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INSTITUTO DE FÍSICA FUNDAMENTAL
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Luca Tagliacozzo

Spatio-temporal tensor networks containing volume law states and their efficient contraction.

Plan de Recuperación,
Transformación y Resiliencia

PGC2018-095862-B-C22
PID2021-127968NB-I00
TED2021-130552B-C22

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

Entangle This VI

Invited speakers:

M. C. Bañuls
I. Bloch
I. Bouchole
A. Capel
M. Cerezo
G. Chan
J. I. Cirac
J. De Nardis
J.J. García-Ripoll
N. Goldman
M. Kliesch
B. Kraus
J. I. Latorre
M. A. Martín-Delgado
M. Oszmaniec
E. Rico
D. Stilck-França
F. Surace
L. Tarruel
R. Trivedi

100 Years of Quantum

Real Jardín Botánico, Madrid
15-19 September 2025

Organizers:

A. M. Alhambra   A. Bermúdez   E. López   G. Sierra   L. Tagliacozzo

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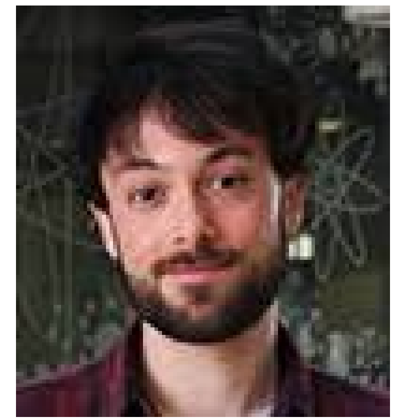
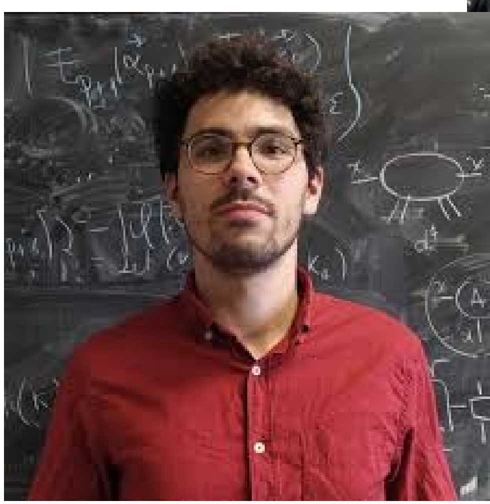



Esperanza Lopez (CSIC),
Sergio Cerezo (CSIC),
Aleix Bou (CSIC).
Jan Schneider (CSIC),
Stefano Carignano (BSC)
Guglielmo Lami (CYU)
Jacopo De Nardis (CYU)
Mari Carmen Bañuls (MPQ)
Miguel Frias (ICFO)
Jacopo Surace (Marseille)
Carlos Ramos (Quobly)

Outline

- Entanglement barrier out-of-equilibrium
- Temporal MPS and entanglement
- Physical Interpretation
- Scaling of temporal entanglement at criticality
- Scaling away from criticality
- New tensor network frameworks for out-of-equilibrium dynamics

Breaking
the entanglement barrier in 1D

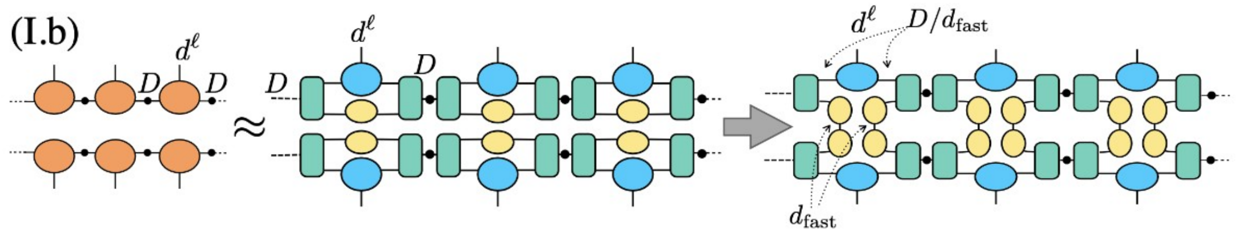
→ Local equilibration
trading LR correlations with mixture

Surace, J., Piani, M. & Tagliacozzo, L. .
Phys. Rev. B **99**, 235115 (2019).
Frías-Pérez, M., Tagliacozzo, L. & Bañuls, M. C.
Phys. Rev. Lett. **132**, 100402 (2024).

→ Sampling spatio-temporal TN using Markov Chain Montecarlo

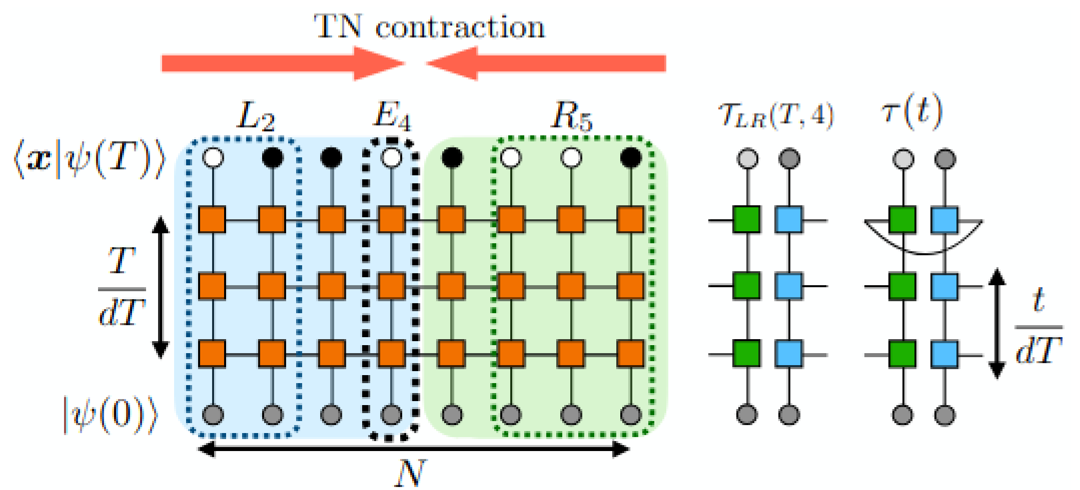
S. Carignano, G. Lami, J. De Nardis, Luca Tagliacozzo, arXiv:2505.09714

- TN algorithm for local equilibration



M Frias-Perez LT and MC Bañuls ,   arXiv:2308.04291

- MC and transverse contractions



S. Carignano, G. Lami, J. De-Nardis, L.T. arXiv:2505.09714

Outline

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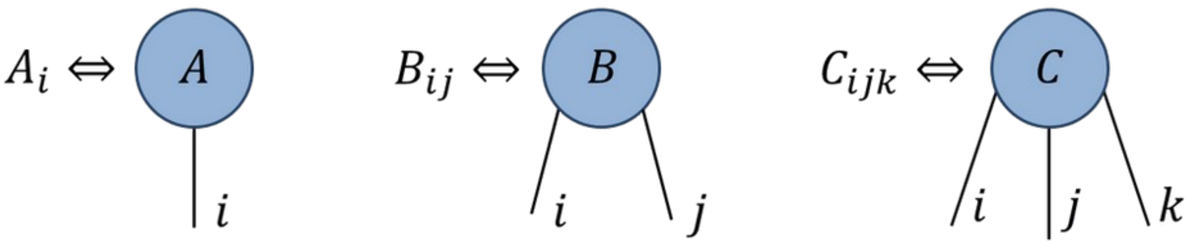
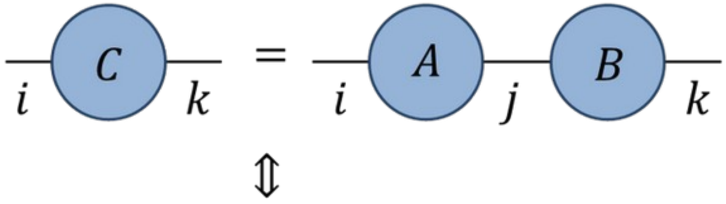
Entanglement at equilibrium

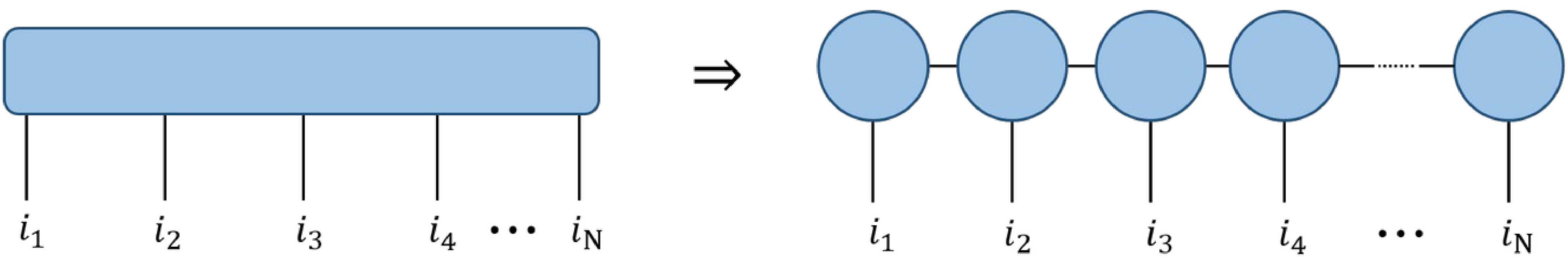
Tensor Networks

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix} \qquad \mathrm{C} = \left[\begin{bmatrix} C_{111} & \cdots & C_{1n1} \\ \vdots & \ddots & \vdots \\ C_{m11} & \cdots & C_{mn1} \end{bmatrix}^{1}_{2}\right]_{3}$$

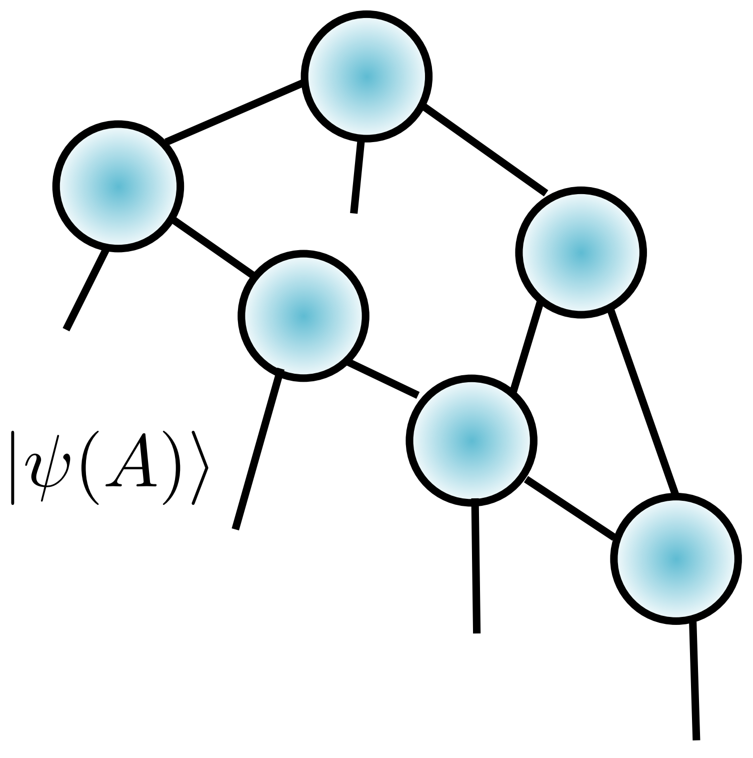


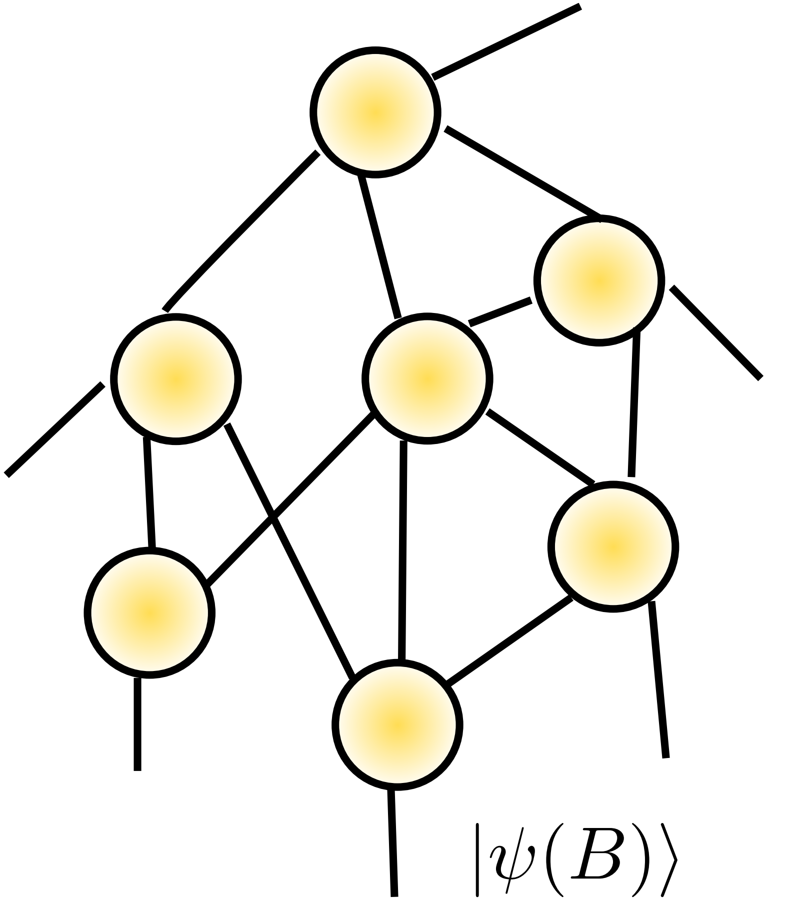


$$C_{ik} = \sum_j A_{ij} B_{jk}$$

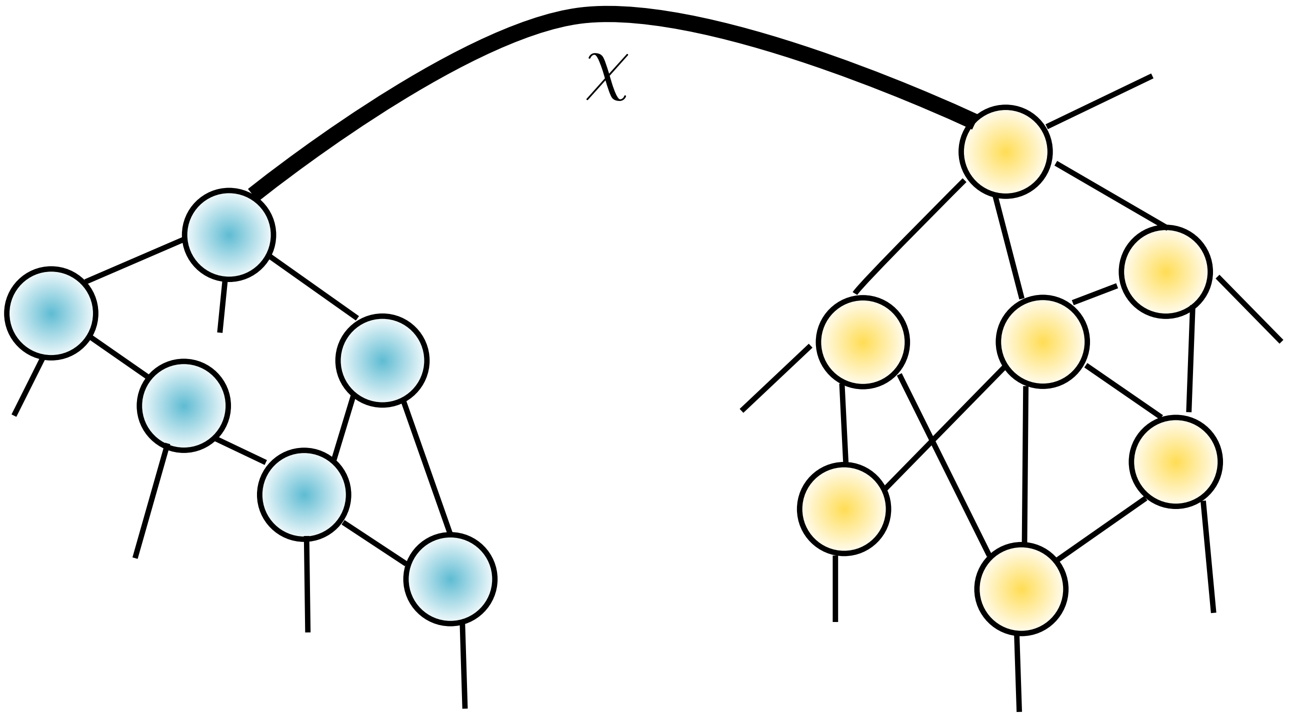


$$|\psi(A,B)\rangle = |\psi(A)\rangle \otimes |\psi(B)\rangle \,.$$





$$|\psi(A,B)\rangle = \sum_{i=1}^{\chi} |\psi(A)\rangle^i \, |\psi(B)\rangle^i \,.$$

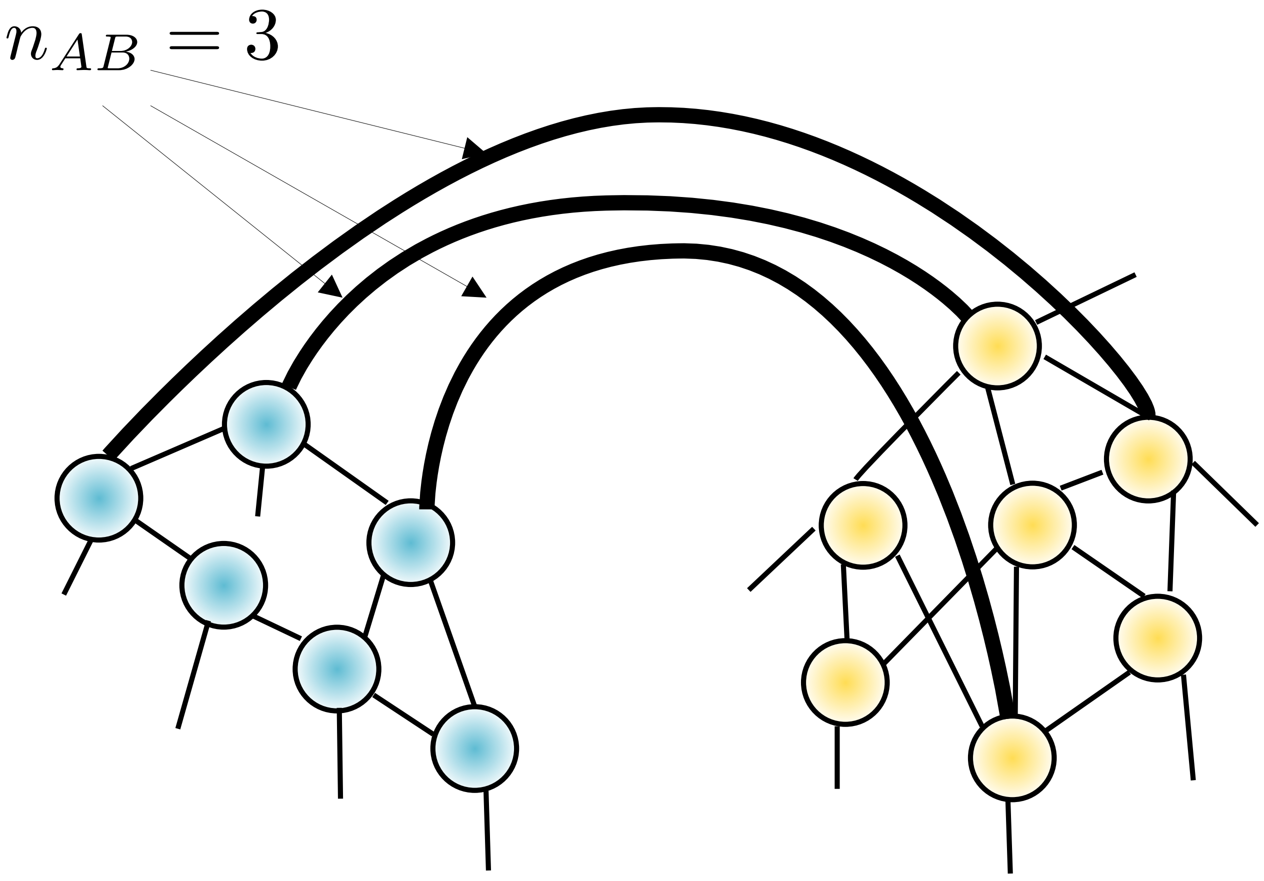


$$\rho_A = \mathrm{tr}_B \left(|\psi_{AB}\rangle\langle\psi_{AB}| \right)$$

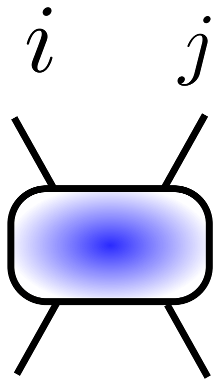
$$\mathrm{S}_n = \frac{1}{1-n} \log(\mathrm{tr}\left(\rho_A^n\right))$$

Entanglement measures
correlations among different constituents

$$S_A \leq n_{AB} \log \chi,$$



Equilibrium

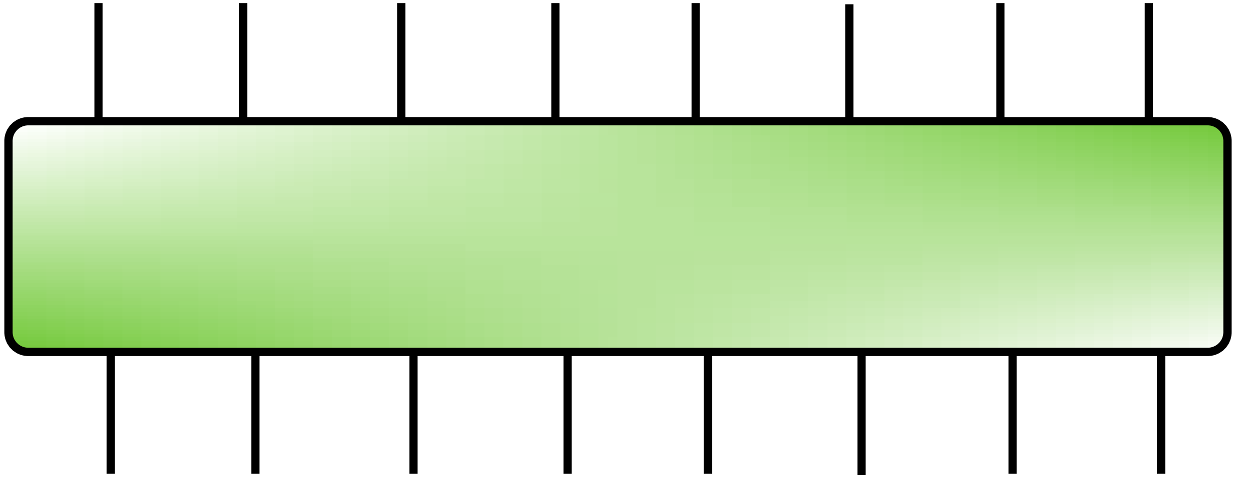


$$\hat{H} = \sum_{\langle i,j \rangle} \hat{h}_{i,j},$$

$$\hat{h}_{i,j} = \hat{h} = \sigma_x^i \sigma_x^j + 0.5\lambda \left(\sigma_z^i + \sigma_z^j \right)$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

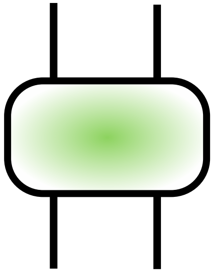
$$\rho(\beta) = \frac{\exp(-\beta \hat{H})}{Z}, \quad Z = \operatorname{tr} \exp(-\beta \hat{H}).$$



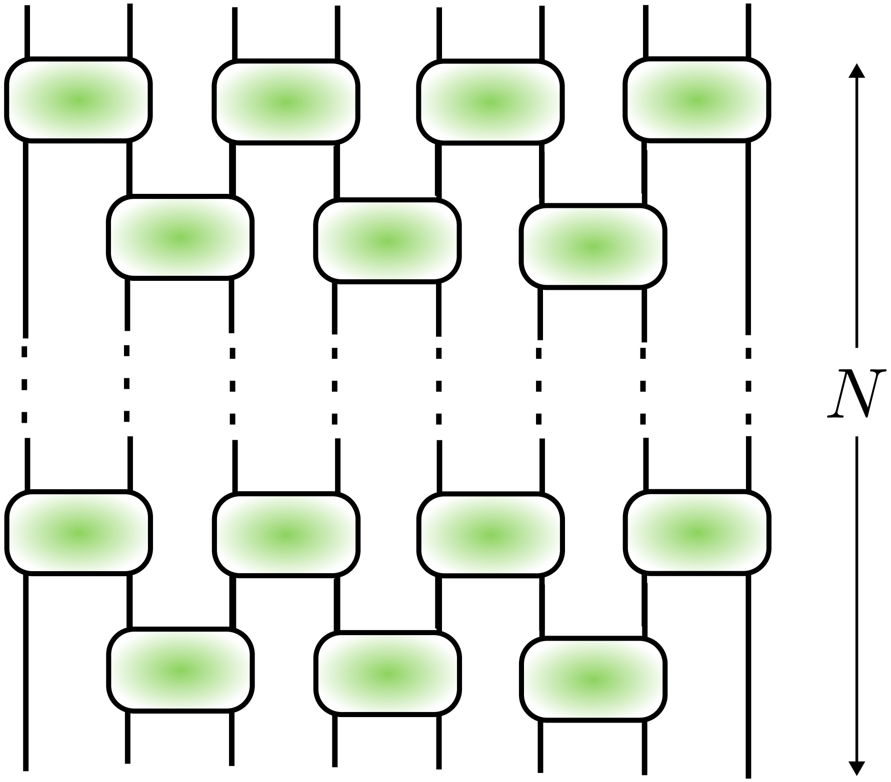
$$\delta\beta = \beta/N$$

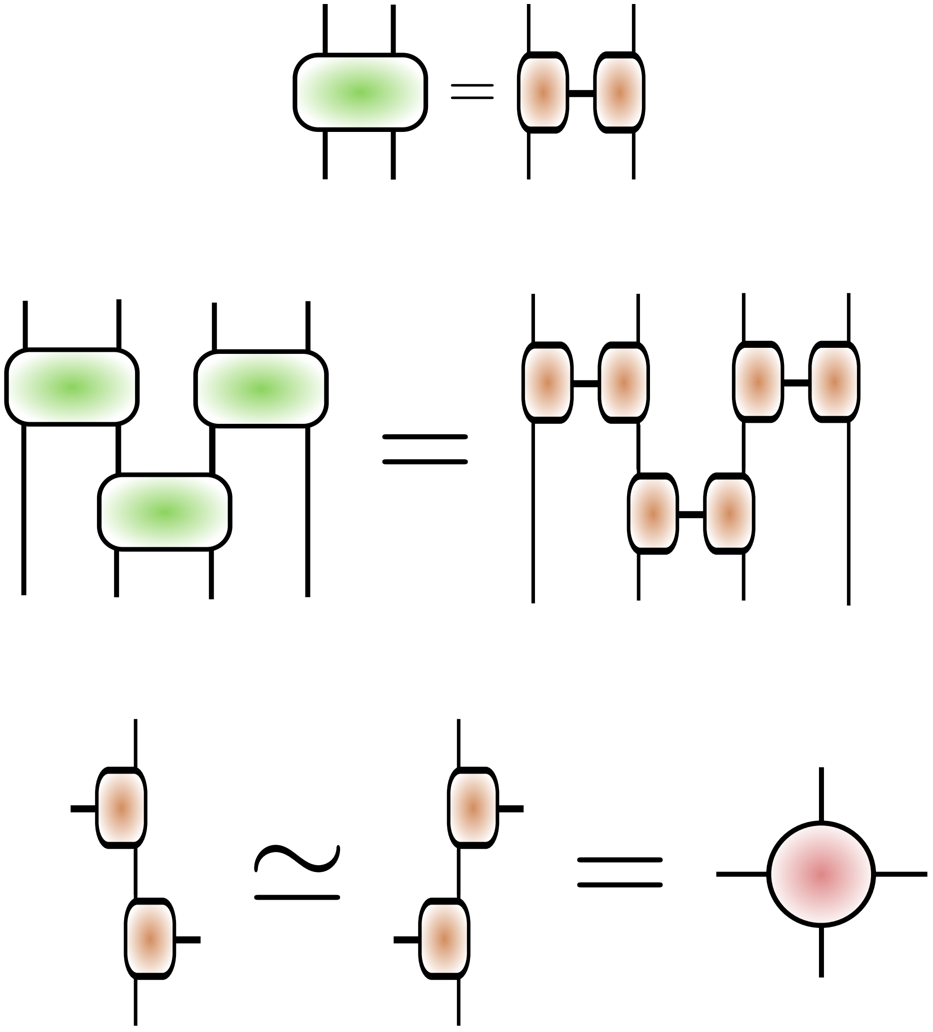
$$\exp(-\beta\hat{H}) = \left[\exp(-\delta\beta\hat{H})\right]^N \simeq \left[\prod_{ij} u(\delta\beta)\right]^N$$

$$u(\delta\beta) = \exp(-\delta\beta\hat{h})$$

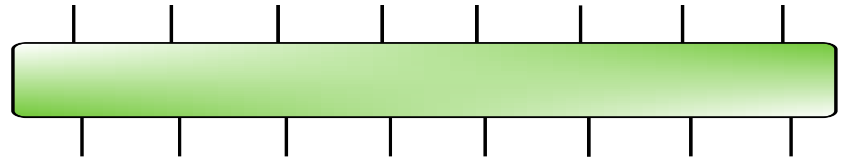


$$\left[\exp(-\delta\beta\hat{H})\right]^N \simeq$$



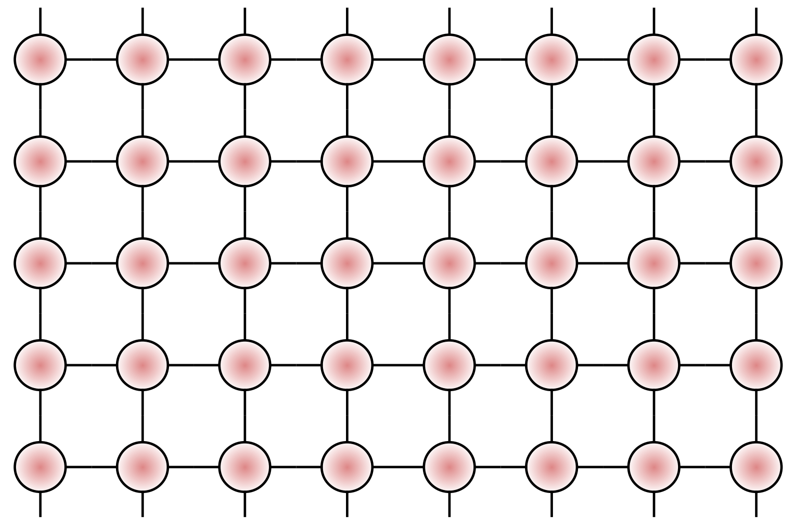


$$\exp(-\beta\hat{H}) =$$

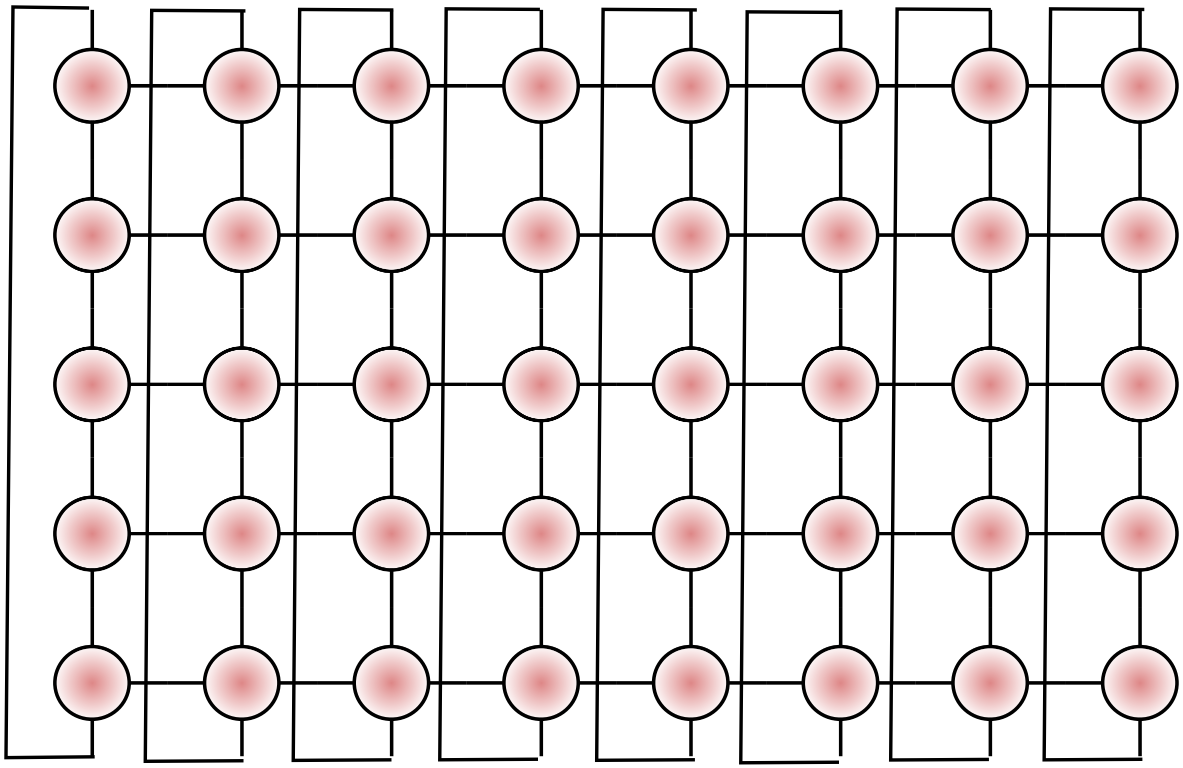


$$\simeq$$

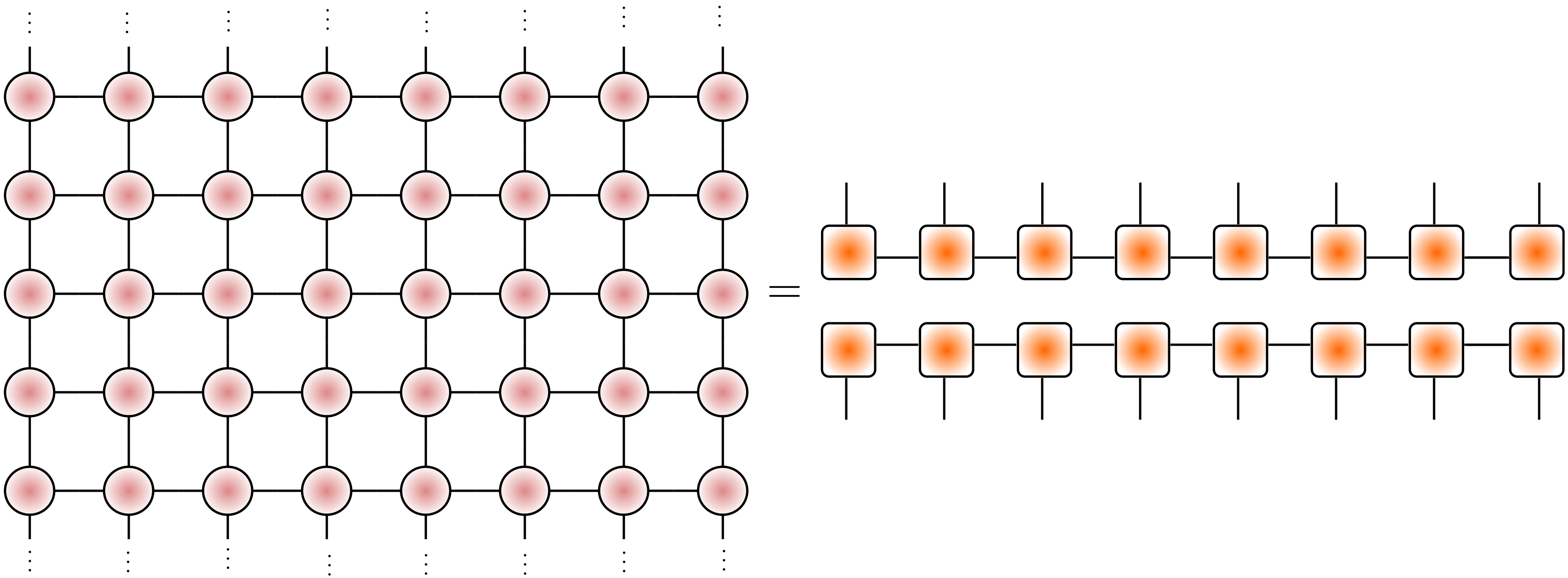
$$\left[\exp(-\delta\beta\hat{H})\right]^N \simeq$$



$$Z(\beta) =$$

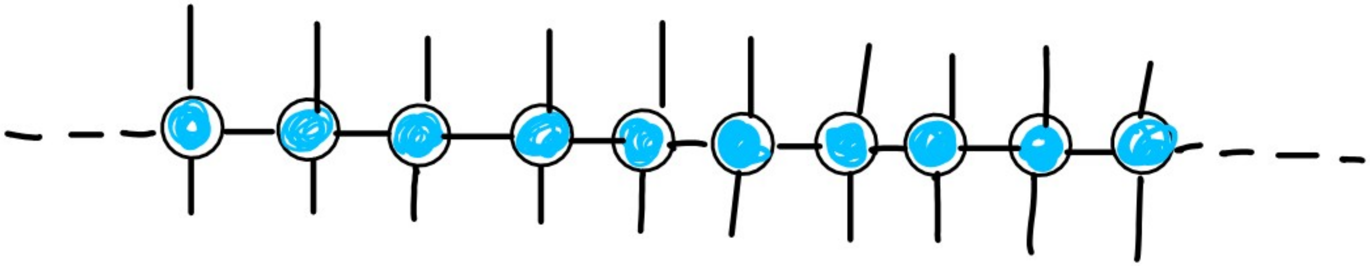


$$|E_0\rangle \langle E_0| = \lim_{\beta \to \infty} \rho(\beta),$$



Gapped transfer matrix

- Given a transfer matrix T (Hermitian)



- Gap implies largest eigenvalue t_1

$$T = \sum_{i=1}^{d^N} t_i |t_i\rangle\langle t_i| \qquad t_1 > t_2 \cdots$$

- The projector on the largest eigenvector is well approximate by the powers

$$T^k \simeq t_1^k |t_1\rangle\langle t_1| + \mathcal{O}(t_2/t_1)^k$$

Properties of the largest eigenvector

- Exp decaying correlation functions

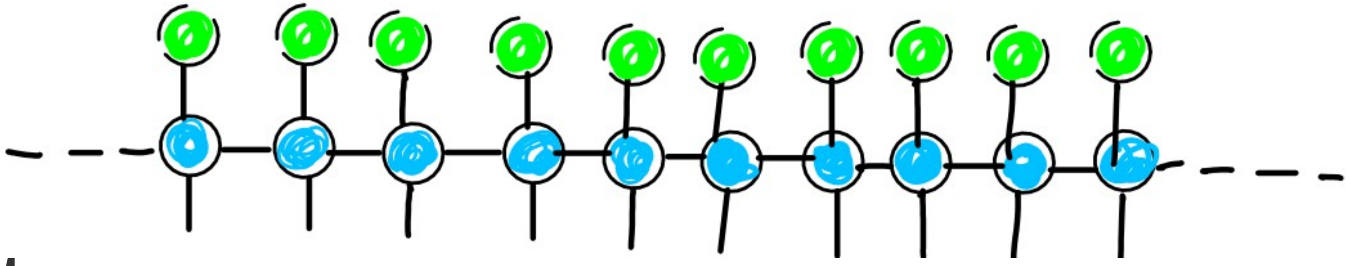
$$\langle O(0)O(r)\rangle \propto \exp(-r\Delta)$$

- Area law of entanglement

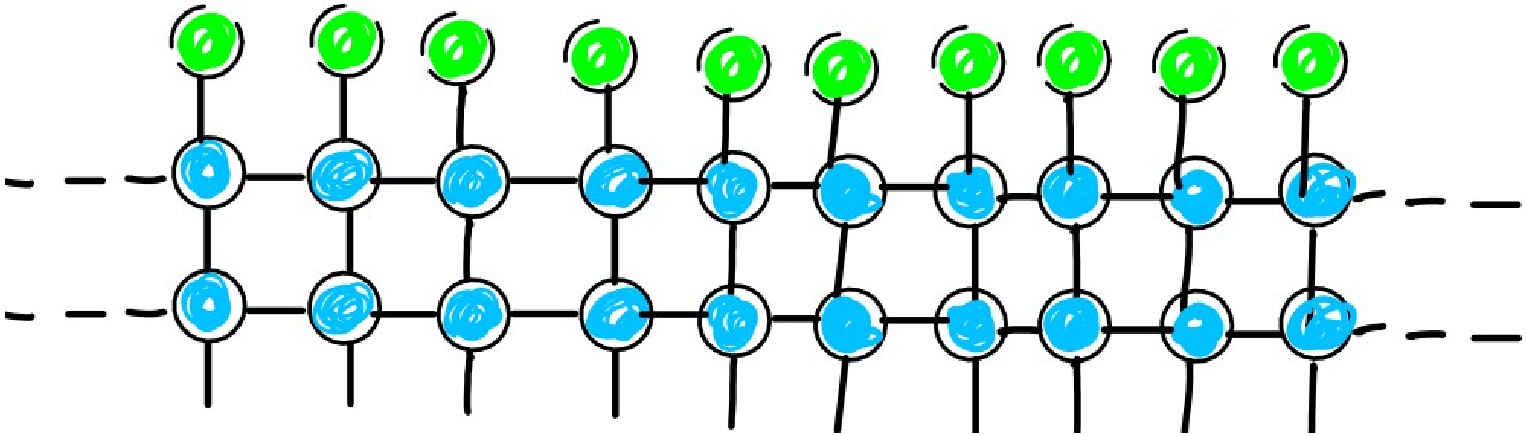
$$S \propto const$$

Largest eigenvector

- Apply T to a random vector

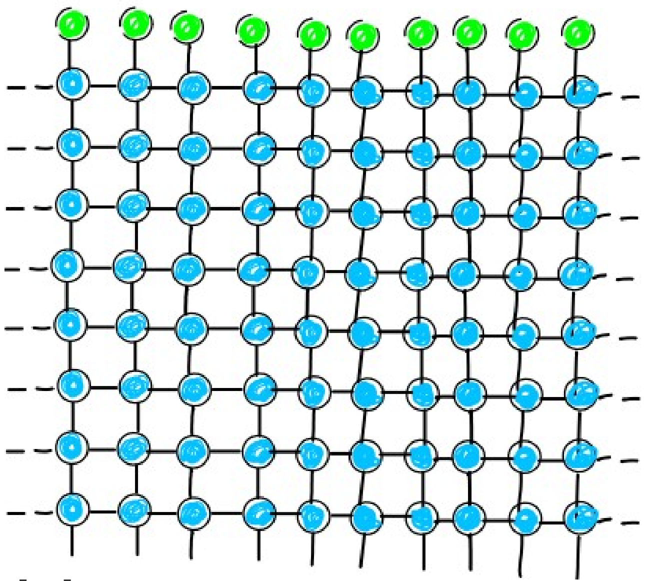


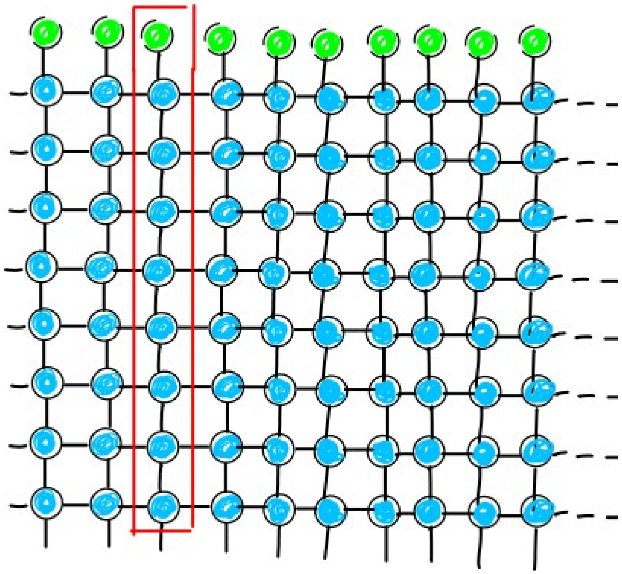
- Iterate



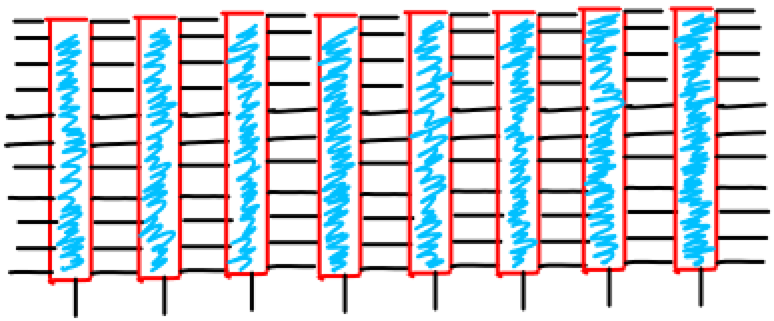
Largest eigenvector

- Keep iterating until it does not change anymore



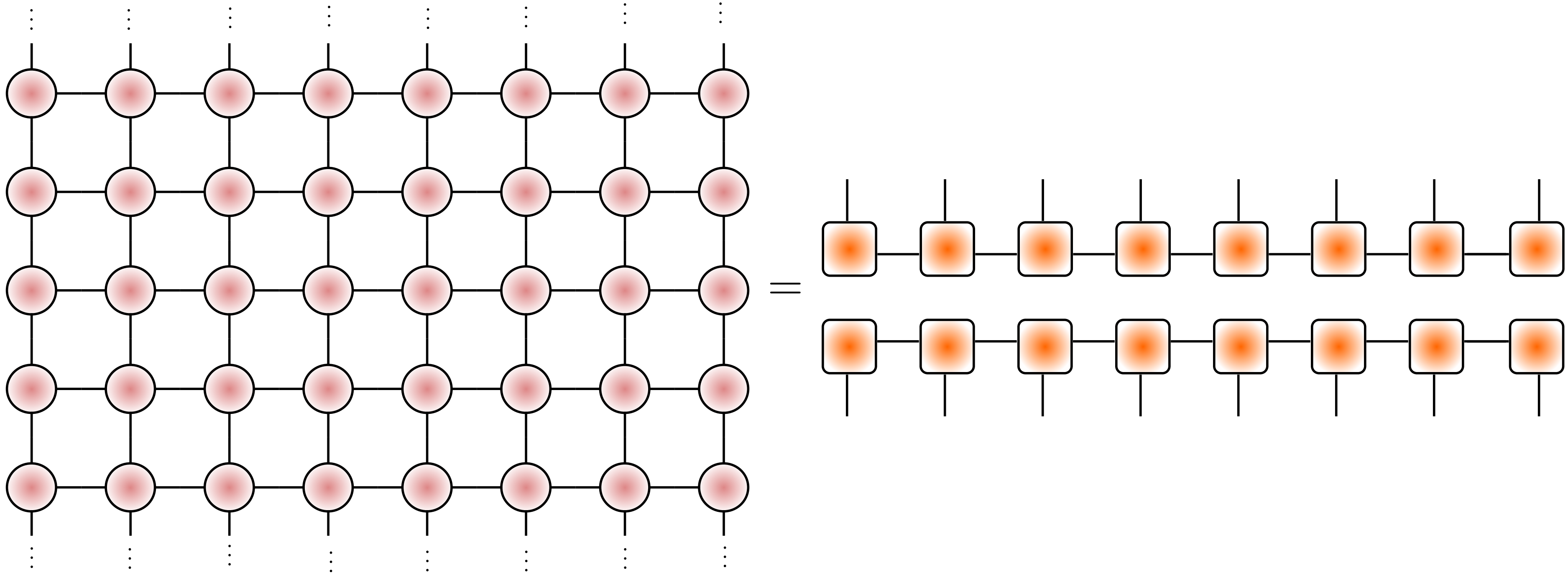


- At this stage you can identify the MPS structure

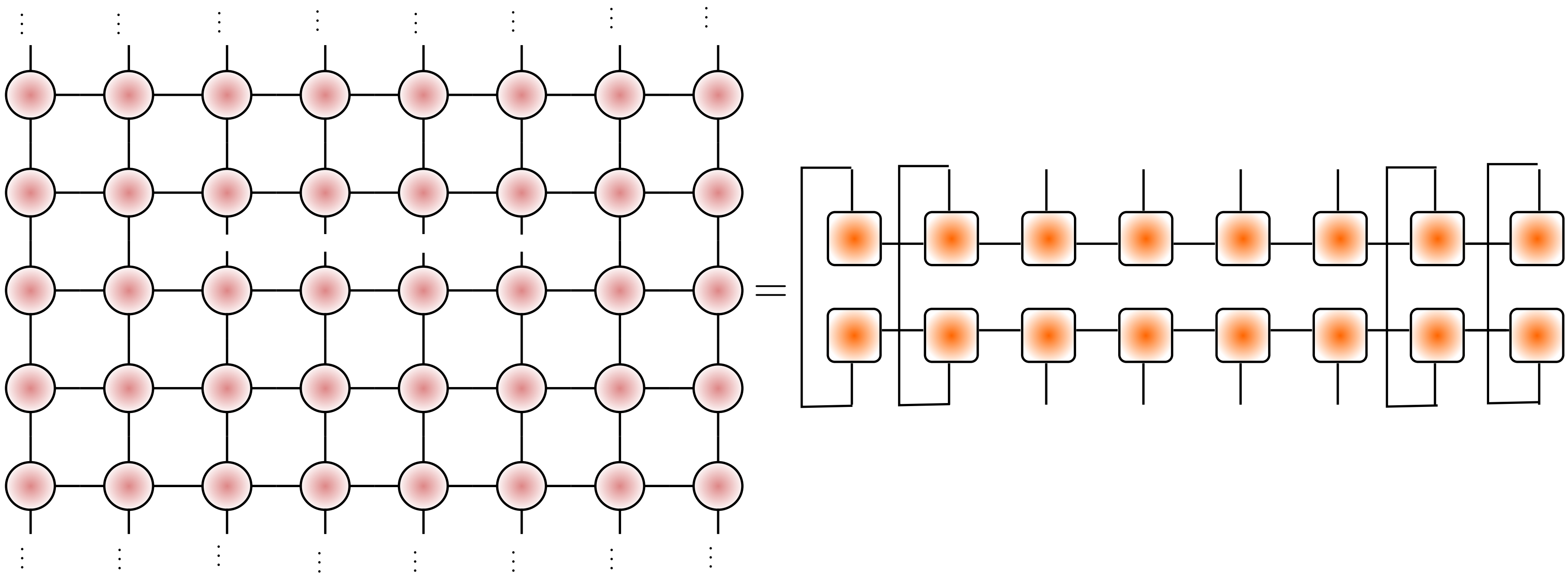


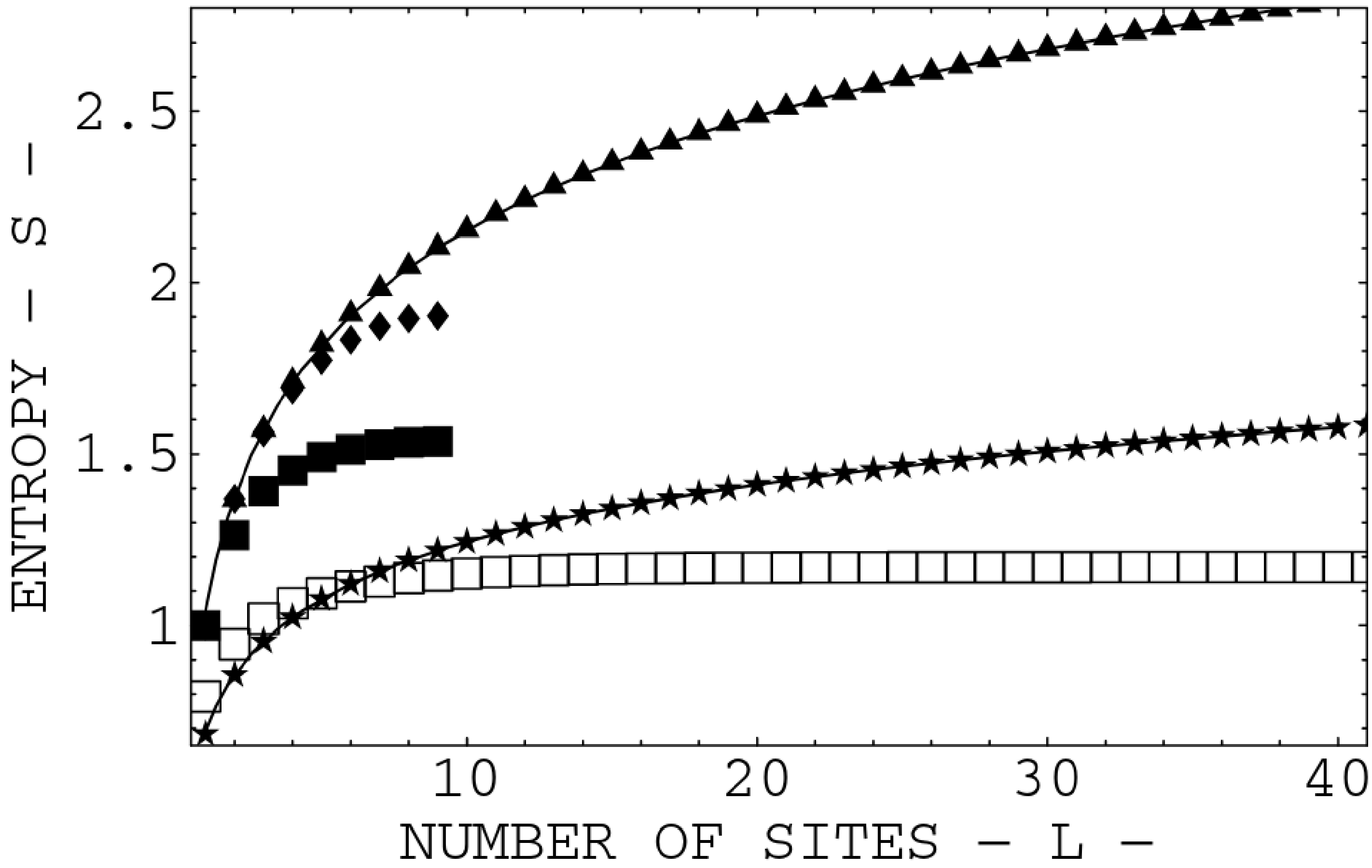
$$\Delta = -\log(t_2/t_1) > 0$$

$$|E_0\rangle \langle E_0| = \lim_{\beta\to\infty} \rho(\beta),$$



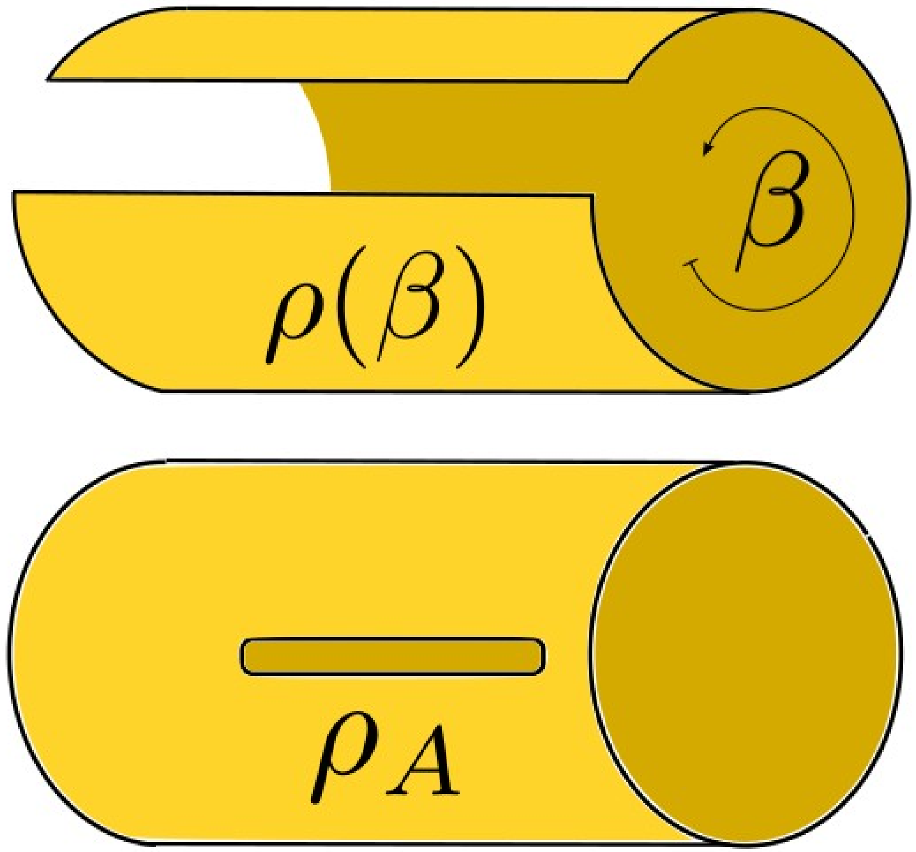
$$\rho_A = \mathrm{tr}\rho(\beta)$$





Vidal+ 2003

$$\rho_\beta = \frac{1}{Z}e^{-\beta H},$$



$$\rho_A = \mathrm{tr}_B\,\rho(\beta)$$

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Esperanza Lopez (CSIC-IFT),
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Jan Schneider (CSIC-IFF),
Stefano Carignano (BSC).

Review on Spatio-Temporal Tensor Networks

arXiv:2502.20214

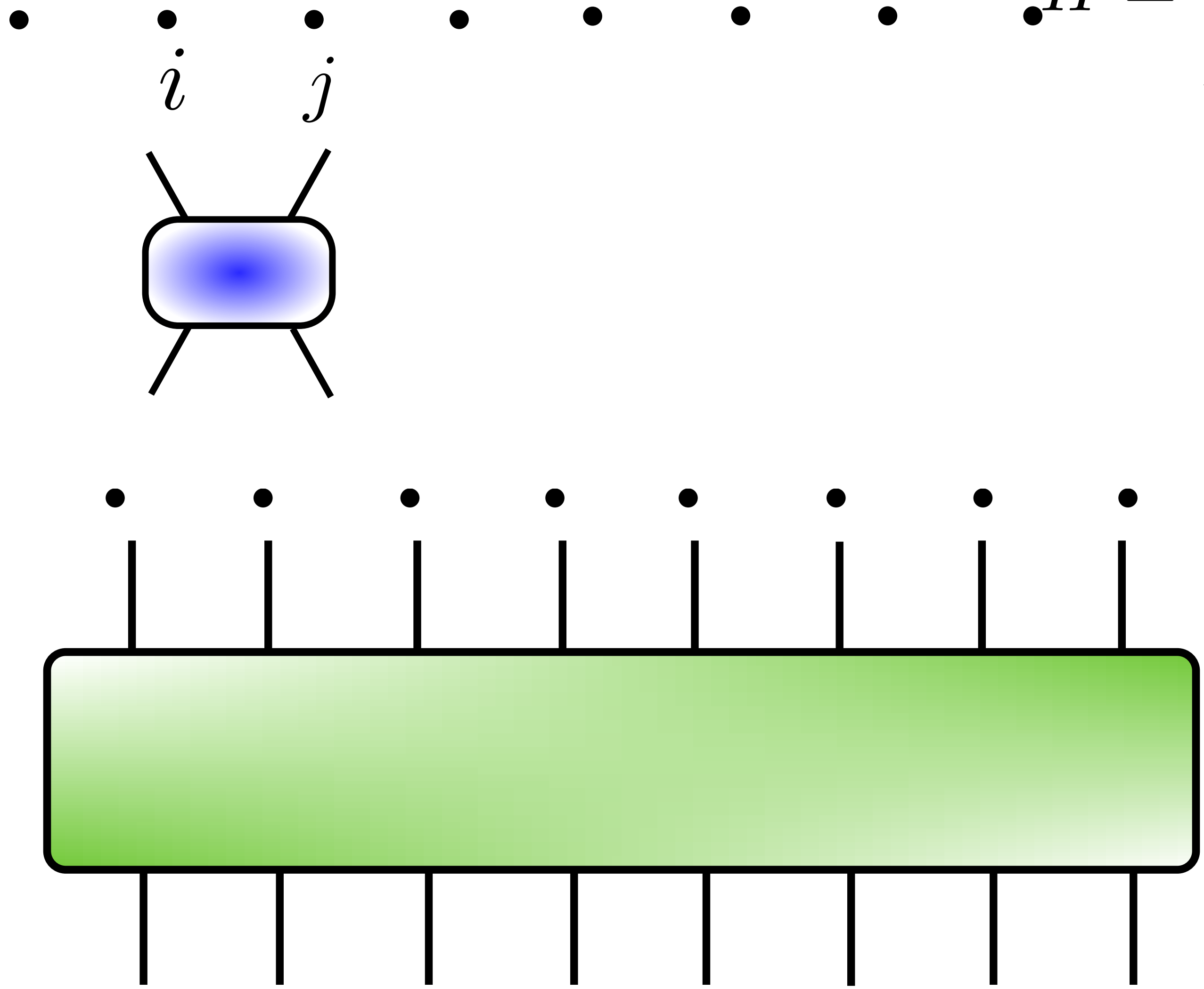
$$\langle\psi|O(T)|\psi\rangle$$

$$O(T) = U(T) O U^\dagger(T)$$

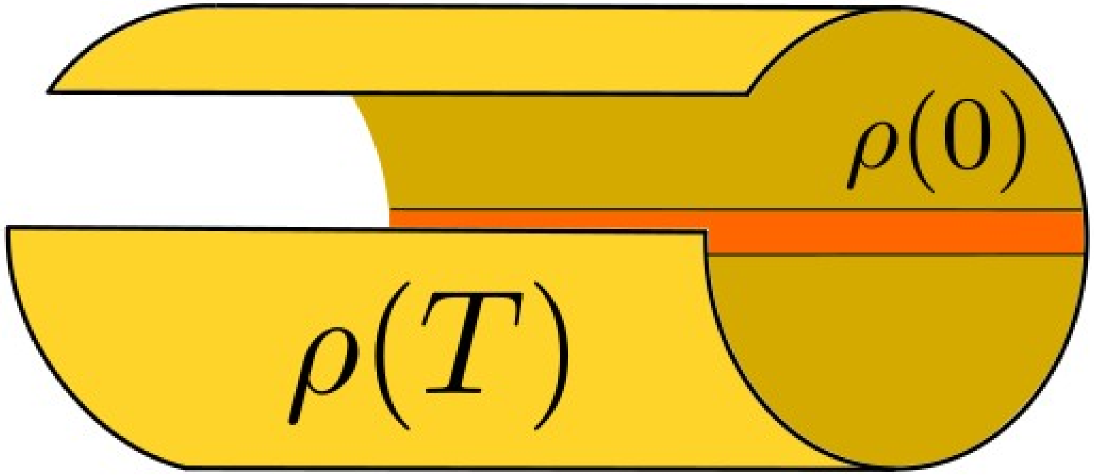
$$U(T) = \exp(-iHT)$$

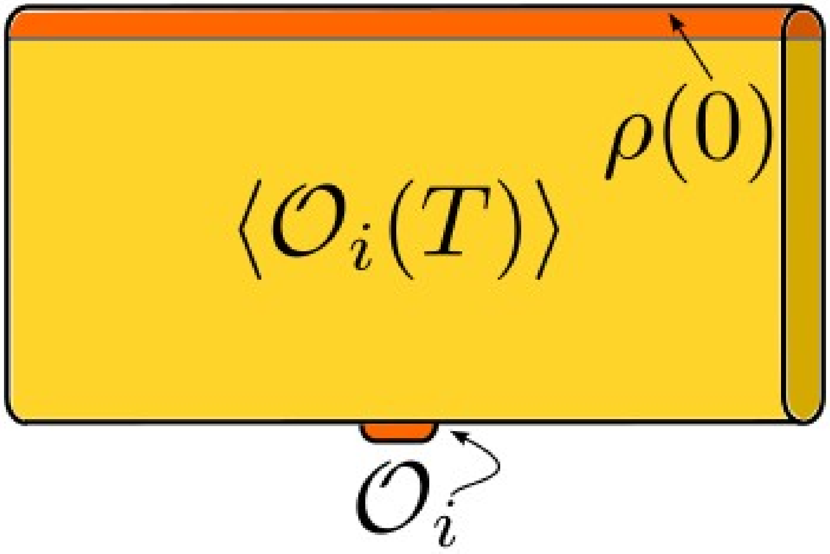
$$H = \sum_{<ij>} h_{ij}$$

$$\hat{H} = \sum_{\langle i,j \rangle} \hat{h}_{i,j},$$



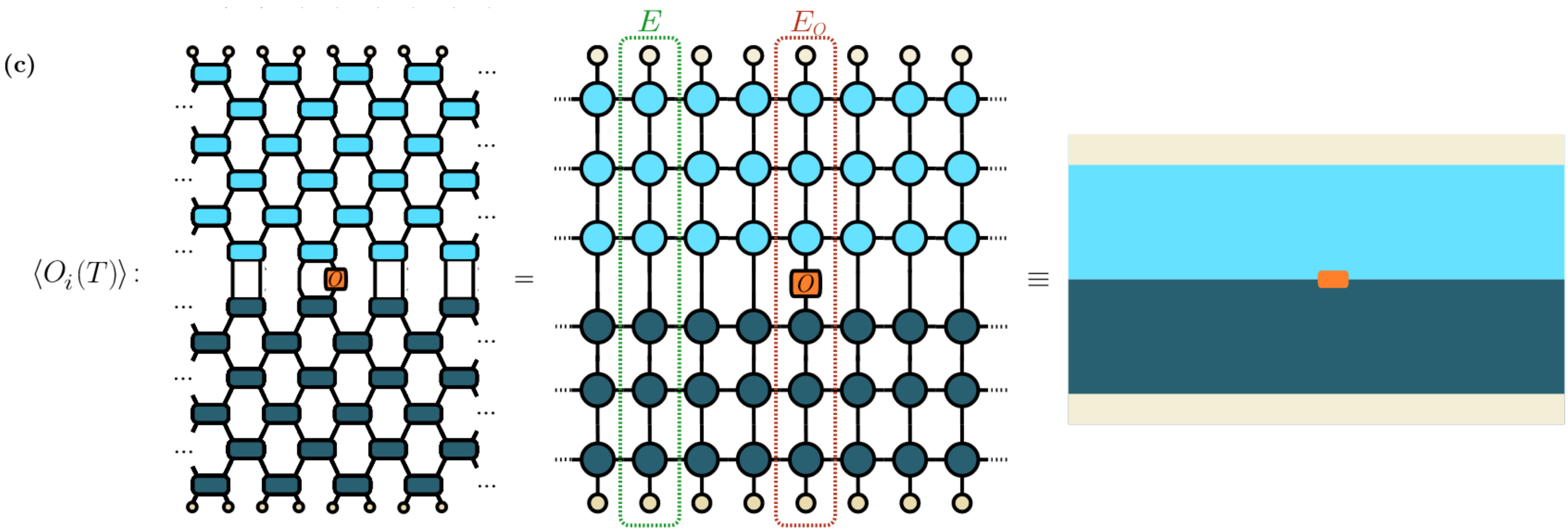
$$\rho(T) = U^{\dagger}(T)\rho_0 U(T)$$





$$U(T) = \exp\left(-iHT\right)$$

$$\langle \mathcal{O}_i \rangle = \mathrm{tr}(\rho(T)\mathcal{O}_i)$$


(c)
$\langle O_i(T) \rangle$:
O
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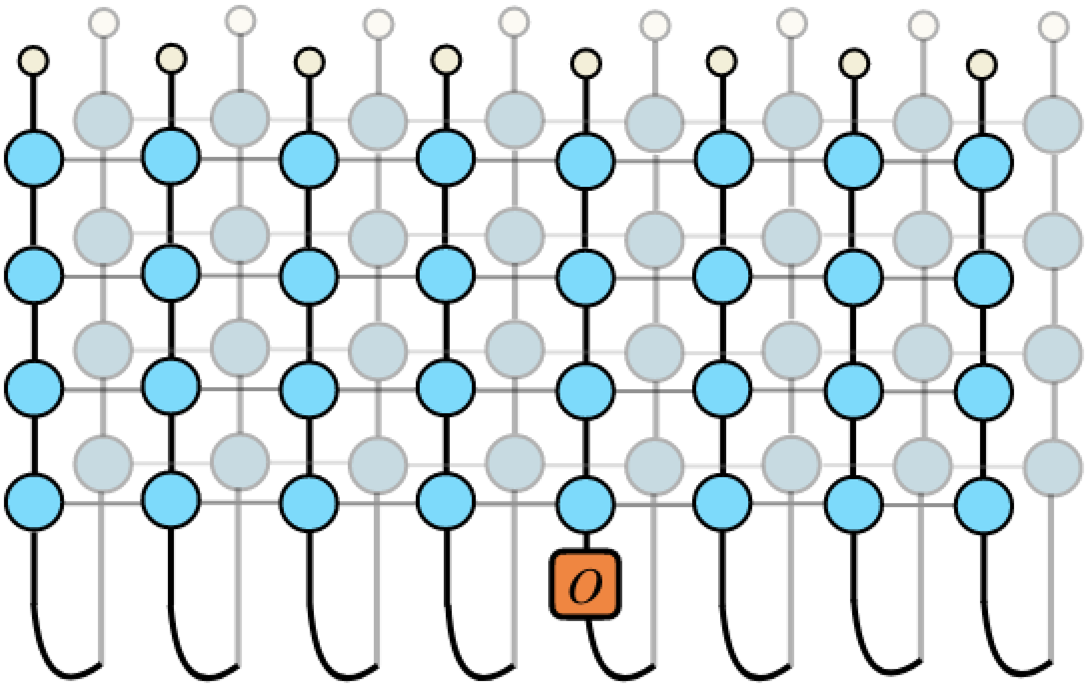
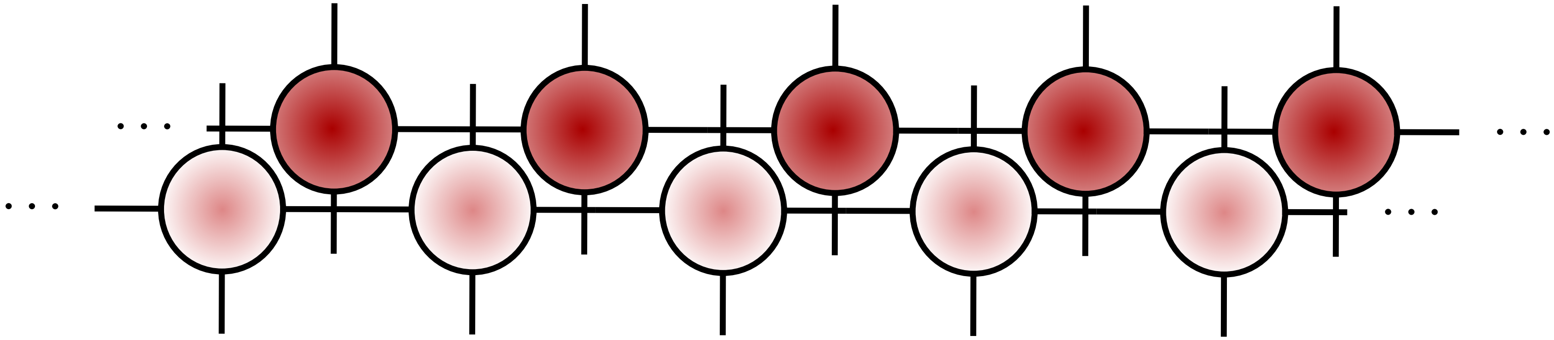
O
≡


O

$$U(\delta t) = \exp(-i\delta t H)$$

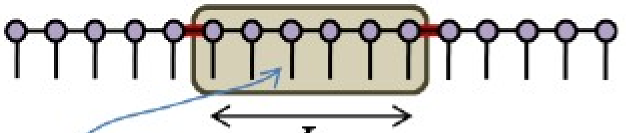
$$U(\delta t) \otimes U^\dagger(\delta_t)$$

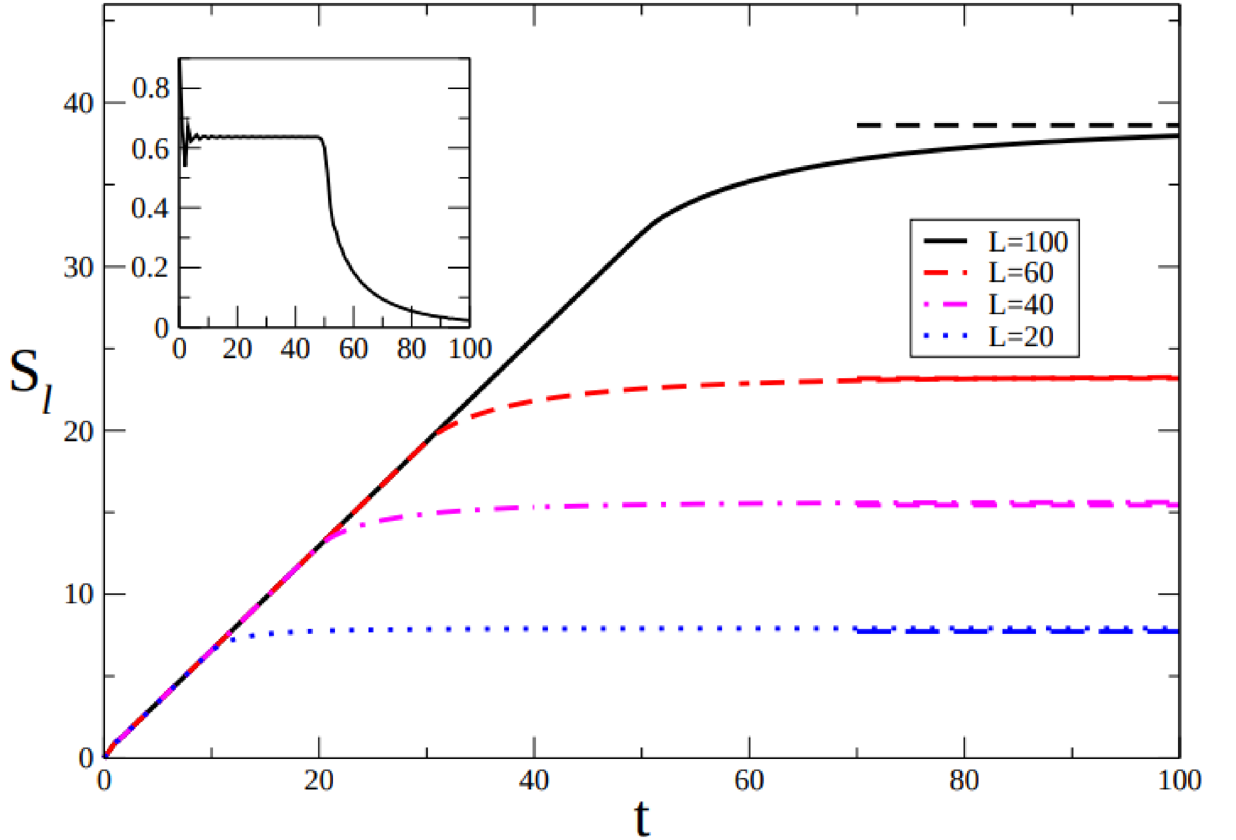
$$H = \sum_i E_i \left|E_i\right\rangle \left\langle E_i\right|$$



$$\Delta_{ij} = -i\delta t(E_i - E_j)$$

$$\Delta_{ii} = 0$$





from Calabrese Cardy 2005

$$S \leq n_{AB} \log(\chi)$$

$$\chi^{n_{AB}} \geq \exp S \propto \exp(t)$$

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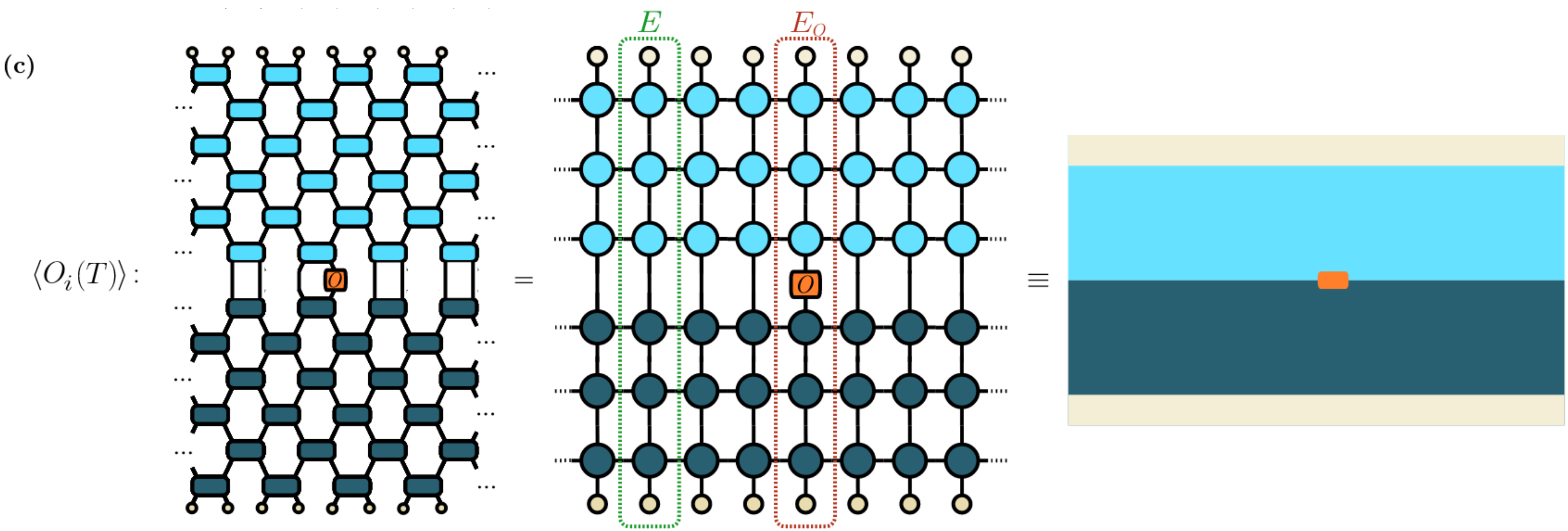
Temporal Entanglement

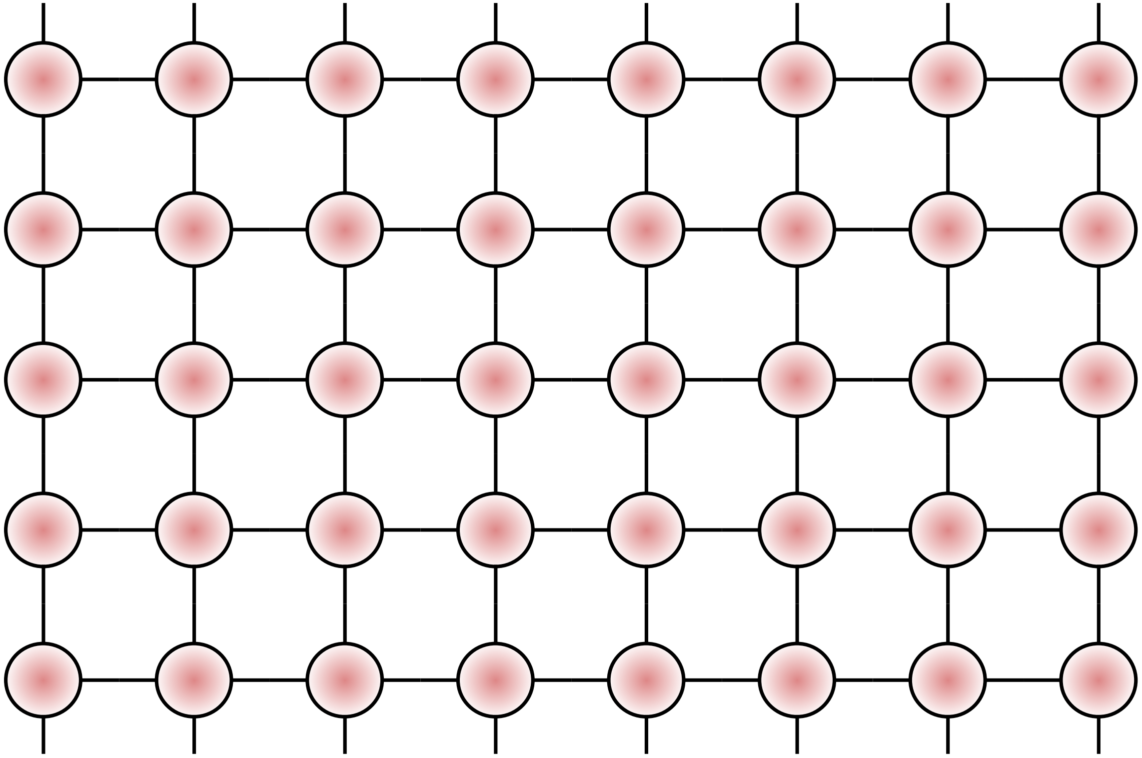
Can we measure the correlations among the same constituents at different times?

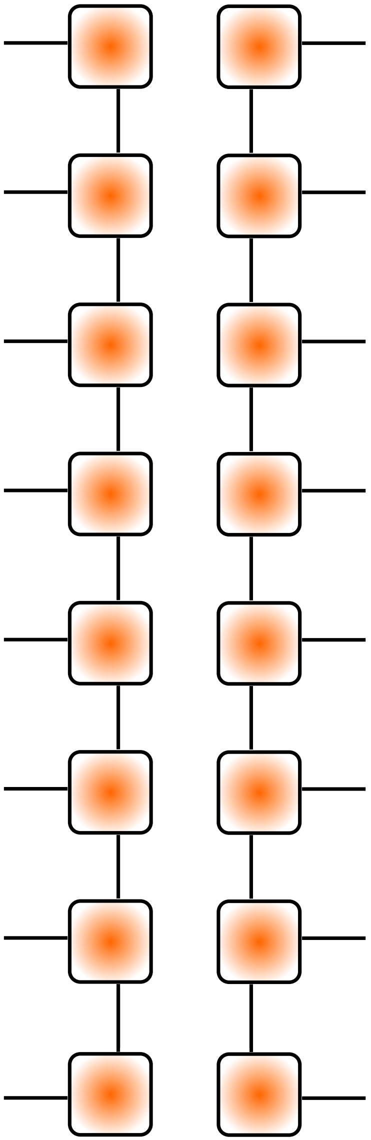
- Computational approaches → quantum complexity (see Bañuls et al, Abanin et al, Chan et al,…) see also our arXiv:2307.11649, arXiv:2505.09714
- Holographic field theories, dS/CFT (Takayangi et al, Heller et al...) for CFT see also our arXiv:2405.14706 next chapter
- QM foundation, the role of time (review of Spekkens)


(c)
$\langle O_i(T)\rangle :$
$=$
E
E_O
O
$\equiv$

Temporal MPS

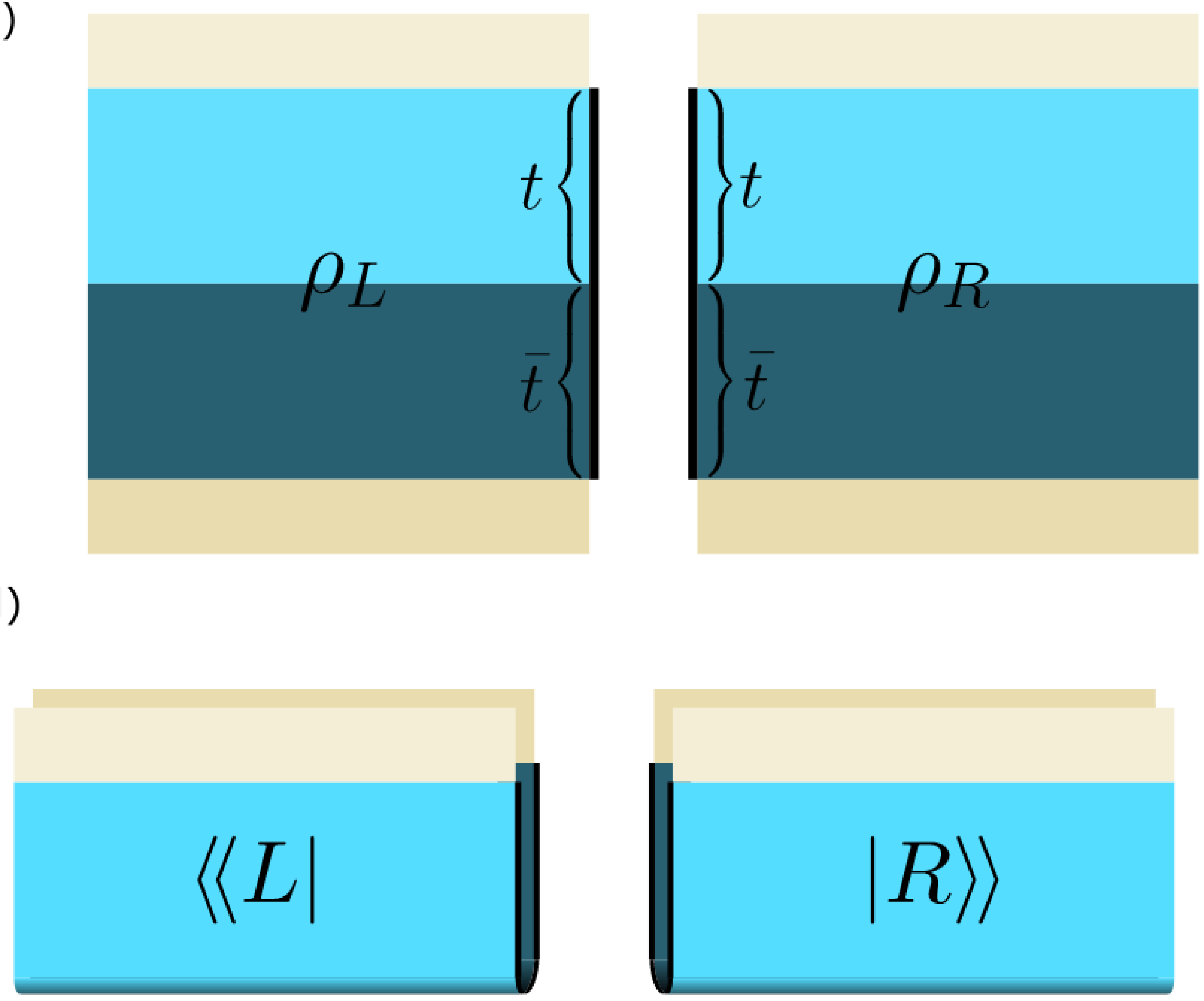
$$\langle\psi| O(T) |\psi\rangle$$



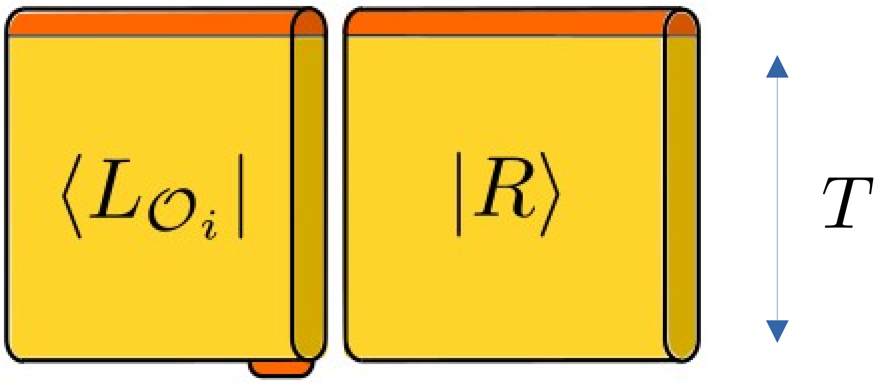


(c)

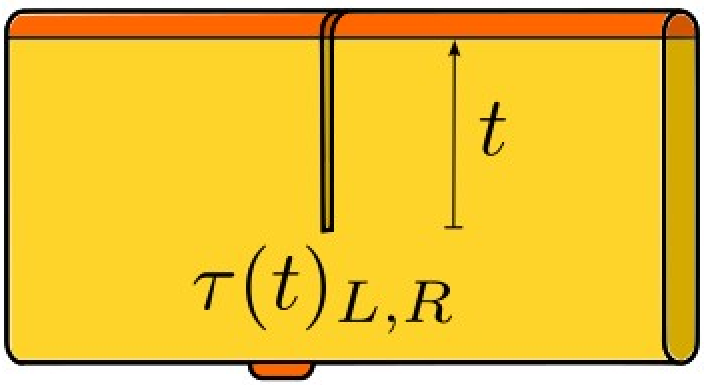
(d)



$$\langle O_i(T)\rangle \equiv \left\langle L_{r\,O_i}\middle|R_r\right\rangle$$



L, R are temporal states, states of constituents over time



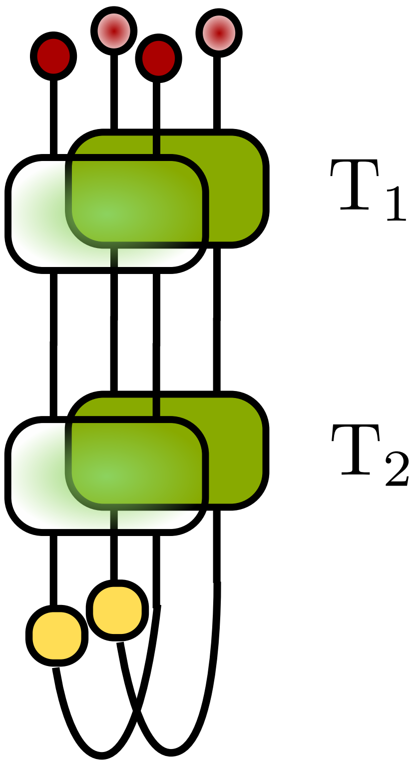
$$\tau(t)_{O_i} = \frac{\mathrm{tr}_{T-t}\,|R\rangle\langle L_{O_i}|}{\langle L_{O_i}|R\rangle}\,.$$

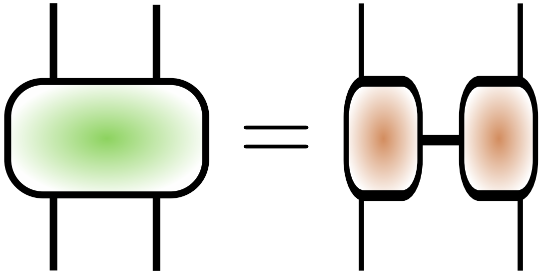
Reduced transition matrices,
Not a legitimate quantum state
Not positive definite
Not even Hermitian

$$\tau(t)_{O_i} = \frac{\mathrm{tr}_{T-t} \left|R\right\rangle\!\left\langle L_{O_i}\right|}{\left\langle L_{O_i}\middle|R\right\rangle} .$$

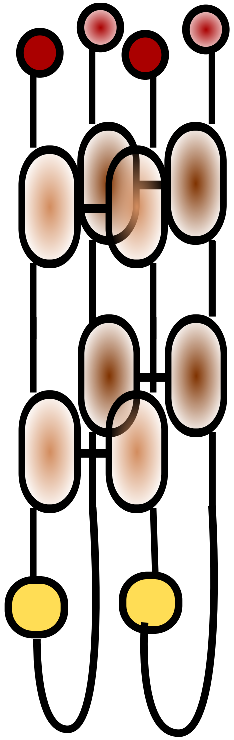
$$\langle\psi| O(T) |\psi\rangle$$

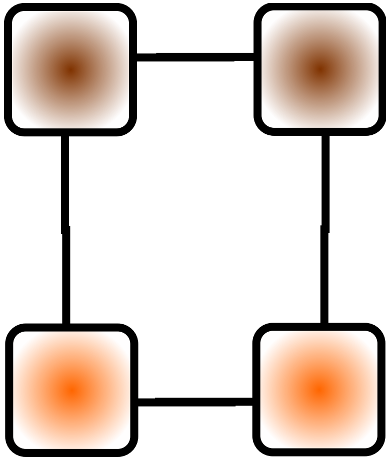
$$T = T_1 + T_2$$



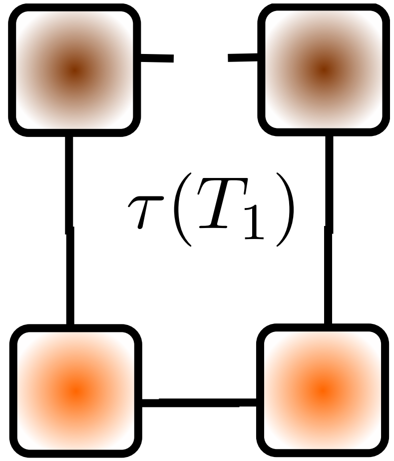


$$U(T_1) = U\Sigma V^\dagger$$

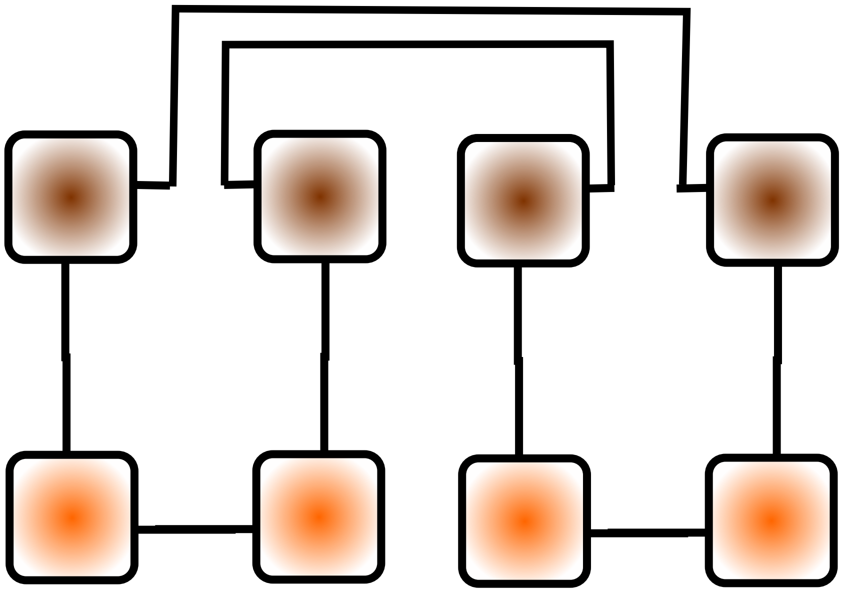




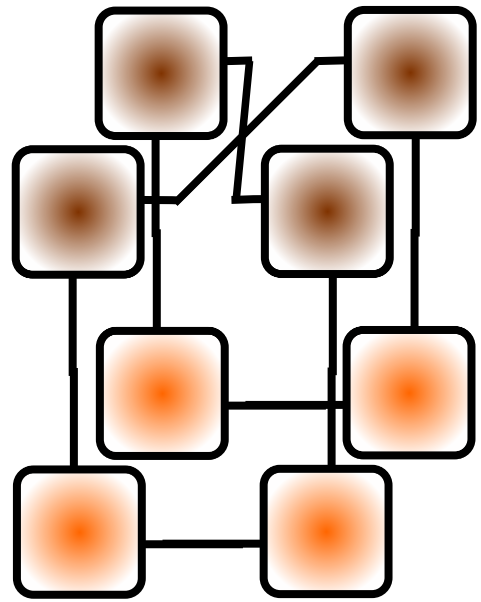
$$\langle\psi|\, O(T)\, |\psi\rangle \equiv \langle\langle L|R\rangle\rangle$$



$$\tau(T_1)$$

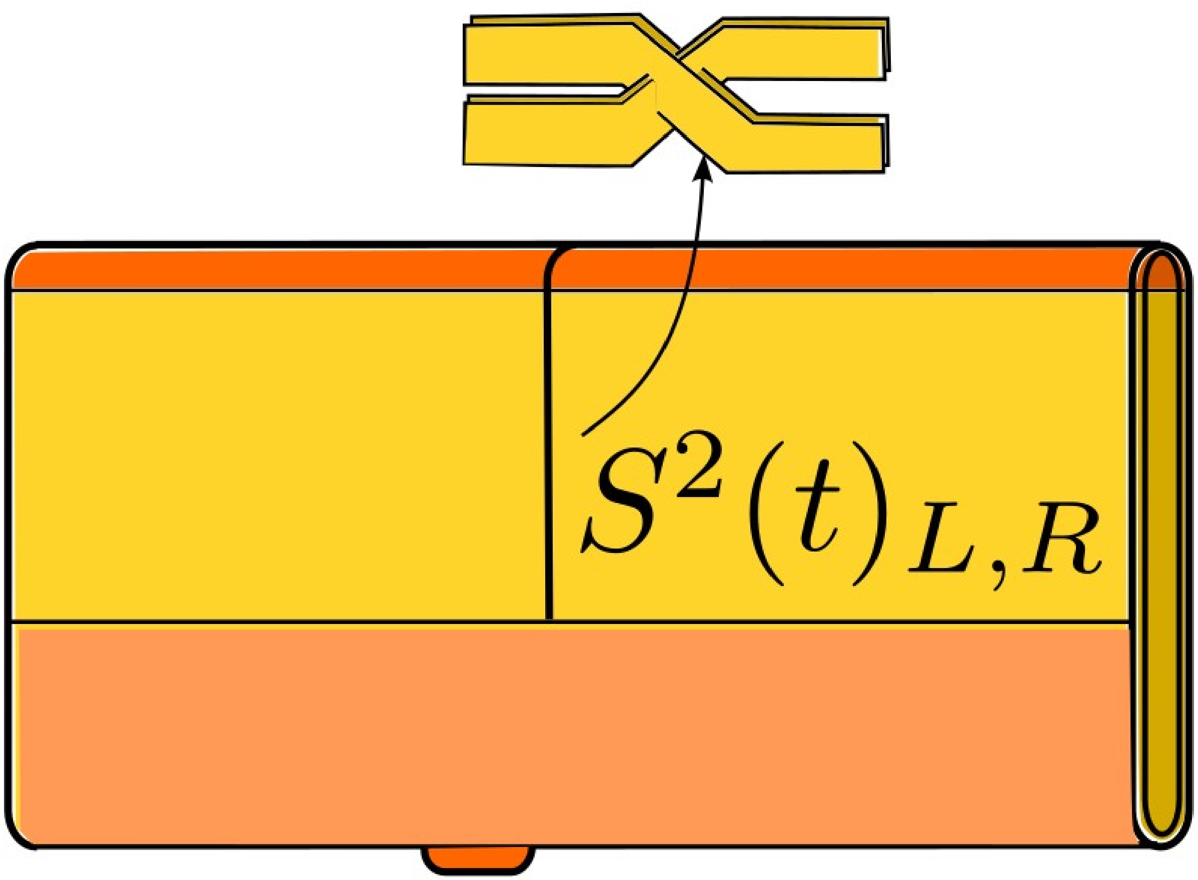


$$\mathrm{tr}\left(\tau(T_1)\right)^2$$



Generalized temporal entropies

$$S^2(t) = \mathrm{tr}\left(\tau(t)^2\right)$$



Outline

- Entanglement barrier out-of-equilibrium
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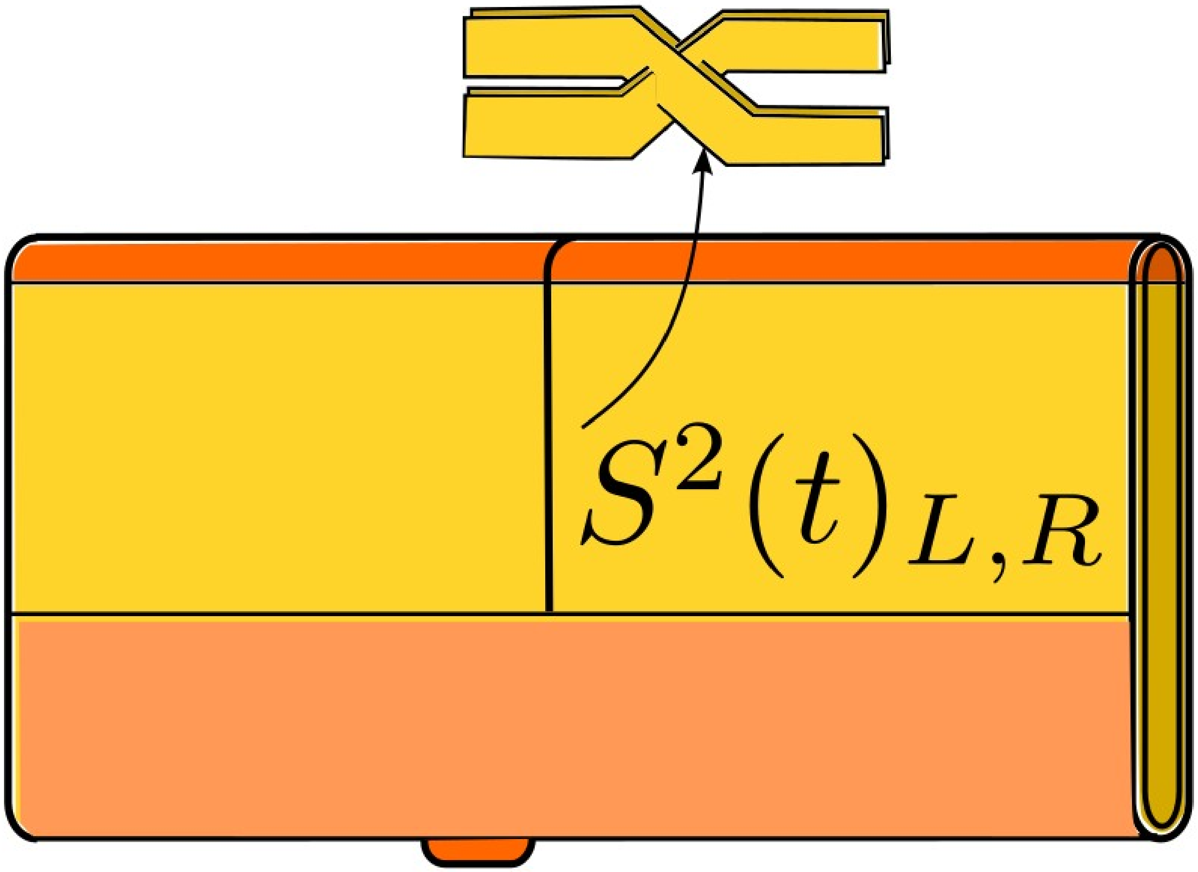
Generalized temporal entropies as the expectation value of Hermitian operators

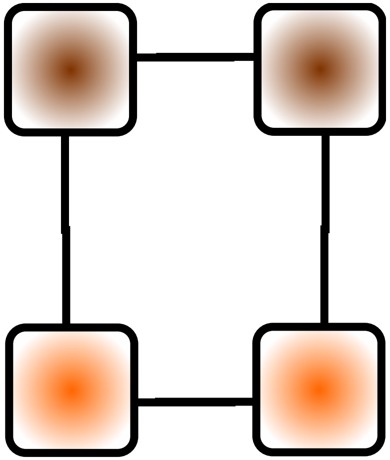
arXiv:2409.05517

Luca Tagliacozzo, YITP 2025

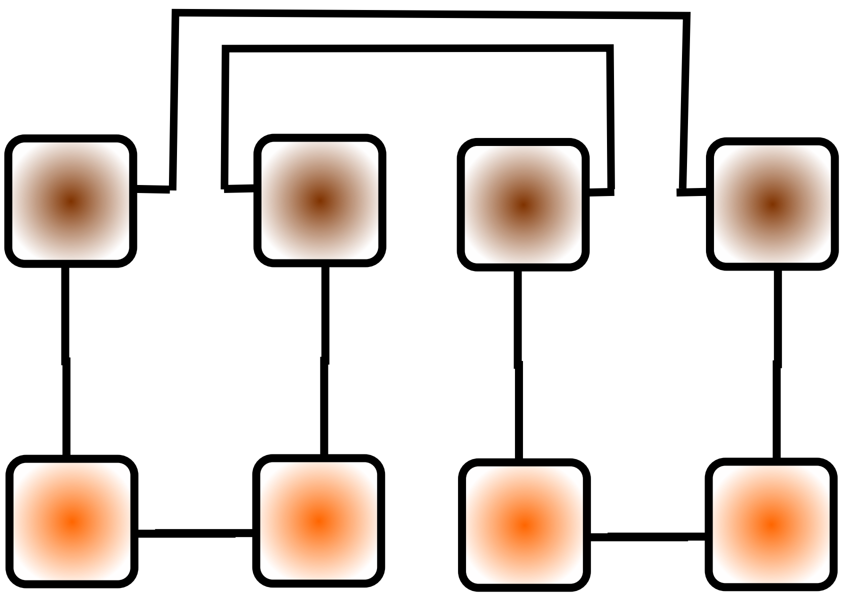
The double quench

$$S^2(t) = \mathrm{tr}\left(\tau(t)^2\right)$$

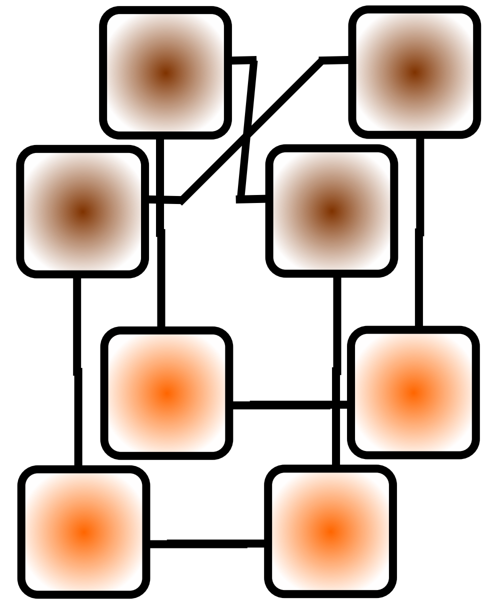




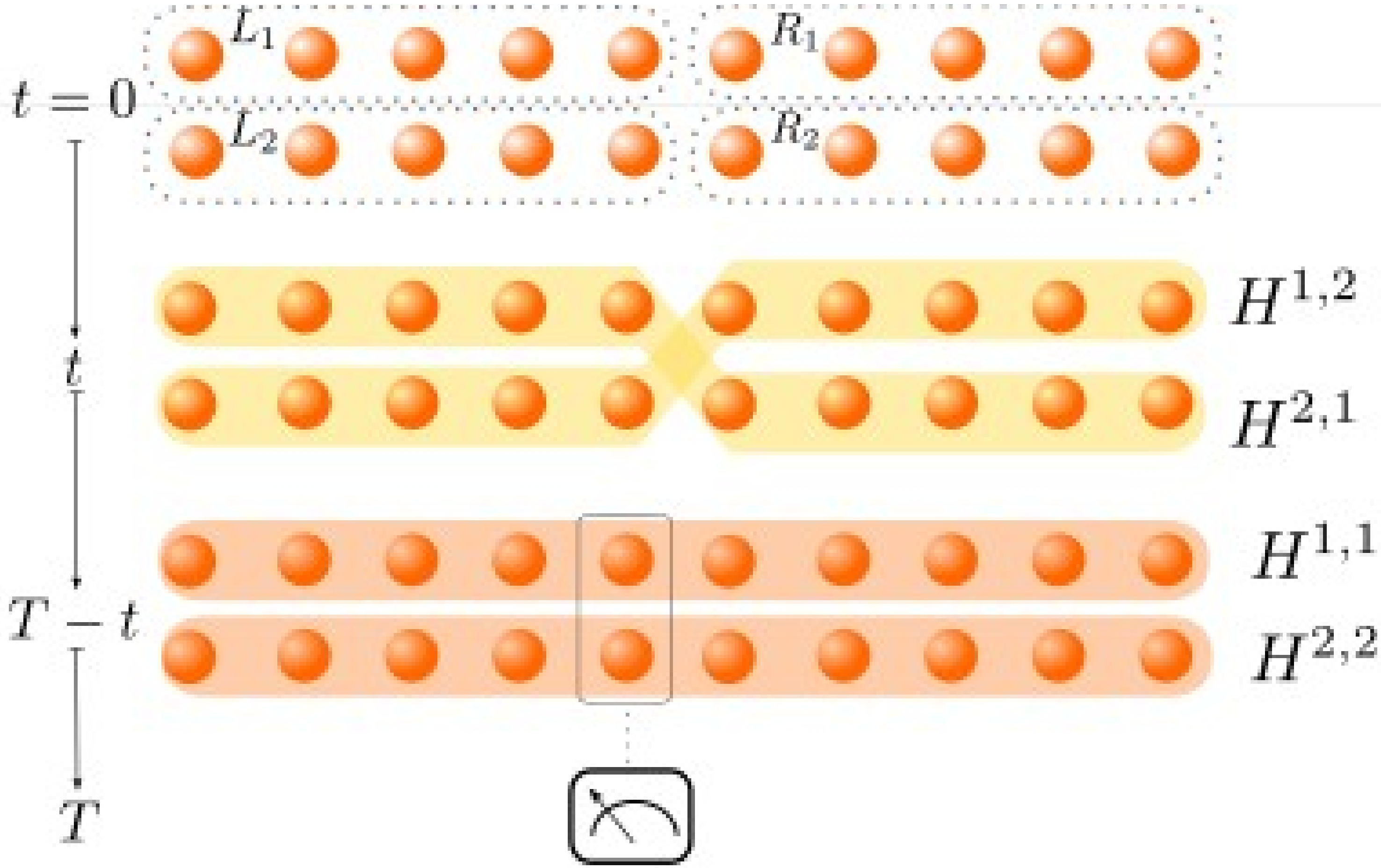
$$\langle\psi|\, O(T)\, |\psi\rangle \equiv \langle\langle L|R\rangle\rangle$$



$$\mathrm{tr}\left(\tau(T_1)\right)^2$$



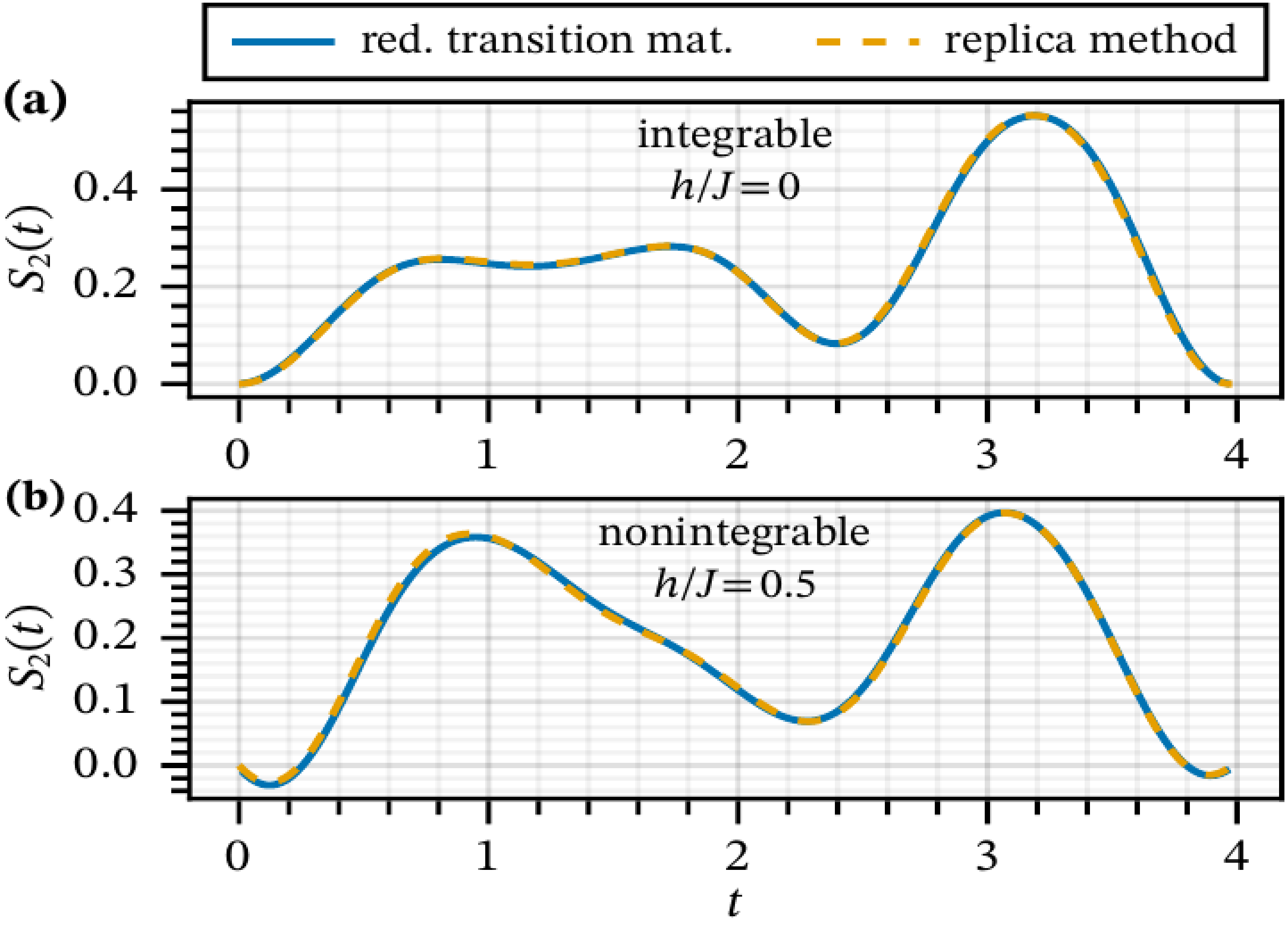
$$\mathrm{tr}\left(\tau(t)^2_{O_i}\right) \equiv \frac{\mathrm{tr}\left(O_i^1 \otimes O_i^2 \rho^{1,2}(T)\right)}{\langle O(T)\rangle^2}$$

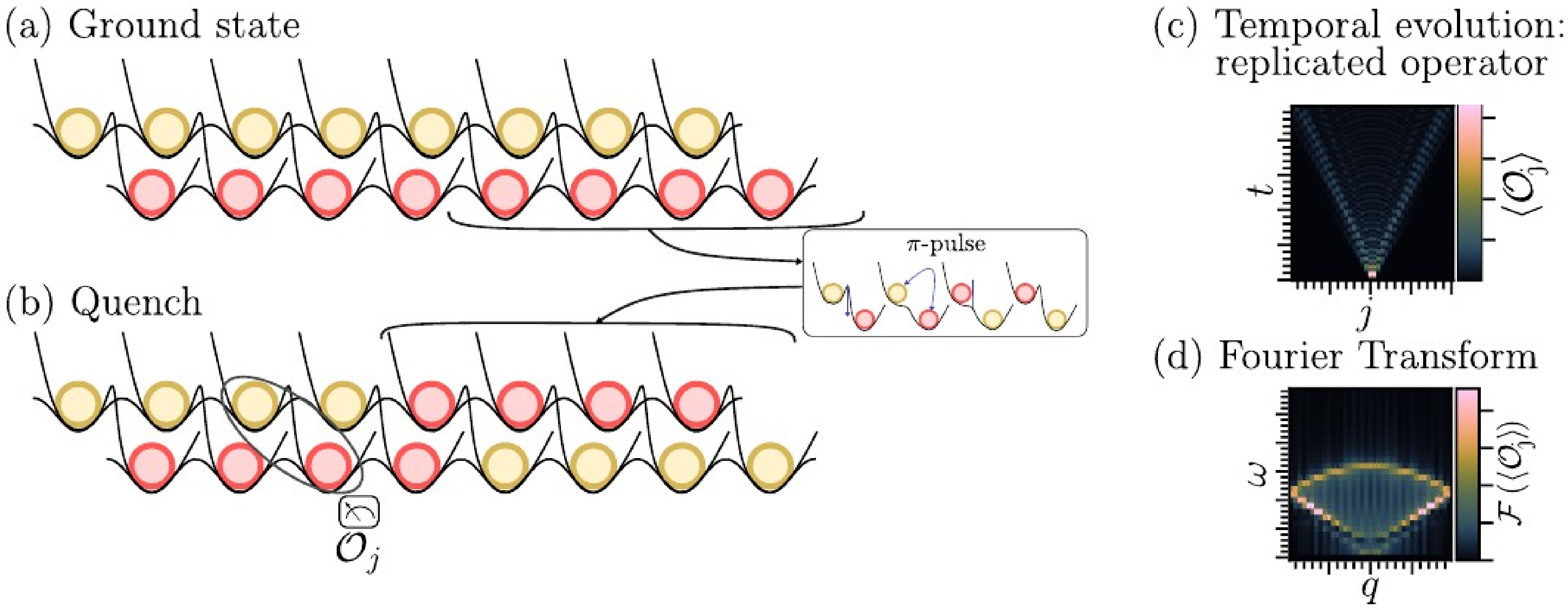


$$\rho^{1,2}(T) \equiv W^{1,2}(T)\left(\rho_0^1 \otimes \rho_0^2\right) W^{1,2}(T)^\dagger$$

$$W^{1,2}(T) = \left(U^1(T-t) \otimes U^2(T-t)\right)\left(U^{1,2}(t) \otimes U^{2,1}(t)\right)$$

$$H = -J \sum_{i=1}^{N-1} \sigma_x^i \sigma_x^{i+1} + \sum_{i=1}^{N} \left(g\sigma_z^i + h\sigma_x^i \right) .$$




(a) Ground state
π-pulse
(b) Quench
$\mathcal{O}_j$
(c) Temporal evolution: replicated operator
t
j
$\langle \mathcal{O}_j \rangle$
(d) Fourier Transform
ω
q
$\mathcal{F}(\langle \mathcal{O}_j \rangle)$

- Pump-probe objects, the pump is a geometric perturbation, the probe is the local operator
- They are real valued
- They can be measured in experiments
- UV finite, for lattice systems

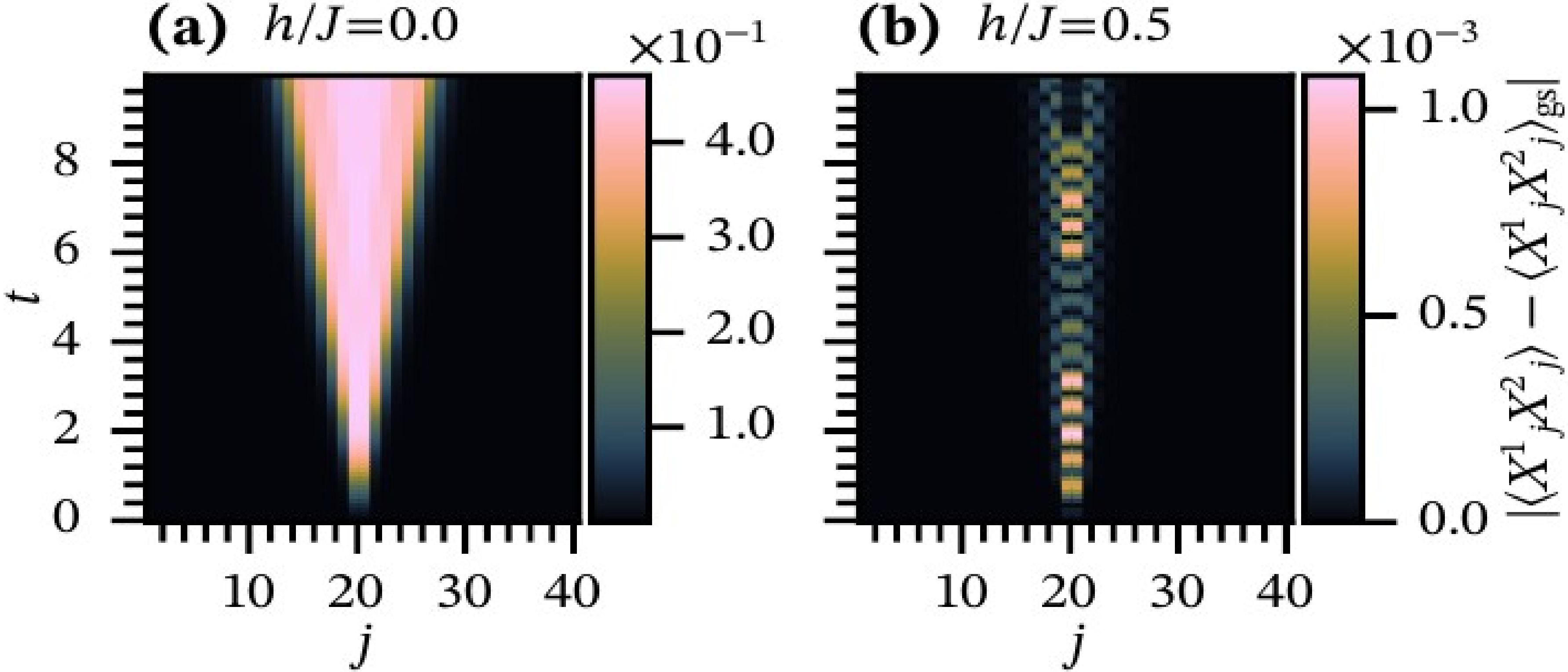
Application, witnessing the nature of the quantum dynamics

Generalized temporal entanglement of the ground state (equilibrium state)

$$\mathrm{tr}\left(\tau(t)^2_{O_i}\right) \equiv \frac{\mathrm{tr}\left(O_i^1 \otimes O_i^2 \rho^{1,2}(T)\right)}{\langle O(T)\rangle^2}$$

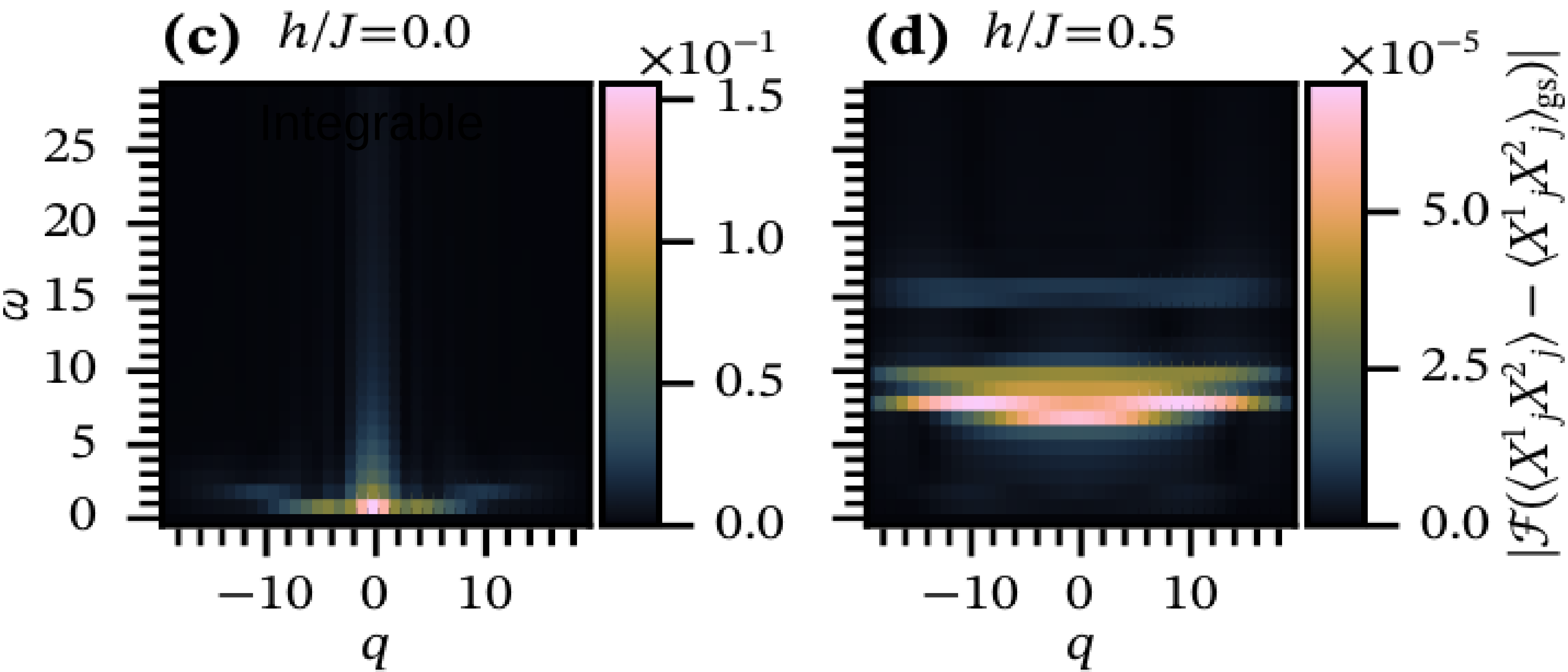
Integrable

Non Integrable



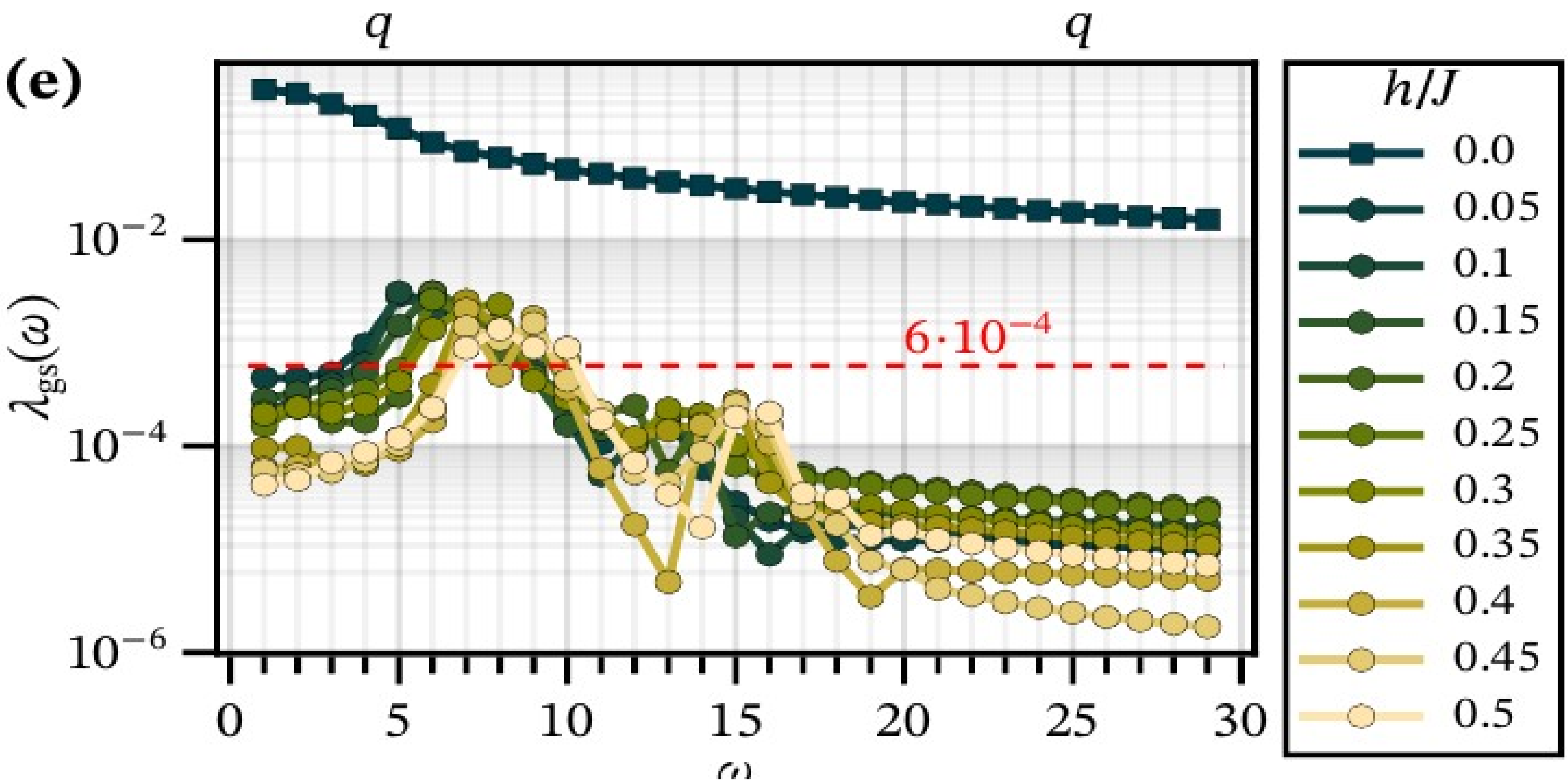
$$\mathcal{F}_{\tau^2}(k,\omega) = \frac{2\pi}{LT}\delta t \sum_{j=1}^{N} e^{-ik(j-\frac{N}{2})} \sum_{n=0}^{t_N} e^{-i\omega t_n} \left(\mathrm{tr}\left(\tau(t_n)^2_{O_j}\right) - 1\right)$$

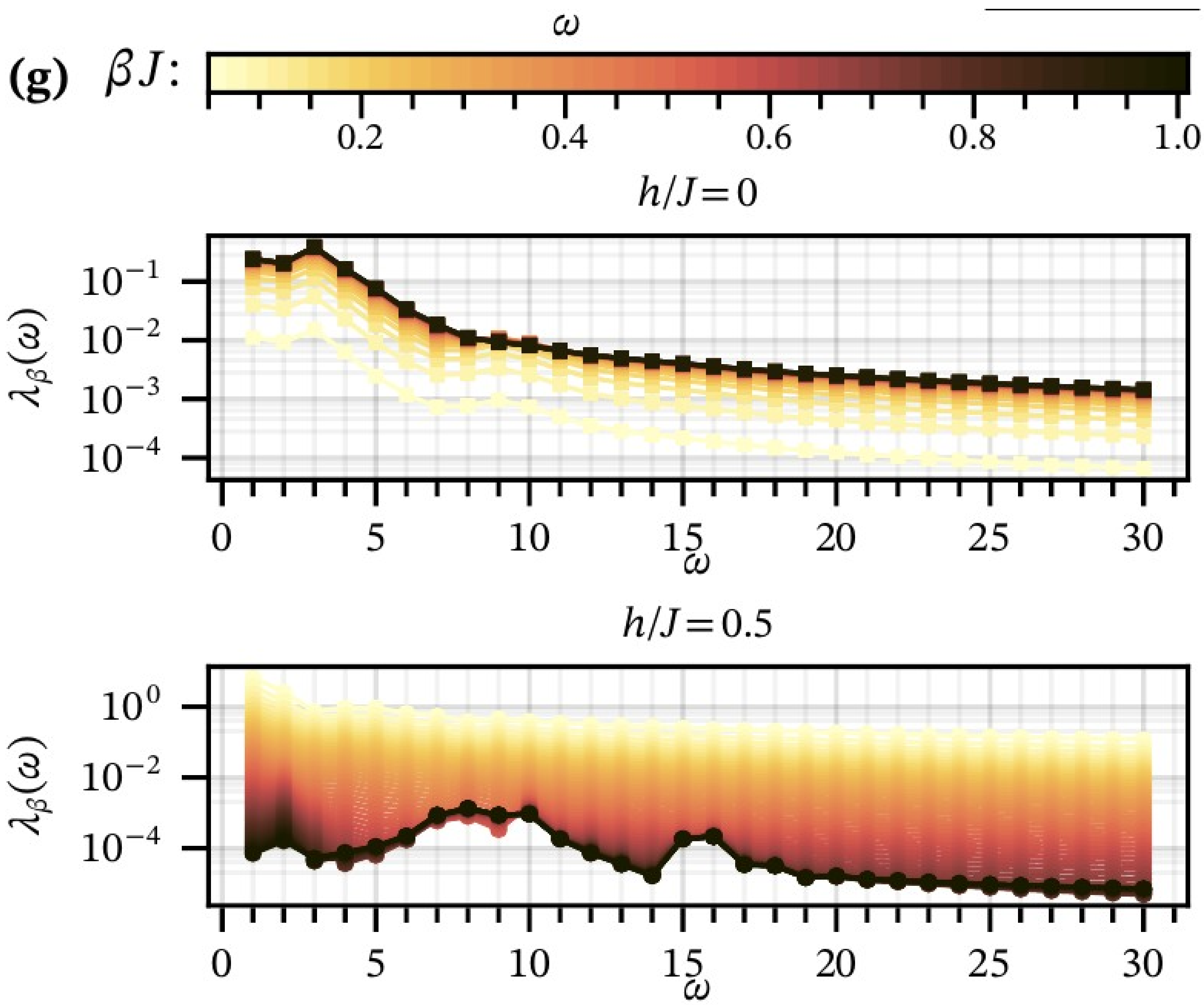
Integrable | Non integrable



Soft mode → integrable dynamics

$$\lambda(\omega) = \frac{2\pi}{T}\delta t \sum_{n=0}^{t_N} e^{-i\omega t_n}\left(\mathrm{tr}\left(\tau(t_n)^2_{O_{N/2}}\right) - 1\right)$$





ω
(g) βJ:
0.2
0.4
0.6
0.8
1.0
h/J = 0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\lambda_\beta(\omega)$
0
5
10
15
20
25
30
ω
h/J = 0.5
10^{0}
10^{-2}
10^{-4}
$\lambda_\beta(\omega)$
0
5
10
15
20
25
30
ω

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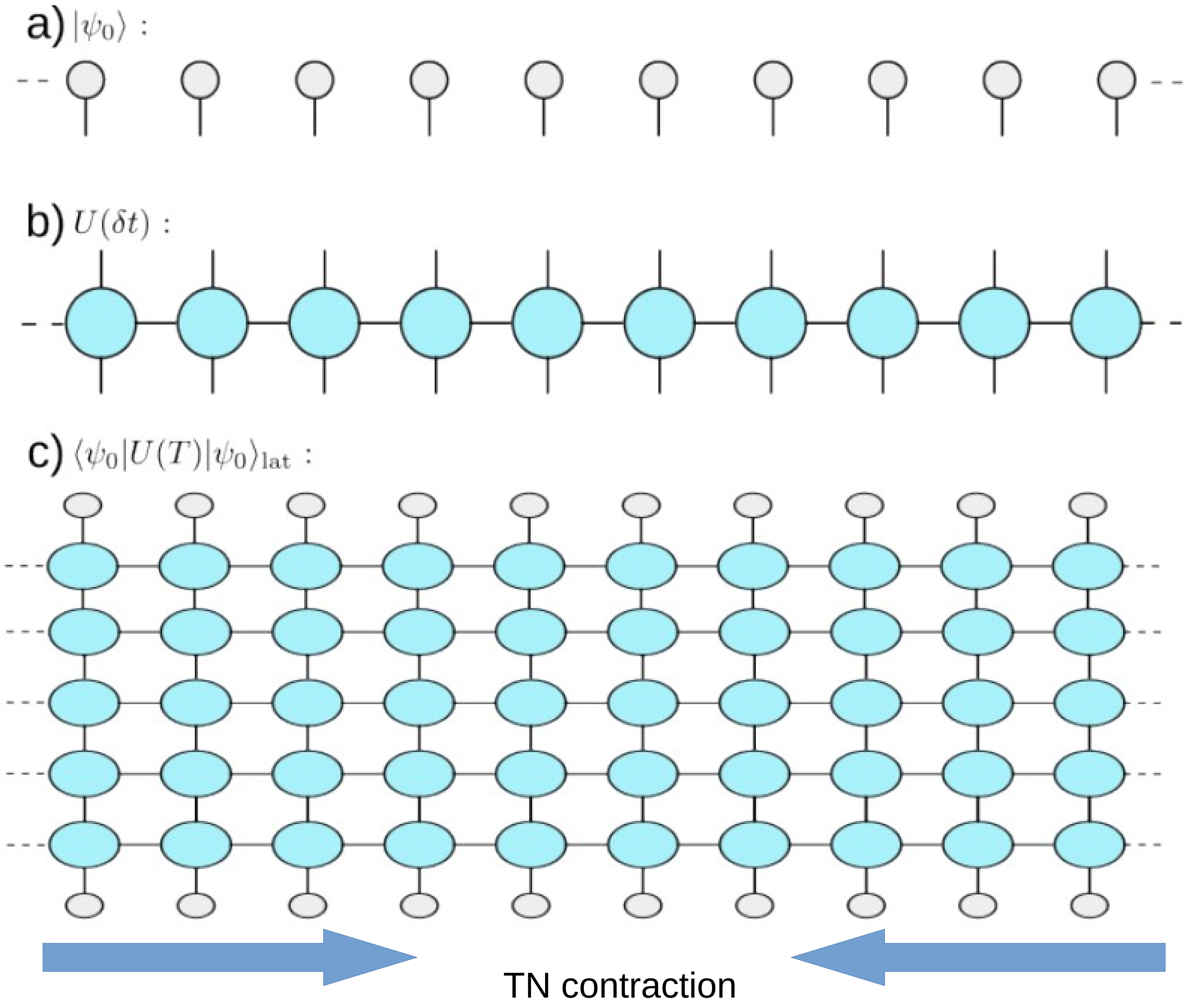
Loschmidt echos and generalized temporal entropies from CFT

with S. Carignano arXiv:2405.14706 (accepted in Quantum)

Luca Tagliacozzo, YITP 2025

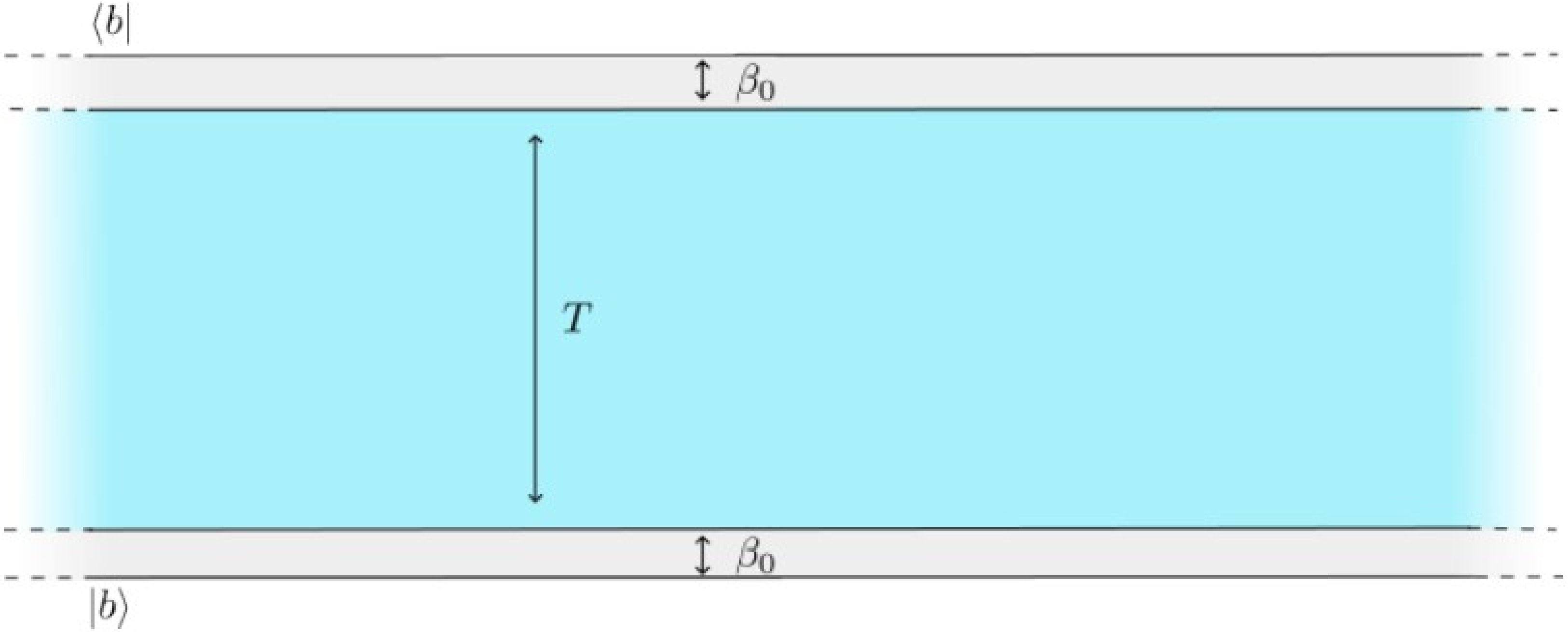
$$\mathcal{L} = | \langle \psi_0 | \, e^{-iHT} \, | \psi_0 \rangle |, \quad l = -\frac{\log \mathcal{L}}{TL}.$$

H is a critical spin model Hamiltonian, e.g. Ising, Potts



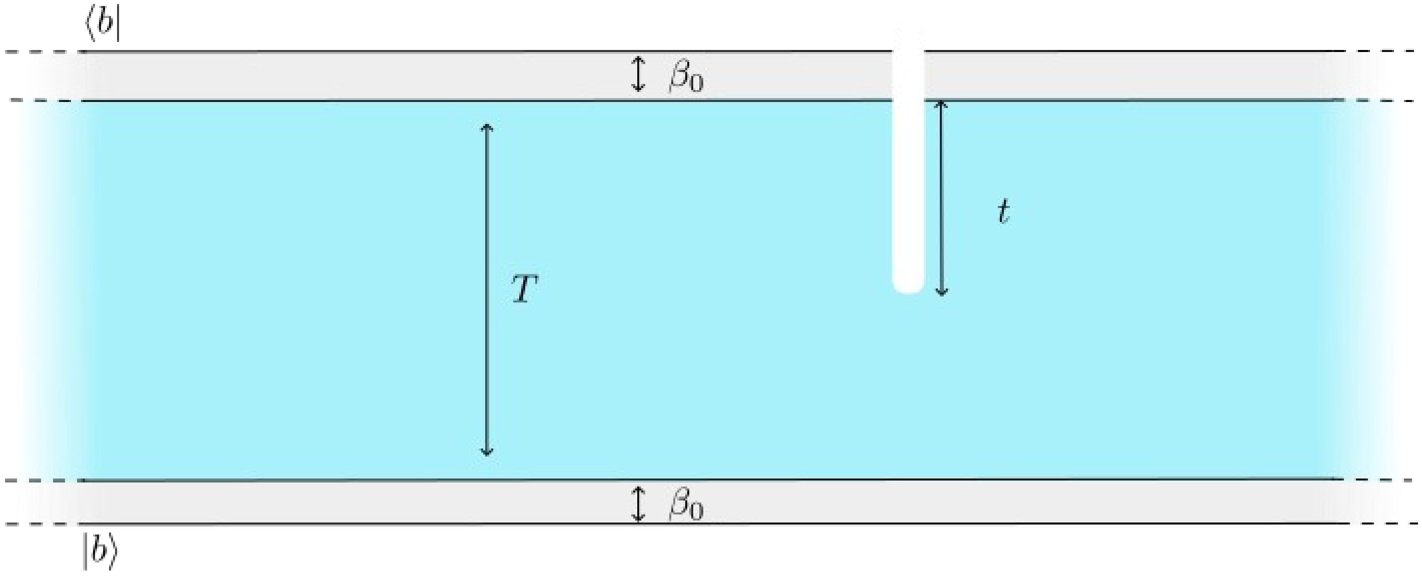
a) $|\psi_0\rangle$:
b) $U(\delta t)$:
c) $\langle\psi_0|U(T)|\psi_0\rangle_{\text{lat}}$:
TN contraction

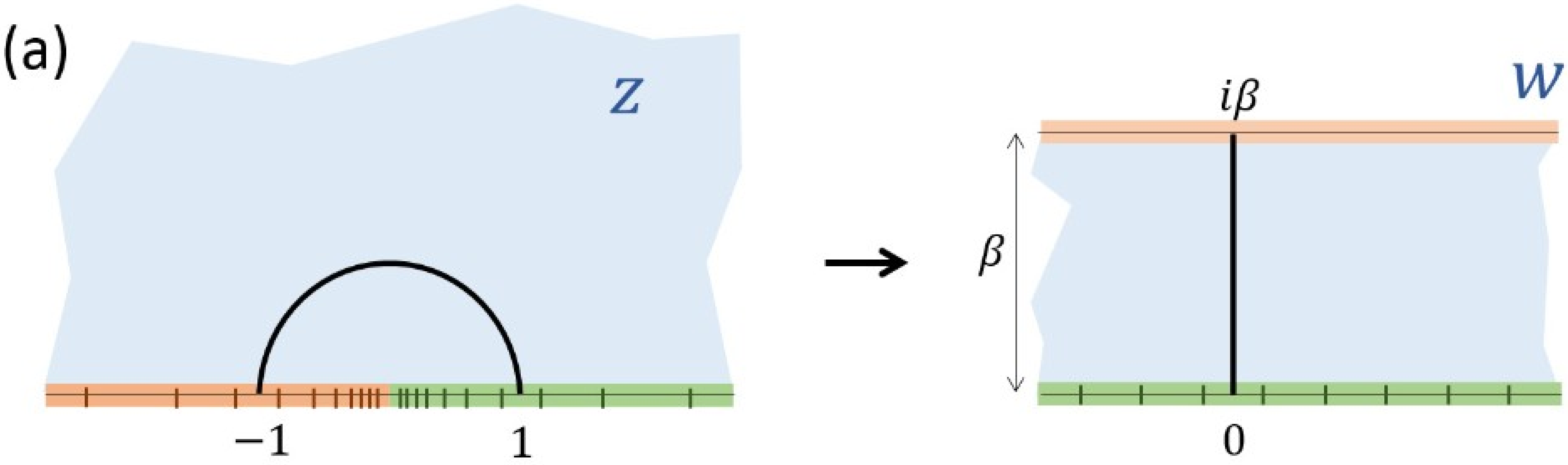
d) $\langle\psi_0|U(T)|\psi_0\rangle_{\mathrm{CFT}}$:



Temporal entropies

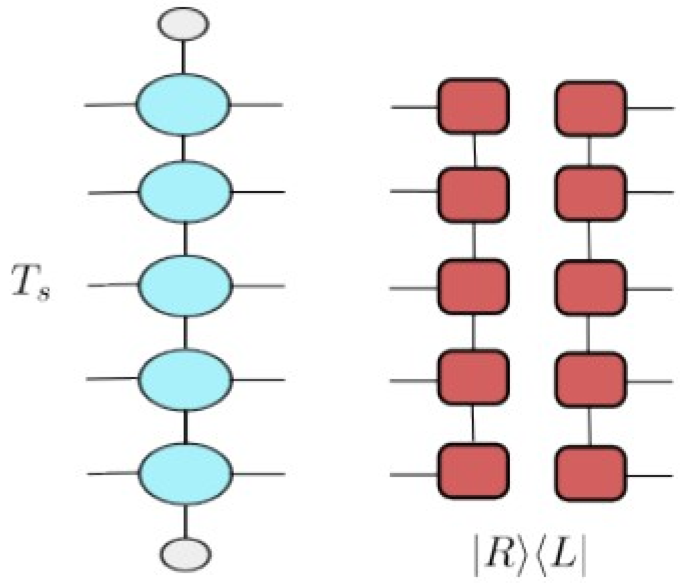
$$\mathcal{T}_t^{L|R} = \mathrm{tr}_{N_t - t}\left[\mathcal{T}^{L|R}\right] ,$$





$$\mathrm{w} = \beta/\pi \log(z)$$

Czech et al 2015

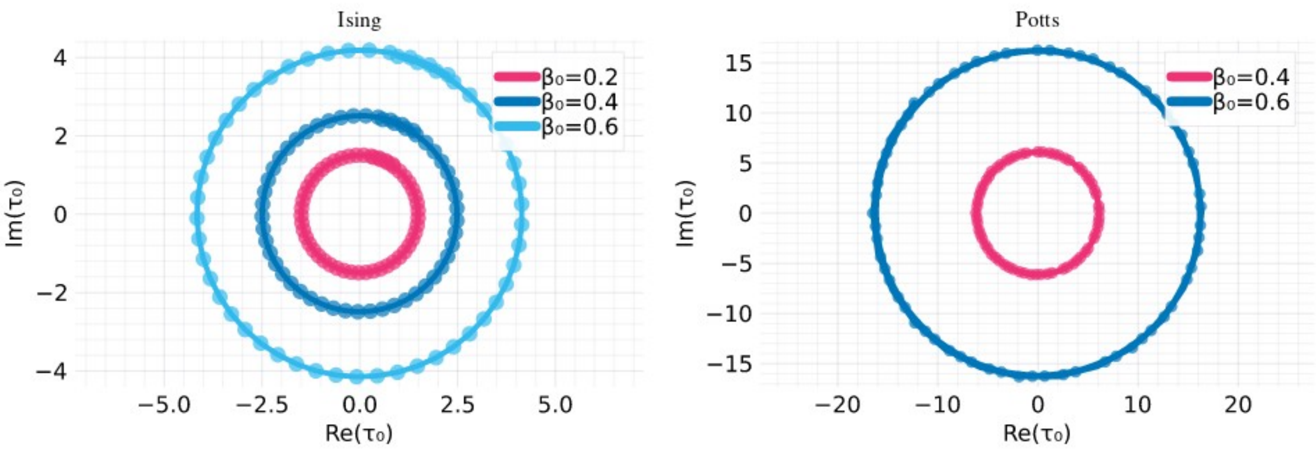


$$\mathrm{T}_s = \sum_i e^{\lambda_i} |R_i\rangle\langle L_i|$$

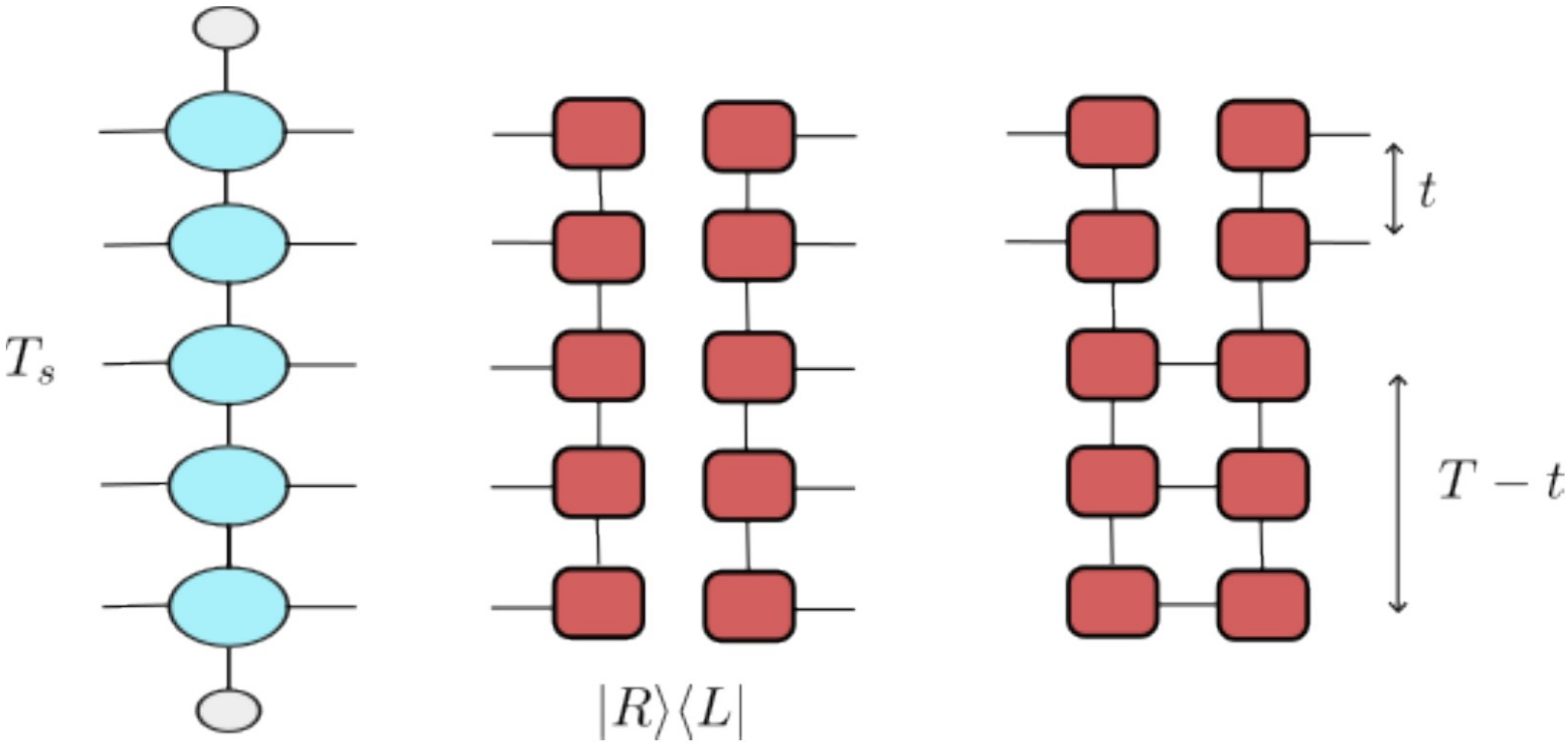
$$l \sim -\frac{|\lambda_0|}{T}$$

$$Re(\lambda_1 - \lambda_0) = \mathcal{O}\Big(\frac{1}{T^2}\Big)\,,$$

$$Im(\lambda_1 - \lambda_0) = -\frac{p\pi x_1}{vT} + \mathcal{O}\Big(\frac{1}{T^2}\Big)\,,$$



$$\mathcal{T}_t^{L|R} = \mathrm{tr}_{N_t - t}\left[\mathcal{T}^{L|R}\right], \quad \mathcal{T}^{L|R} = \frac{|R\rangle\langle L|}{\langle L|R\rangle},$$



$$\langle\phi(\omega_1)\phi(\omega_2)\rangle = \left(\frac{\pi}{p\beta v}\right)^{2x} \sin\left[\frac{\pi}{\beta v}(u_1 - u_2)\right]^{-2x},$$

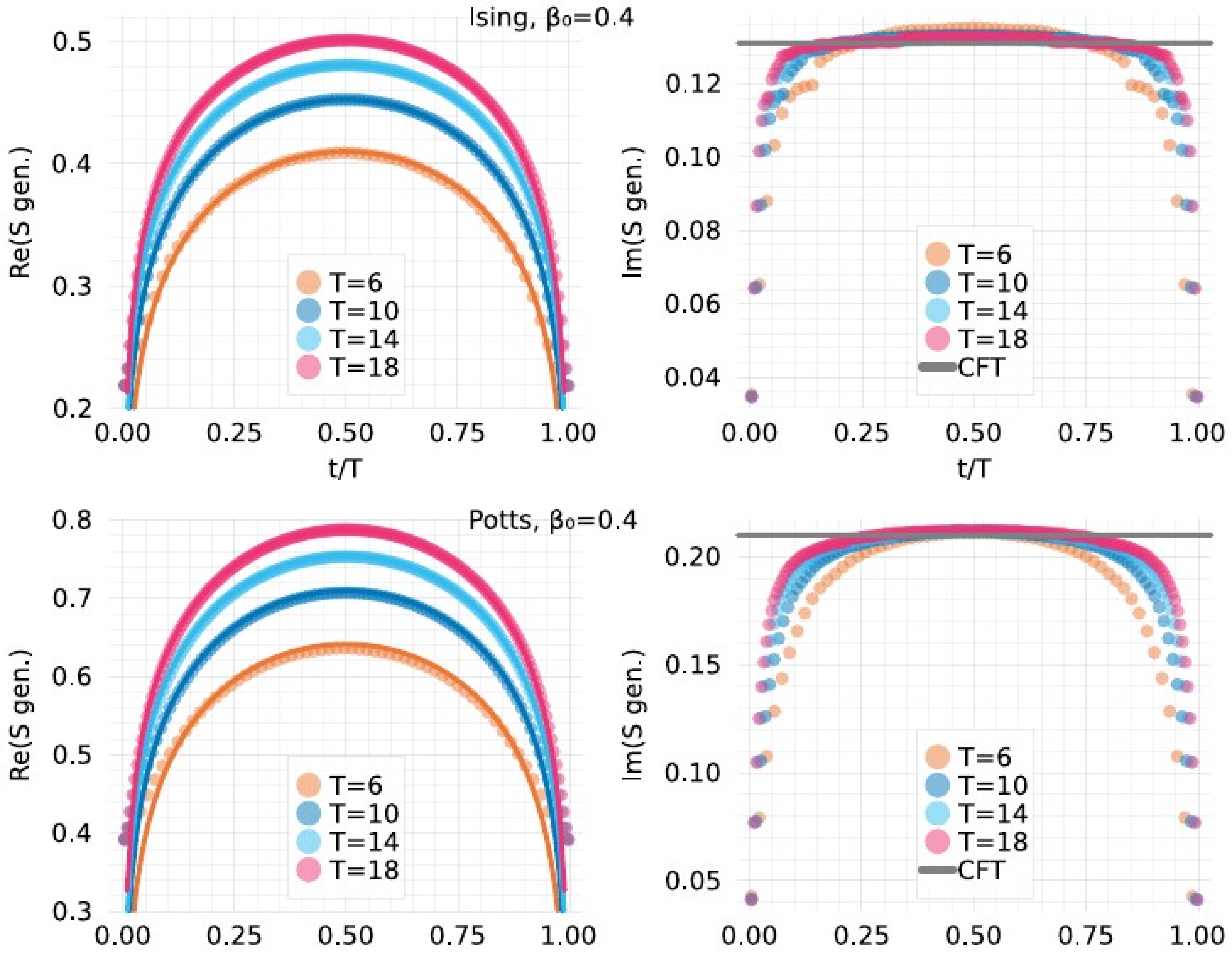
$$\mathrm{x} \to \Delta_n$$

$$\Delta_n = \frac{c}{24}(n - 1/n)$$

$$\beta \to iT \;, u_1 - u_2 \to it, n \to 1$$

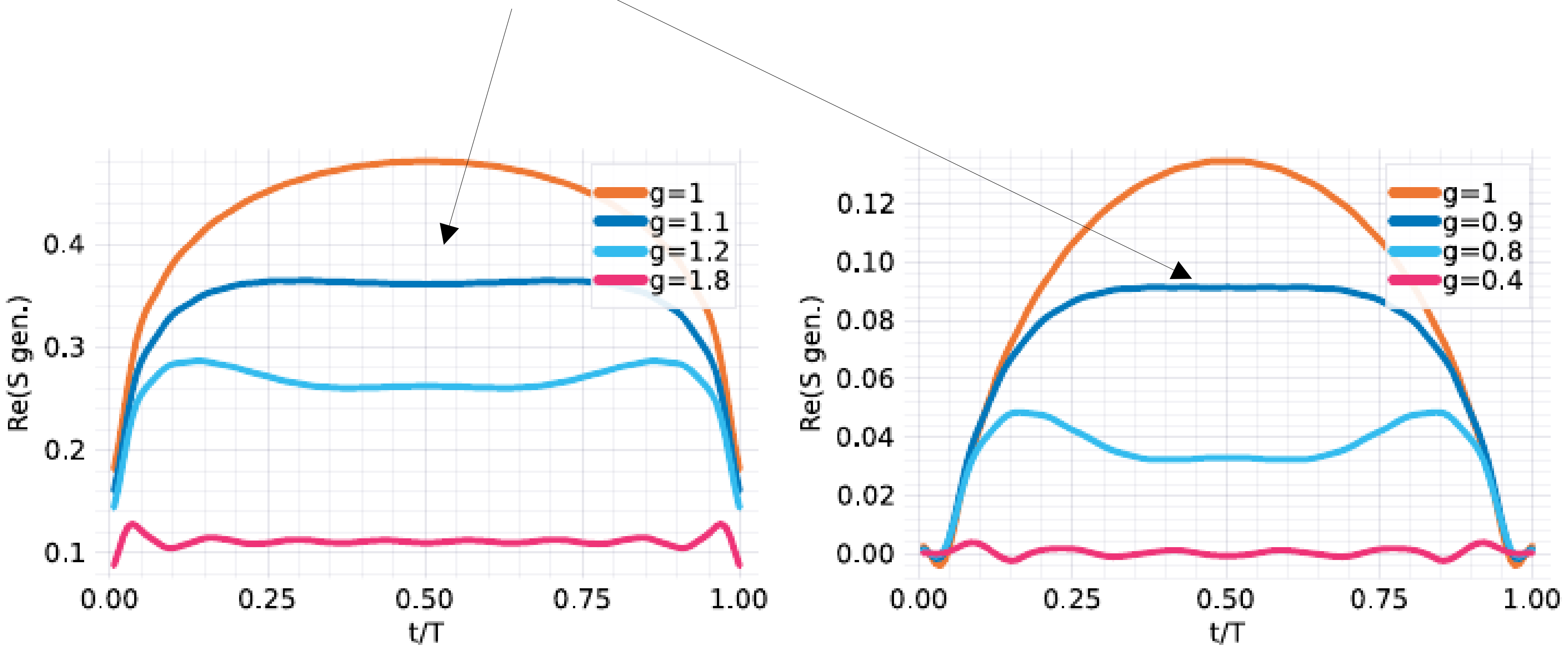
$$S_{gen} = s_0 + \frac{i\pi c}{12} + \frac{c}{6}\log\left[\frac{2T}{\pi}\sin\left(\frac{\pi t}{T}\right)\right],$$

K. Doi, J. Harper, A. Mollabashi, T. Takayanagi, and Y. Taki, JHEP. **2023**, 52 (2023)



$$S_{gen} = s_0 + \frac{i\pi c}{12} + \frac{c}{6}\log\left[\frac{2T}{\pi}\sin\left(\frac{\pi t}{T}\right)\right],$$

AREA LAW in the vicinity of the CFT



t/T t/T

- Generalized temporal entropies grow at most logarithmically → EASY temporal MPS simulations
- Loschmidt echos can be simulated for long time with polynomial resources

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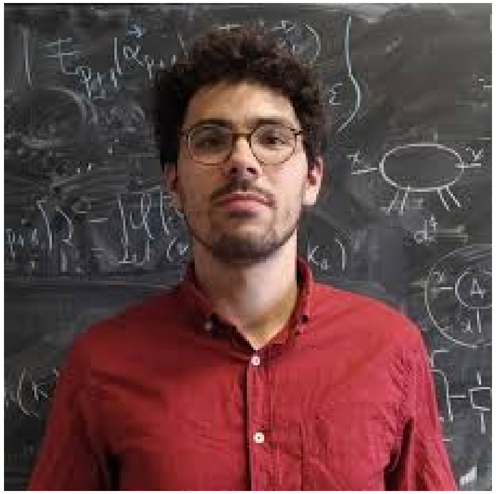
Moving away from the CFT

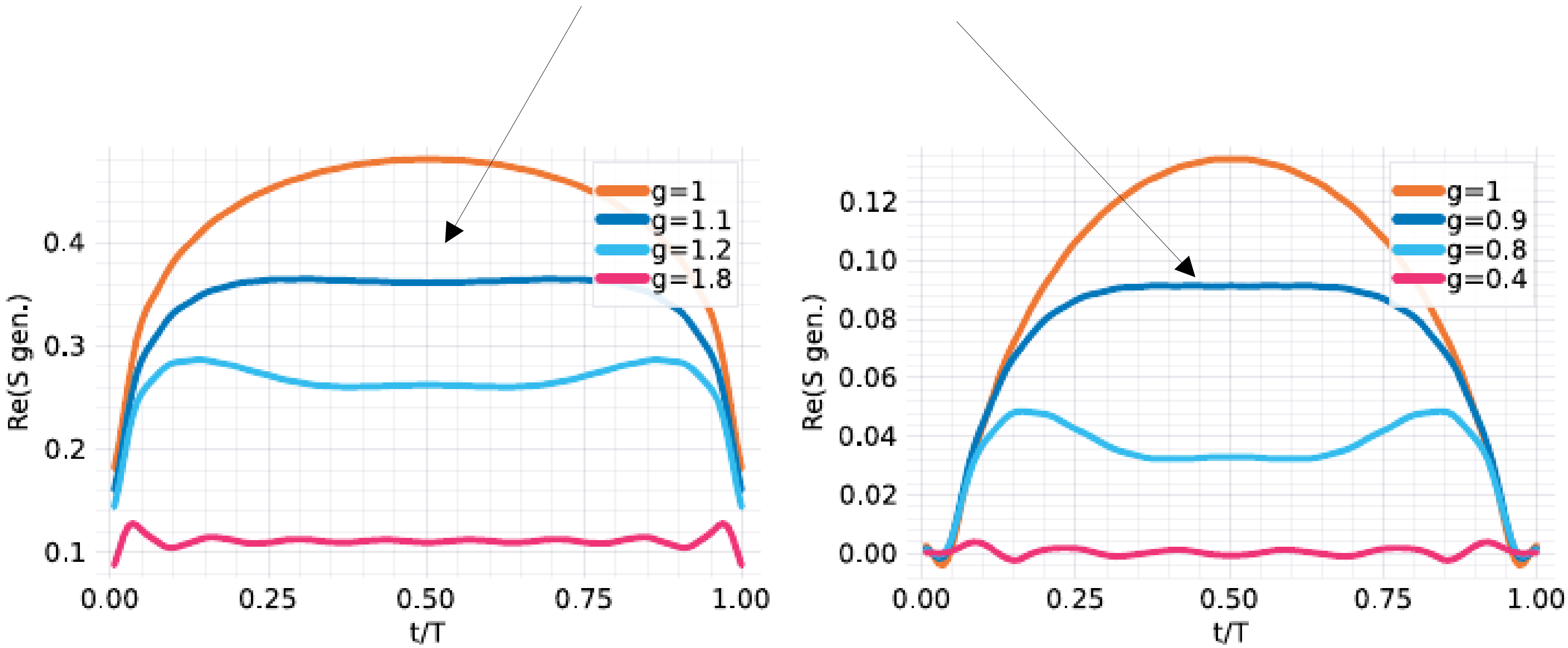
  

Stefano Carignano (BSC), Guglielmo Lami (CYU-FR),
Jacopo De Nardis (CYU-FR)

Dynamical phase diagrams, and the complexity of spatio-temporal tensor networks

arXiv:2505.09714

AREA LAW in the vicinity of the CFT



t/T t/T

- We consider the Euclidean CFT
- and analytically continue the results to real time,
- can we always do this?

DQPT

$$Z(z) = \langle \Psi_i | e^{-zH} | \Psi_i \rangle$$

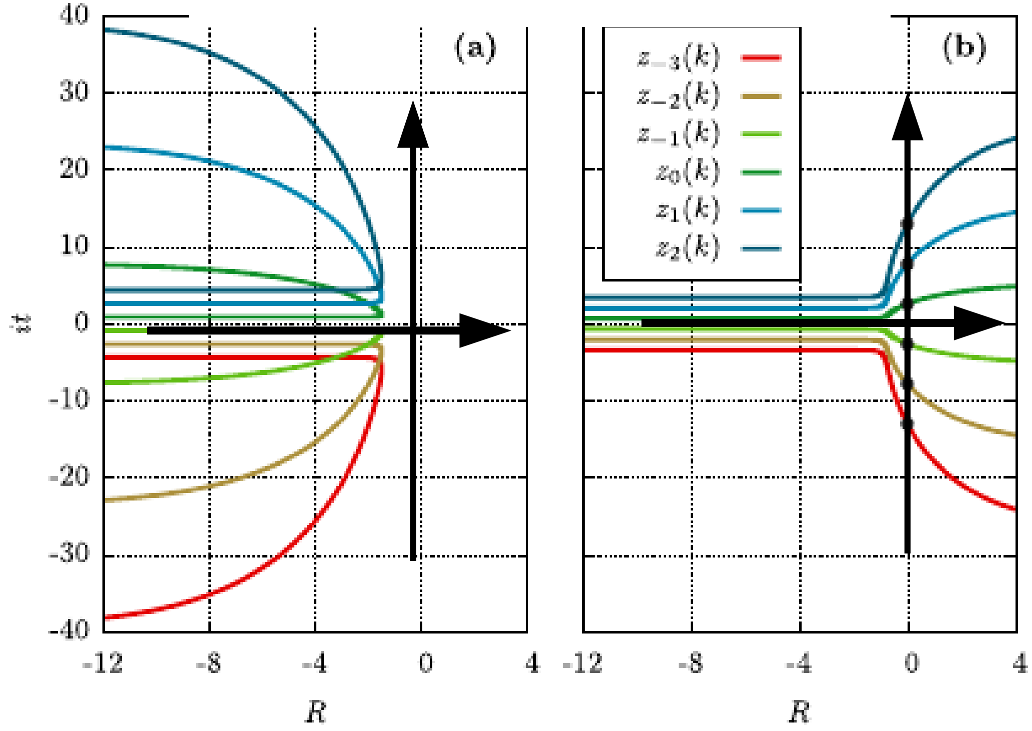
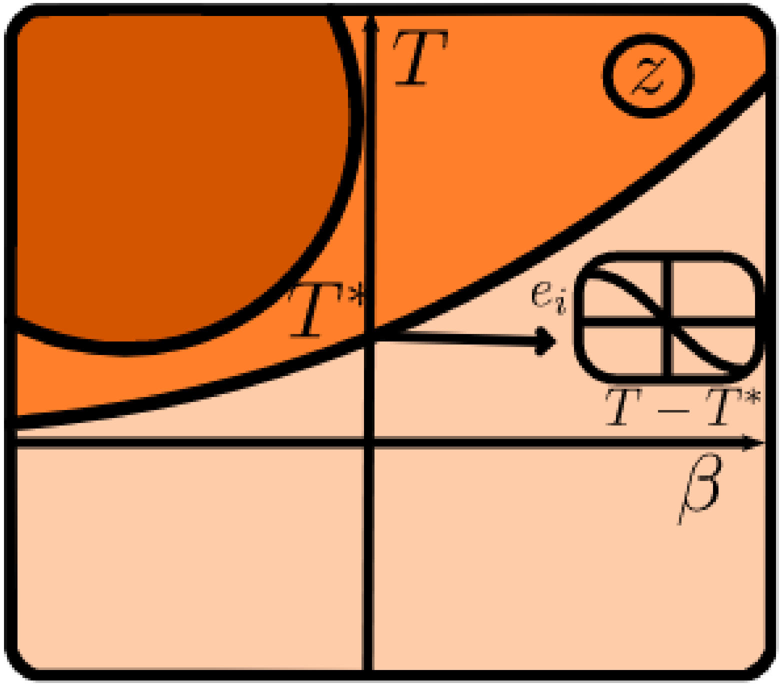


FIG. 1 (color online). Lines of Fisher zeros for a quench within the same phase $g_0 = 0.4 \to g_1 = 0.8$ (left) and across the quantum critical point $g_0 = 0.4 \to g_1 = 1.3$ (right). Notice that the

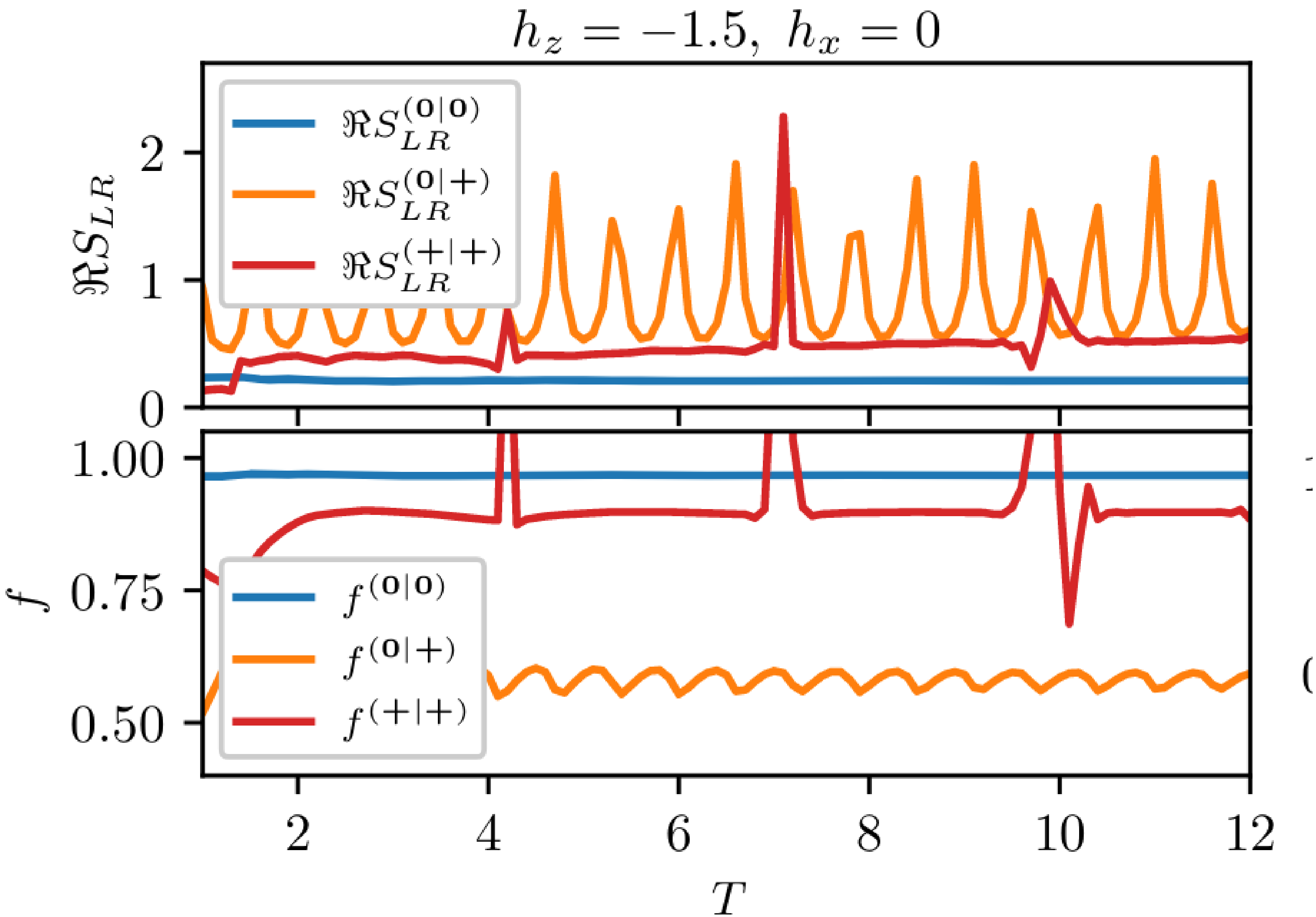
M. Heyl, A. Polkovnikov, and S. Kehrein, Phys. Rev. Lett. **110**, 135704 (2013)

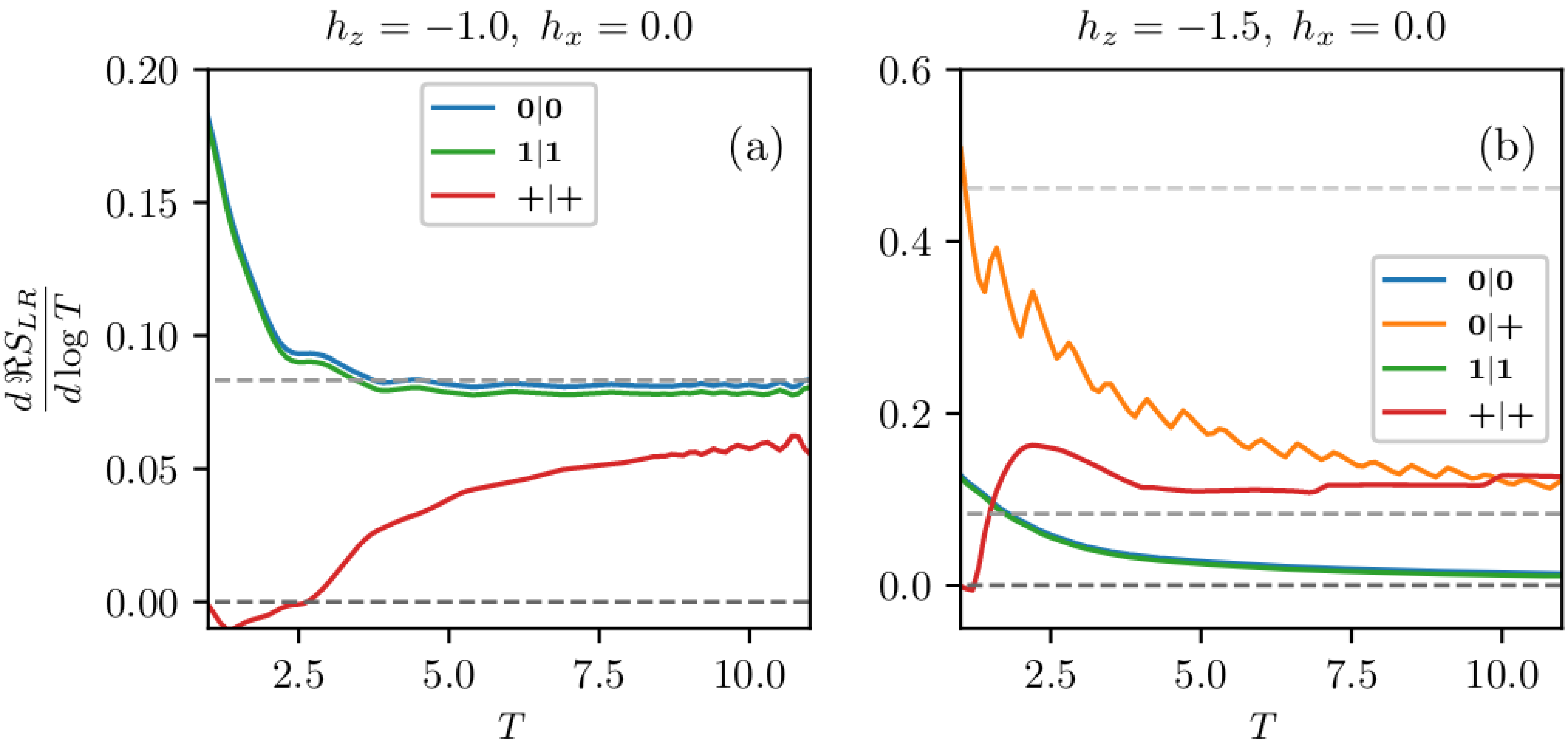
$$\hat{H} = \sum_j \left(X_j X_{j+1} + h_z Z_j + h_x X_j \right),$$

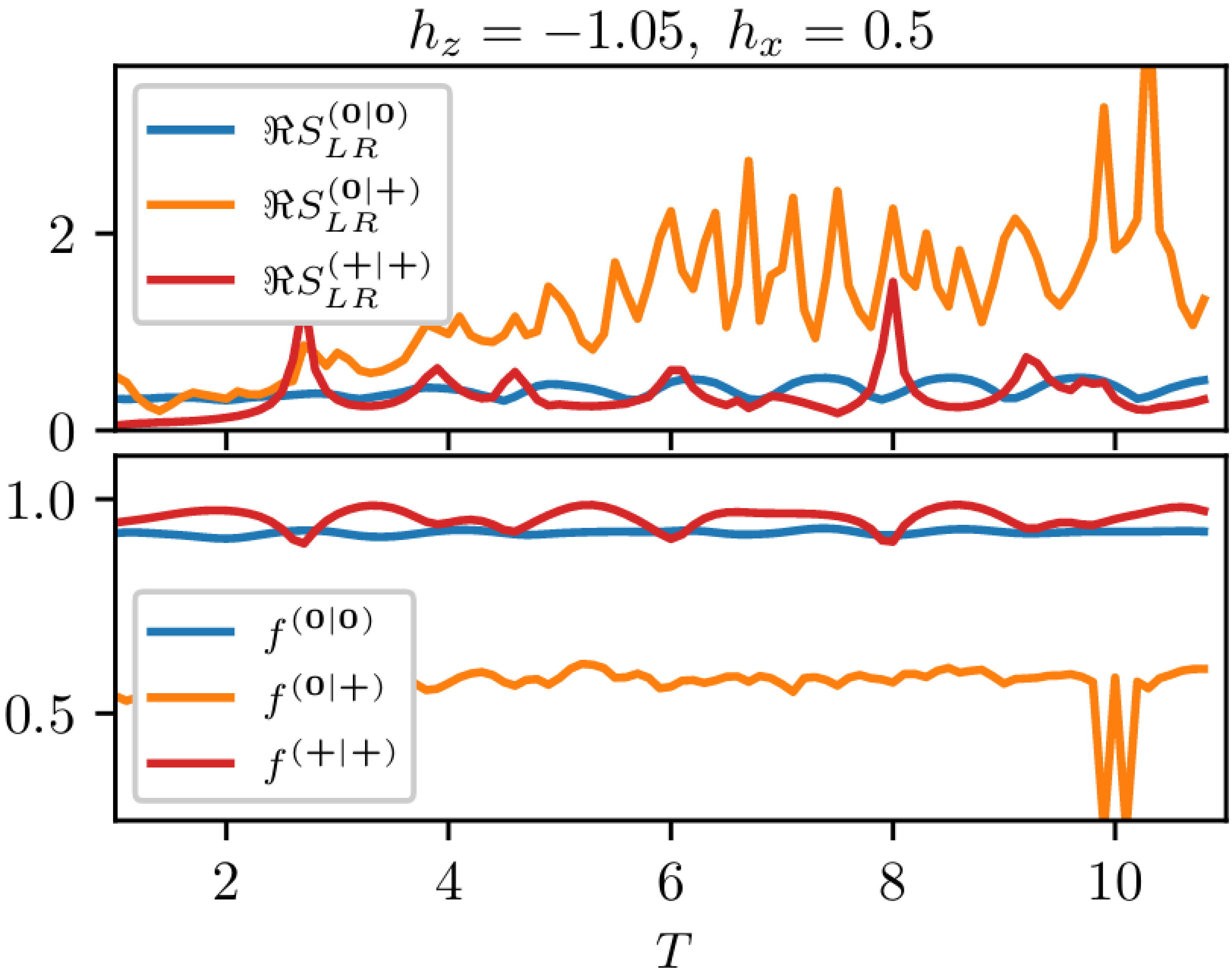
$$f^{(\boldsymbol{x}|\boldsymbol{y})}(T) = \frac{1}{NT} \log \left| \langle \boldsymbol{x} | \mathrm{e}^{-i\hat{H}T} | \boldsymbol{y} \rangle \right|, \qquad \boldsymbol{x}, \boldsymbol{y} \in \{ \mathbf{0}, \mathbf{1}, \mathbf{+} \},$$

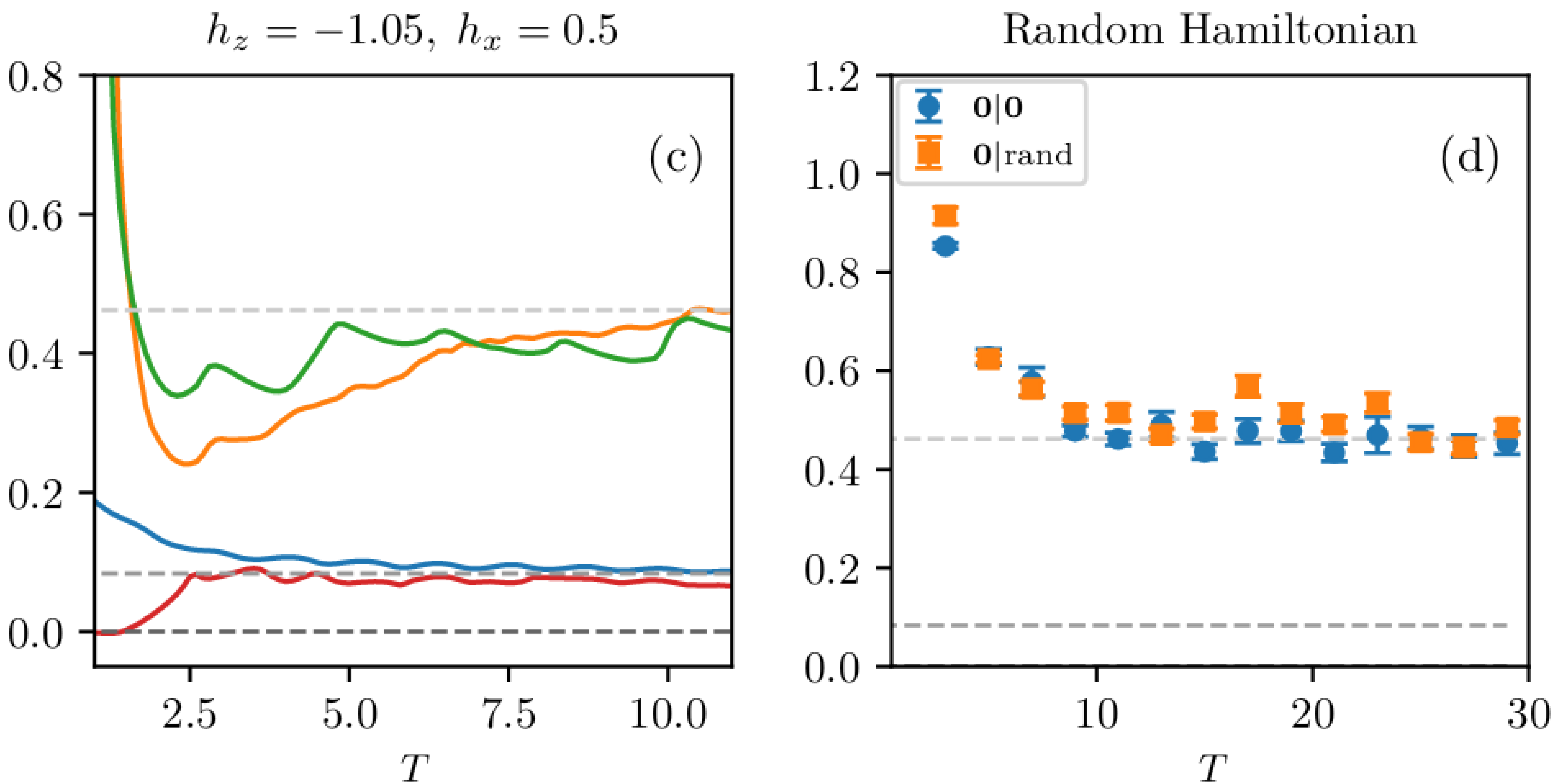


$$f^{(\boldsymbol{x}|\boldsymbol{y})}(T) = \frac{1}{NT} \log\left|\langle \boldsymbol{x}|\mathrm{e}^{-i\hat{H}T}|\boldsymbol{y}\rangle\right|, \qquad \boldsymbol{x},\boldsymbol{y} \in \{\mathbf{0},\mathbf{1},+\},$$




$h_z = -1.0,\ h_x = 0.0$
$h_z = -1.5,\ h_x = 0.0$
$\frac{d\,\Re S_{LR}}{d \log T}$
0|0
1|1
+|+
0|+
(a)
(b)
0.20
0.15
0.10
0.05
0.00
0.6
0.4
0.2
0.0
2.5
5.0
7.5
10.0
T


$h_z = -1.05,\ h_x = 0.5$
$\Re S_{LR}^{(\mathbf{o}|\mathbf{o})}$
$\Re S_{LR}^{(\mathbf{o}|+)}$
$\Re S_{LR}^{(+|+)}$
$f^{(\mathbf{o}|\mathbf{o})}$
$f^{(\mathbf{o}|+)}$
$f^{(+|+)}$
2
0
1.0
0.5
2
4
6
8
10
T


$h_z = -1.05,\ h_x = 0.5$
0.8
0.6
0.4
0.2
0.0
(c)
2.5
5.0
7.5
10.0
T
Random Hamiltonian
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\mathbf{0}|\mathbf{0}$
$\mathbf{0}|$rand
(d)
10
20
30
T

- Generalized temporal entropies grow at most logarithmically → even away from the CFT, DQPT are not transitions in complexity, just the locus of the failure of the analytic continuation.
- Loschmidt echos can be simulated for long time with polynomial computational resources

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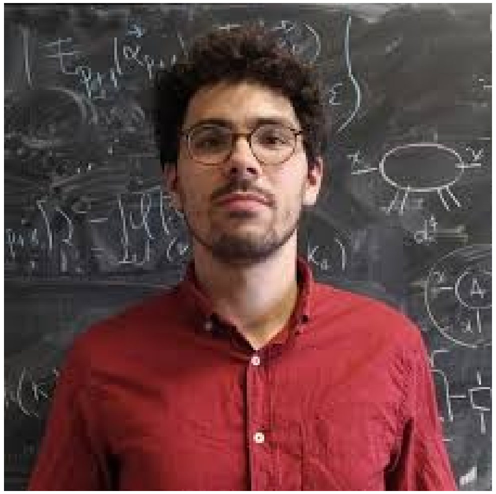
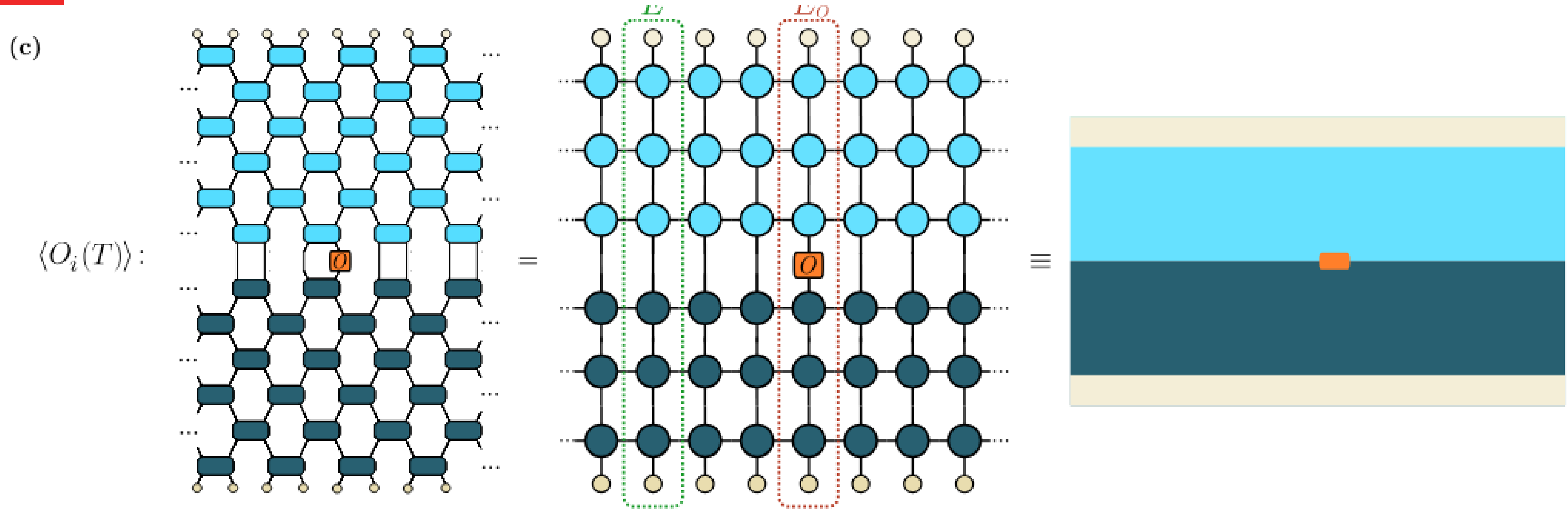
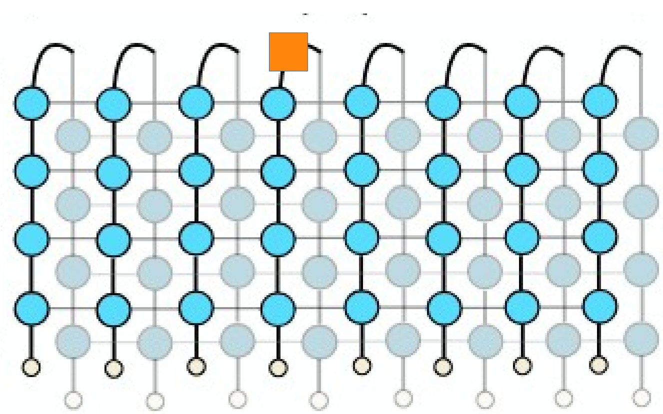
  

Stefano Carignano (BSC), Guglielmo Lami (CYU-FR),
Jacopo De Nardis (CYU-FR)

Overcoming the entanglement barrier, sampling spatio-temporal tensor networks

arXiv:2505.09714


(c)
$\langle O_i(T) \rangle$:
O
=
E
E_O
O
≡



$$\langle O \rangle = \langle \psi(T) | \hat{O} | \psi(T) \rangle.$$

$$\langle O \rangle = \sum_{\boldsymbol{x}} p(\boldsymbol{x})\, O_{\mathrm{loc}}(\boldsymbol{x}), \qquad p(\boldsymbol{x}; T) = \left| \langle \boldsymbol{x} | \psi(T) \rangle \right|^2,$$

where

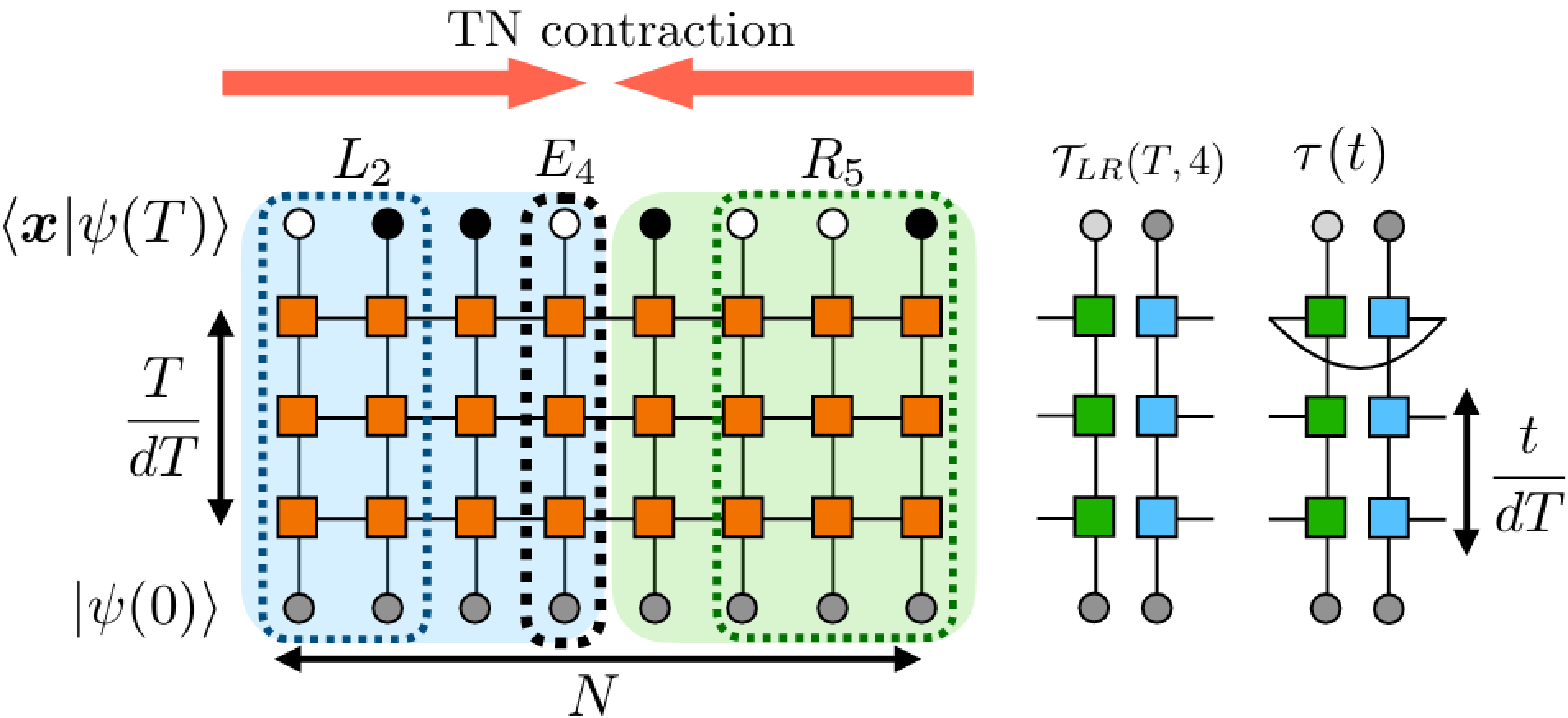
$$O_{\mathrm{loc}}(\boldsymbol{x}) = \sum_{\boldsymbol{x}'} \frac{\langle \boldsymbol{x}' | \psi(T) \rangle}{\langle \boldsymbol{x} | \psi(T) \rangle} \, \langle \boldsymbol{x} | \hat{O} | \boldsymbol{x}' \rangle \, ,$$

Markov chain Monte-Carlo

$$p(\boldsymbol{x};T) = \left|\langle \boldsymbol{x}|\psi(T)\rangle\right|^2,$$

$$\boldsymbol{x}^{\,m} \to \boldsymbol{x}^{m+1}$$

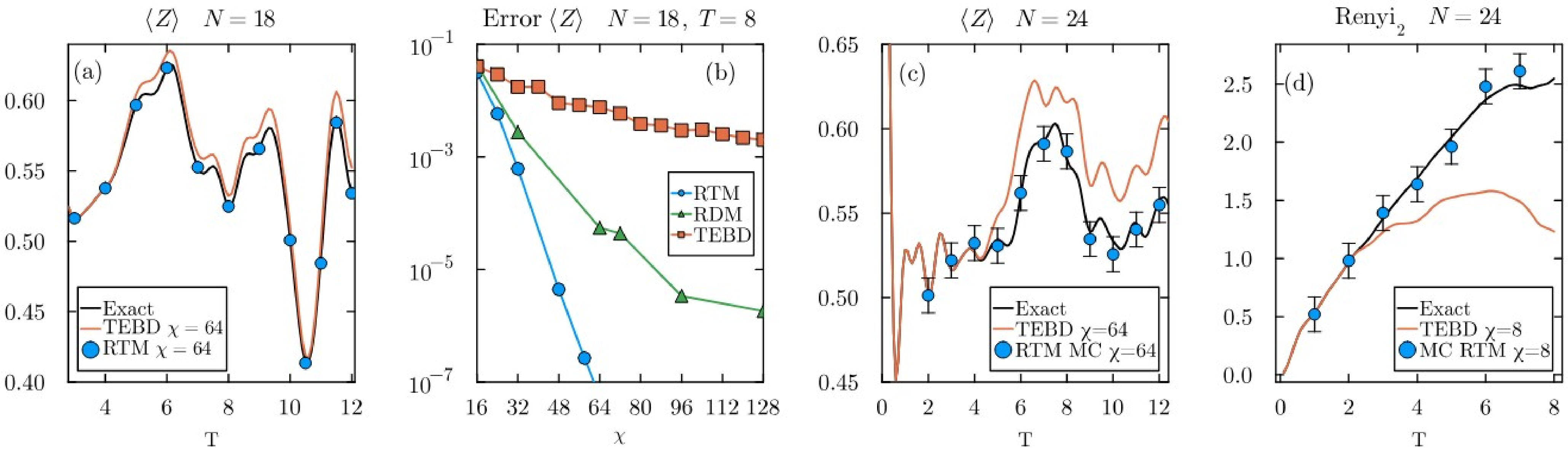
$$p_{\mathrm{acc}} = \min\Big(1,\, p(\boldsymbol{x}^{(m+1)};T)/p(\boldsymbol{x}^{(m)};T)\Big).$$



Row 1 Row 2 Row 3 Row 4

Numerical results

$$\hat{H} = \sum_j \left(X_j X_{j+1} + h_z Z_j + h_x X_j \right),$$



$$h_z = -1.05 \text{ and } h_x = 0.50.$$

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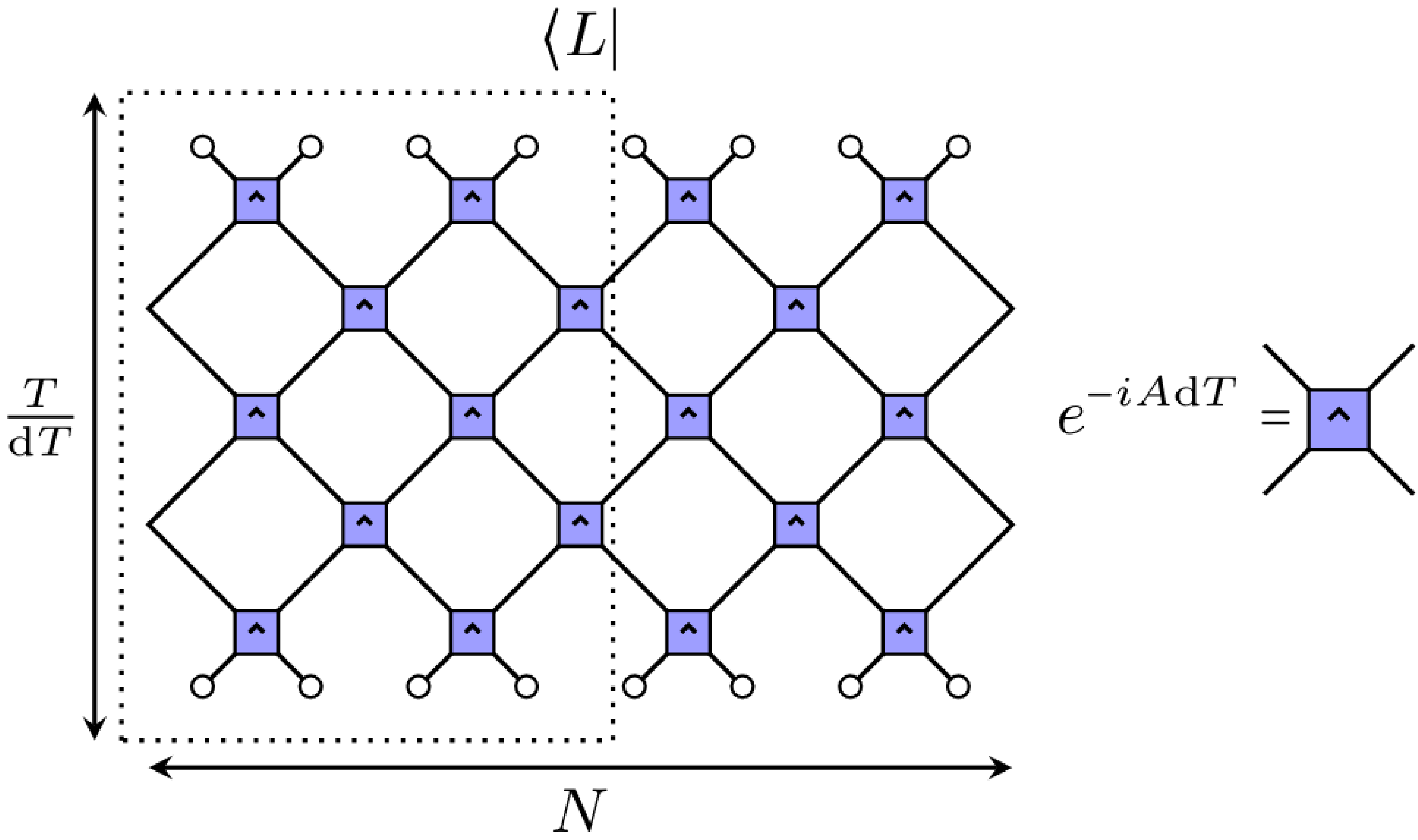
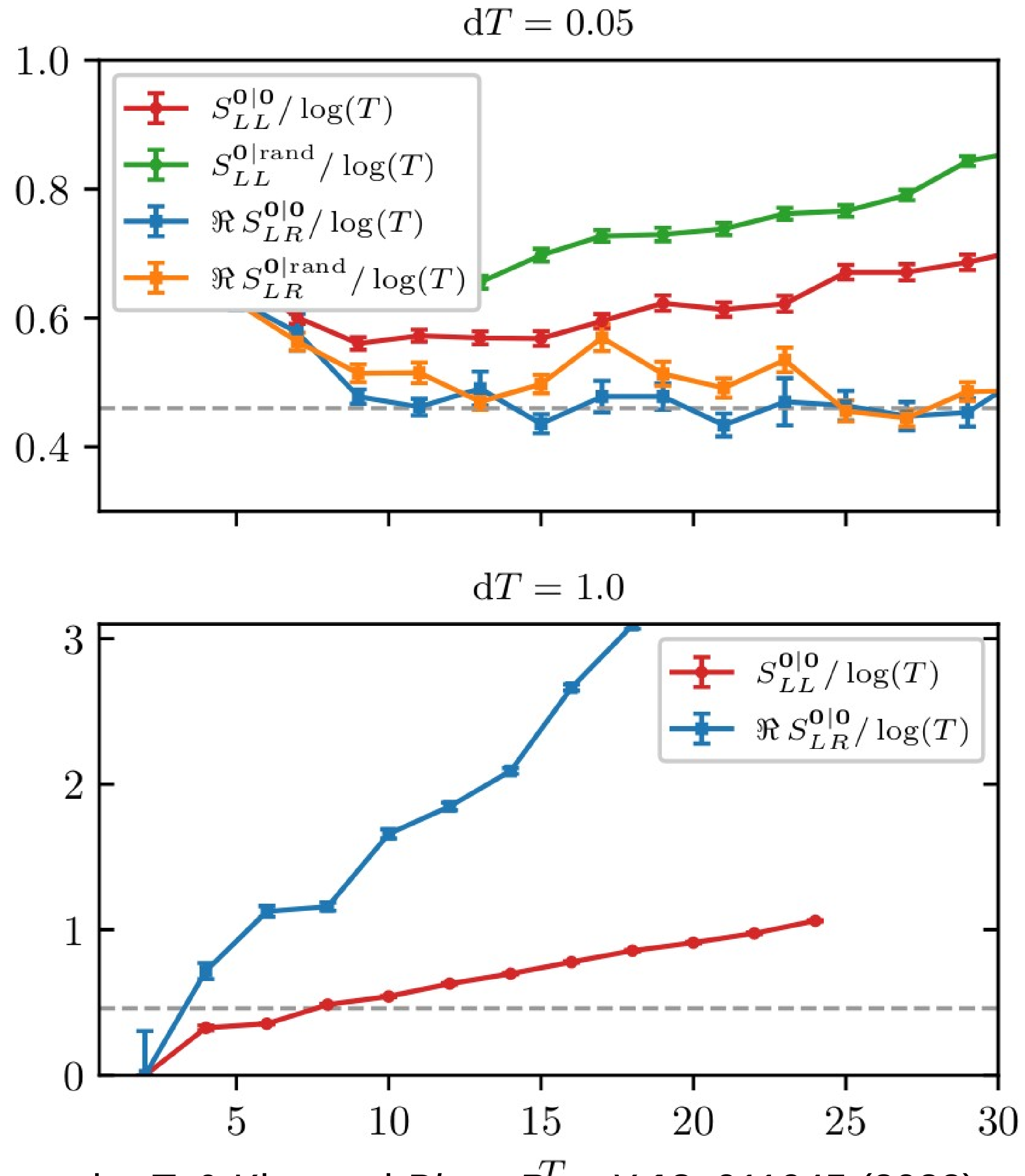
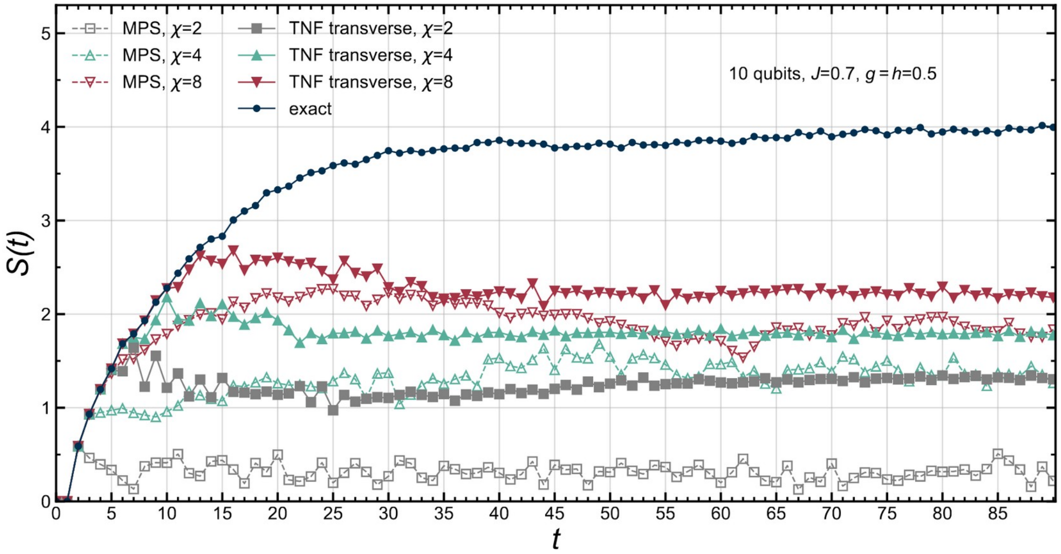
Comments



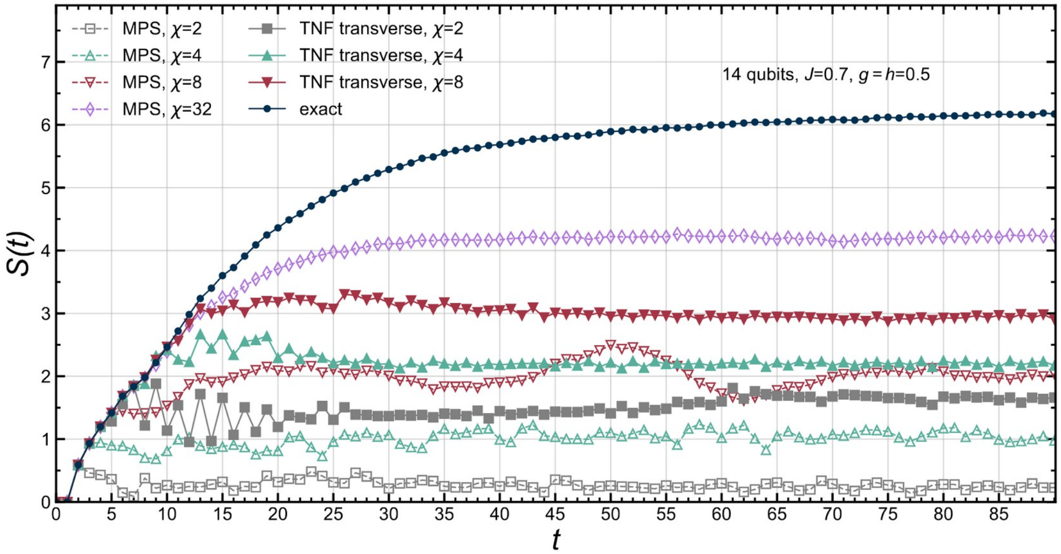
with A a generic GUE matrix



Ippoliti, M., Rakovszky, T. & Khemani *Phys. Rev. X* **12**, 011045 (2022).
Lu, T.-C. & Grover *PRX Quantum* **2**, 040319 (2021).



(a)



(b)

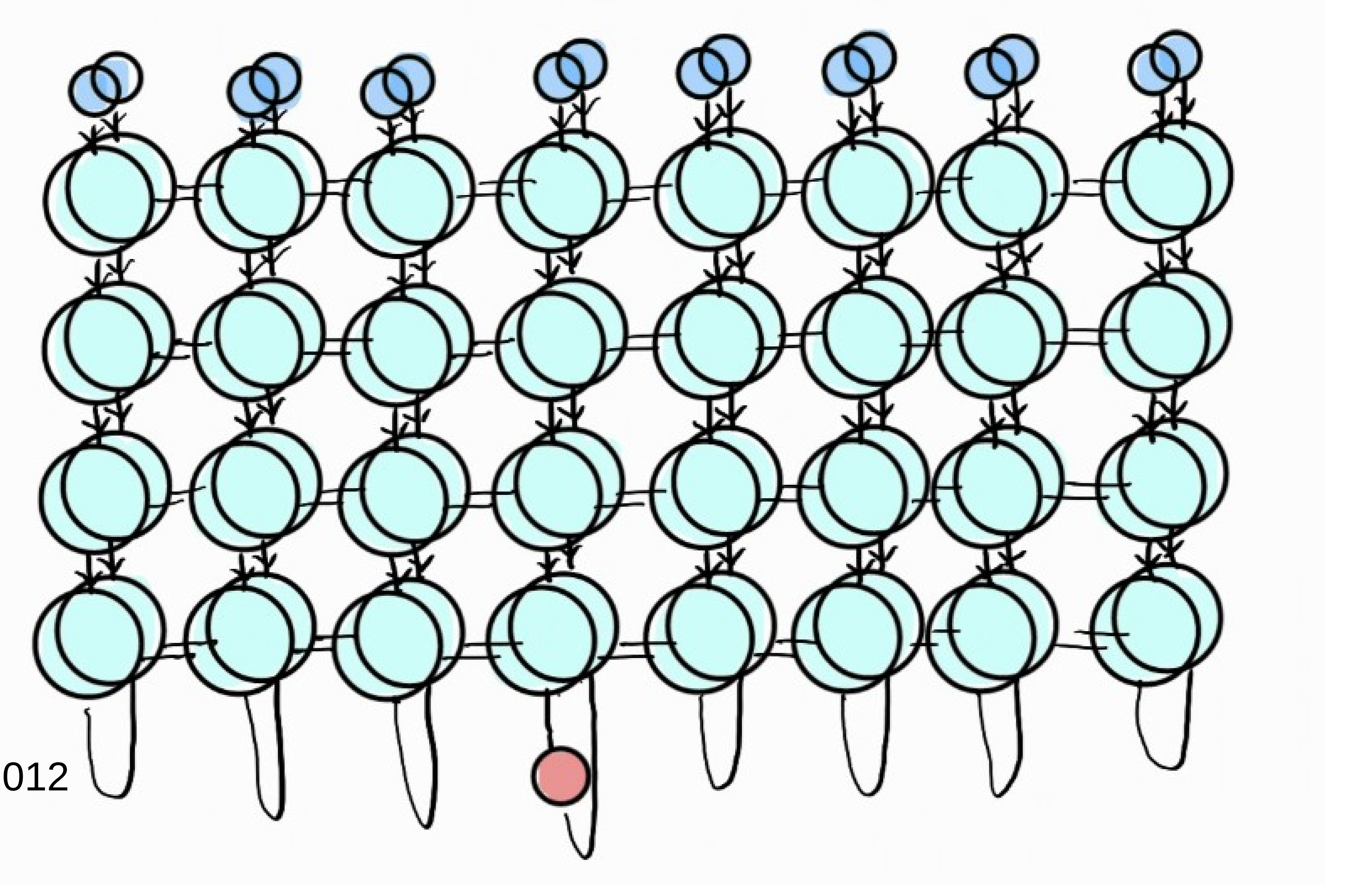
Liu, W.-Y., Du, S.-J., Peng, R., Gray, J. & Chan, G. K.-L. *Phys. Rev. Lett.* **133**, 260404 (2024)

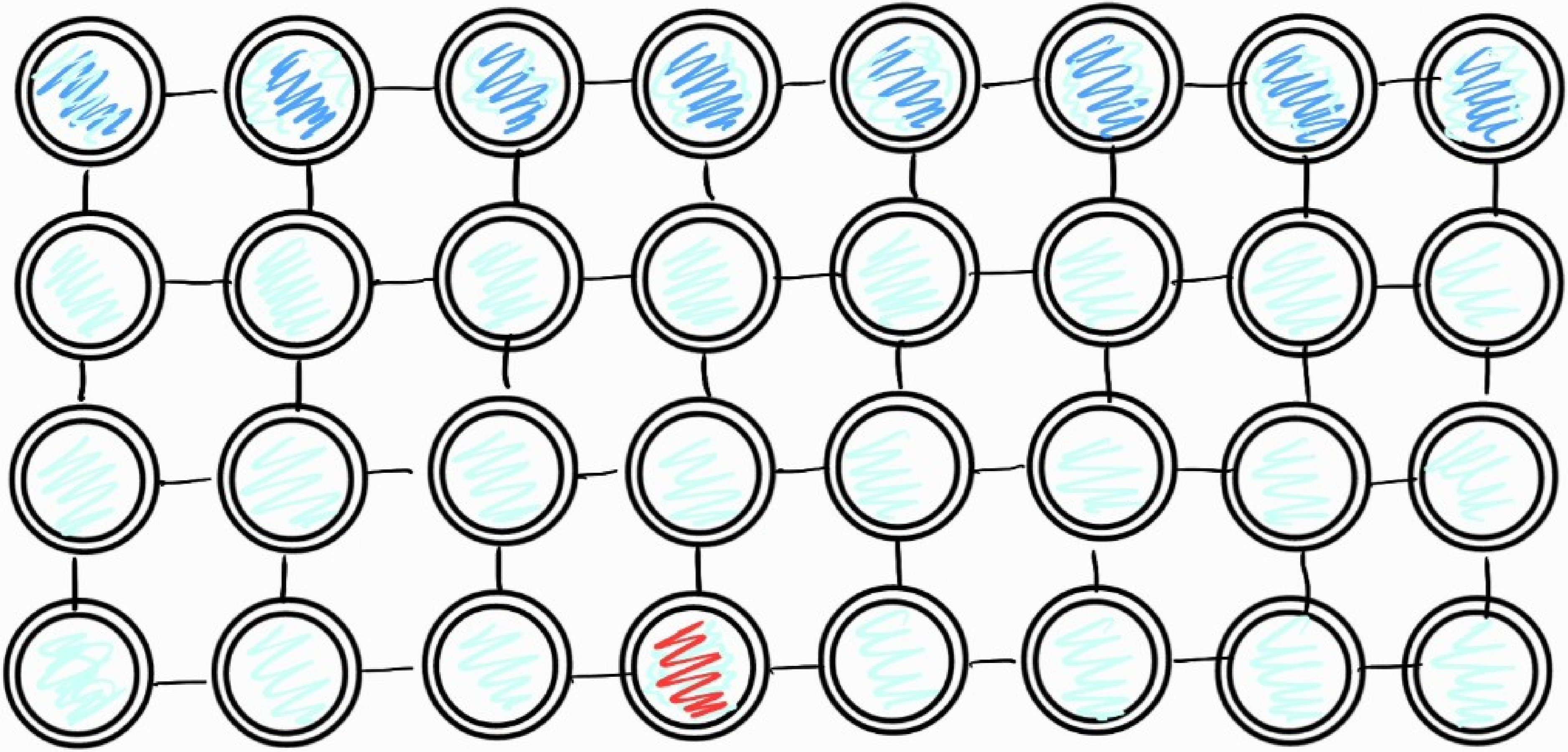
Upper bounding the rank of the reduced transition matrices

with S. Carignano C. Ramos arXiv:2307.11649

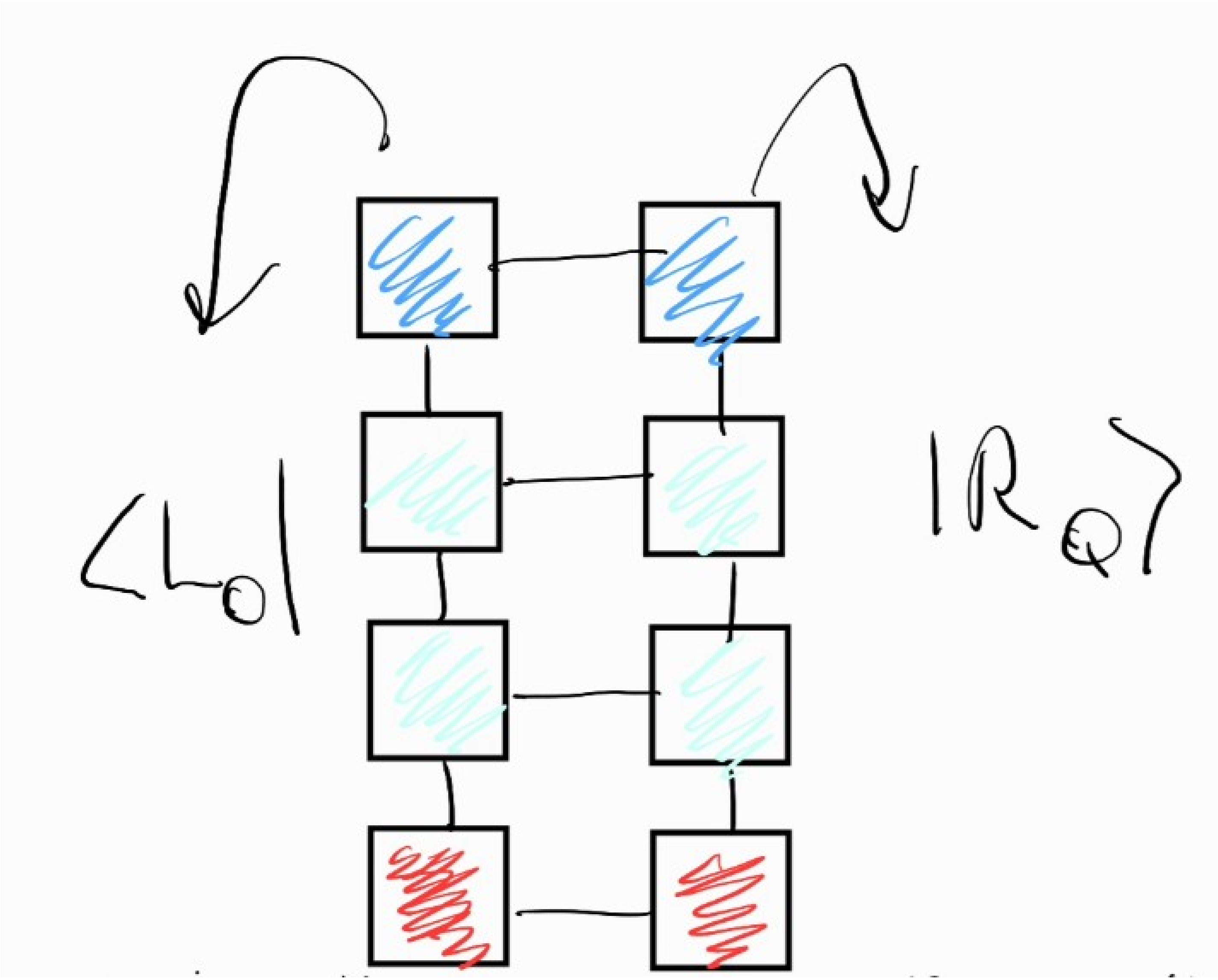
Luca Tagliacozzo, YITP 2025



Bañuls et al. 2009
Muller Hermes et al 2012
Hastings et al. 2015
Tirrito et al (LT) 2018
Lerose et al 2021
Giudice et al 2021….

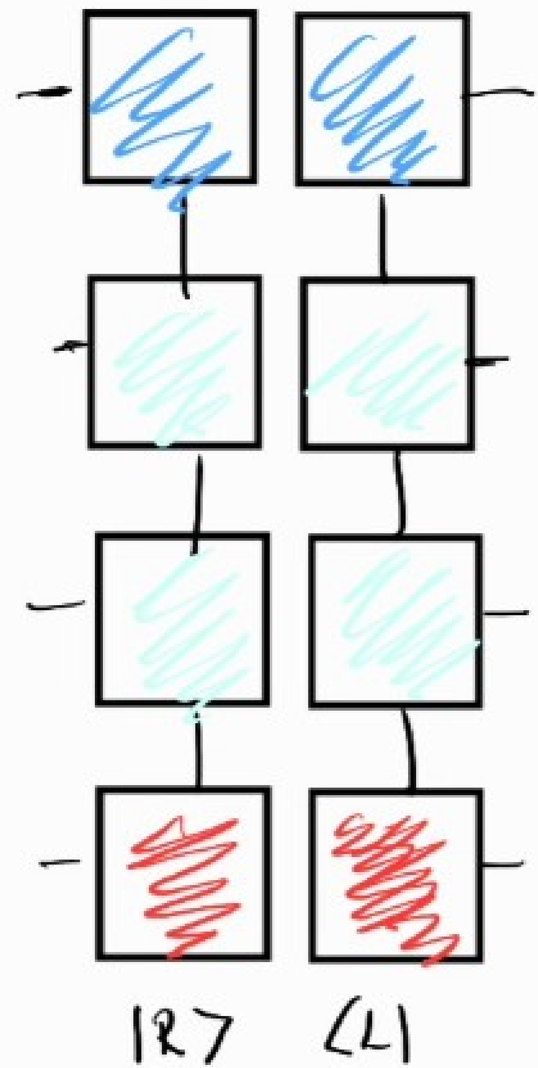


$$\langle \psi_0 | U^\dagger(t) O_i U(t) | \psi_0 \rangle$$

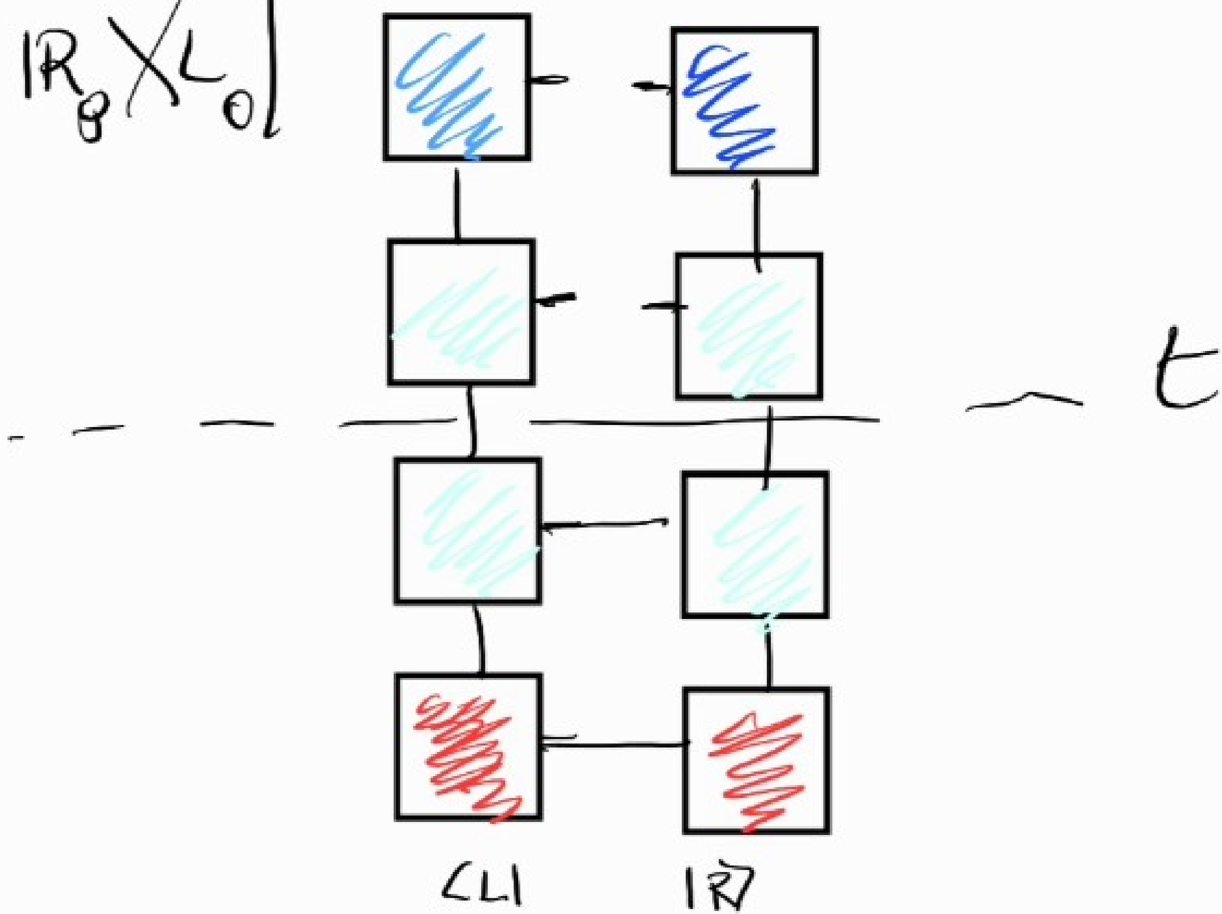


temporal MPS, tMPS

$Z_{\mathbb{R}_{p_{itt}}} X L_{oc} | =$



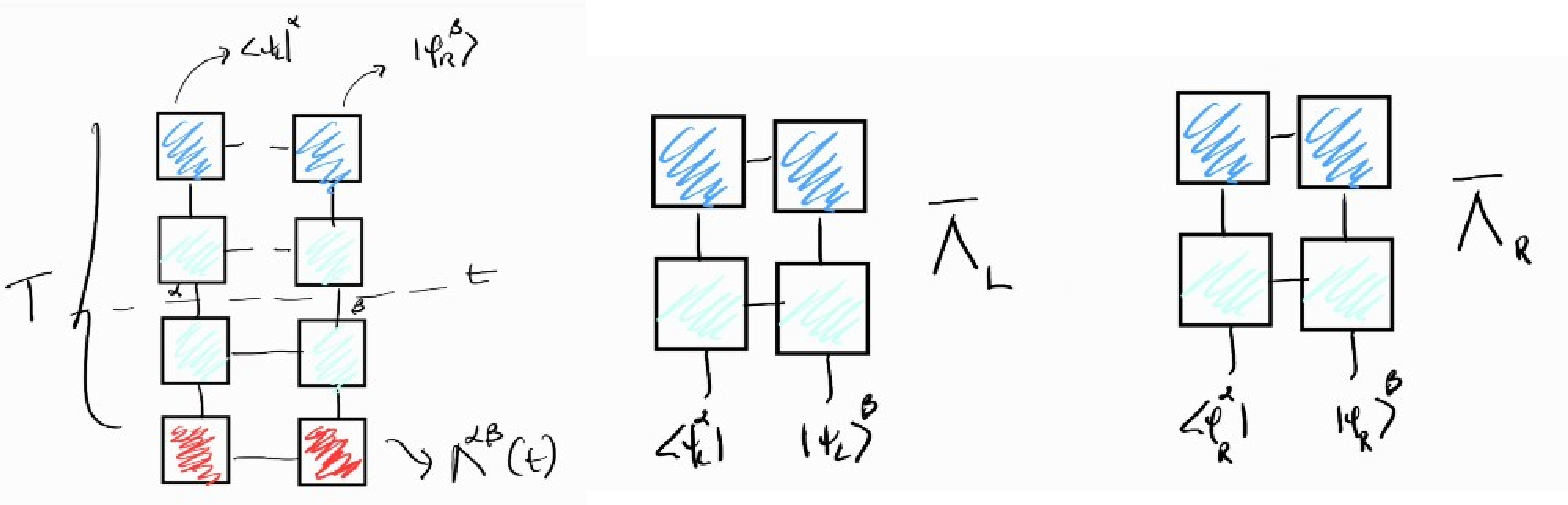
$$\tau^t = \mathrm{tr}_{(T-t)}\, \tau\, |R_0\rangle\langle L_0|$$



Simulation cost is dictated by
Cost of low rank approximation
of

$$\mathcal{T}^t \qquad \forall t \in \{0...T\}$$

Since operators are included,
we need the consider the worst
case scenario

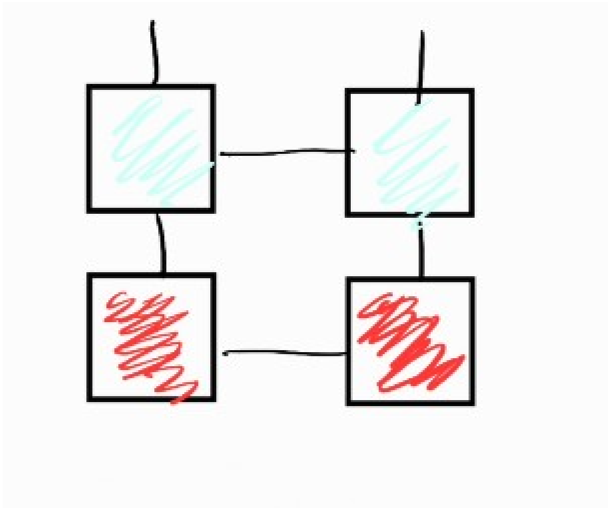


$$\mathcal{T}^t \simeq \sqrt{\bar{\Lambda}_L}\Lambda_t\sqrt{\bar{\Lambda}_R}$$

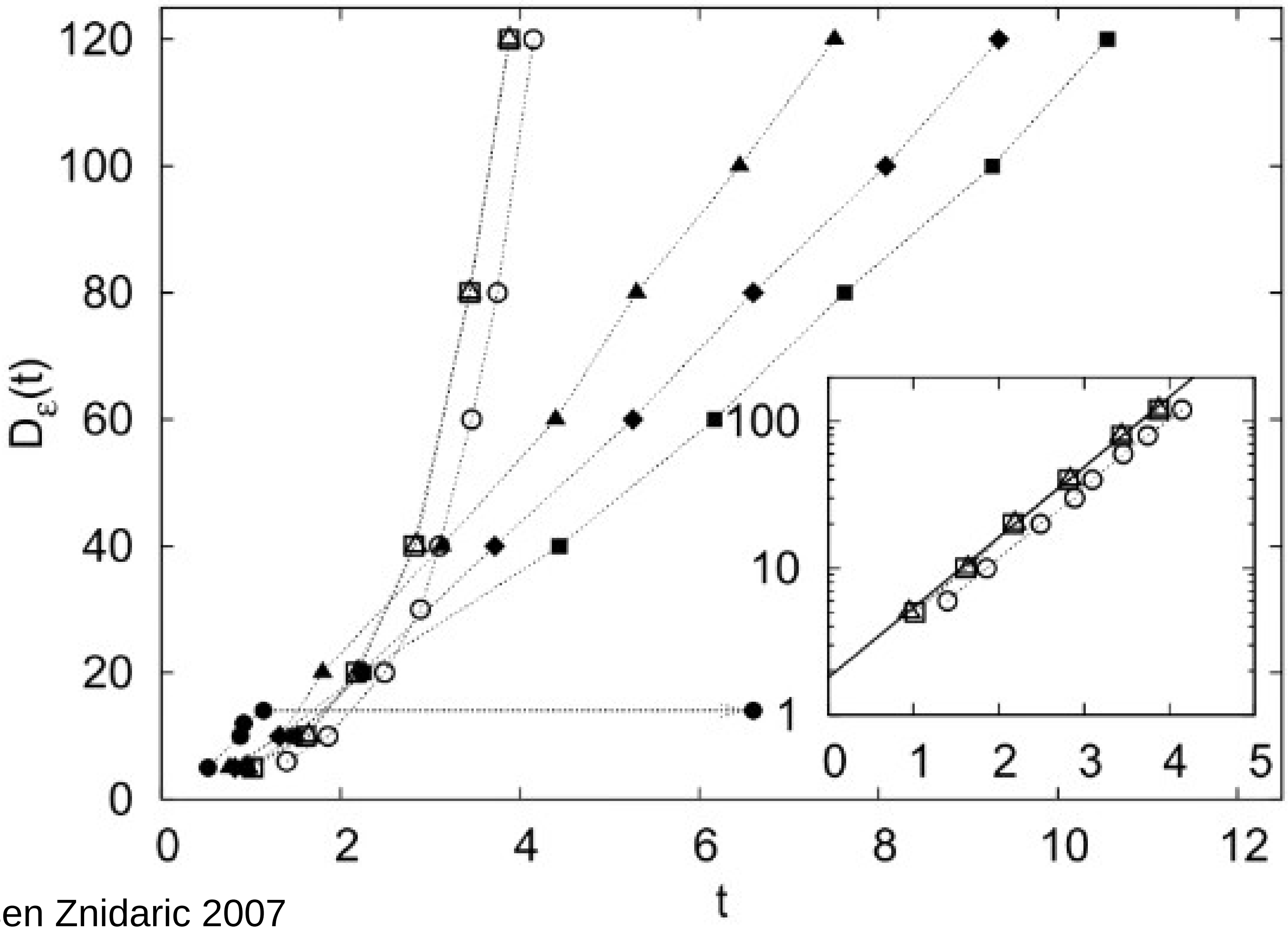
$$\bar{\Lambda}_L = \bar{\Lambda}_R$$

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \min\left\{\mathrm{rank}\left(\Lambda_t\right), \mathrm{rank}\left(\bar{\Lambda}_L\right)\right\}$$

$$\Lambda_t$$



Heisenberg evolution of operator



Prosen Znidaric 2007

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \min\left\{\mathrm{rank}\left(\Lambda_t\right), \mathrm{rank}\left(\bar{\Lambda}_L\right)\right\}$$

$$\mathrm{rank}\left(\Lambda^t\right) \propto \exp\left(t\right) \qquad \mathrm{rank}\left(\bar{\Lambda}_L\right) \propto \exp(t)$$

$$\Downarrow$$

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \exp(t)$$

$$H = \sum_i (-X_i X_{i+1} - g Z_i - h X_i)$$

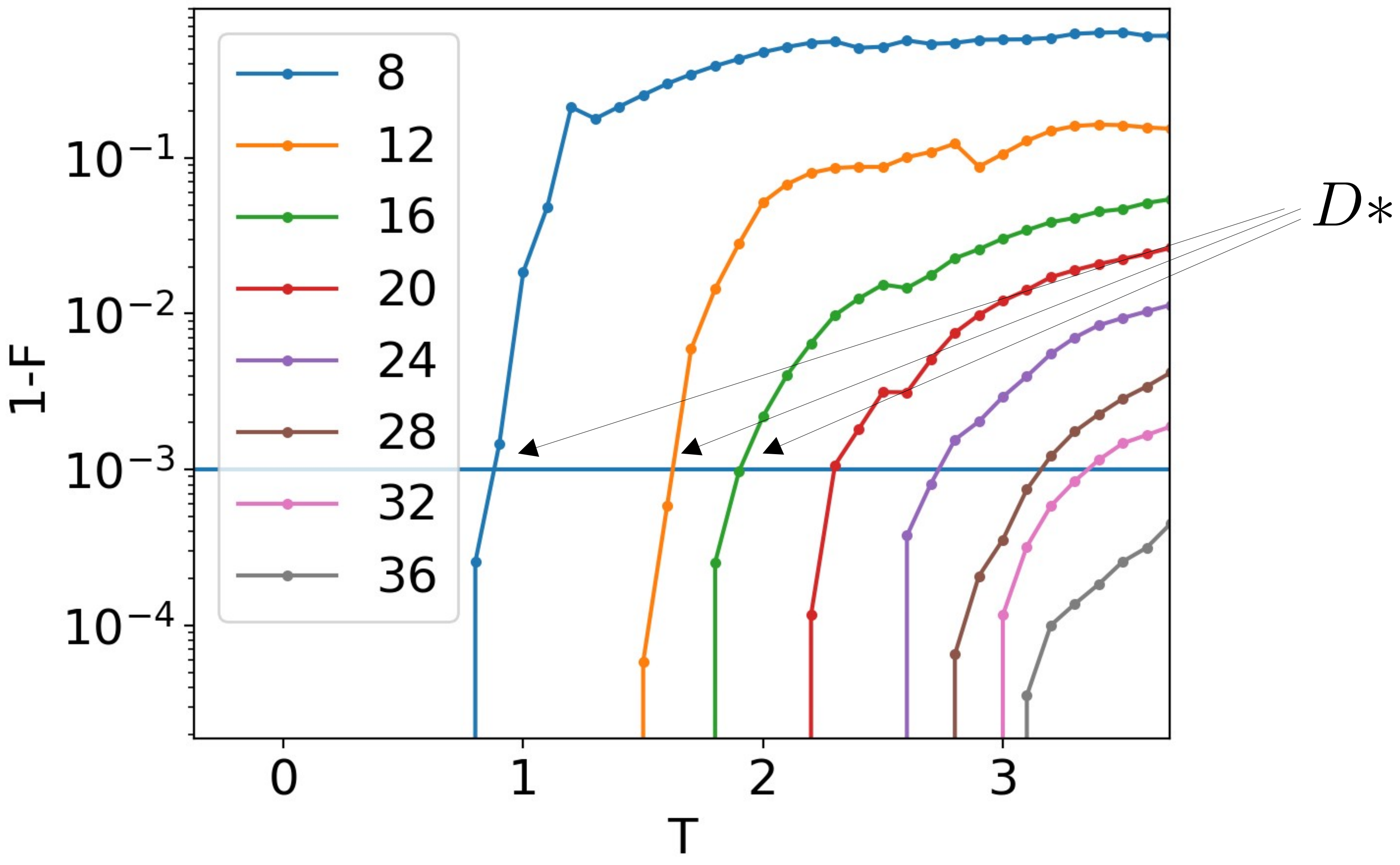
Integrable quench:

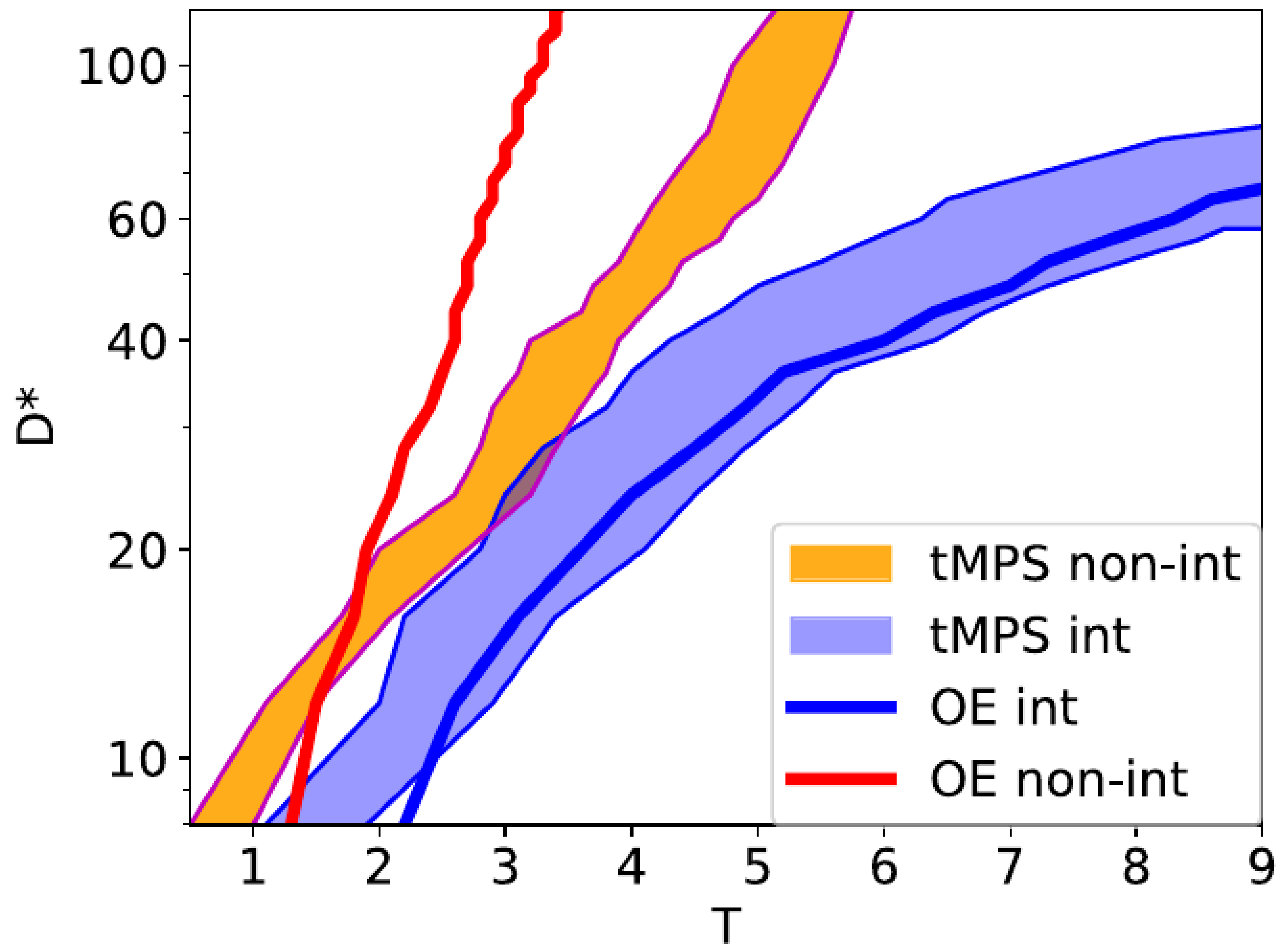
$$\{g = \infty, h = 0\} \to \{g = 0.7, h = 0\}$$

Non-integrable quench:

$$\{g = \infty, h = 0\} \to \{g = -1.05, h = 0.5\}$$

$$F = \frac{\langle L_X^D | L_X \rangle^2}{\langle L_X^D | L_X^D \rangle \langle L_X | L_X \rangle}$$




100
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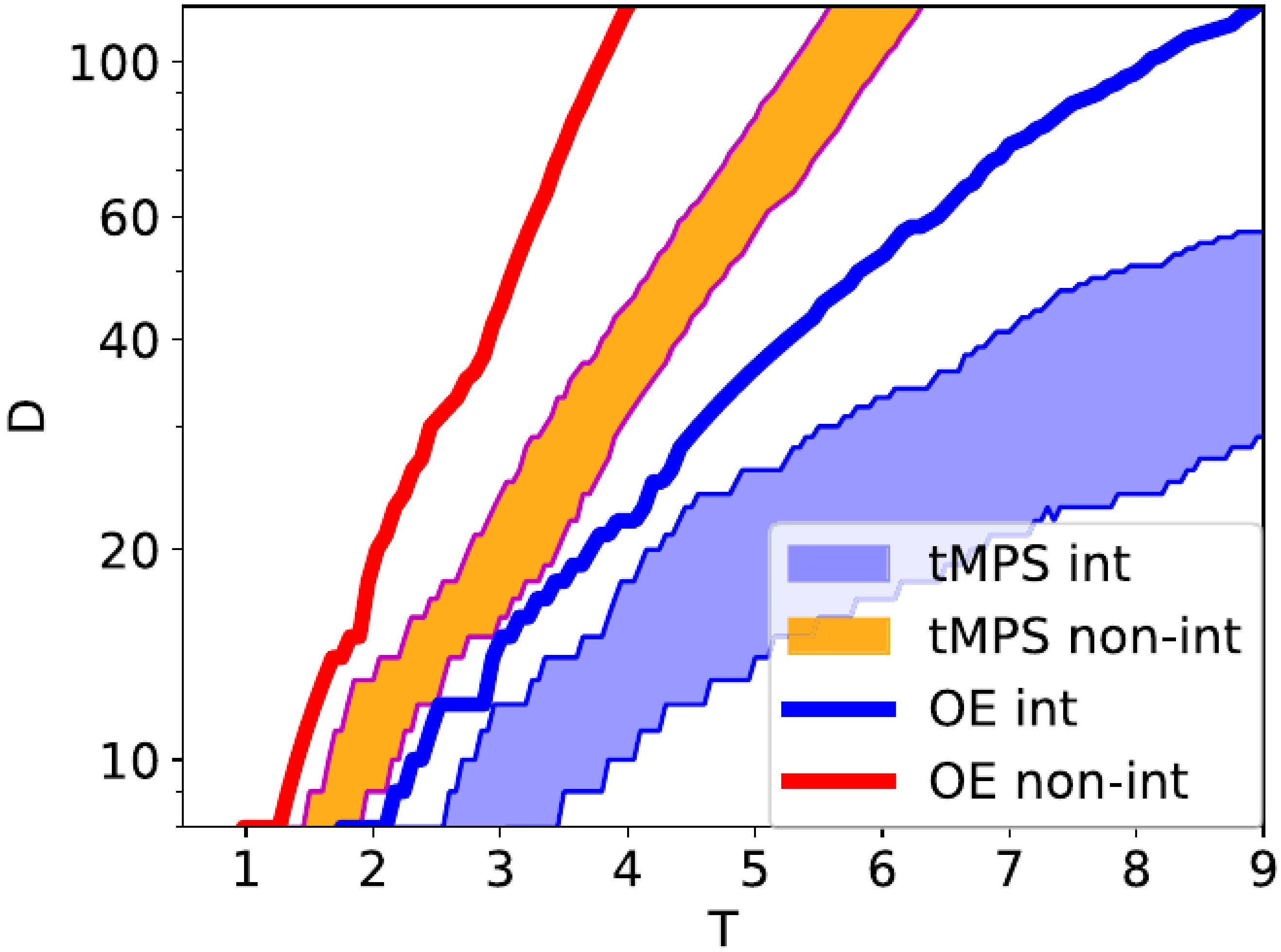
T
tMPS non-int
tMPS int
OE int
OE non-int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tMPS non-int
OE int
OE non-int

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PhD Fellowship available to start in 2026