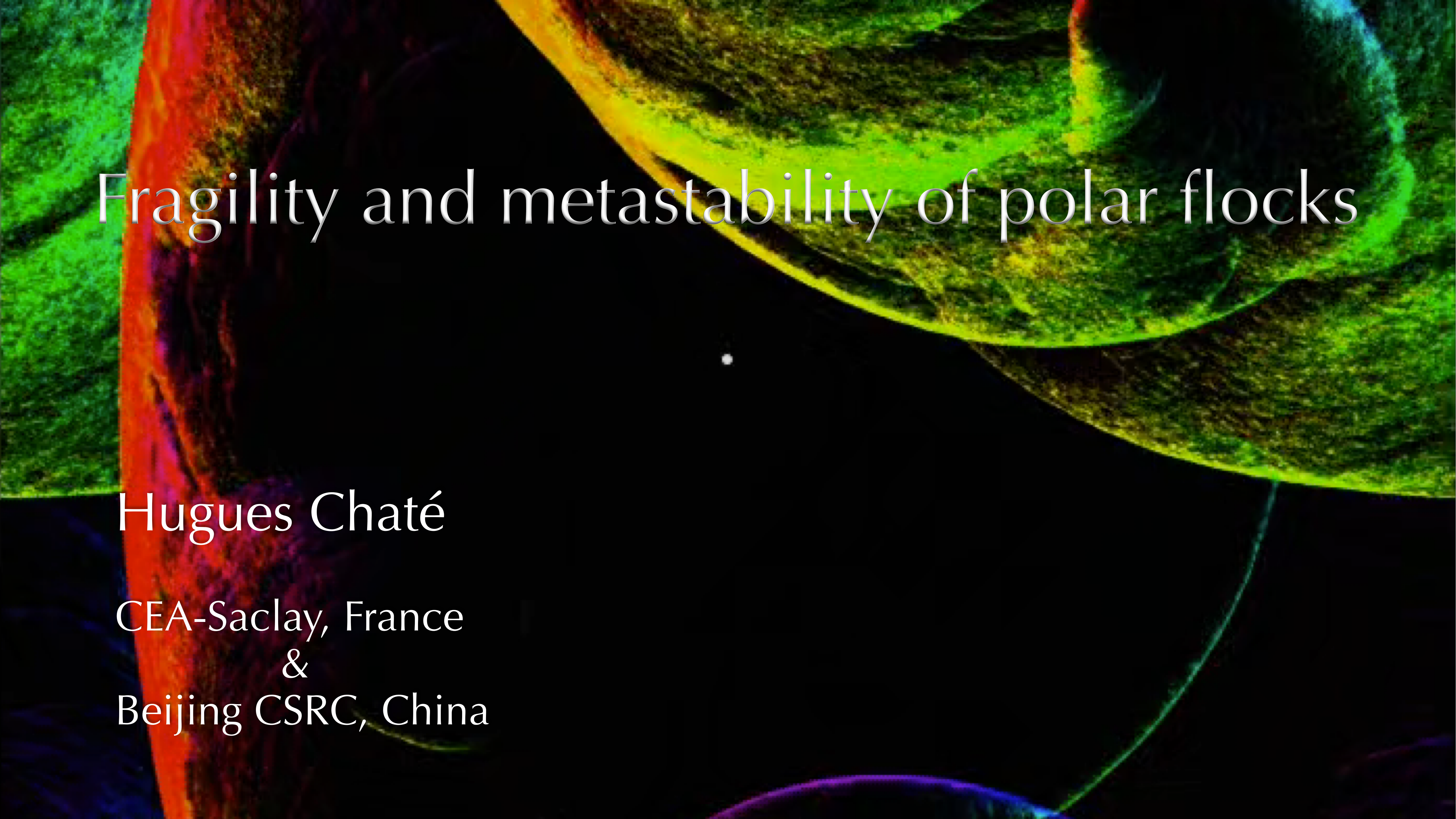

Fragility and metastability of polar flocks

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John Toner

Active matter

- Definition: contains constituents that transform energy, stored internally or gathered from their environment, into mechanical work
- Examples abound at all scales:
  - animal groups, cell colonies and tissues, molecular motors, enzymes
  - swarms of robots, swimmers, phoretic colloids
- Detailed balance and time-reversal symmetry broken 'in the bulk'

  → qualitatively 'new' collective phenomena, e.g.:
  - true long-range orientational order from spontaneous breaking of continuous symmetry in 2D
  - phase separation without attractive interactions

Long-range orientational order in 2D

- Long-range orientational order (arising from spontaneously broken continuous symmetry) impossible in equilibrium 2D systems (Mermin-Wagner theorem)

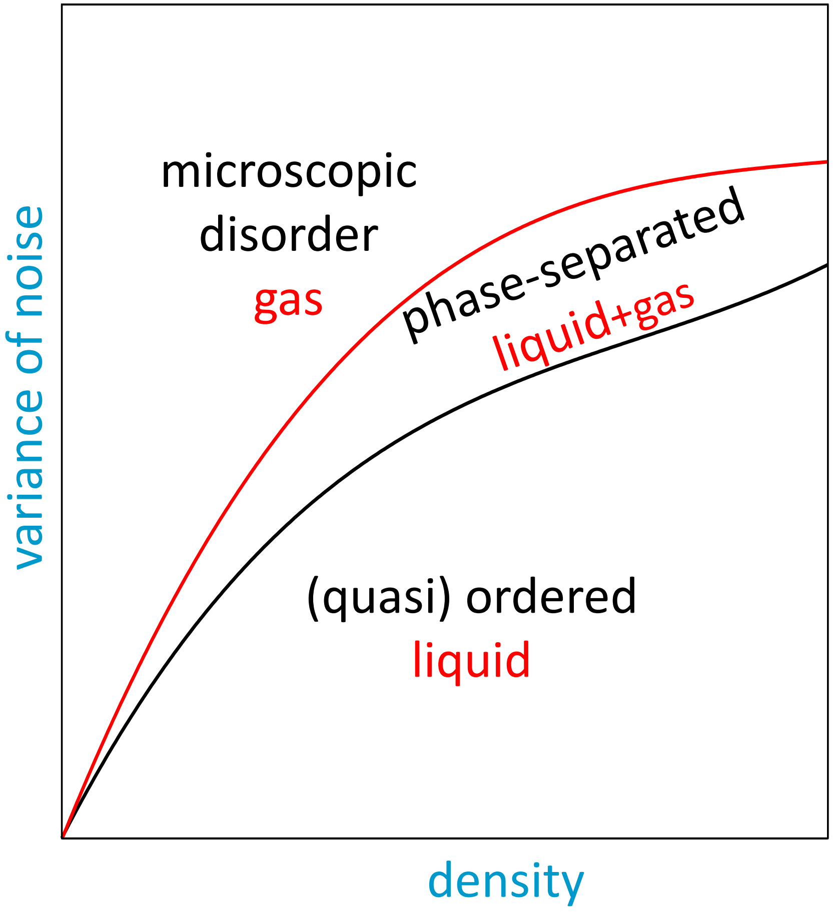
- Example: XY model with either ferromagnetic or nematic interactions. Only QLRO (correlations decay algebraically).

- But in 1995, Vicsek et al. (implicitly) and Toner & Tu (directly) argued for true LRO in 2D for minimal model and theory of collective motion

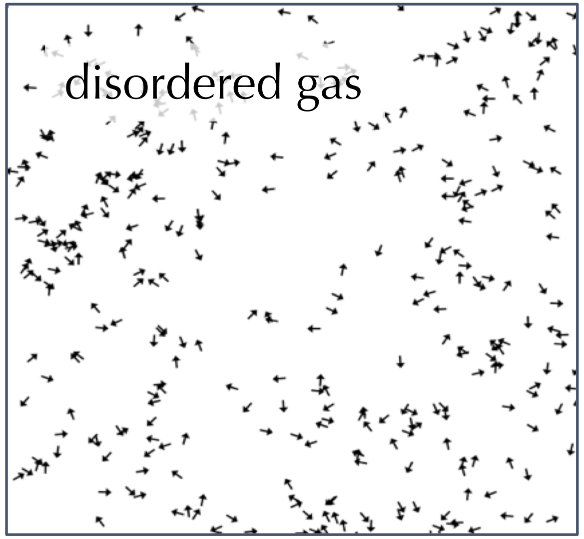
- Vicsek model and Toner-Tu theory emblematic example of DADAM: dry aligning dilute active matter

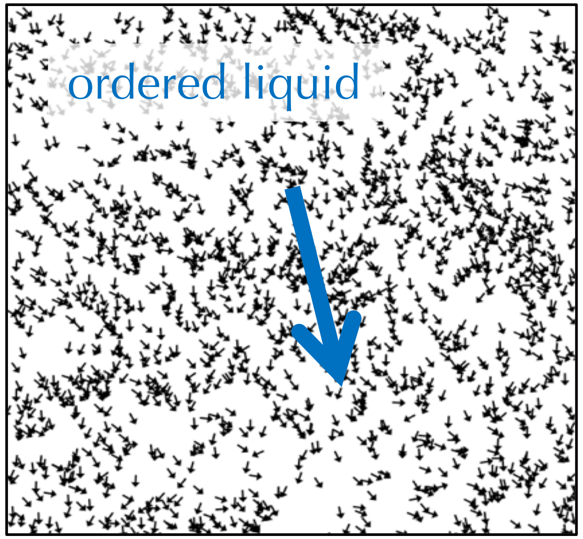
Generic phase diagram of DADAM

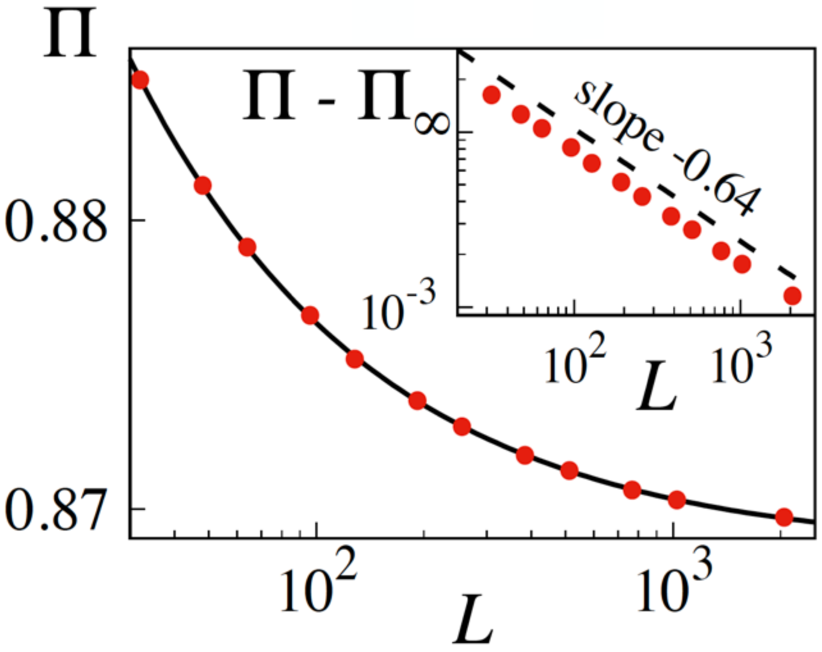
- Vicsek model: point particles, constant speed, local (ferromagnetic) alignment of velocities vs. noise (flying XY model)
- Contrary to initial belief, no direct order-disorder transition, but phase separation:
    - Disordered gas phase at low density/strong noise
    - At high density/low noise, polar or nematic ordered liquid phase, with long-range correlations
    - In between: phase-separated inhomogeneous phase with dense and ordered patches in gas background



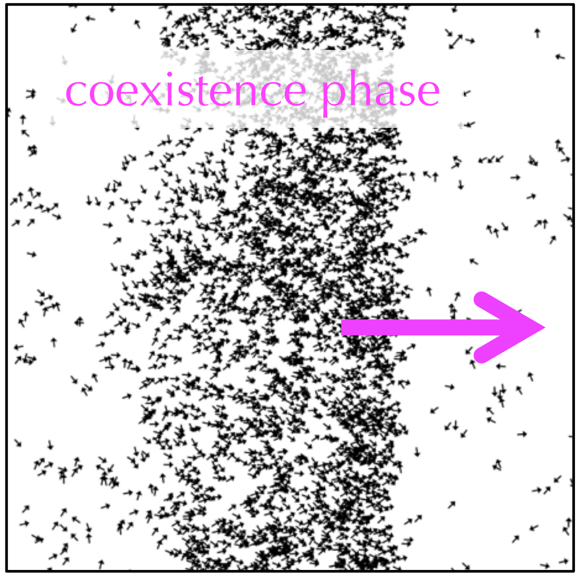
Vicsek model (in 2D)

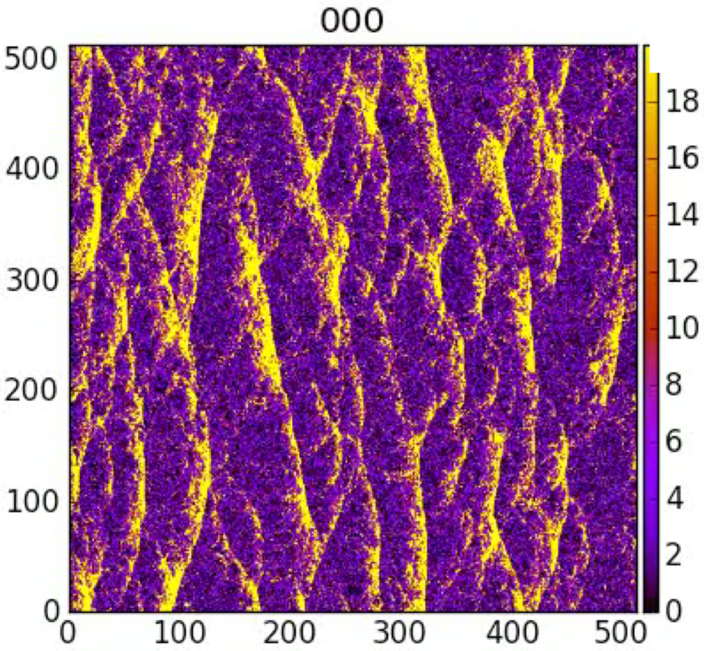






polar order parameter vs system size in loglog scales





- true long-range polar order with universal exponents (generic scaling) in liquid phase
- coexistence phase also shows true long-range polar order (and smectic order as well)

Toner-Tu theory

In 1995, John Toner and Yuhai Tu, inspired by Vicsek, write down, from symmetry arguments, a (fluctuating) hydrodynamic description of the VM

$$\partial_t \vec{v} + \lambda_1 (\vec{v} \cdot \vec{\nabla}) \vec{v} + \lambda_2 (\vec{\nabla} \cdot \vec{v}) \vec{v} + \lambda_3 \vec{\nabla}(|\vec{v}|^2)$$
$$= \alpha \vec{v} - \beta |\vec{v}|^2 \vec{v} - \vec{\nabla} P_1 - \vec{v}(\vec{v} \cdot \vec{\nabla} P_2) + D_1 \vec{\nabla}(\vec{\nabla} \cdot \vec{v})$$
$$+ D_T \nabla^2 \vec{v} + D_2 (\vec{v} \cdot \vec{\nabla})^2 \vec{v} + \vec{f},$$

$$\partial_t \rho + \nabla \cdot (\vec{v} \rho) = 0,$$

- only terms needed from power-counting arguments
- Ginzburg-Landau + compressible Navier-Stokes features
- TT only considered homogeneous ordered phase; use perturbative, one-loop 'dynamic RG' (à la Forster-Stephen-Nelson)

Toner-Tu theory

	$d=2$		$d=3$		$d \geq 4$
	TT95	numerics	TT95	numerics	mean-field
χ	-0.20	$-0.31(2)$	-0.60	$\simeq -0.62$	$1-d/2$
ξ	0.60	0.95(2)	0.80	$\simeq 1$	1
$\zeta \equiv d-1+2\chi+\xi$	1.20	1.33(2)	1.60	1.77(3)	2
z	1.20	1.33(2)	1.60	$\simeq 1.77$	2
GNF	1.60	1.67(2)	1.53	1.59(3)	$1+2/d$

Toner & Tu, considering only the liquid phase, predicted:

- breakdown of linear hydrodynamics in *d*<4
- anisotropic long-range correlations of space-time fluctuations
- true long-range order even in 2D

Comments:

- Toner-Tu predictions essentially qualitatively correct (numerics)
- 2D exponent values 'exact' at all orders of perturbation theory thanks to extra symmetry
- but many important vertices 'missed', mostly involving density field

➢ theory, even at perturbative level, remains incomplete

Variants considered by Toner et al.

A number of variants considered by Toner and collaborators over the years, in part in attempts to find 'exact' cases:

- Incompressible: imposing incompressibility of velocity/polarity field suppresses density.
  Results: continuous (2^{nd} order) transition fixed point studied in $d=(4-\varepsilon)$ at 1-loop, mapping of ordered phase onto (1+1)d KPZ in 2D, exact exponents in higher dimensions
- Malthusian: fast birth-and-death process allows to enslave density field, leaving essentially constant-density but not incompressible v-field equation.
  Results: TT95 2D results for ordered phase really exact in this case, $d=(4-\varepsilon)$ 1-loop otherwise.
- Various other cases: random-field-type quenched disorder with and without incompressibility, 2D flocks in contact with bulk fluid, etc.

Rest of talk:

How robust is all this?

(spoiler: not much!)

Fragility and metastability of ordered phases

Fragility:

Any amount of

- quenched disorder induces ergodicity breaking and changes TT phase qualitatively. Weaker forms of QD induce LRO phase, stronger forms yield 'strong-disorder' regimes
- rotational anisotropy changes TT phase into discrete symmetry one with Ising-like order and short-range fluctuations
- chirality disorder destroys TT phase

Metastability:

The orientationally-ordered phase of

- constant-density flocks is destroyed everywhere by spontaneous nucleation of an active foam of asters
- Toner-Tu/Vicsek flocks is metastable to the nucleation and ballistic growth of counter-going fronts in large parameter domain
- discrete rotational symmetry (Ising) flocks is *always* metastable

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Chirality disorder: Kuramoto-Vicsek model

- Some effects of *population* disorder (individuals with distinct properties) reported in active matter
- We revisited chirality disorder for continuous-time Vicsek model: akin to noisy Kuramoto model with oscillator phase giving direction of motion
- Kuramoto-Vicsek model:

$$\partial_t \mathbf{r}_i = v_0 \mathbf{u}_i(\theta_i) \tag{1}$$

$$\partial_t \theta_i = \omega_i + \frac{k}{n_i} \sum_{j \in \partial i} \sin(\theta_j - \theta_i) + \sqrt{2D_r}\eta_i \tag{2}$$

with ω_i assigned from zero-mean distribution of width ω_0

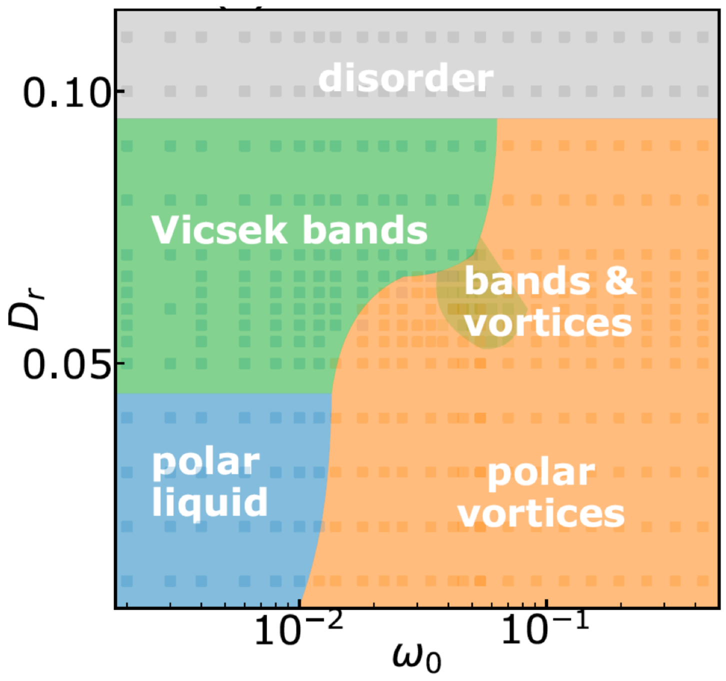
- Main question: can LRO/synchronization survive frequency disorder?

Chirality disorder: Kuramoto-Vicsek

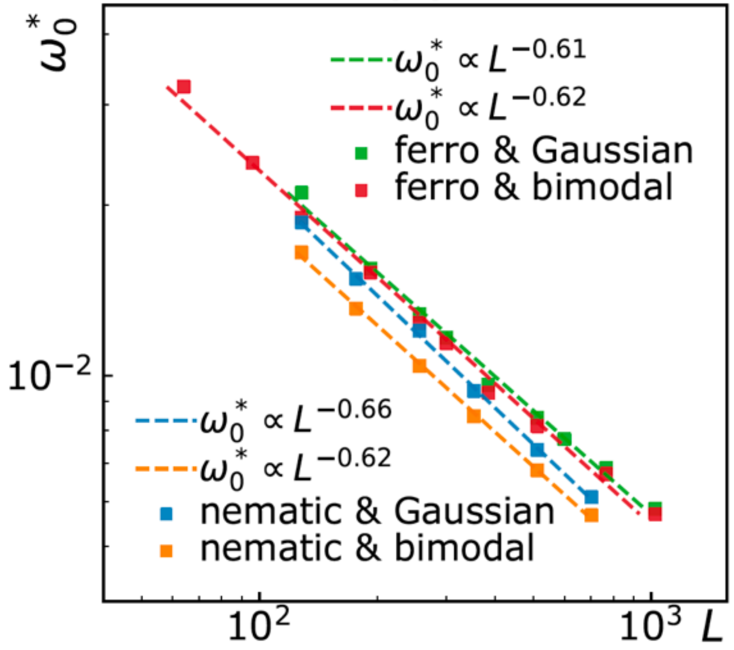
The answer is no for the polar liquid (asymptotically)!

but coexistence phase (bands) can survive finite amount of disorder

phase diagram at fixed size, density



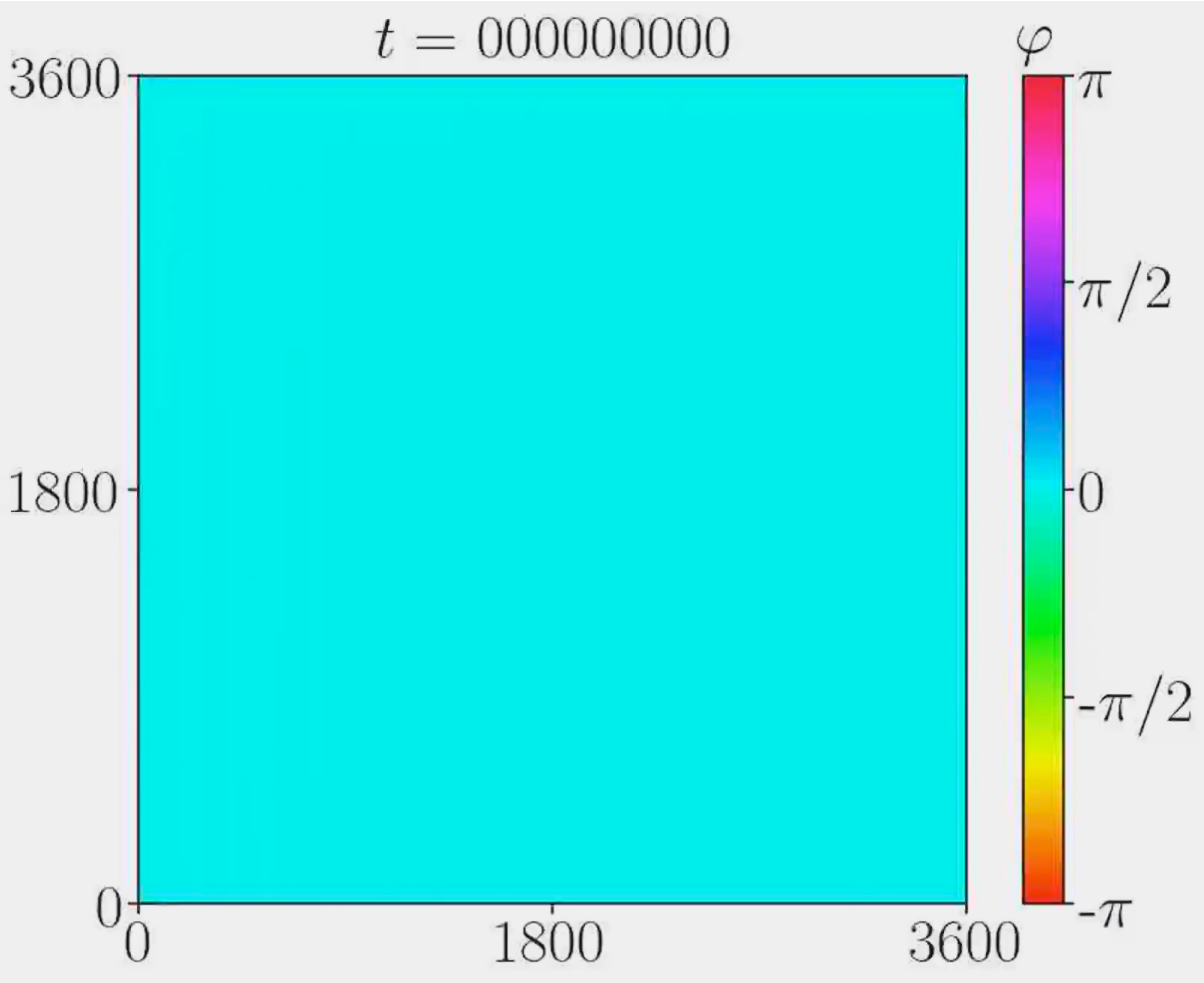
max ω_0 for existence of liquid vs L



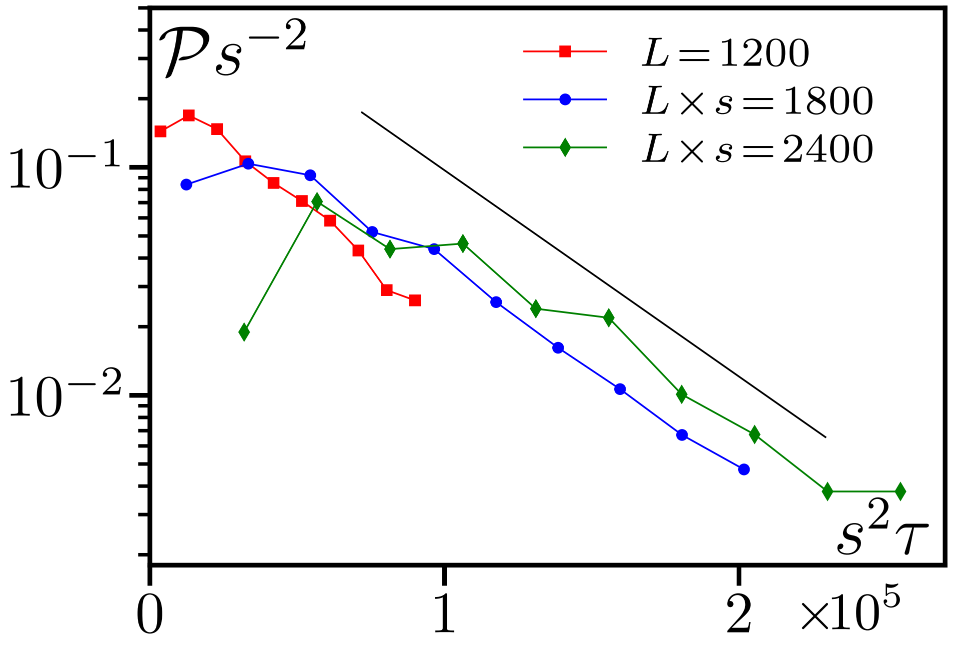
Same answer at hydrodynamic level due to linear instability of polar liquid

Metastability of constant-density flocks

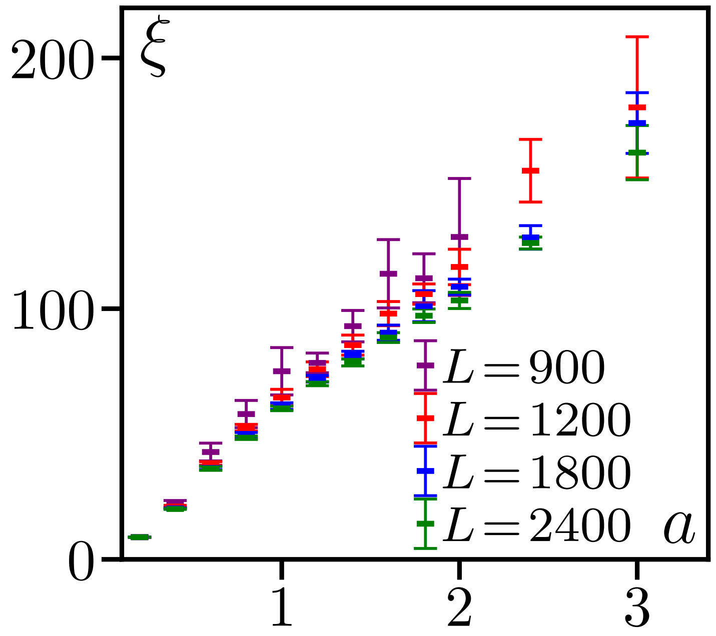
Simulation of Toner-Tu equation with constant density, noise, and no incompressibility



Metastability of constant-density flocks

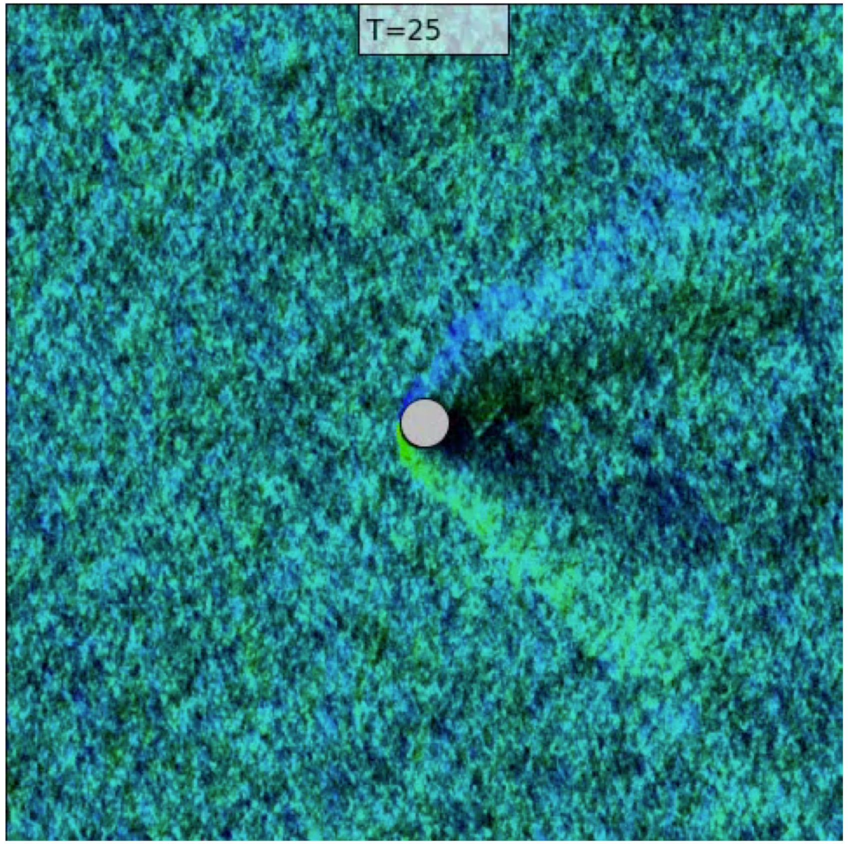


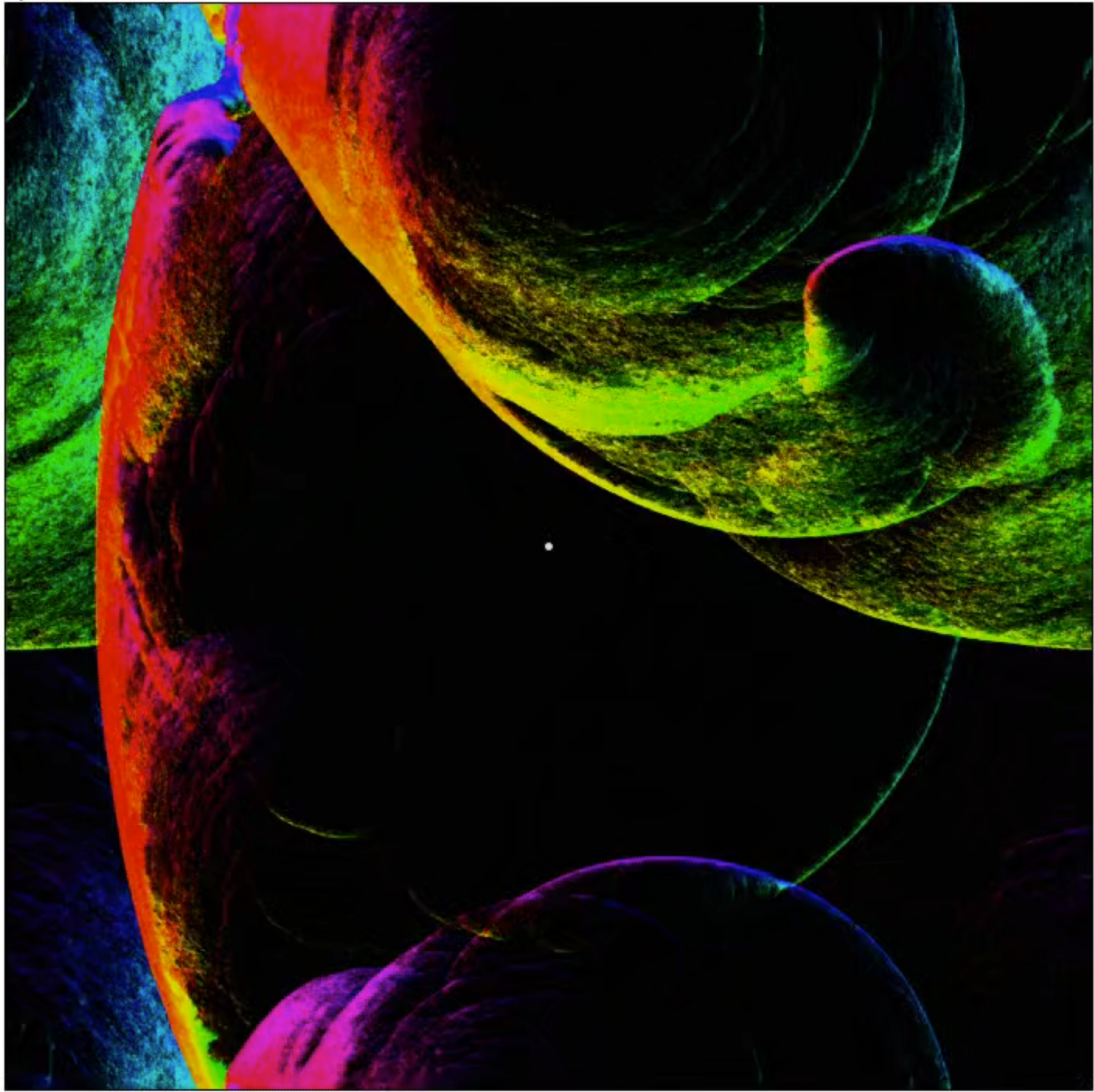
PDF of nucleation time scales like $1/L^2$, suggesting local event.



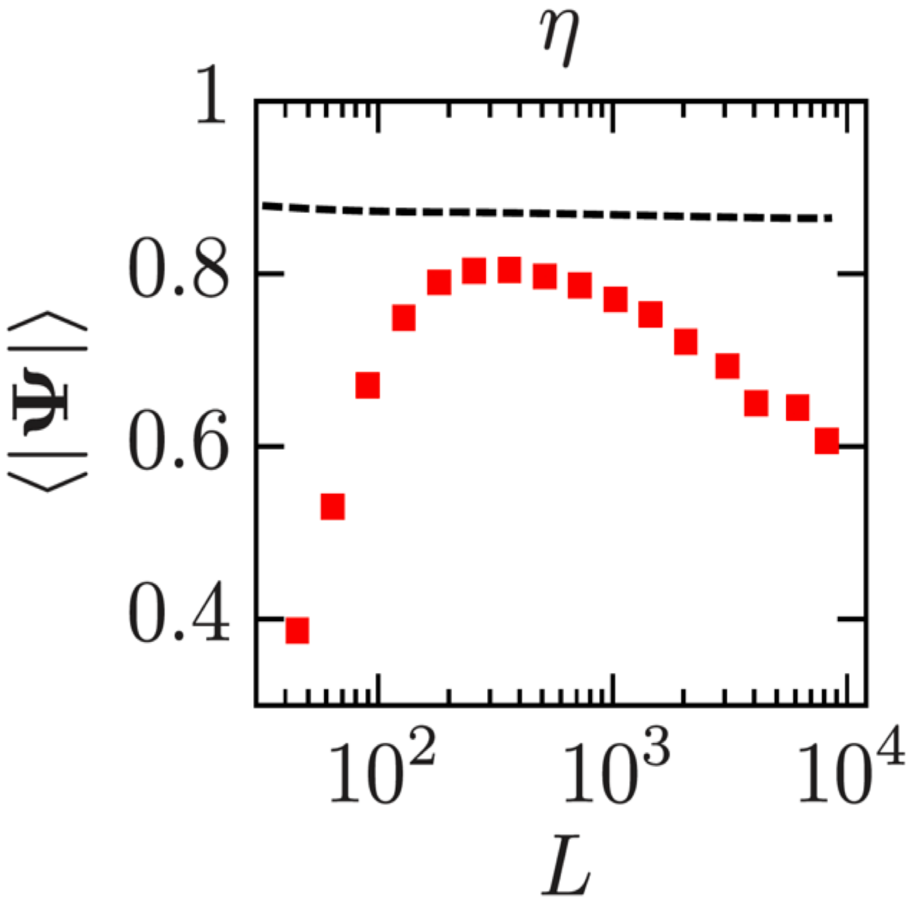
Correlation length in active foam steady state does not diverge as one goes deeper into ordered phase: metastability everywhere

Metastability of Vicsek flocks

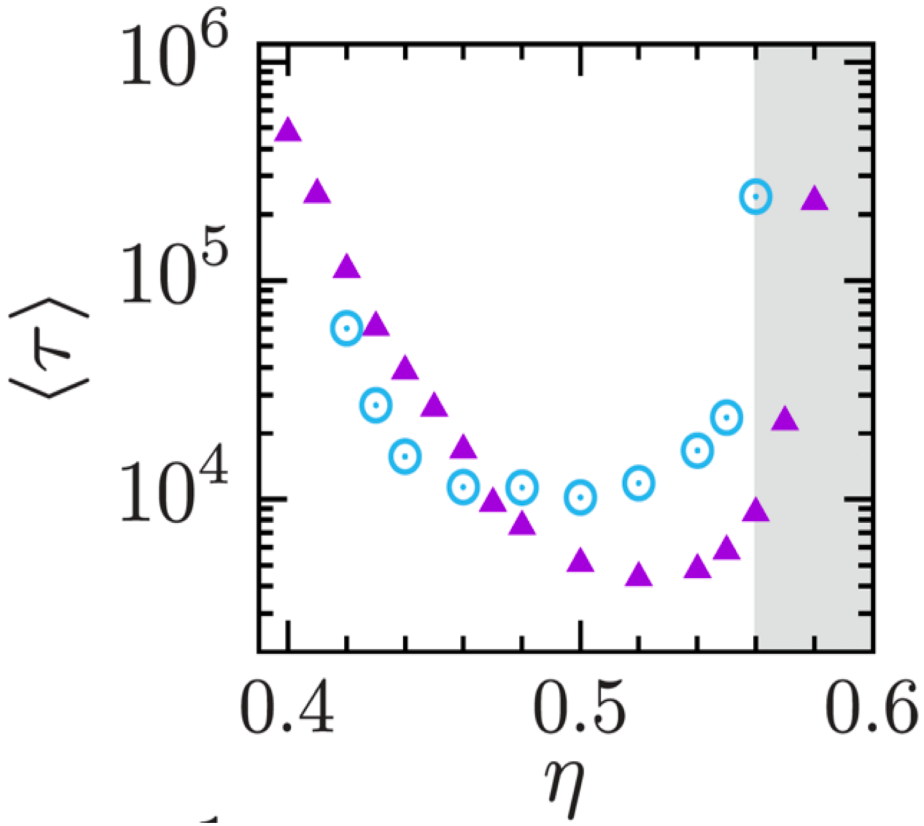




Metastability of Vicsek flocks

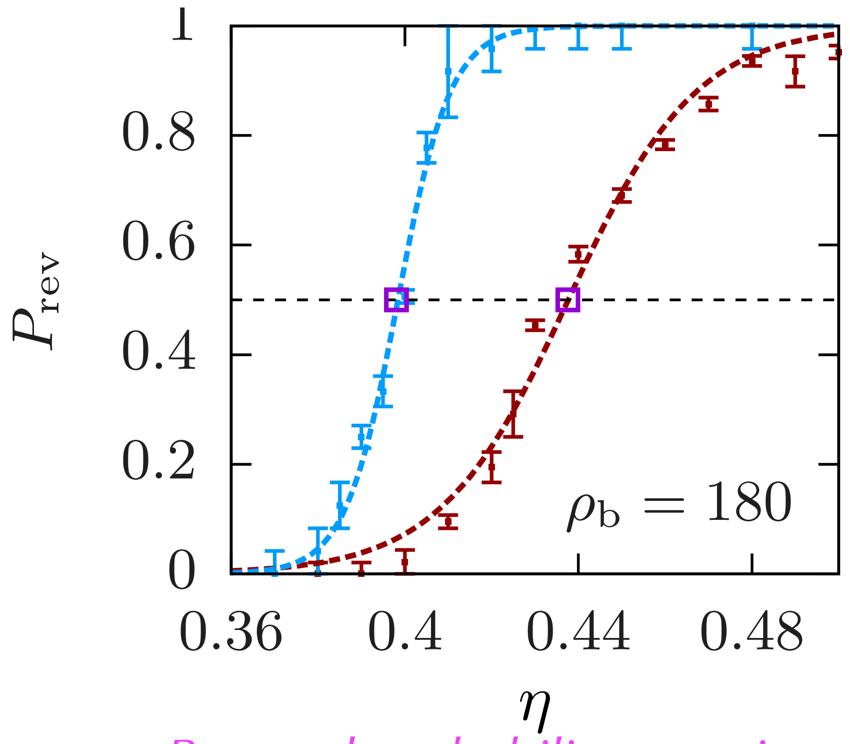


Increasing system size, order breaks down (less and less time to relax to liquid)

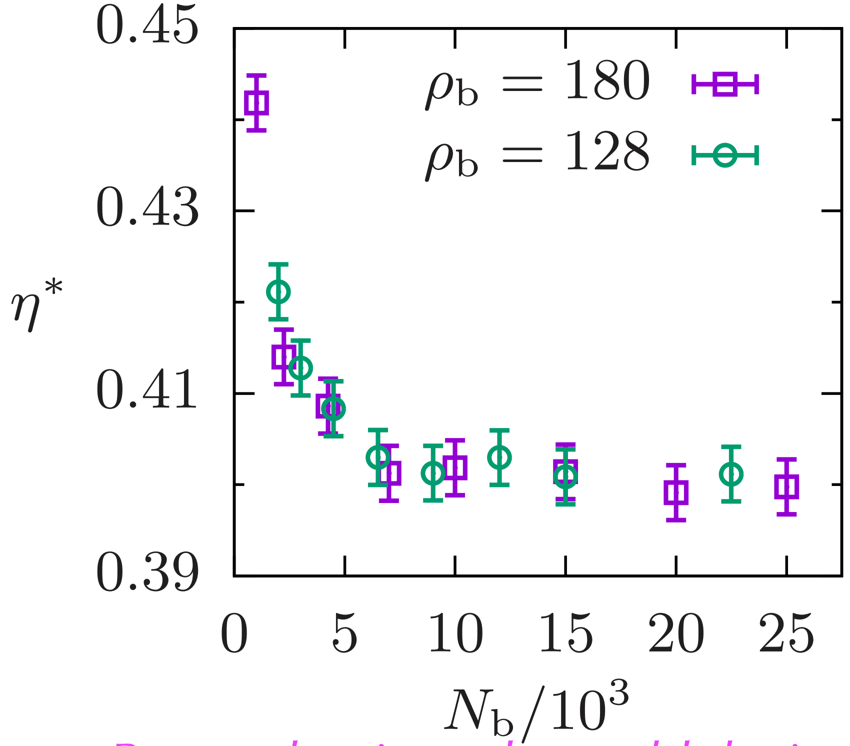


Mean time between reversals diverges when going deeper into liquid phase

Metastability of Vicsek flocks



Reversal probability vs noise



Reversal noise value vs blob size

- Flow reversals when introducing a blob of particles going against the main flow ➛ metastability
- Same results in low-order truncation of Boltzmann eq, but not in standard TT hydrodynamic theory

Summary

- Homogeneous polar flocks both fragile and metastable
- Still major surprises in simple active matter models
- Beware of perturbative arguments / RG methods
- Other cases?

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