Quantum Mpemba effect in a quantum dot with reservoirs

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[**Ref:** Amit Kumar Chatterjee, Satoshi Takada, and Hisao Hayakawa, arXiv:2304.02411]

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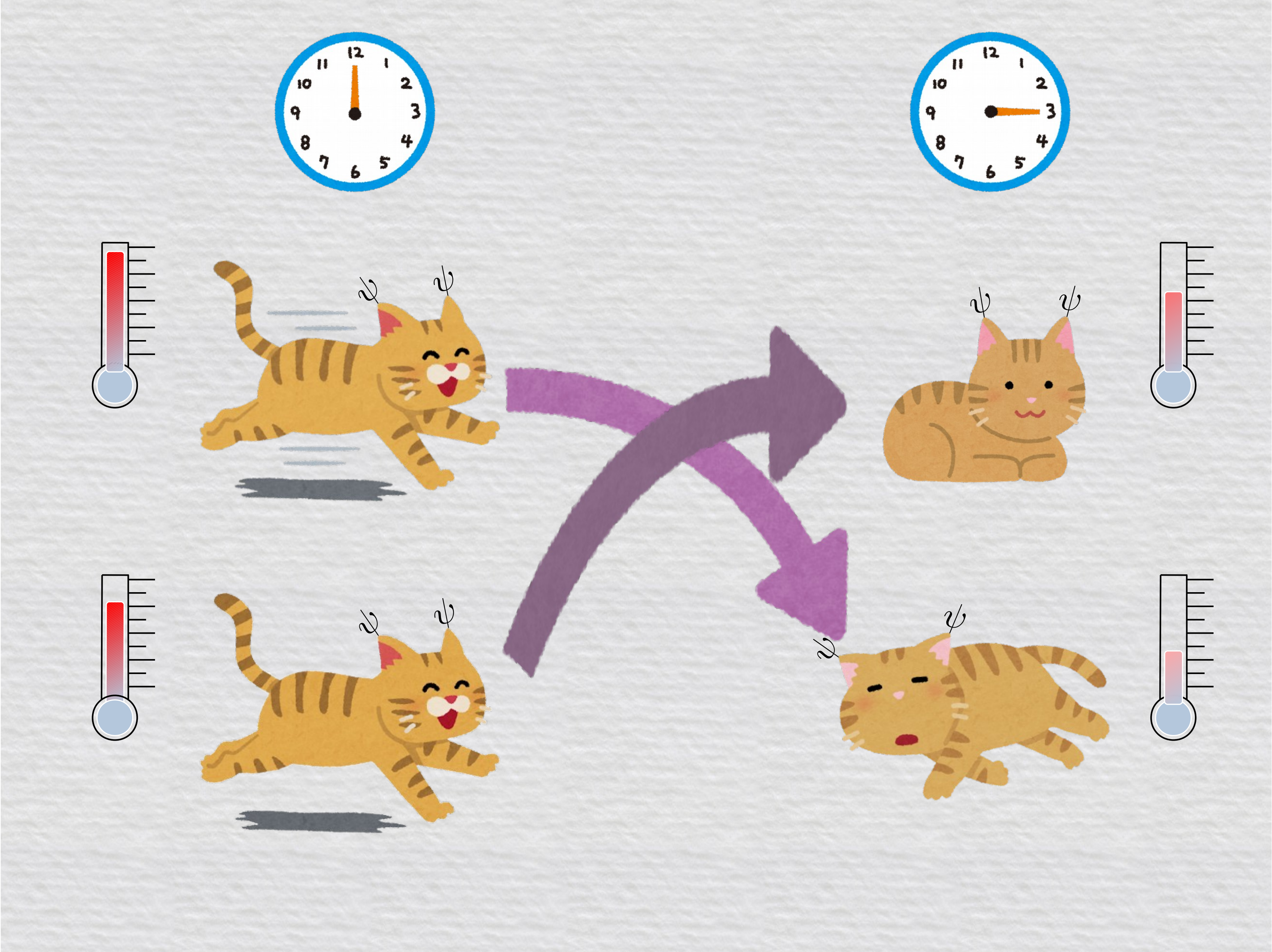


A quantum system (cat) can cool faster when starting from initially hotter rather than colder temperature.

[Image credit: S. Takada, A. K. Chatterjee and H. Hayakawa]

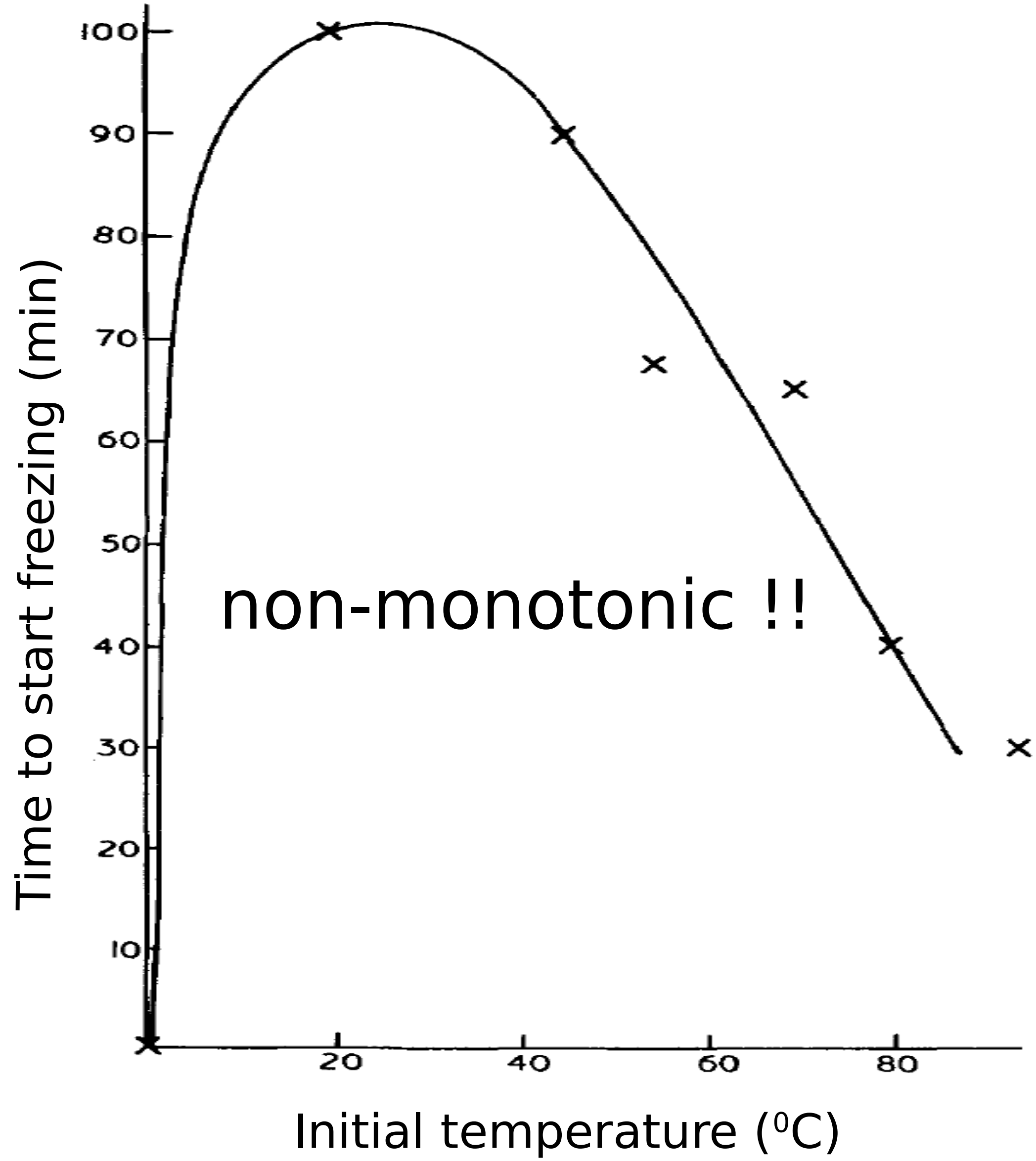
Mpemba effect

Experiment (Mpemba & Osborne):

freezing equal volumes
of water samples at different
initial temperatures

Hot water can freeze
faster than cold water

[* *thermal* pheomena *]



[Cool ? *by* E. B. Mpemba and D. G. Osborne, Phys. Educ. 4, 172 (1969)]

Mpemba effect as Generalized anomalous relaxation

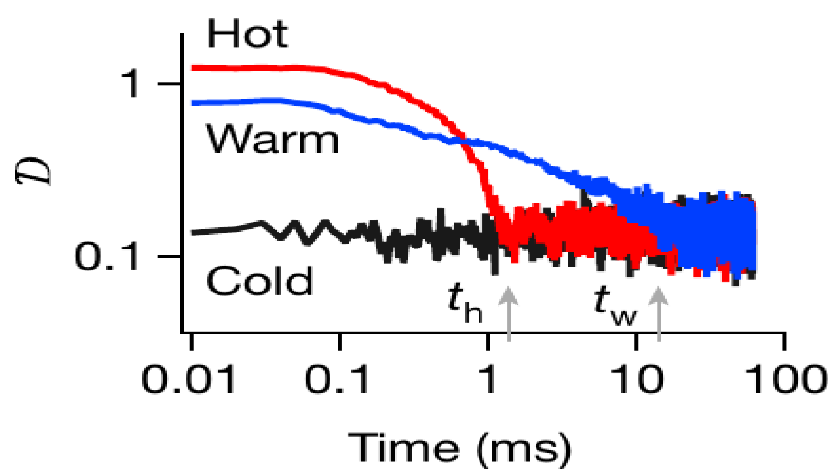
"time to start freezing"

↓ replaced by

"crossing of trajectories" before reaching same steady state

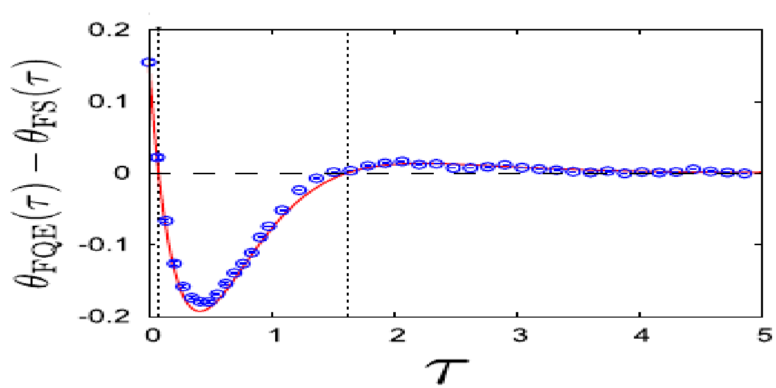
- Colloids

[Kumar and Bechhoefer, Nature 584, 64 (2020)]



- Inertial suspensions

[Takada, Hayakawa and Santos, PRE 103, 032901 (2021)]



- Markovian models: $$\frac{\mathrm{d}p_i(t)}{\mathrm{d}t} = \sum_j R_{ij}(T_b)p_j(t)$$

$$\overrightarrow{p}(t) = \overrightarrow{\pi}(T_b) + \underbrace{e^{\lambda_2 t}a_2\overrightarrow{v_2}}_{\text{slowest relaxation}} + \underbrace{e^{\lambda_3 t}a_3\overrightarrow{v_3} + \cdots + e^{\lambda_n t}a_n\overrightarrow{v_n}}_{\text{other relaxation modes}} \qquad \lambda_2 > \lambda_3 > \cdots > \lambda_n$$

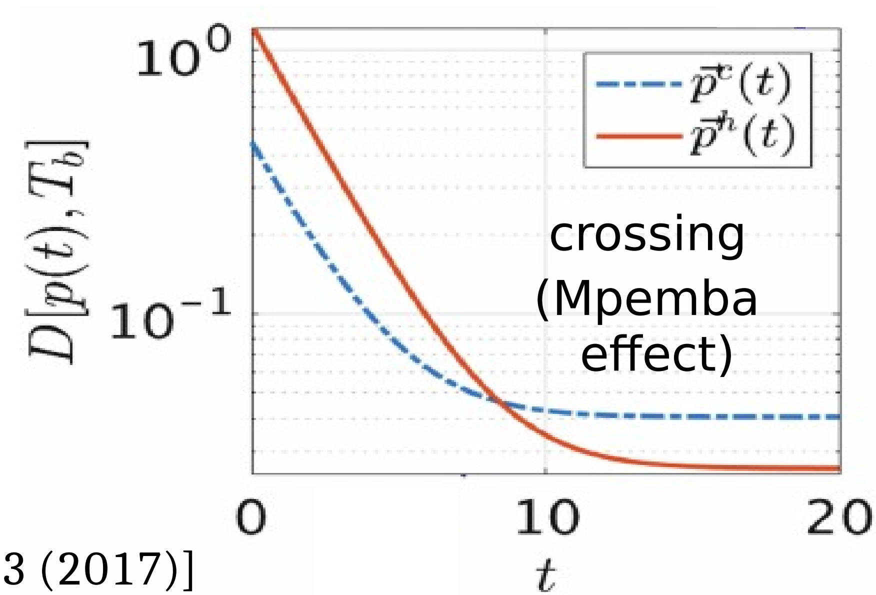
Hot copy: $\vec{p}^h(t) \approx \vec{\pi}(T_b) + a_2^h \vec{v}_2 e^{\lambda_2 t}$

Cold copy: $\vec{p}^c(t) \approx \vec{\pi}(T_b) + a_2^c \vec{v}_2 e^{\lambda_2 t}$

} ignores initial relaxation

Distance function from equilibrium:

$$D_e[\vec{p}; T_b] = \sum_i \left(\frac{E_i \Delta p_i}{T_b} + p_i \ln p_i - \pi_i^b \ln \pi_i^b \right)$$



[Lu and Raz, PNAS 114, 5083 (2017)]

$$\vec{p}^{h}(t) \approx \vec{\pi}(T_b) + a_2^h \vec{v}_2 e^{\lambda_2 t} \qquad \vec{p}^{c}(t) \approx \vec{\pi}(T_b) + a_2^c \vec{v}_2 e^{\lambda_2 t}$$

Criterion for Mpemba effect (proposed by Lu & Raz):

$$|a_2^c| > |a_2^h|$$

But, what about the other relaxation modes ?

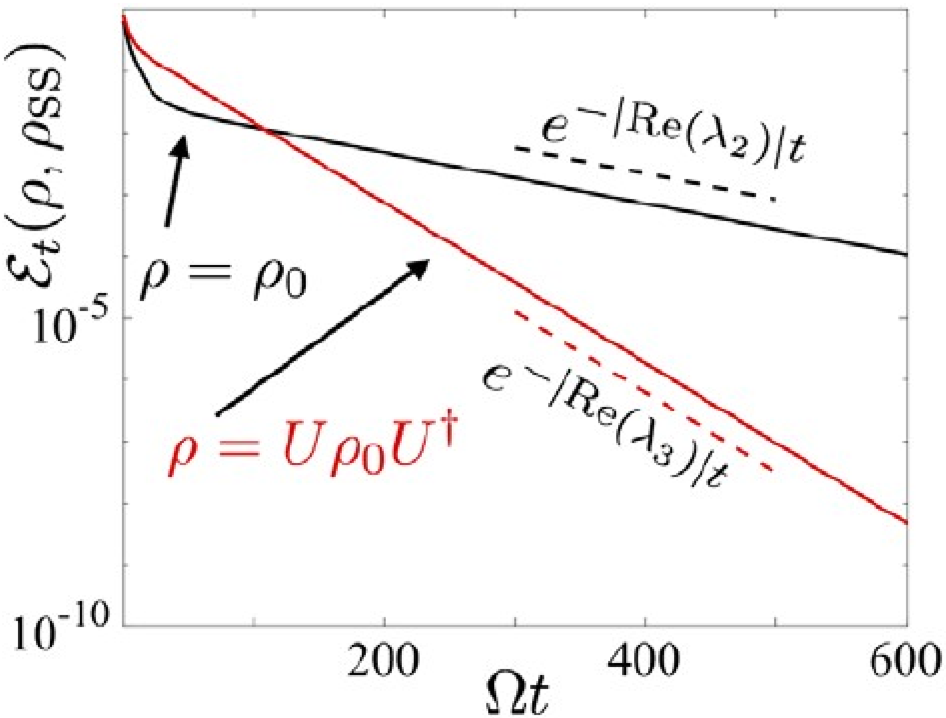
The co-efficients $a_3, \dots, a_n$ have no role to play ?

⇓

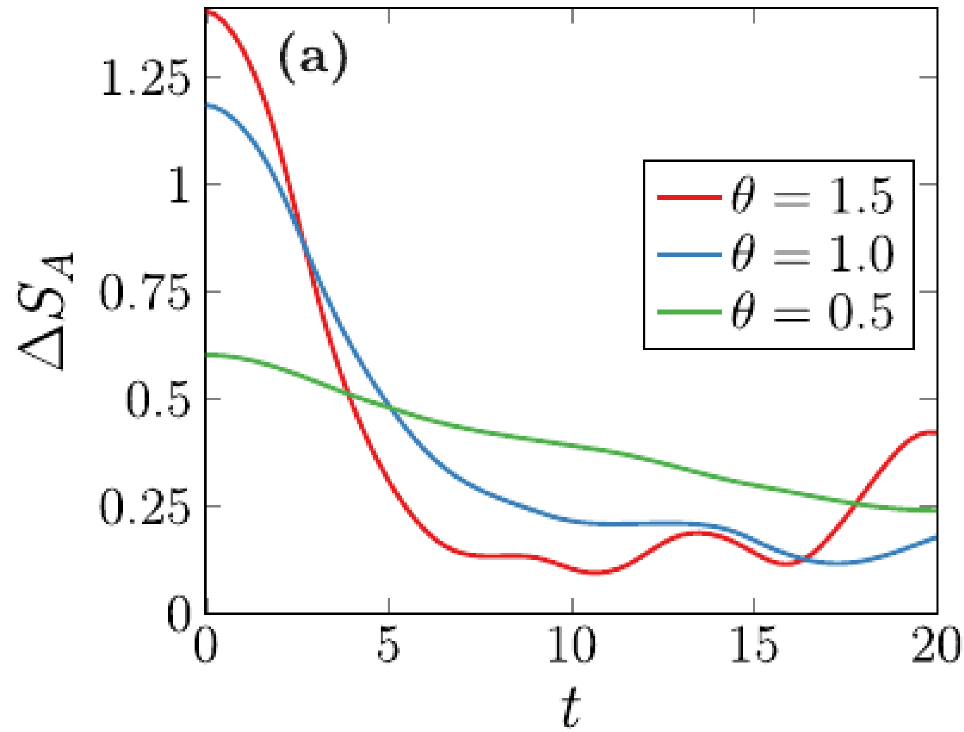
Question: Role of initial relaxation on Mpemba effect ?

Quantum Mpemba effect (QMPE)

vastly unexplored field



Distance from equilibrium in dissipative Dicke model



Entangle asymmetry in XXZ spin chain

[Carollo, Lasanta and Lesanovsky, PRL 127, 060401 (2021)]

[Ares, Murciano and Calabrese, Nature Communications 14, 2036 (2023)]

Analysis of temperature missing

⇓ Question

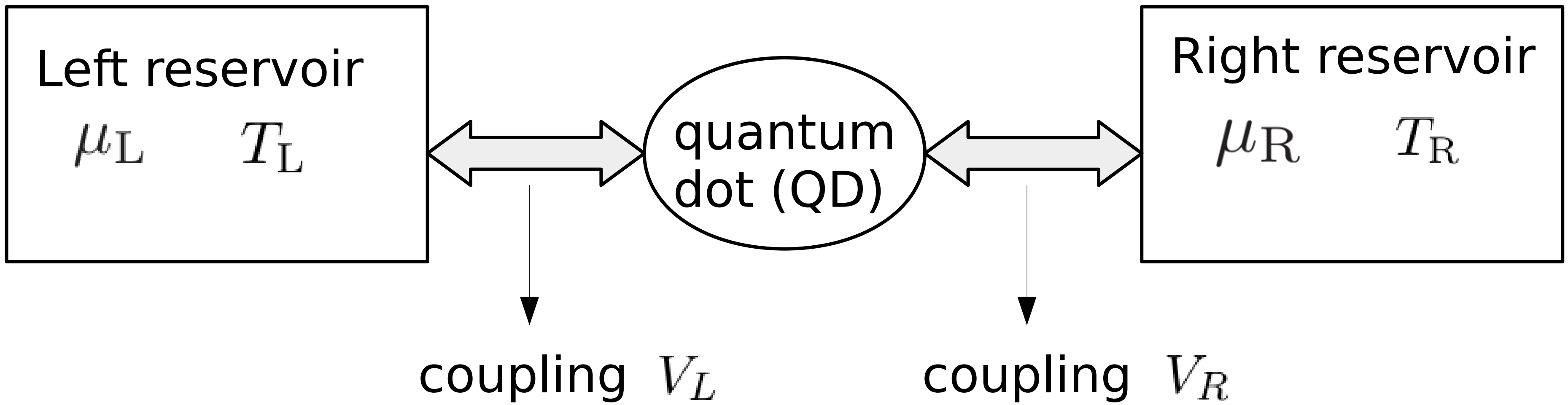
What about “thermal” Quantum Mpemba effect ?
(*closer to original classical Mpemba effect*)

Our motivations

- Show the existence of *thermal* quantum Mpemba effect

- Explore the roles of relaxation modes other than the slowest mode

Quantum dot coupled to two reservoirs



QD states: $\uparrow\downarrow$, $\uparrow$, $\downarrow$, vacant

Total Hamiltonian: $\hat{H}_{\rm tot} = \hat{H}_{\rm s} + \hat{H}_{\rm r} + \hat{H}_{\rm int}$

- $\hat{H}_{\rm s}$: System Hamiltonian
- $\hat{H}_{\rm r}$: Reservoir Hamiltonian
- $\hat{H}_{\rm int}$: System-reservoirs interaction

Anderson model:

$$\hat{H}_{\mathrm{s}} = \sum_{\sigma} \epsilon_0 \hat{d}_\sigma^\dagger \hat{d}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow$$

$$\hat{H}_{\mathrm{r}} = \sum_{\gamma,k,\sigma} \epsilon_k \hat{a}_{\gamma,k,\sigma}^\dagger \hat{a}_{\gamma,k,\sigma}$$

$$\hat{H}_{\mathrm{int}} = \sum_{\gamma,k,\sigma} V_\gamma \hat{d}_\sigma^\dagger \hat{a}_{\gamma,k,\sigma} + \mathrm{h.c.}$$

ϵ_0: energy of electron in quantum dot

ϵ_k: energy of electron corresponding to wave number k in reservoirs

U: electron-electron interaction in quantum dot

V_L, V_R: coupling strength between quantum dot and reservoirs

$\hat{d}^\dagger, \hat{d}$: creation and annihilation operators in quantum dot

$\hat{a}^\dagger, \hat{a}$: creation and annihilation operators in reservoirs

$\hat{n}$: number operator $(= \hat{d}^\dagger \hat{d})$

γ: reservoir indices L, R σ: up-spin $(\uparrow)$ or down-spin $(\downarrow)$

Quantum Master equation:

$$\frac{d}{d\tau}\hat{\rho} = \hat{K}\hat{\rho}$$

$\hat{\rho}$: four possible states: $\uparrow\downarrow$, $\uparrow$, $\downarrow$, vacant

(wide band approximation)

Transition matrix:
$$\hat{K} = \begin{pmatrix} -2f_-^{(1)} & f_+^{(1)} & f_+^{(1)} & 0 \\ f_-^{(1)} & -f_-^{(0)} - f_+^{(1)} & 0 & f_+^{(0)} \\ f_-^{(1)} & 0 & -f_-^{(0)} - f_+^{(1)} & f_+^{(0)} \\ 0 & f_-^{(0)} & f_-^{(0)} & -2f_+^{(0)} \end{pmatrix}$$

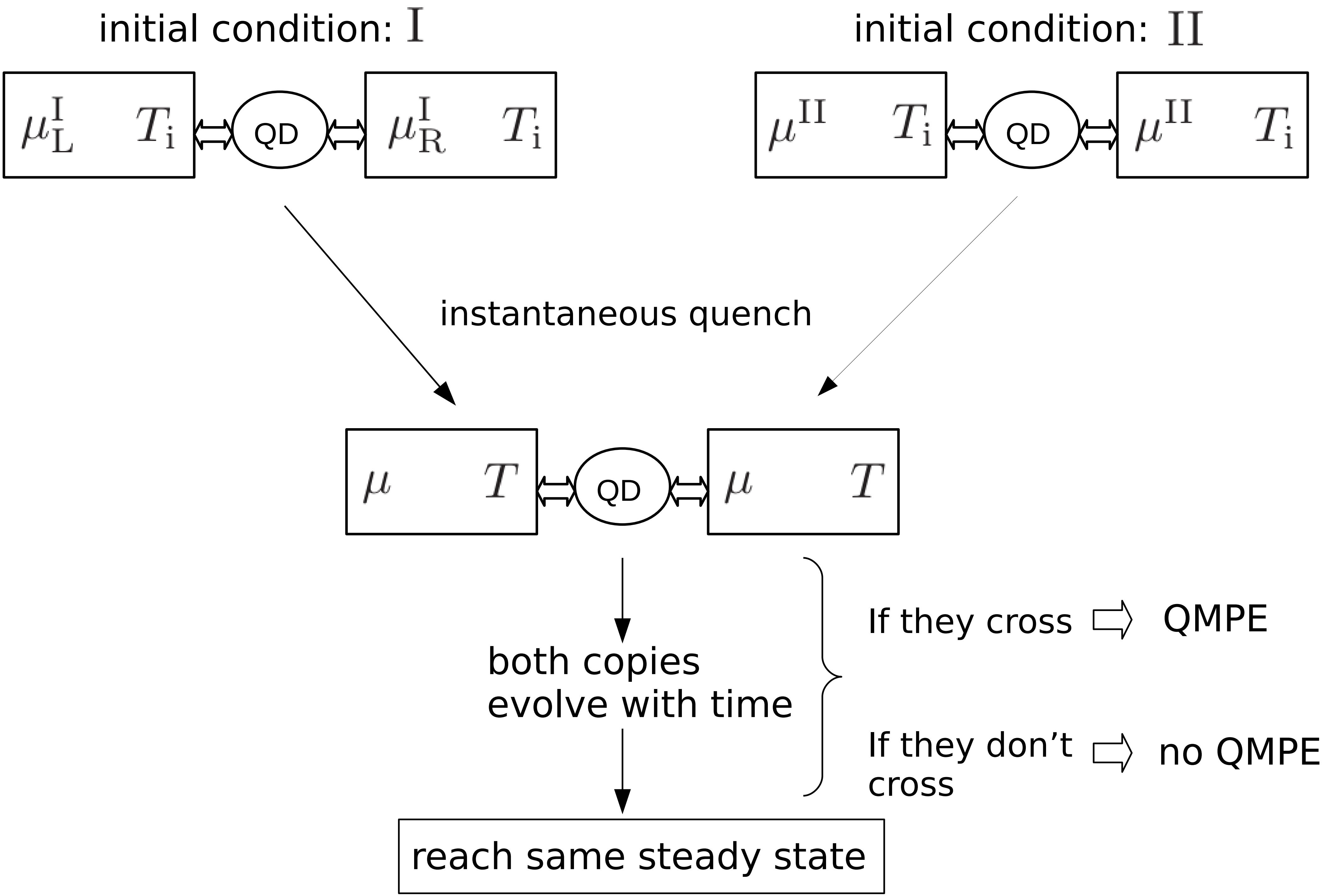
where
$$f_+^{(j)} := f_{\mathrm{L}}^{(j)}(\mu_{\mathrm{L}}, U, \epsilon_0, T) + f_{\mathrm{R}}^{(j)}(\mu_{\mathrm{R}}, U, \epsilon_0, T), \quad j = 0, 1$$

$$f_-^{(j)} = 2 - f_+^{(j)}$$

$$f_\gamma^{(j)}(\mu_\gamma, U, \epsilon_0, T) := \frac{1}{1 + e^{(\epsilon_0 + jU - \mu_\gamma)/T}}$$: Fermi-Dirac distribution

$\gamma = \mathrm{L}, \mathrm{R}$

Protocol



initial condition: I
initial condition: II
$\mu_{\mathrm{L}}^{\mathrm{I}}$ T_{i}
QD
$\mu_{\mathrm{R}}^{\mathrm{I}}$ T_{i}
μ^{II} T_{i}
QD
μ^{II} T_{i}
instantaneous quench
μ T
QD
μ T
both copies
evolve with time
If they cross
QMPE
If they don’t
cross
no QMPE
reach same steady state

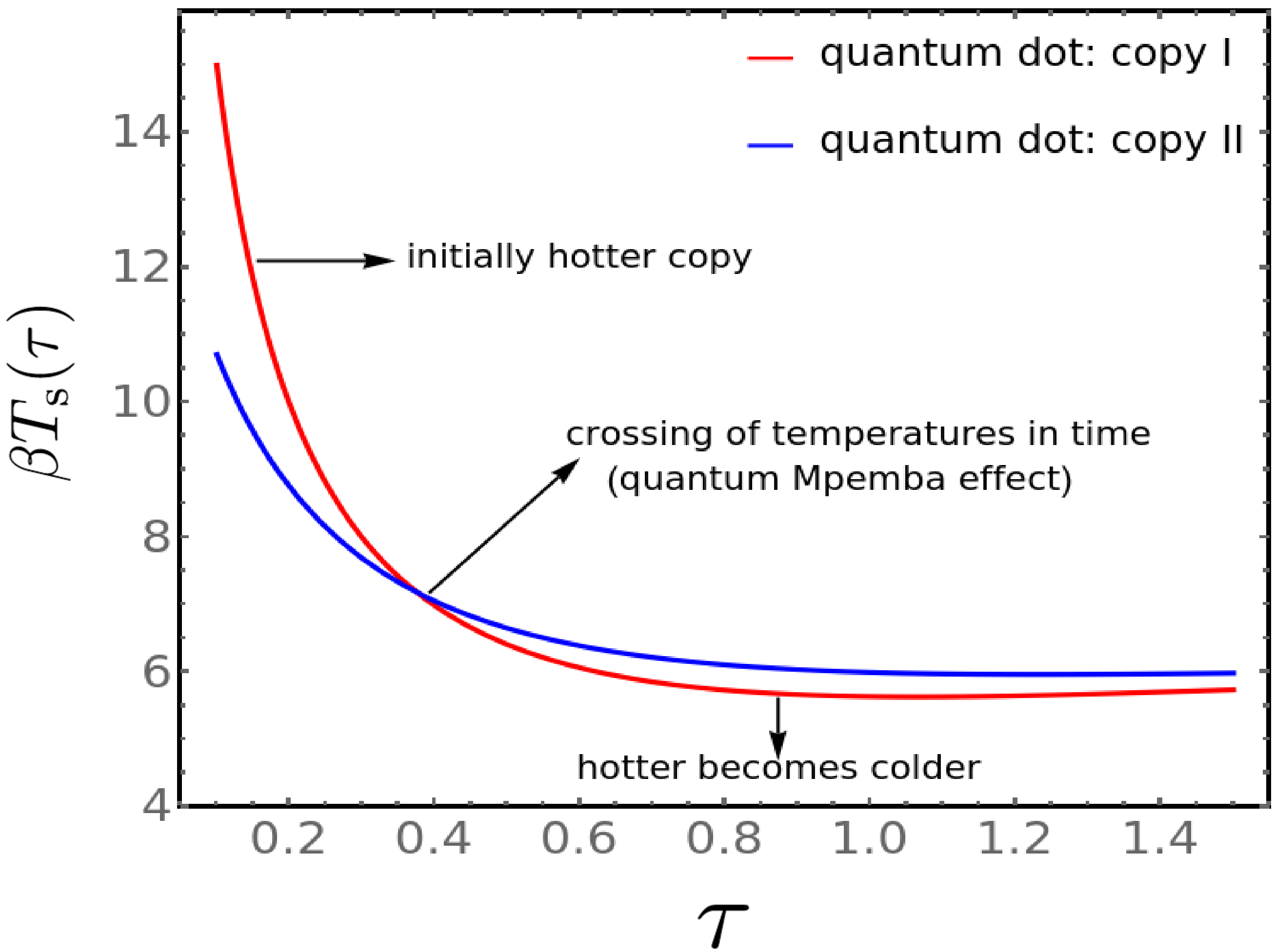
Thermal Quantum Mpemba effect

Temperature: $T_{\rm s}(\tau) := \dfrac{\partial E_{\rm s}(\tau)}{\partial S_{\rm vN}(\tau)} = \dfrac{\partial E_{\rm s}(\tau)}{\partial \tau} \Big/ \dfrac{\partial S_{\rm vN}(\tau)}{\partial \tau}$

$$E_{\rm s}(\tau) = {\rm Tr}[\hat{\rho}(\tau)\hat{H}_{\rm s}]$$

$$S_{\rm vN}(\tau) = -\sum_\alpha \rho_\alpha(\tau)\ln(\rho_\alpha(\tau))$$

$$\beta = 1/T$$



$$\beta\epsilon_0 = 2.0, \beta U = 1.25, \beta\mu_{\rm L}^{\rm I} = 4.5, \beta\mu_{\rm R}^{\rm I} = 1.0, \beta\mu^{\rm II} = 2.43, \beta T_{\rm i} = 1.15, \beta\mu = 2.0.$$

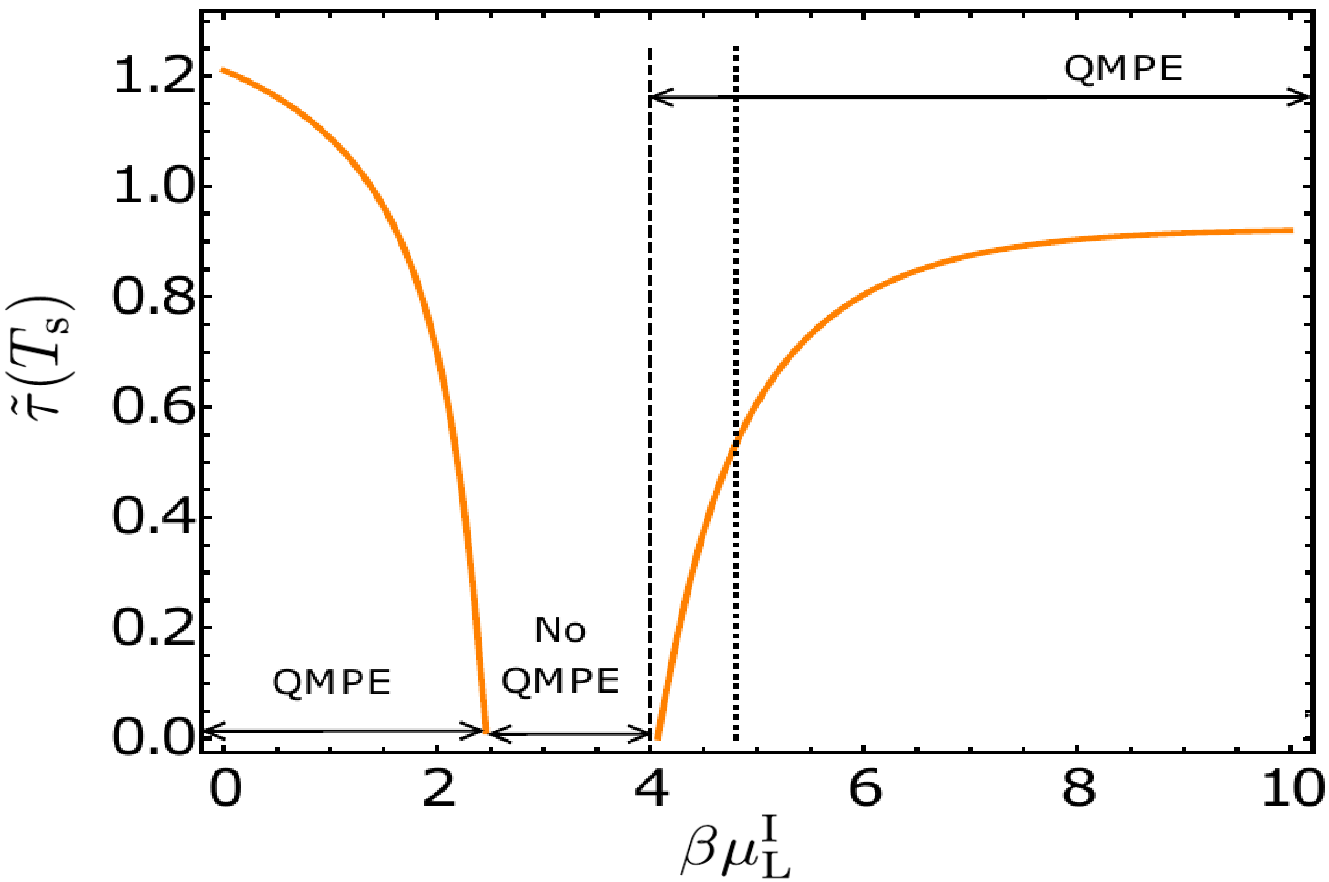
$\Delta T_s := T_s^{\rm I} - T_s^{\rm II}$

$\tau = \tilde{\tau}(T_S)$: solution for $\Delta T_S = 0$

crossing time *(indicator of QMPE)*

$0 < \tilde{\tau}(T_s) < \infty$: thermal QMPE

$\tilde{\tau}(T_s) \to \infty$: no QMPE



thermal QMPE is rather generic than occasional

$\beta\epsilon_0 = 2.0, \beta U = 1.25, \beta\mu_R^{\rm I} = 1.0, \beta\mu^{\rm II} = 2.43, \beta T_{\rm i} = 1.15, \beta\mu = 2.0.$

Role of relaxation modes

- a_2 is zero $\Longrightarrow$ No contribution from slowest relaxation mode

- To show QMPE in density matrix elements:

$$\Delta\rho_\alpha(\tau) := \rho^{\mathrm{I}}_\alpha(\tau) - \rho^{\mathrm{II}}_\alpha(\tau), \qquad \alpha = 1,2,3,4 \quad (\equiv \uparrow\downarrow, \uparrow, \downarrow, \text{vacant})$$

$$= e^{\lambda_3 \tau} \hat{R}_{\alpha,4} \Delta a_4 \left[S_\alpha + e^{-(\lambda_3 - \lambda_4)\tau} \right]$$

$$S_\alpha := (\hat{R}_{\alpha,3}\Delta a_3)/(\hat{R}_{\alpha,4}\Delta a_4)$$

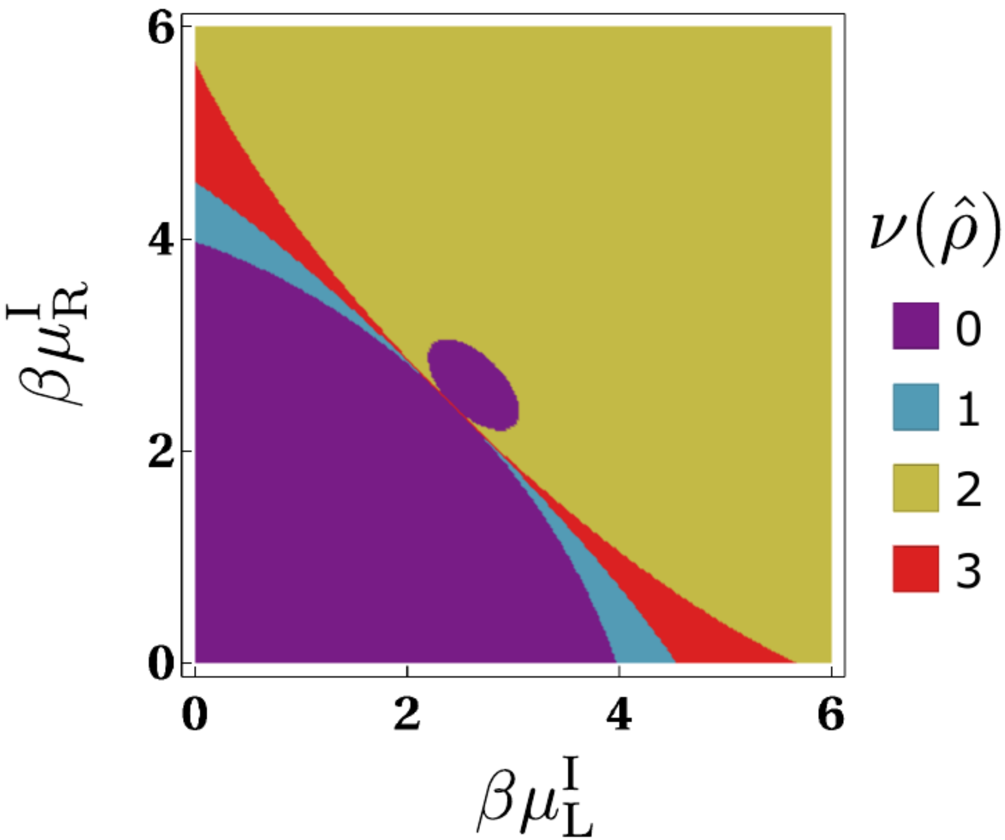
combined role of the surviving relaxation modes

$\Downarrow$

Necessary criterion for QMPE:

$$S_\alpha < 0 \quad \& \quad |S_\alpha| < 1$$

$\nu(\hat{\rho})$: Number of density matrix elements showing QMPE



$\beta\epsilon_0 = 2.0, \beta U = 1.25,$

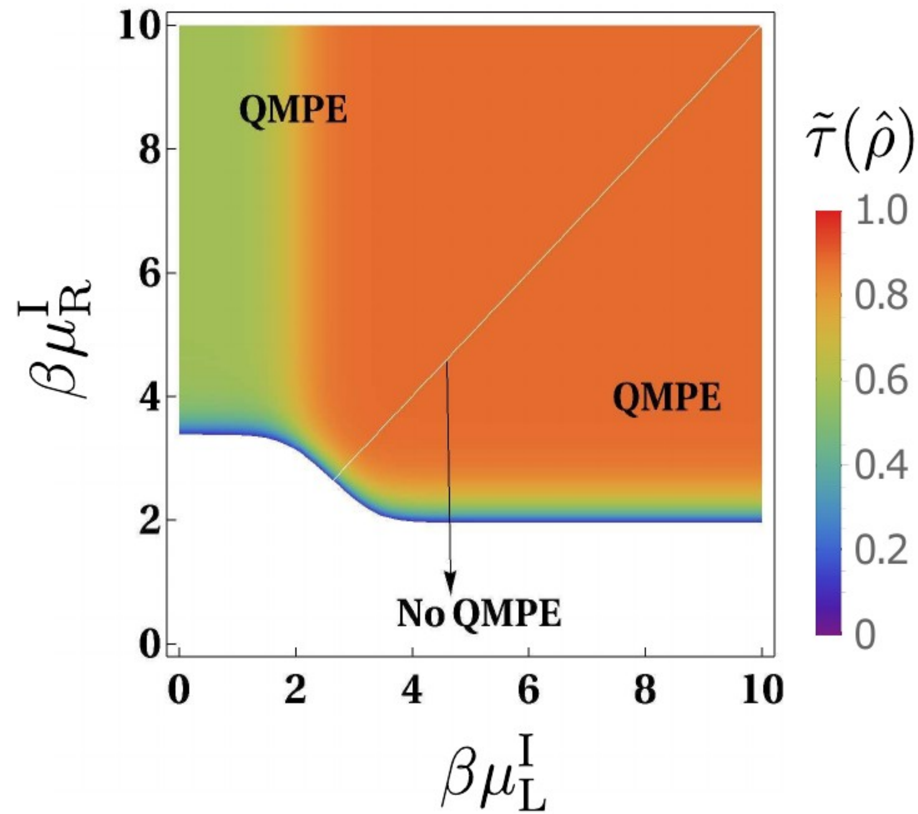
$\beta\mu^{II} = 2.43, \beta T_i = 1.15, \beta\mu = 2.0.$

To get QMPE in density matrix elements:

$$\Delta\rho_\alpha(\tau_\alpha) = 0$$

$$\tilde{\tau}(\hat{\rho}) = \max[\tau_1, \tau_2, \tau_3, \tau_4] \quad \text{if } 0 < \tau_\alpha < \infty$$

$$\tilde{\tau}(\hat{\rho}) \rightarrow \infty \text{ if no finite } \tau_\alpha \text{ exists } \forall\alpha$$



$\beta\epsilon_0 = 2.0, \beta U = 1.25,$

$\beta T_i = 0.25, \beta\mu = 2.0$ and $\beta\mu^{II} = \beta\mu_R^I$

Summary

- A quantum system can cool faster when it starts from hotter initial temperature rather than colder initial temperature. This is the *thermal quantum Mpemba effect*.

  (demonstrated in a single level quantum dot with reservoirs)

- The criterion for quantum Mpemba effect explicitly depends on the combined effect of relaxation modes other than the slowest relaxation mode.

(different from previously proposed criterion for classical Markovian systems)

[**Ref:** Amit Kumar Chatterjee, Satoshi Takada, and Hisao Hayakawa, arXiv:2304.02411]

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[Selected for American Physical Society's outreach to the press]

THANK YOU