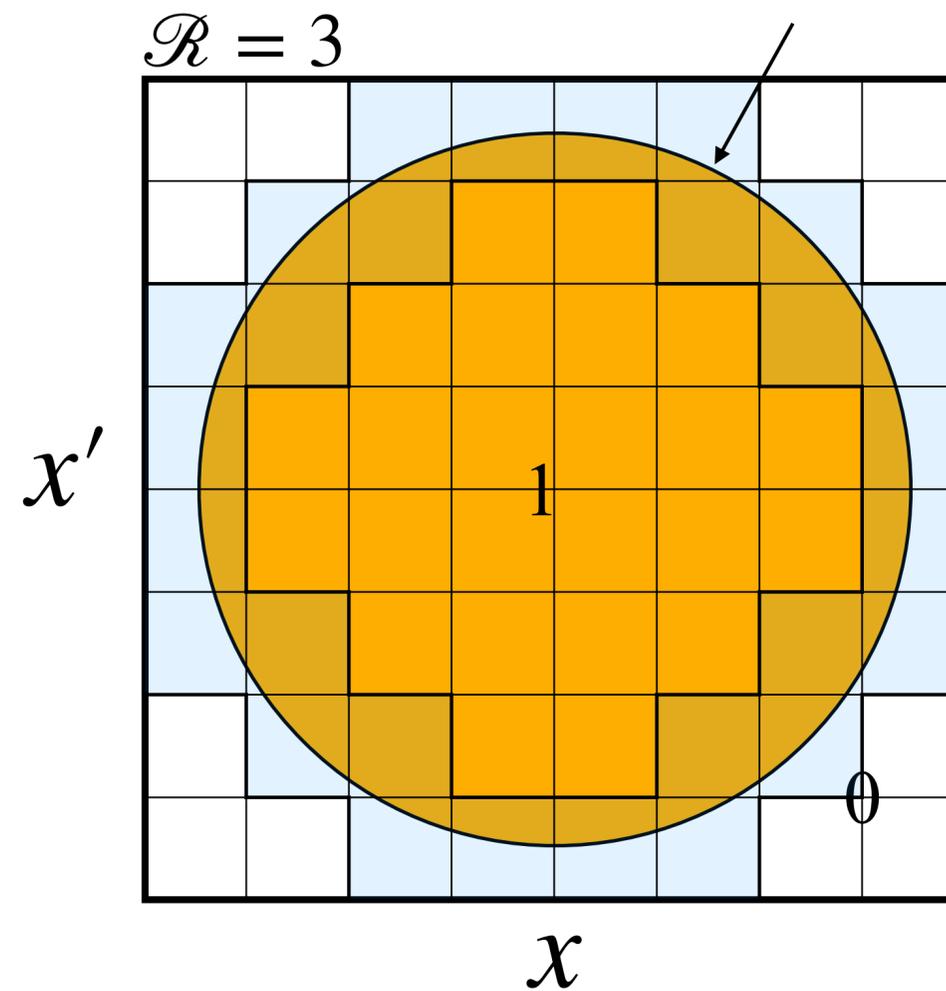
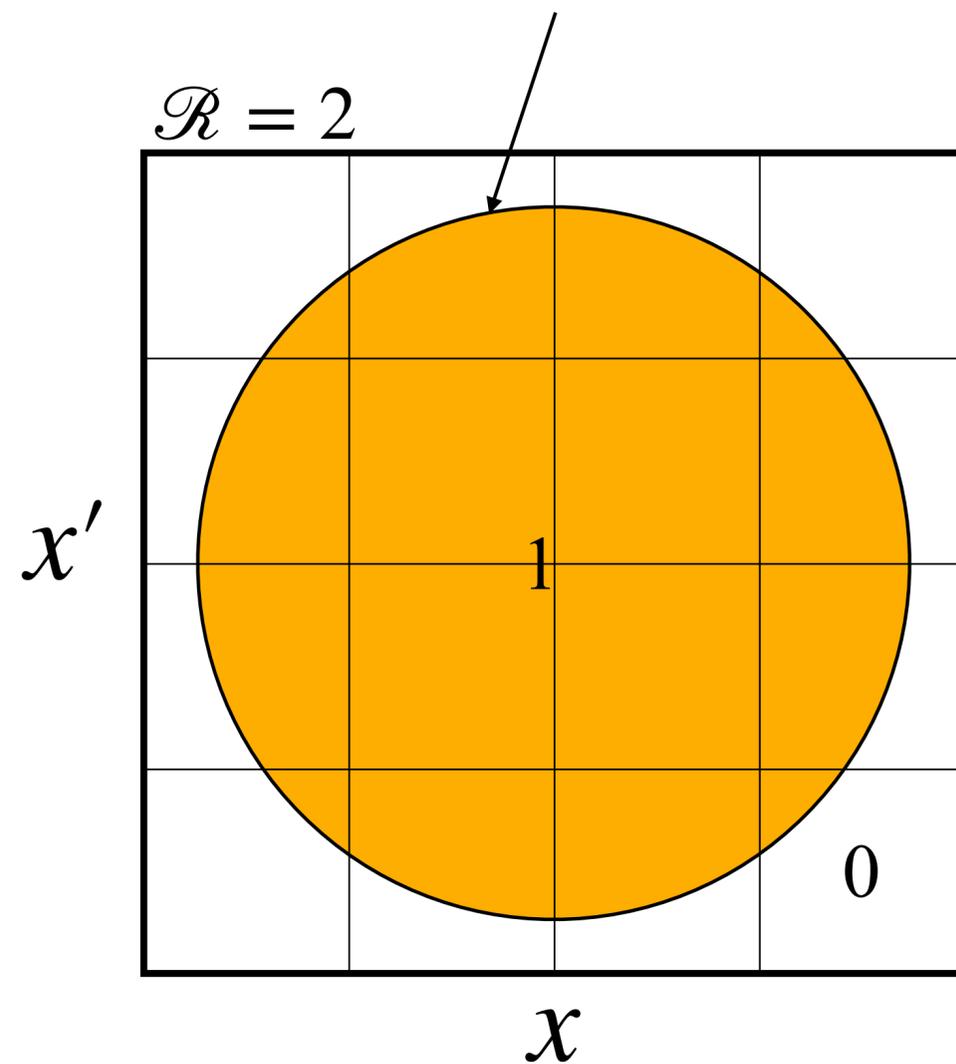
Less compressible functions

χ increases exponentially with \mathcal{R} .

I. V. Oseledets and E. E. Tyrtyshnikov, SIAM J. Sci. Comput. **33**, 1315 (2011)

\because # of linearly independent patches increases exponentially.

Discontinuity on curved boundary



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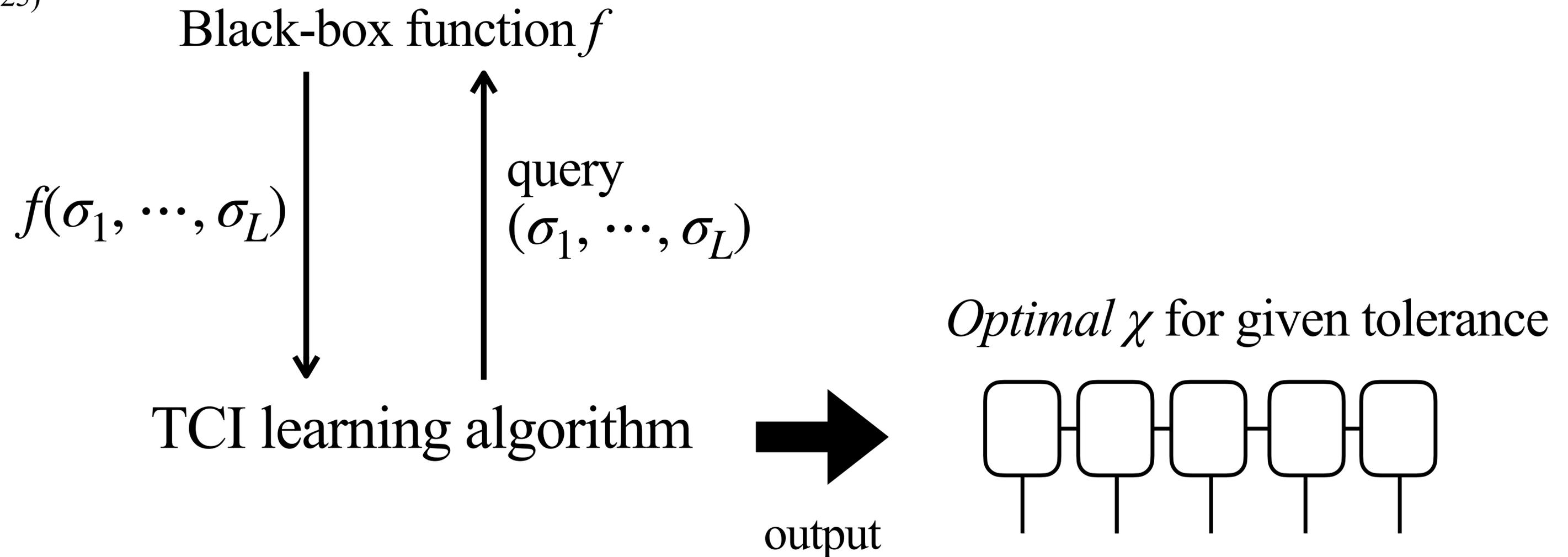
Tensor Cross Interpolation (TCI)

I. V. Oseledets, SIAM Journal on Scientific Computing **33**, 2295 (2011)

S. Dolgov and D. Savostyanov, Computer Physics Communications **246**, 106869 (2020)

Y. Núñez Fernández *et al.*, PRX **12**, 041018 (2022)

Y. Núñez Fernández, M. K. Ritter, M. Jeannin, J.-W. Li, T. Kloss, T. Louvet, S. Terasaki, O. Parcollet, J. von Delft, **HS**, X. Waintal, SciPost Phys. **18**, 104 (2025)



Adaptive learning for TT without gradient descent

Some details in Part II

Pioneer work on TCI + quantum field theory

Y. Núñez Fernández *et al.*, PRX **12**, 041018 (2022)

Replacing diagrammatic Monte Carlo sampling

$$H = H_0 + UH_{\text{int}} \quad Q(U) = \sum_n Q_n U^n \quad \text{Charge, Green's function, etc.}$$

$$Q_n = \int dv_1 \dots dv_n \tilde{Q}_n(v_1, \dots, v_n)$$

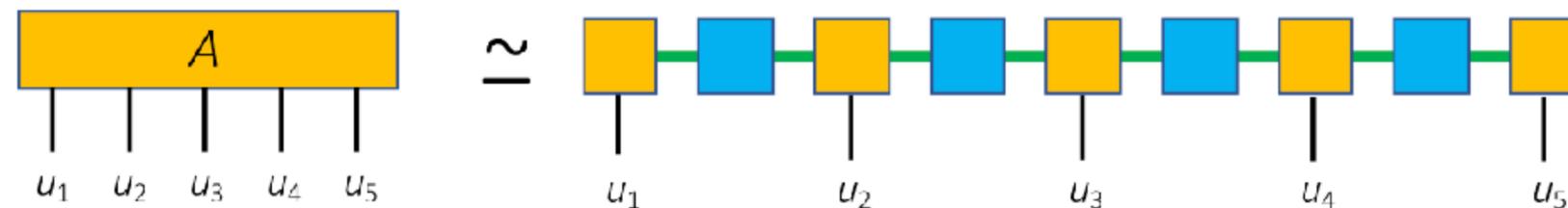


Low-rank approximation by Tensor Cross Interpolation (TCI)

$$\tilde{Q}_n(v_1, \dots, v_n) \approx M_1(v_1) \cdots M_n(v_n)$$

$$Q_n \approx \left(\int dv_1 M_1(v_1) \right) \cdots \left(\int dv_n M_n(v_n) \right)$$

TCI

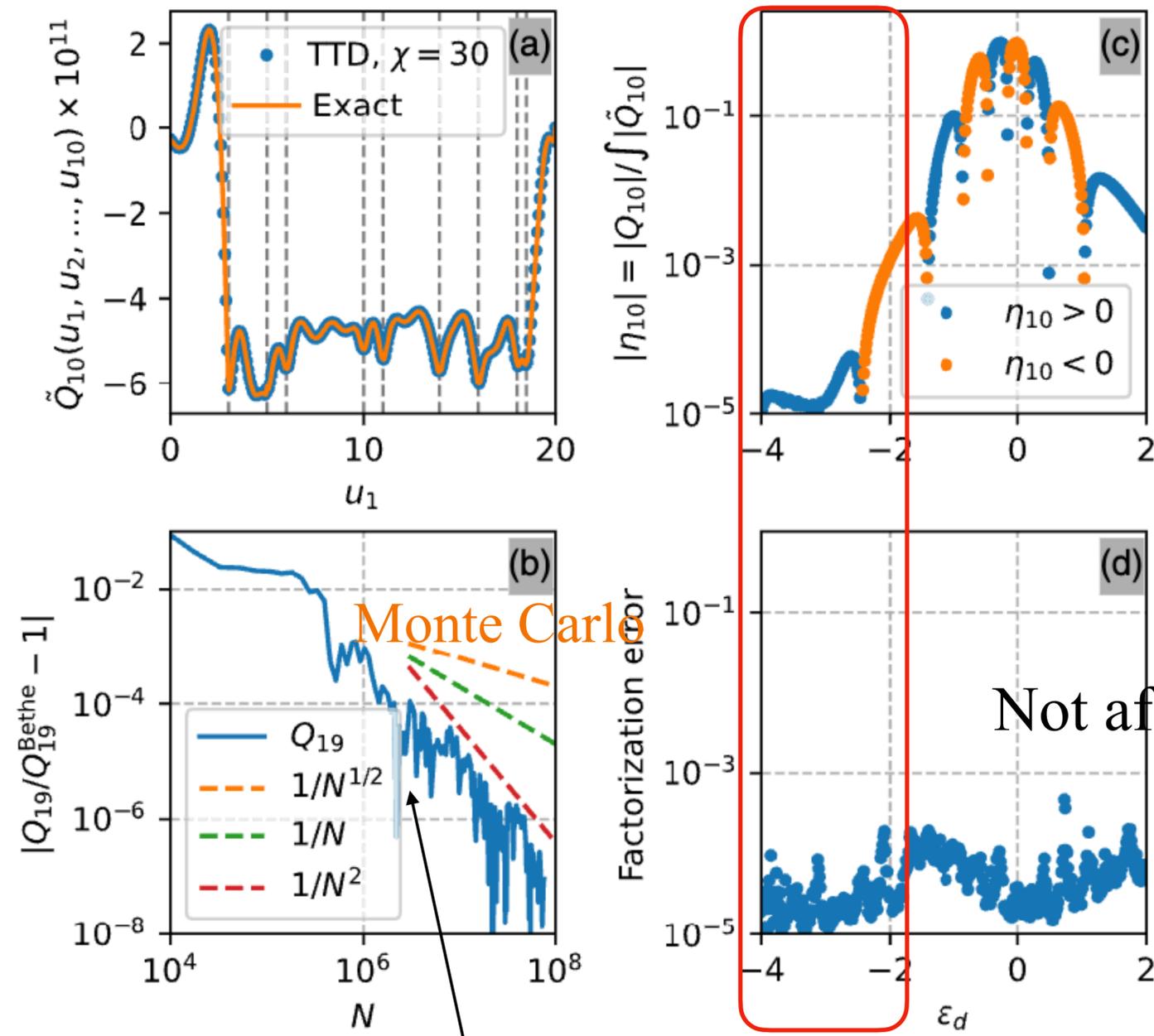


(without quantics)

Pioneer work on TCI + quantum field theory (without quantics)

Y. Núñez Fernández *et al.*, PRX **12**, 041018 (2022)

Single-orbital Anderson impurity model in real-time formalism
Small average sign



Alternative/complementary approaches
to Monte Carlo sampling

Not affected by a sign problem

Faster (seemingly algebraic) convergence

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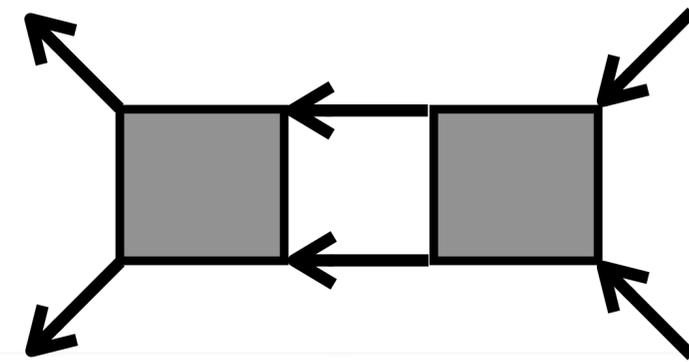
Part II

Short lecture on TCI

Quantum field theories

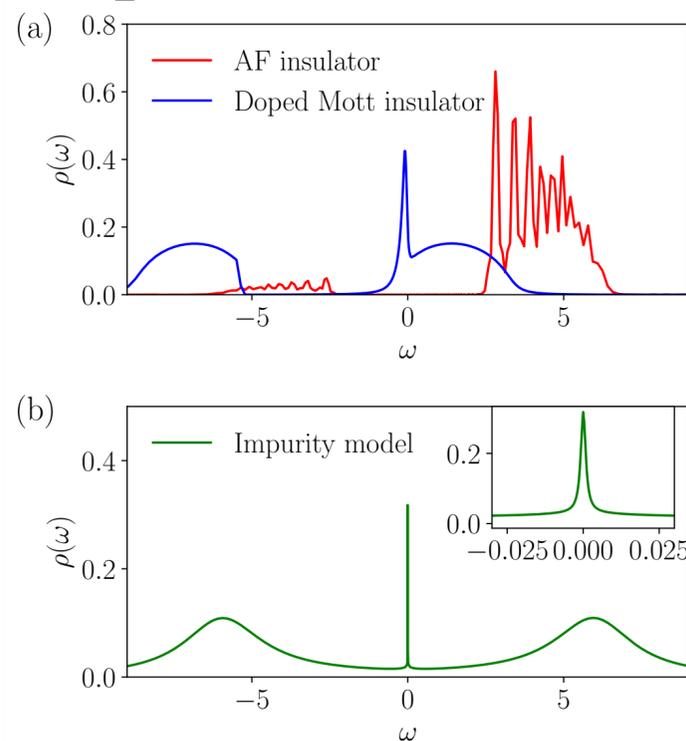
HS *et al.*, PRX 13, 021015 (2023)

Space-time dependence of correlation functions are compressible!

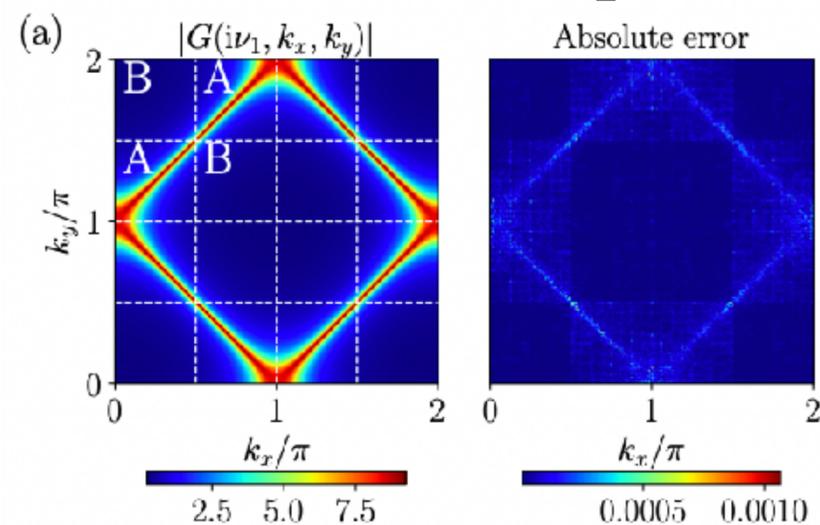


2P vertex functions

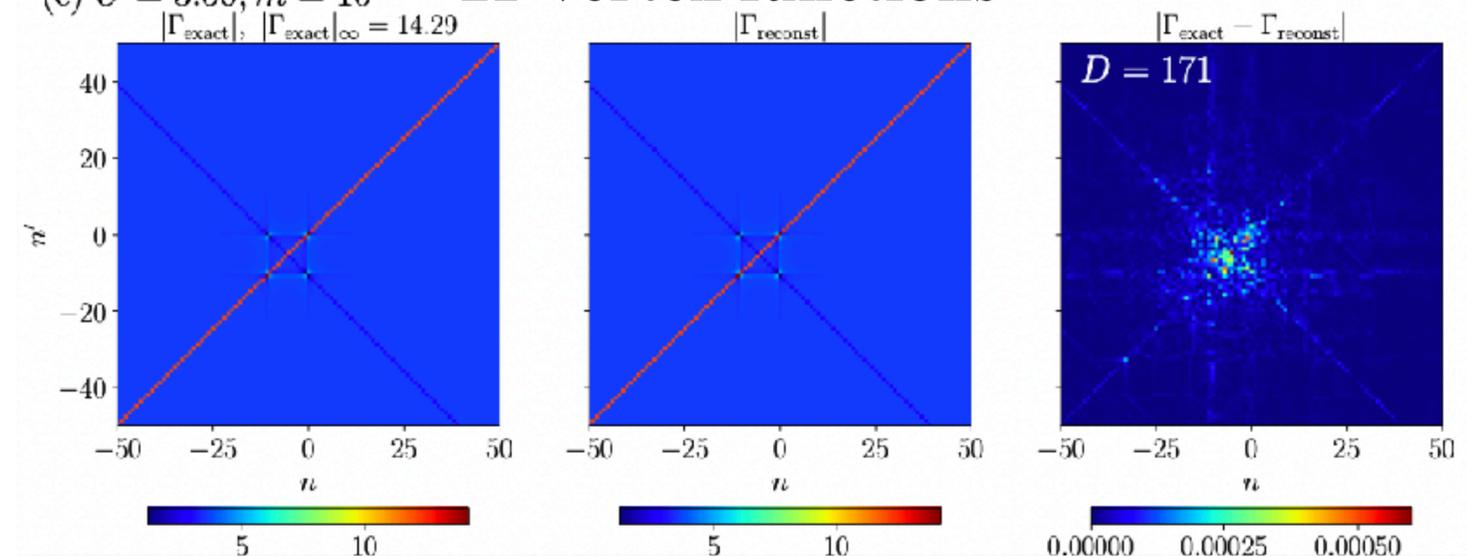
Spectral function



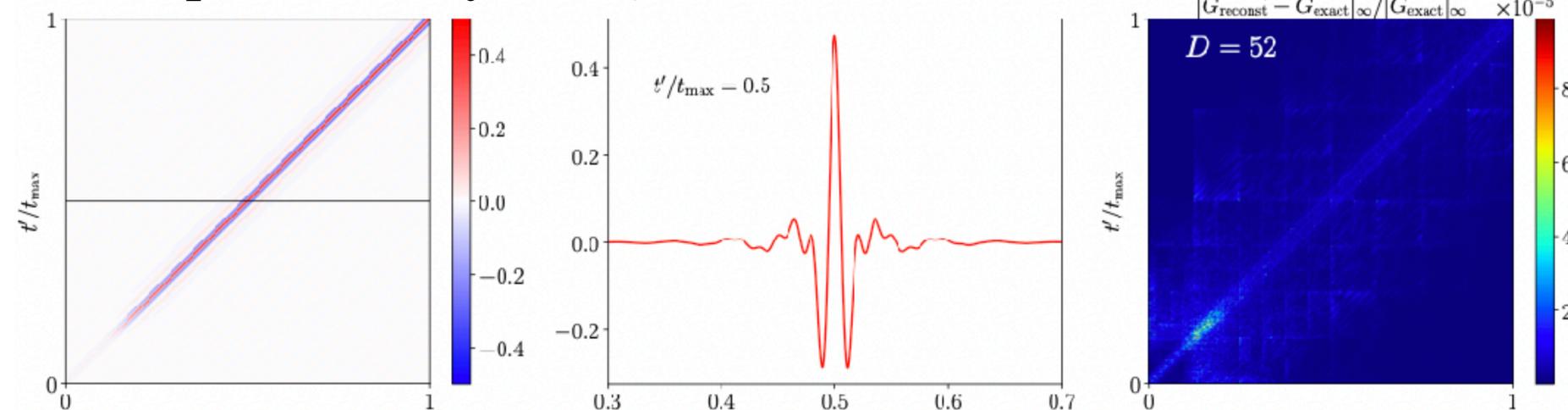
1P momentum space



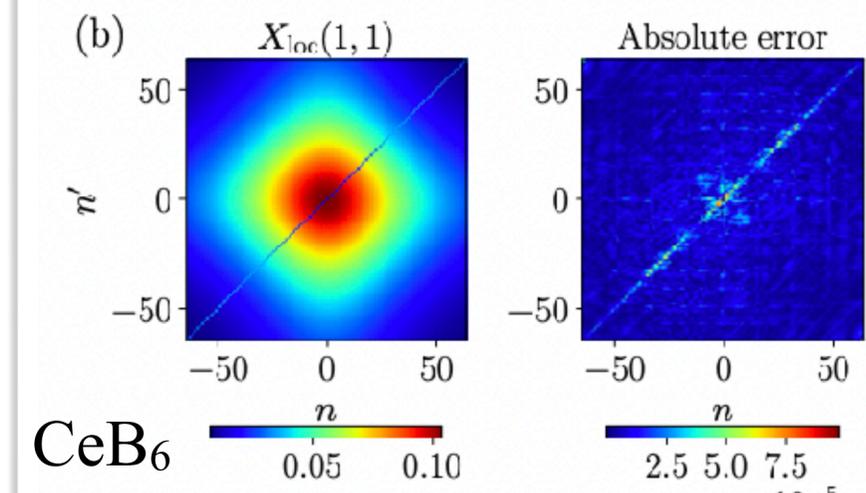
(c) $U = 3.56, m = 10$



Nonequilibrium system (real-time Green's function)



Multipolar susceptibility



CeB₆

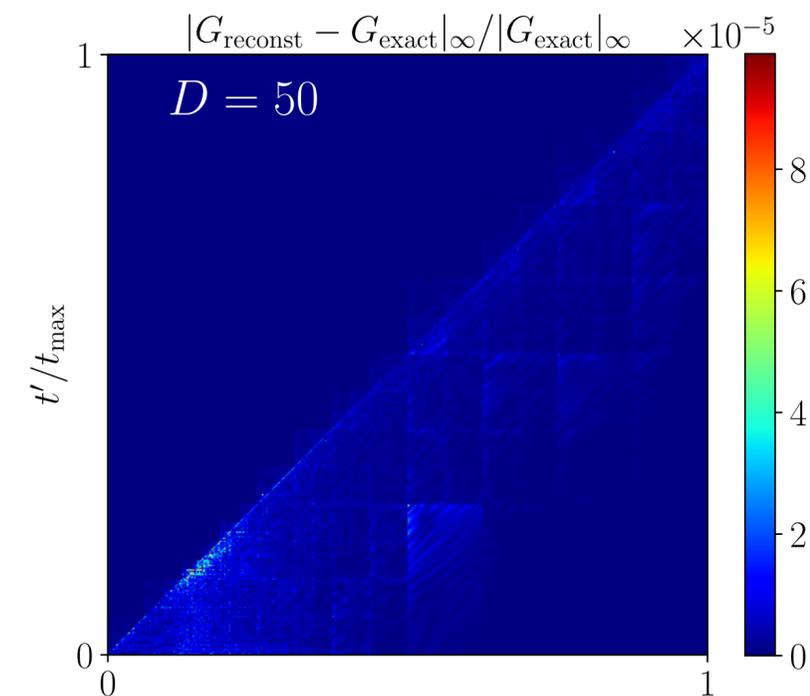
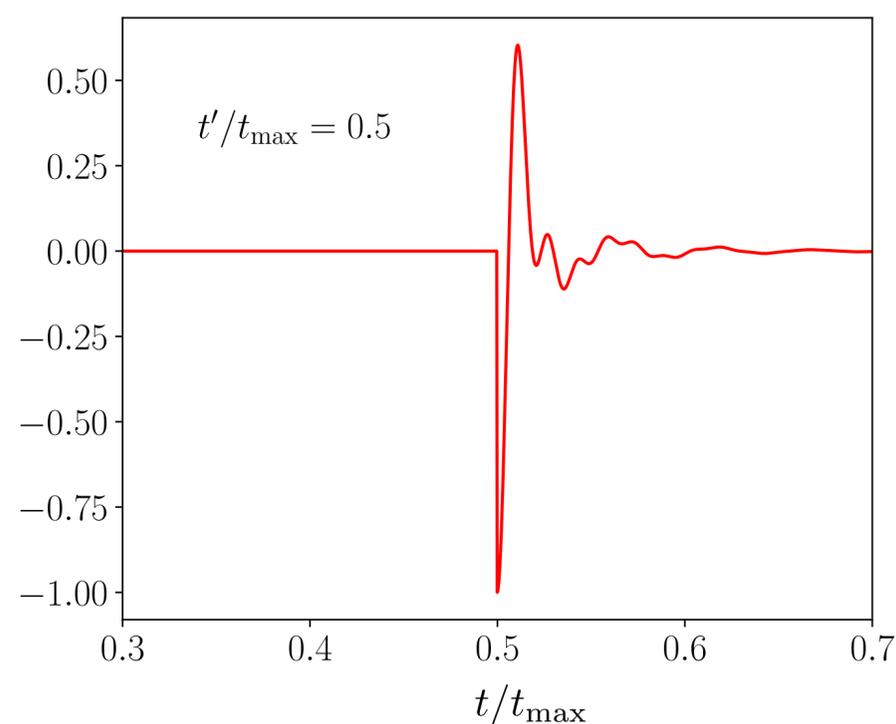
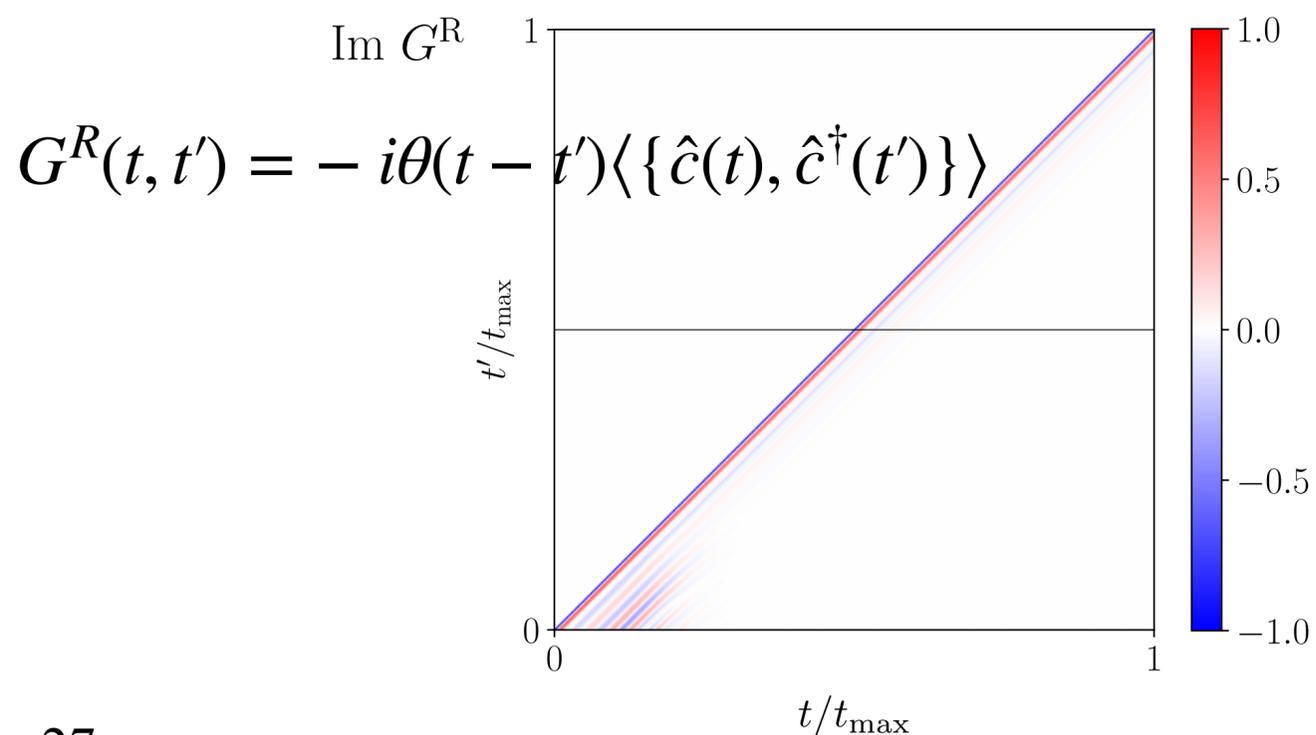
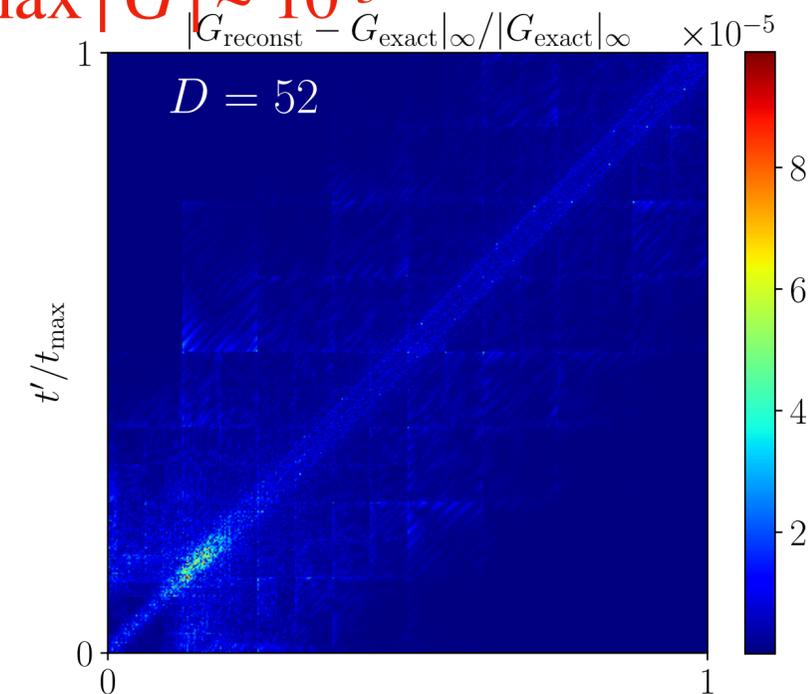
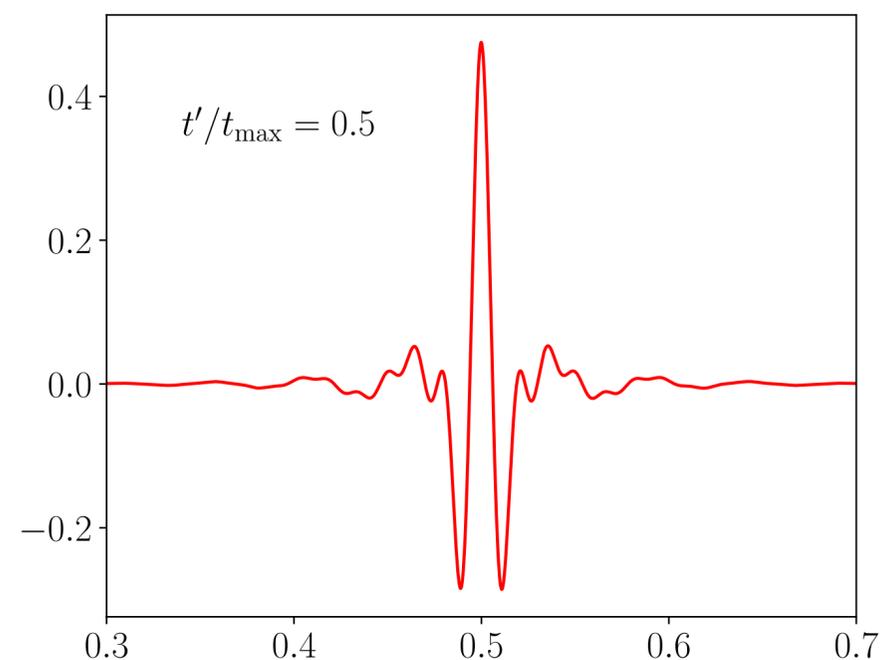
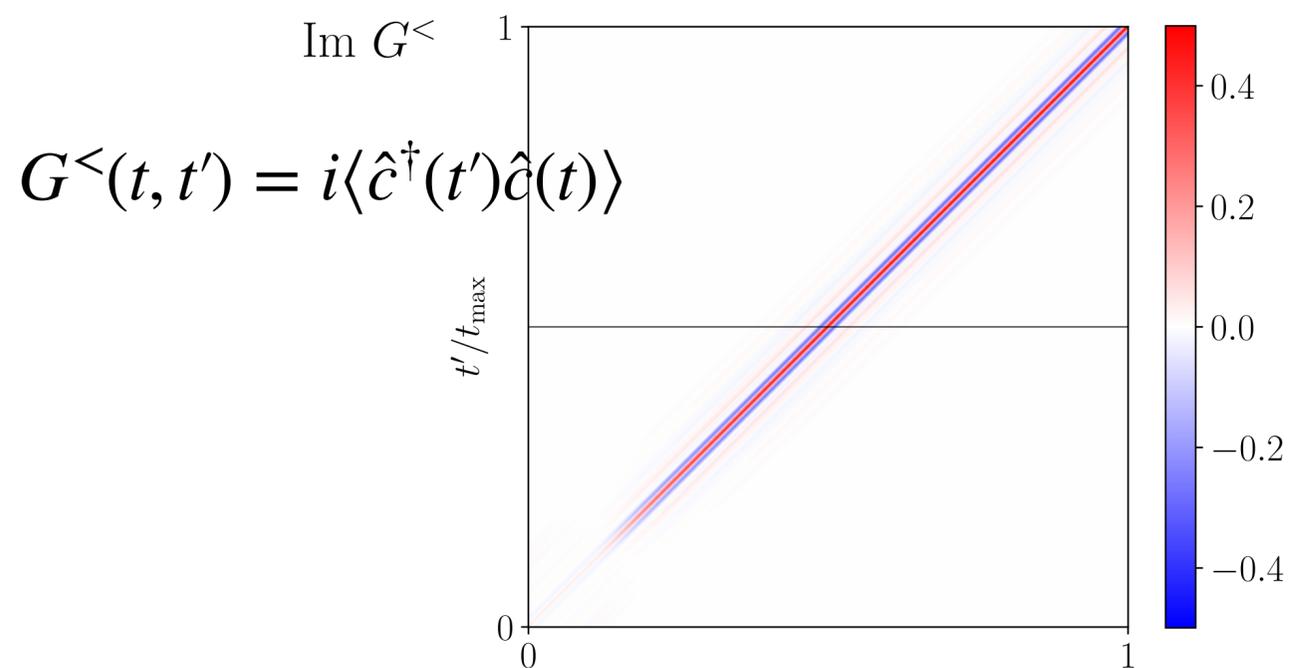
Real-time Green's function: Non-equilibrium case

Low- T AF Mott phase excited by a short electric field pulse, Bethe lattice, $T = 0.05$

Compression ratio $\sim 10^3$

$|\delta G|/\max |G| \sim 10^{-3}$

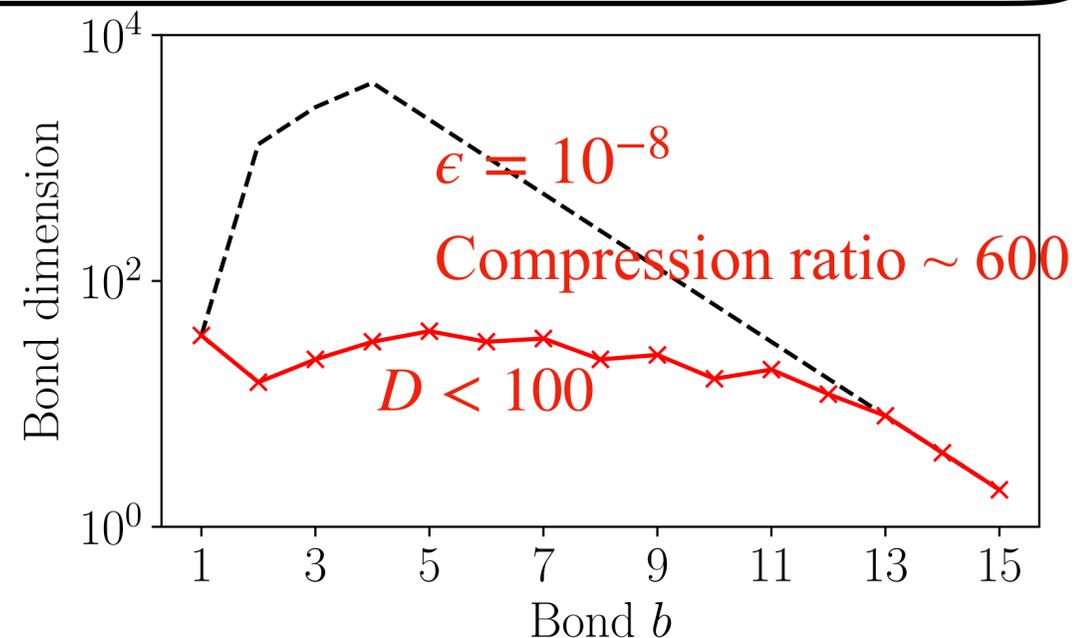
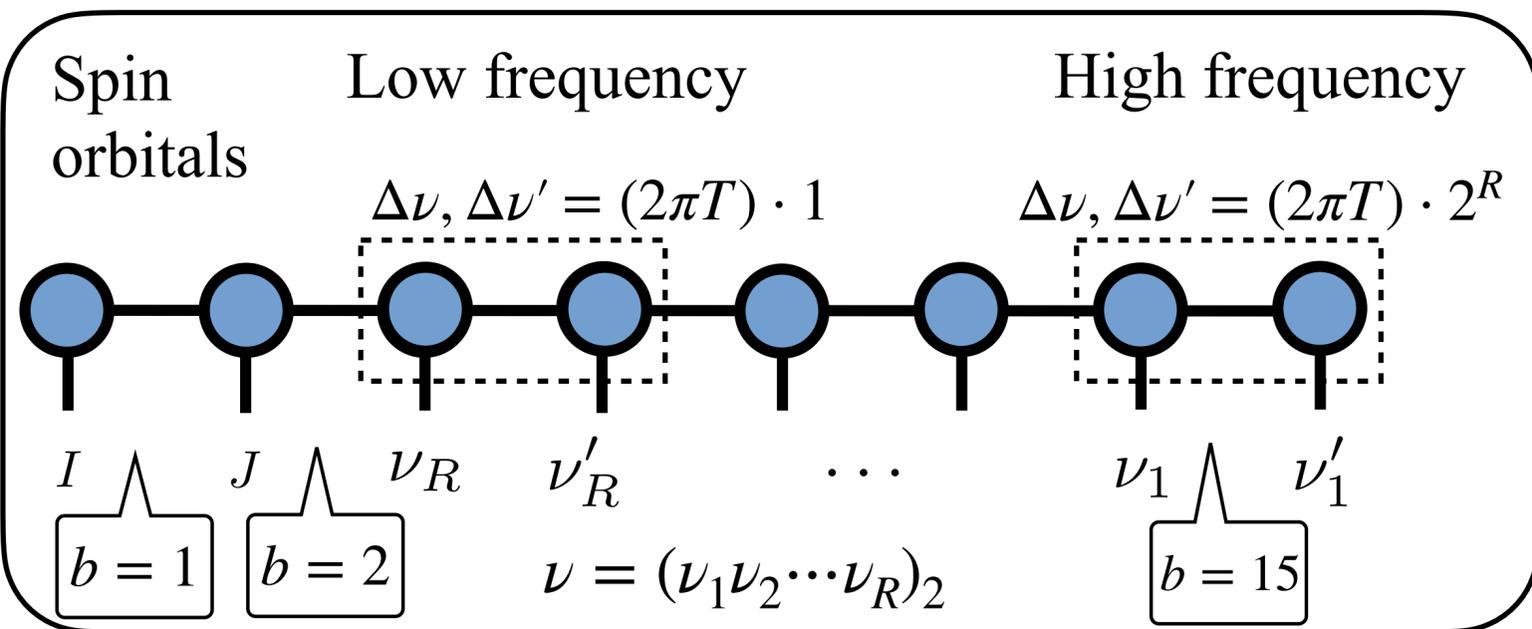
$|G_{\text{reconst}} - G_{\text{exact}}|_{\infty}/|G_{\text{exact}}|_{\infty} \times 10^{-5}$



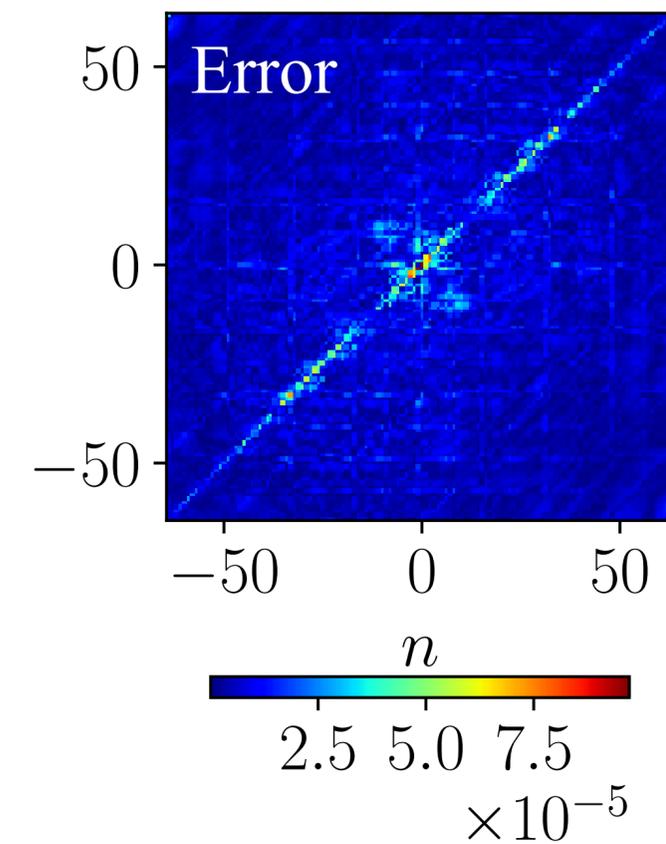
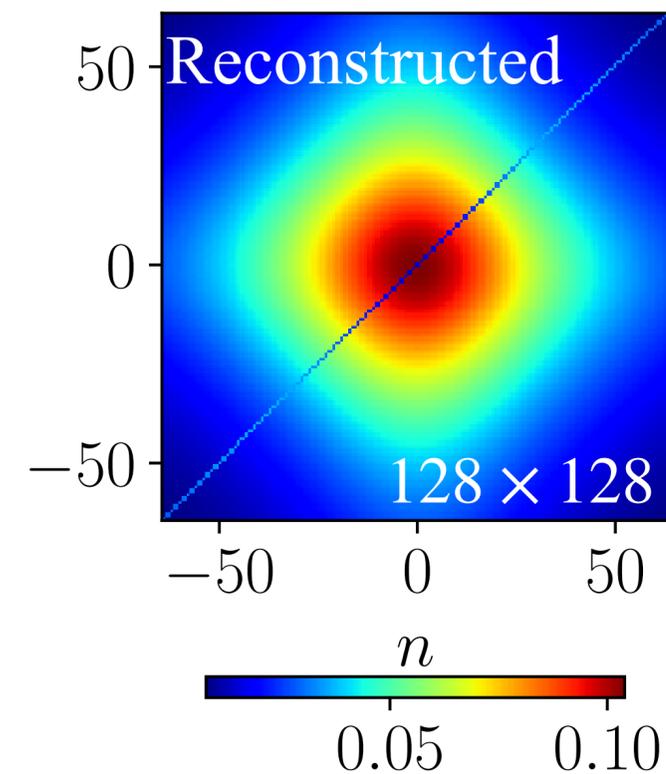
Multipolar susceptibility of an f -electron system: CeB₆

J. Otsuki, K. Yoshimi, **HS**, and H. O. Jeschke, arXiv:2209.10429v1

- Six correlated states ($j=5/2$)
- DFT+DMFT using the Hubbard-I approximation
- Static multipolar susceptibility computed by solving Bethe-Salpeter equation



$(I, J) = (1, 1)$



$$\nu = (2n + 1)\pi T, \nu' = (2n' + 1)\pi T$$

What we have done in the last two years...

BZ integration

M. K. Ritter, Y. Núñez Fernández, M. Wallerberger, J. von Delft, **HS**, X. Waintal, PRL **132**, 056501 (2024)

New Tensor Cross Interpolation algorithms and software

Y. Núñez Fernández, M. K. Ritter, M. Jeannin, J.-W. Li, T. Kloss, T. Louvet, S. Terasaki, O. Parcollet, J. von Delft, **HS**, X. Waintal, SciPost Phys. **18**, 104 (2025)

Solving multiorbital impurity models coupled with phonons

H. Ishida, N. Okada, S. Hoshino, and **HS**, PRL **135**, 046502 (2025)

Simulating nonequilibrium dynamics of correlated systems

M. Murray, **HS**, P. Werner, PRB **109**, 165135 (2024)

M. Środa, K. Inayoshi, **HS**, P. Werner, arXiv:2412.14032v1

Solving parquet equations at the two-particle level

S. Rohshap, M. K. Ritter, **HS**, J. von Delft, M. Wallerberger, A. Kauch, Phys. Rev. Research **7**, 023087 (2025)

Tensor cross interpolation approach for quantum impurity problems based on the weak-coupling expansion

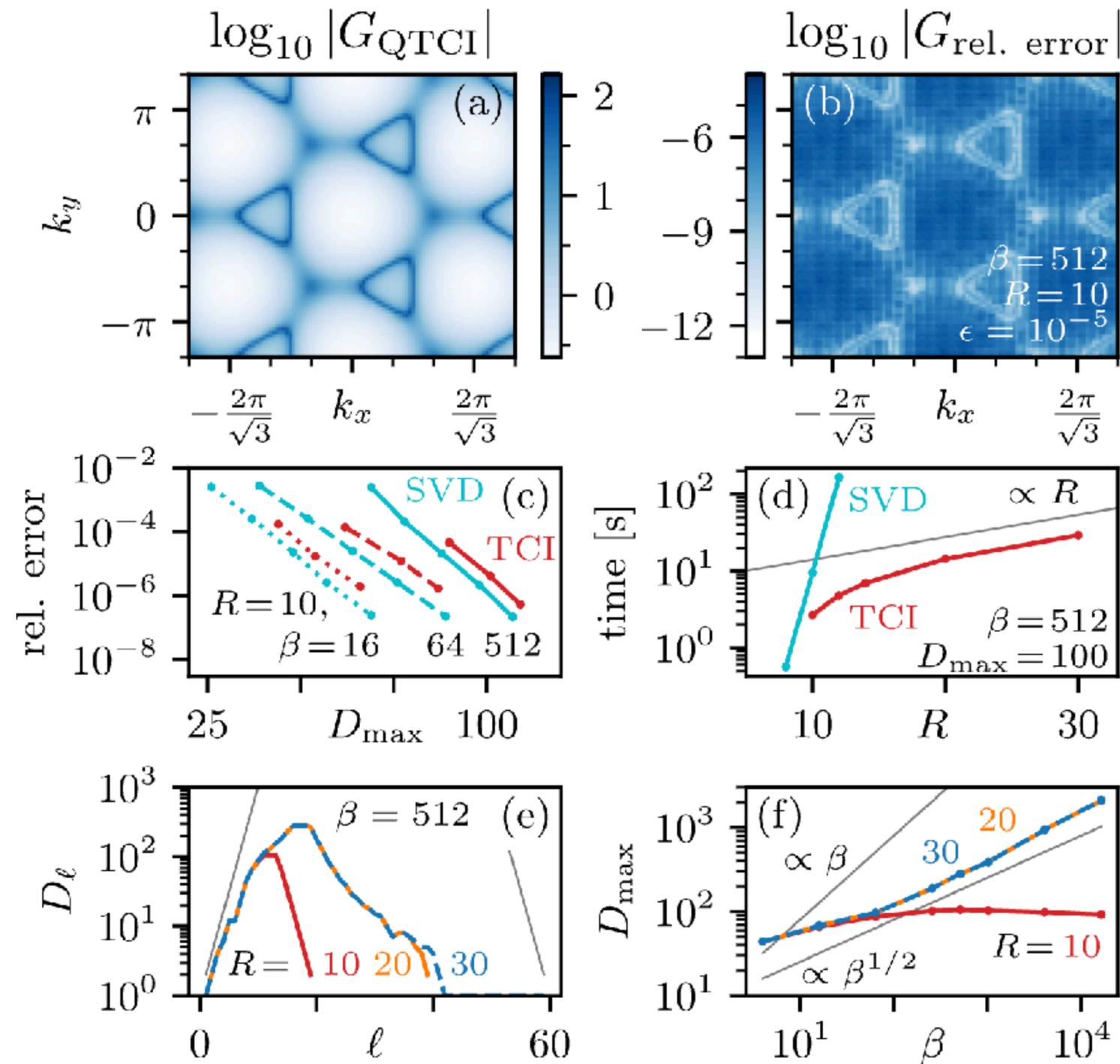
S. Matsuura, **HS**, P. Werner, N. Tsuji, Phys. Rev. B **111**, 155150 (2025)

Diagnosing phase transitions through time scale entanglement

S. Rohshap, H. Ishida, F. Bippus, A. Kauch, K. Held, **HS**, M. Wallerberger, arXiv:2507.11276v1

QTT + TCI: Integration with exponentially high resolution

M. K. Ritter, Y. Núñez Fernández, M. Wallerberger, J. von Delft, **HS**, X. Waintal, PRL **132**, 056501 (2024)



1. Construct a QTT for $f(k_x, k_y)$ by adaptive sampling

2. Integrate the QTT

- Exponentially fine resolution with the number of bits R
- Not need to evaluate the function on a dense grid
- # of function evaluations $\propto D_{\text{max}}^2 R$

Multiorbital electron-phonon model

H. Ishida, N. Okada, S. Hoshino, and HS, PRL 135, 046502 (2025)

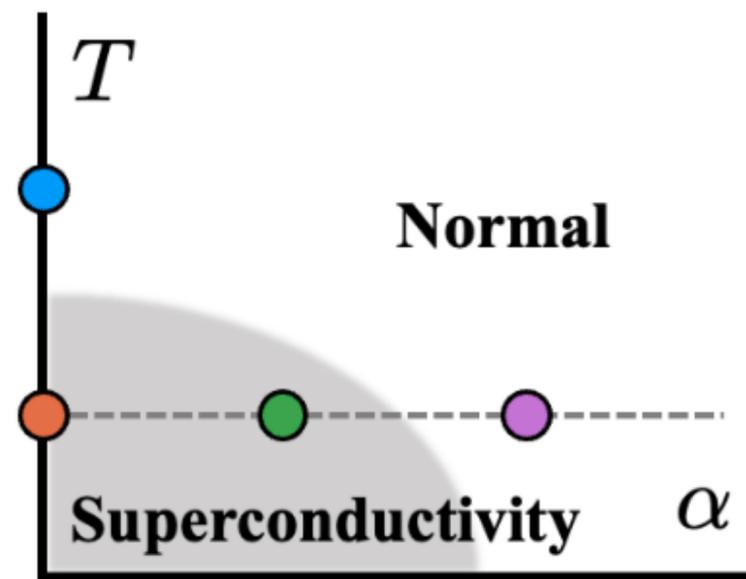
Three orbitals + phonons (for C₆₀ lattice), impurity model

$$\mathcal{H} = \sum_{ij} \sum_{\gamma\gamma'} \left(t_{ij}^{\gamma\gamma'} - \mu \delta_{ij} \delta_{\gamma\gamma'} \right) c_{i\gamma}^\dagger c_{j\gamma'} + \sum_{i\eta} \omega_\eta a_{i\eta}^\dagger a_{i\eta} \\ + \sum_{i\eta} I_\eta : T_{i\eta} T_{i\eta} : + \sum_{i\eta} g_\eta \phi_{i\eta} T_{i\eta}$$

i : site

γ, γ' : spinorbital

η : phonon



orbital field

Electron

$$\Sigma_{\gamma\gamma'}^{(2)}(\tau) = \sum_{\substack{\eta_1 \sim \eta_4 \\ \tau', \tau''}} \eta_1 \eta_2 \eta_3 \eta_4 \\ f(\gamma, \gamma', \eta_1, \eta_2, \eta_3, \eta_4, \tau, \tau', \tau'')$$

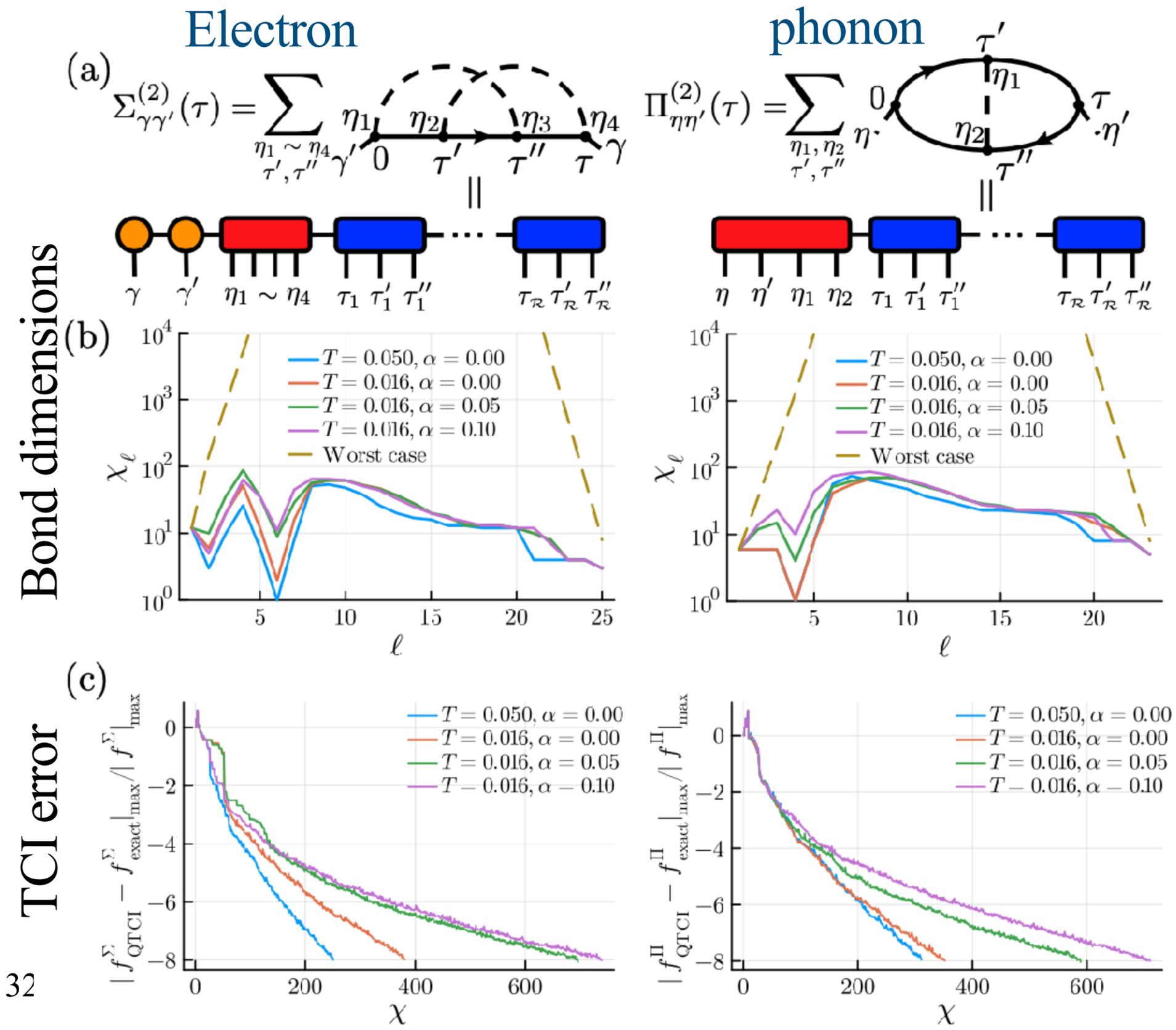
12 12 6 6 6 6 β β β

Phonon

$$\Pi_{\eta\eta'}^{(2)}(\tau) = \sum_{\substack{\eta_1, \eta_2 \\ \tau', \tau''}} \eta_1 \eta_2$$

12 = 3 orbital × 2 spins × 2 Nambu modes

Feynman diagram integration



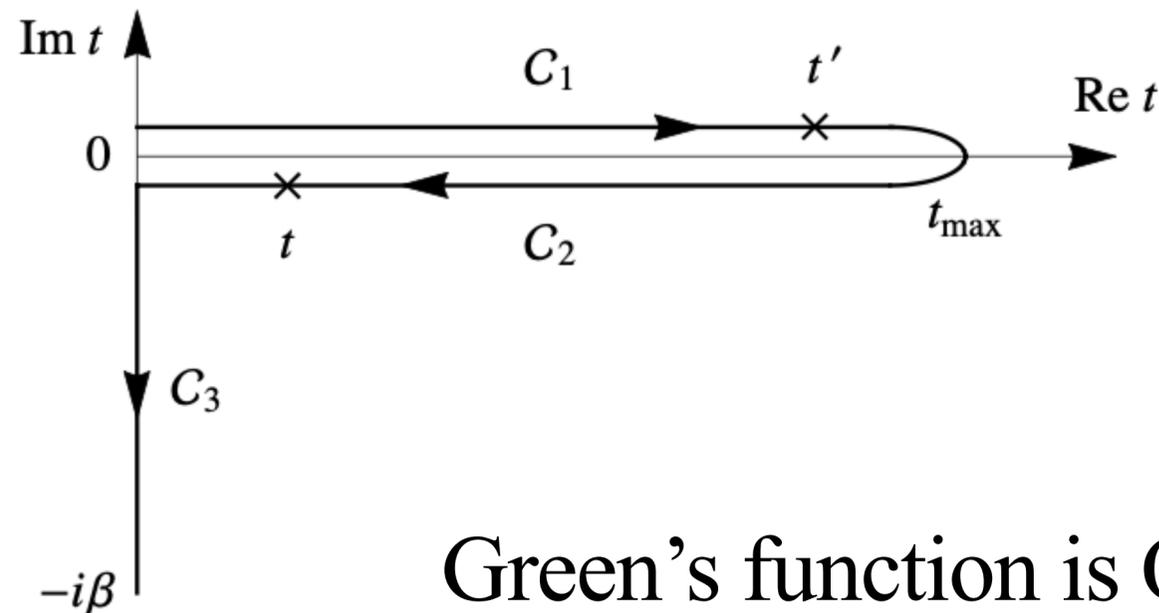
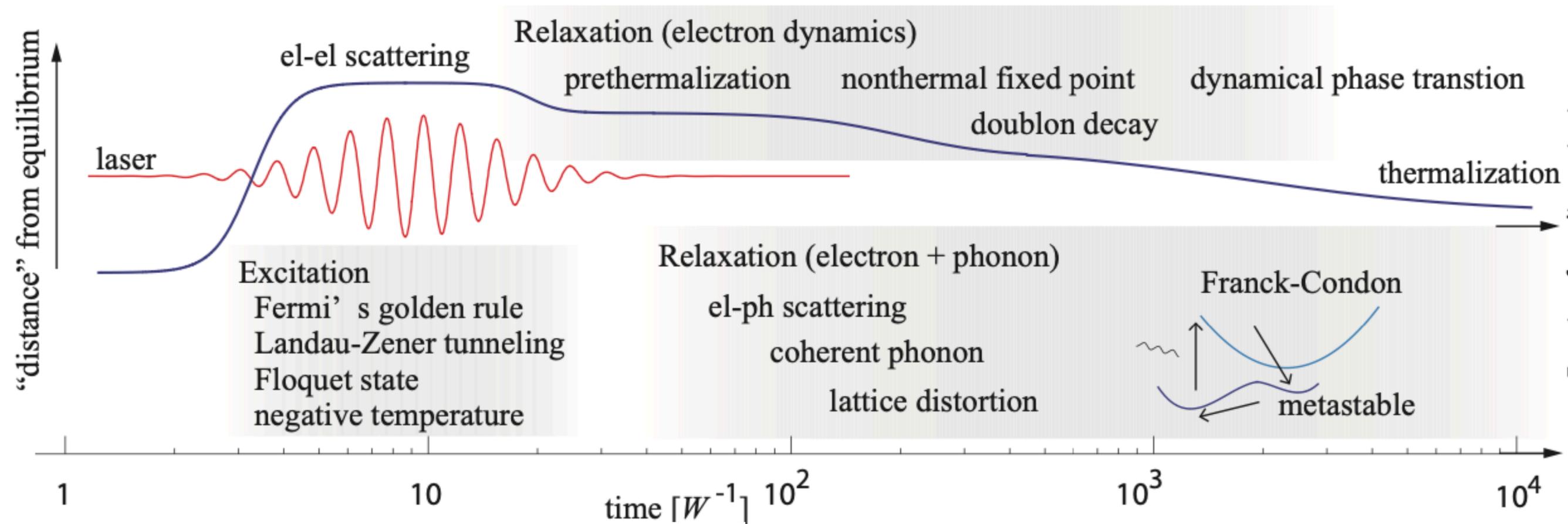
Low rank!

Exponential convergence of
TCI error

Future perspectives: higher
perturbation orders

Simulating nonequilibrium dynamics of correlated systems

H. Aoki, N. Tsuji, M. Eckstein, M. Kollar, T. Oka, P. Werner, RMP **86**, 779 (2014)



Memory bottleneck

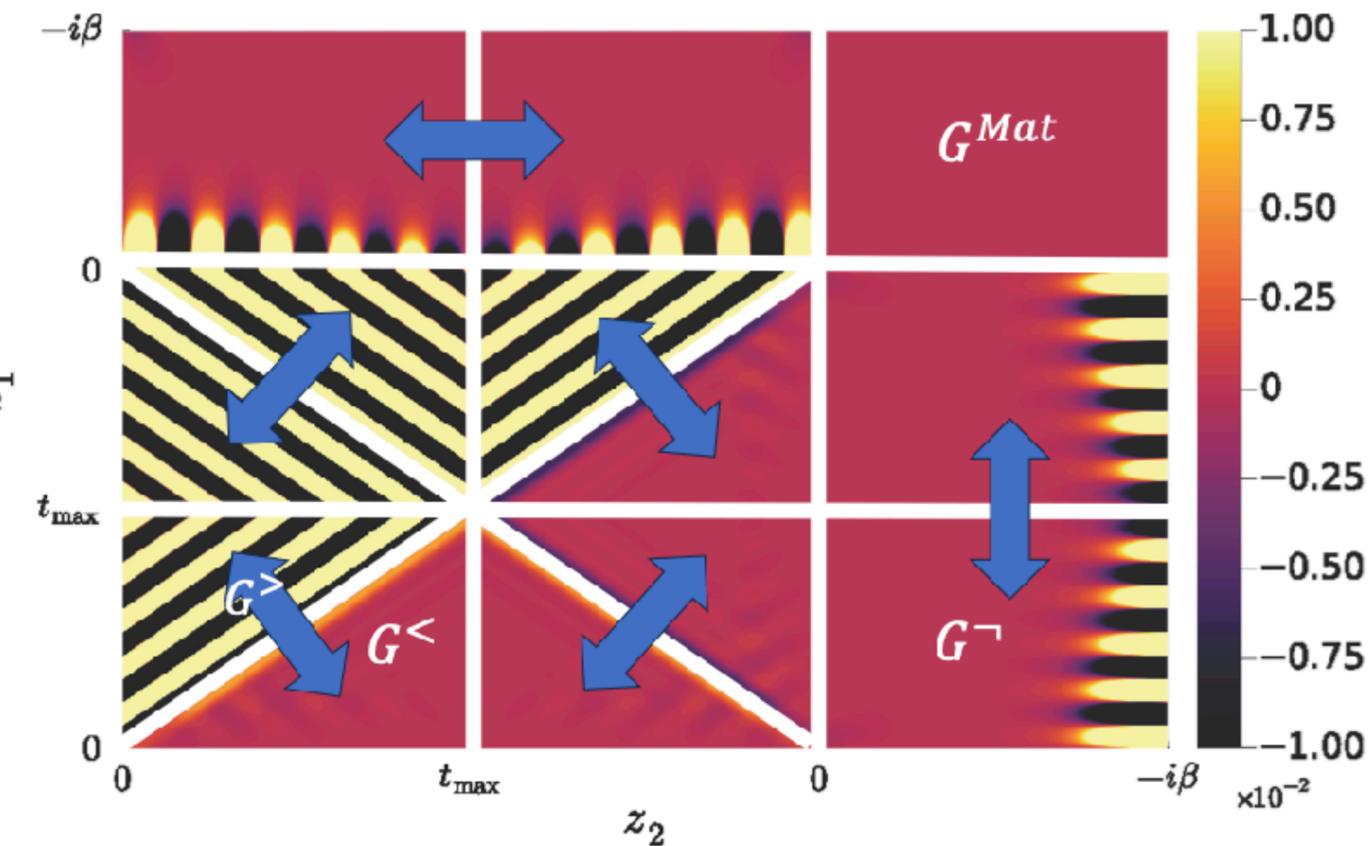
$$G(z, z')$$

Green’s function is QTT compressible! HS *et al.*, PRX **13**, 021015 (2023)

Dyson equation

M. Murray, **HS**, P. Werner, PRB **109**, 165135 (2024)

M. Środa, K. Inayoshi, **HS**, P. Werner, arXiv:2412.14032v1



The Dyson equation $G = G_0 + G_0 \Sigma G$ can be solved iteratively or as a linear system:

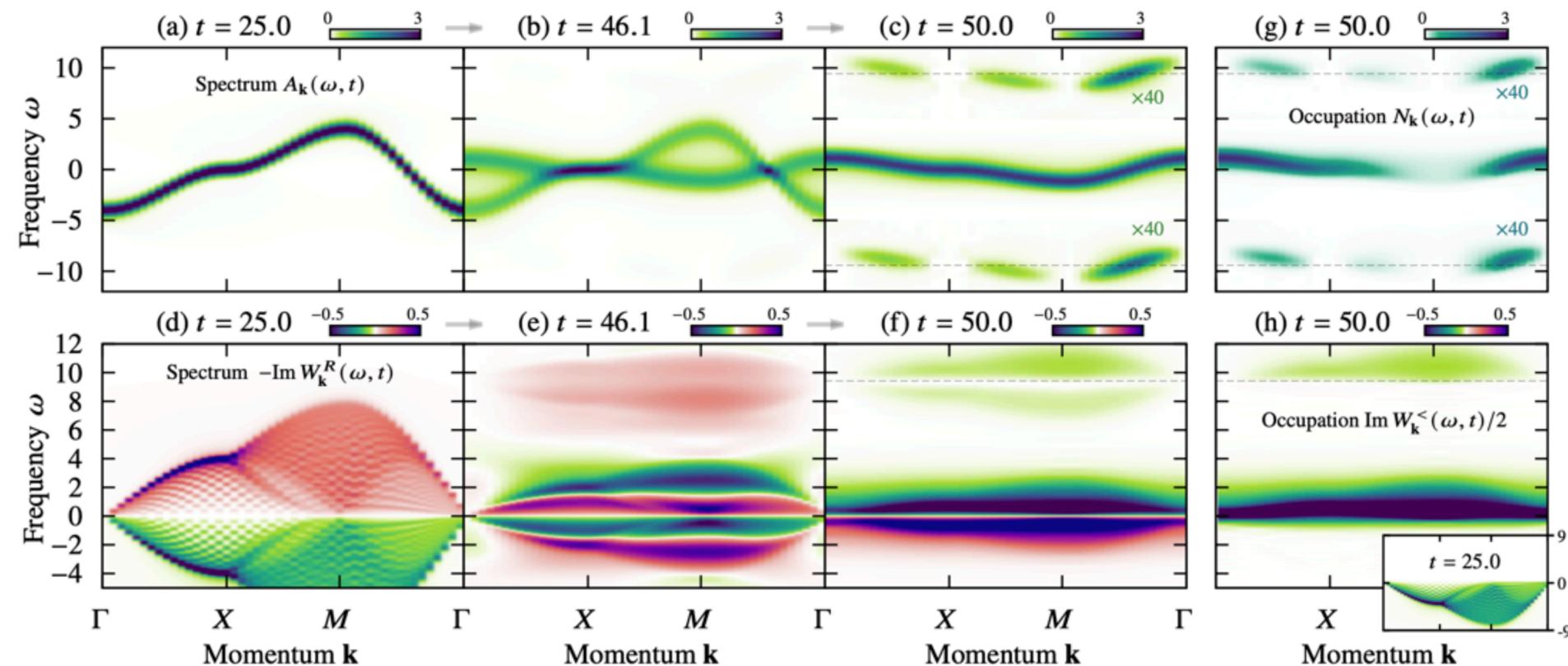
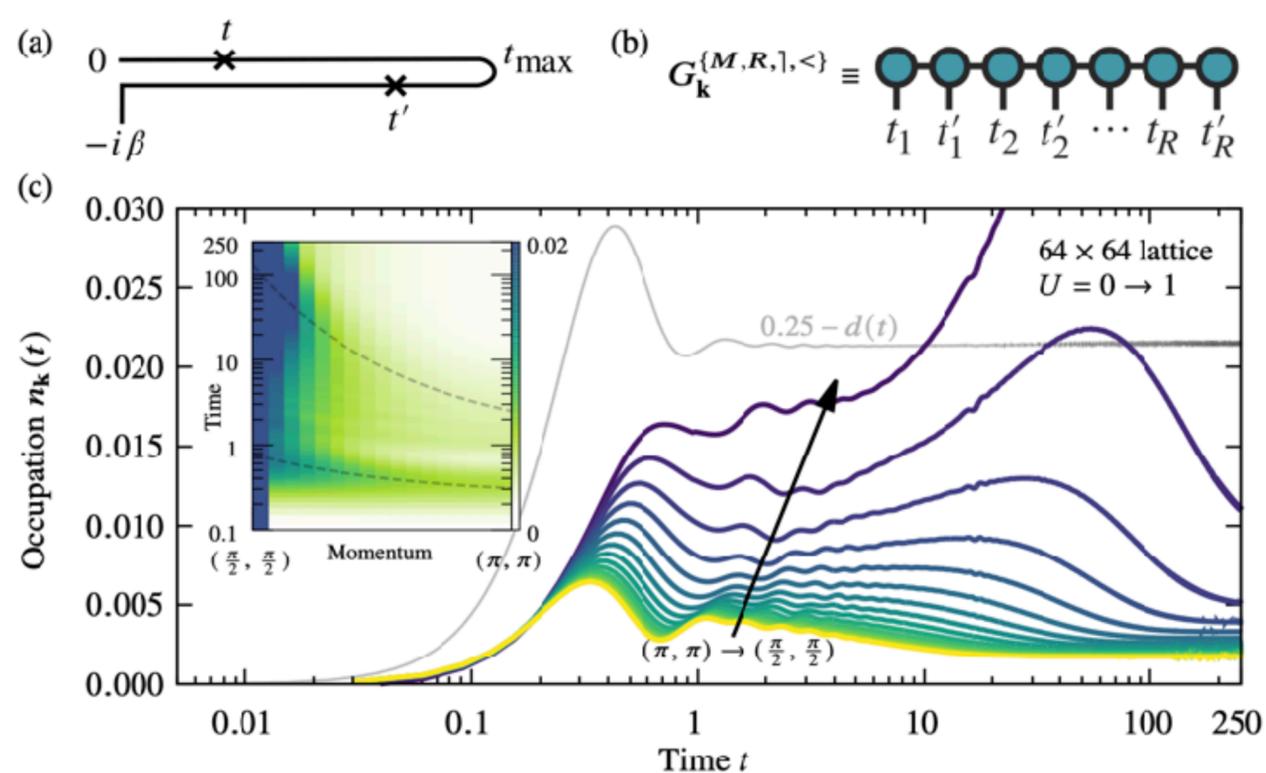
$$(I + G_0 \Sigma) G = G_0$$

in a compressed form!

GW calculations with high k resolution

M. Środa, K. Inayoshi, HS, P. Werner, arXiv:2412.14032v1

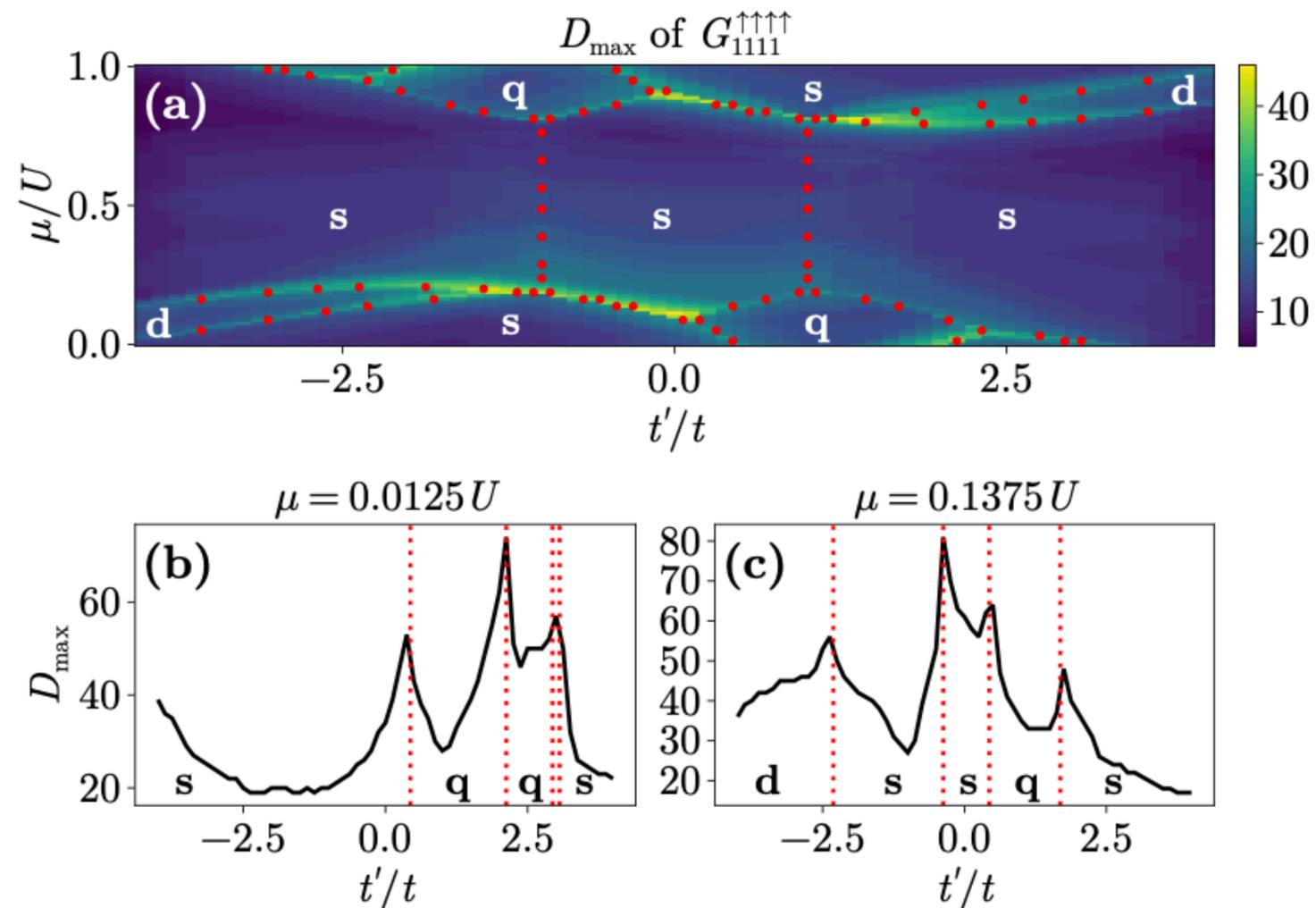
QTT solves the memory bottleneck: 6 TB \rightarrow 1 GB



Interaction quench $U = 0 \rightarrow 1$

QTT as an indicator of crossover and phase transition

S. Rohshap, H. Ishida, F. Bippus, A. Kauch, K. Held, **HS**, M. Wallerberger, arXiv:2507.11276v1



Use the bond dimension of QTT representing imaginary-time dependence of two-point correlators as an indicator of a crossover or a phase transition!

FIG. 3. Four-site Hubbard ring with next nearest-neighbor hopping: Red dots (dotted lines) indicate crossings of s(inglet), d(oublet) and q(uartet) ground states. (a) D_{\max} of QTT of $G_{1111}^{\uparrow\uparrow\uparrow\uparrow}(\tau_1, \tau_2, \tau_3)$ at $\beta = 50$ with $\epsilon = 10^{-5}$ and $R = 6$. (b)-(c) D_{\max} for various μ/U -slices at $\epsilon = 10^{-8}$.

New TCI algorithms and open-source softwares



Y. Núñez Fernández M. K. Ritter

Learning tensor networks with tensor cross interpolation: new algorithms and libraries

Yuriel Núñez Fernández^{1,2*}, Marc K. Ritter^{3,4}, Matthieu Jeannin², Jheng-Wei Li²,
Thomas Kloss¹, Thibaud Louvet², Satoshi Terasaki⁵, Olivier Parcollet^{4,6}, Jan von Delft³,
Hiroshi Shinaoka⁷, and Xavier Waintal^{2*}

SciPost Phys. **18**, 104 (2025)

1. Robust and stable TCI algorithms
2. Open-source C++/Python/Julia libraries
3. Self-contained introduction to (Q)TCI

Open-source libraries

<https://tensor4all.org/>

tensor4all



This website collects information from the tensor4all group which is working on tensor network methods.

Literature

A pedagogical introduction to tensor network methods, which includes an overview of the existing literature and also new algorithms, can be found in:

Yuriel Núñez Fernández, Marc K. Ritter, Matthieu Jeannin, Jheng-Wei Li, Thomas Kloss, Thibaud Louvet, Satoshi Terasaki, Olivier Parcollet, Jan von Delft, Hiroshi Shinaoka, and Xavier Waintal, [arXiv:2407.02454](https://arxiv.org/abs/2407.02454).

Please check the [reference](#) page for more information on TCI and quantics tensor trains.

Code

We provide two software libraries that implement algorithms from the above manuscript for computing low-rank tensor representations. The code focuses on recent applications of tensor networks to objects that do not necessarily involve many-body quantum mechanics. It also contain known and new variants of the tensor cross interpolation (TCI) algorithm for unfolding tensors into tensor trains. One code is called Xfac (written in C++ with Python bindings), and a second implementation with similar functionality is based on Julia:

- [Xfac \(C++ / Python\)](#)
- [Julia code](#)

Available in C++, Python, Julia

- TensorCrossInterpolation.jl **TCI**
 - QuanticsGrids.jl **Quantics representation**
 - QuanticsTCI.jl **QTCI**
 - Interfacing with ITensors.jl ...
- Superfast Fourier transform, convolution...**

Julia example

$$f(x) = \cos\left(\frac{x}{B}\right) \cos\left(\frac{x}{4\sqrt{5}B}\right) e^{-x^2} + 2e^{-x} \text{ with } B = 2^{-30} \approx 10^{-9}.$$

Fast oscillations Slow decay

```

using QuanticsTCI
import QuanticsGrids: DiscretizedGrid, origcoord_to_grididx

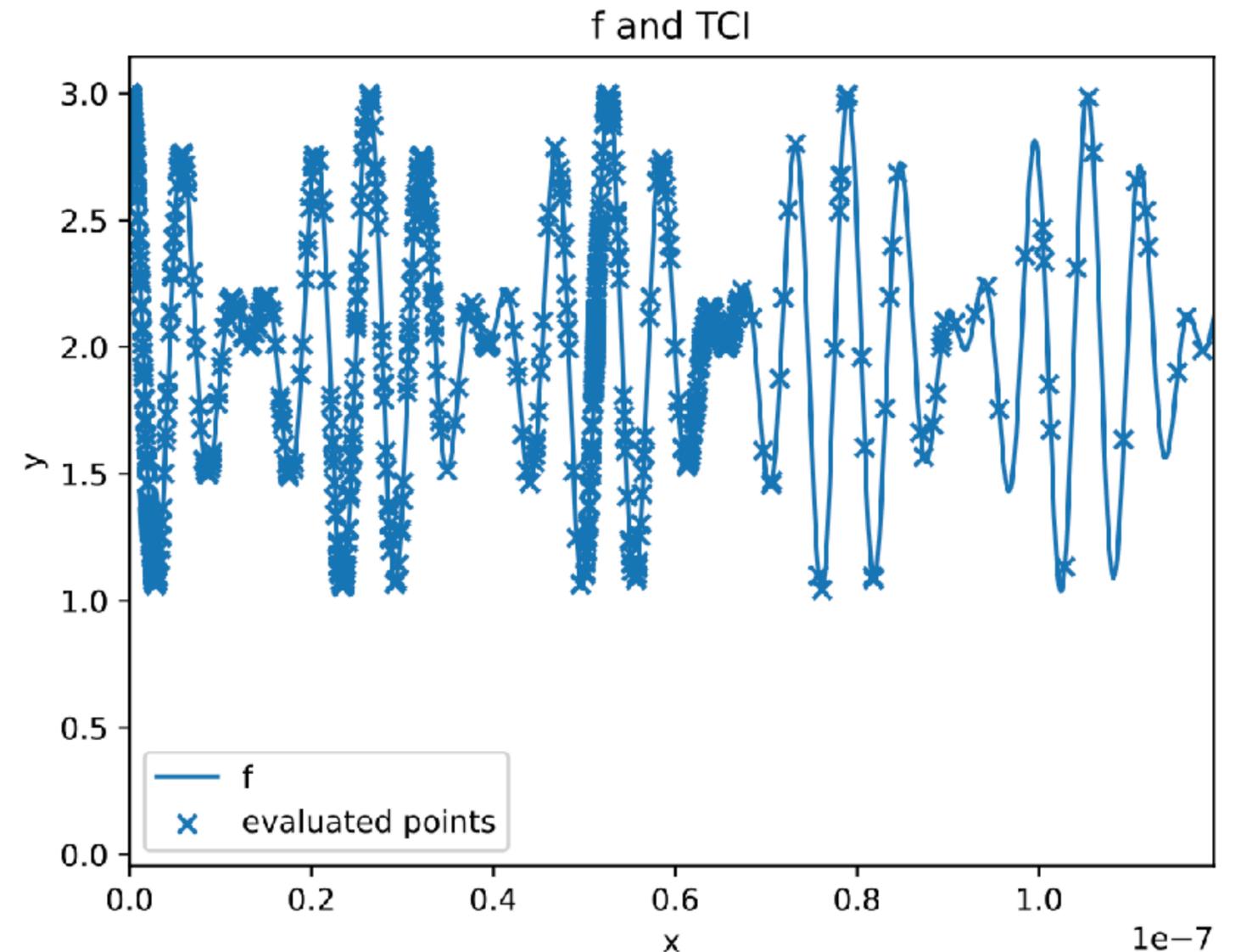
R = 50      # number of bits
M = 2^R    # size of the grid
grid = DiscretizedGrid{1}(R, 0.0, 3.0; includeendpoint=true)

# Function of interest
function f(x)
    B = 1/2^30
    cos(x/B) * cos(x/(4*sqrt(5)*B)) * exp(-x^2) + 2 * exp(-x)
end

# Convert to quantics format and sweep
tci, ranks, errors = quanticscrossinterpolate(Float64, f, grid)

# Plot
using Plots
p = plot(label="tci")
xs = range(0, 1/2^23, length=1000)
plot!(p, xs, f.(xs), label="f(x)")
plot!(p, xs, [tci(origcoord_to_grididx(grid, x)) for x in xs], label="QTCI", marker=:x)
savefig(p, "1D_quantics.png")

```



Summary (of Part I) & Outlook

QTT + TCI could serve as a general framework for addressing various problems.

We are establishing computational theories:

- Divide-and-conquer algorithms G. Grosso, ..., **HS** in preparation
- TT + coordinate transformation with L. Wang, Y. Michishita, S. Ishida
- Tree tensor networks with R. Watanabe

(Possible) applications:

- Quantum field calculations at the two-particle level (non-local extension of DMFT)
- Simulating nonequilibrium dynamics of correlated systems
- Perturbative *ab initio* calculations (e.g., Eliashberg theory for superconductors)
- ...

→ Part II (if time permits)

Join in us @ tensor4all