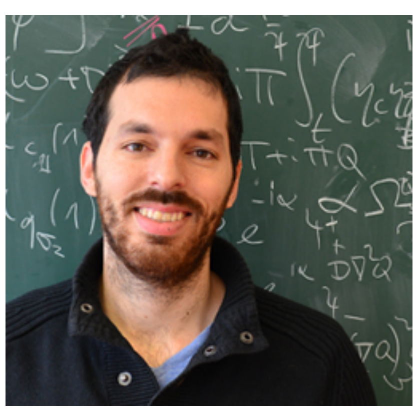
YITP-YSF Symposium
Perspectives on Non-Equilibrium
Statistical Mechanics
August 03, 2023
Kyoto, Japan (via zoom)

Universität zu Köln

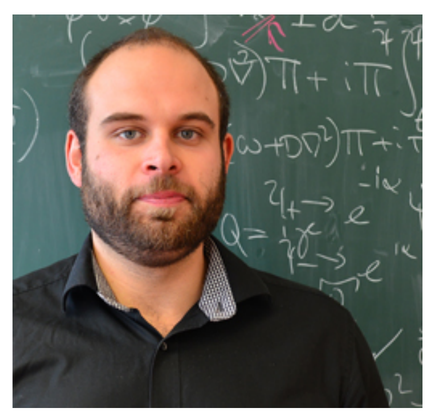
Measurement induced phase transitions: New perspectives for non-equilibrium statistical mechanics?

Sebastian Diehl

Institute for Theoretical Physics, University of Cologne



Ori Alberton

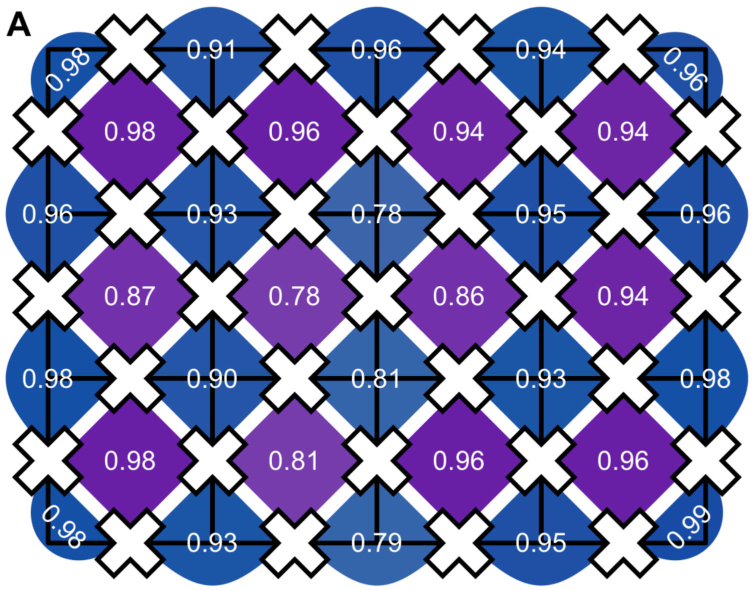


Michael Buchhold

Thomas Müller

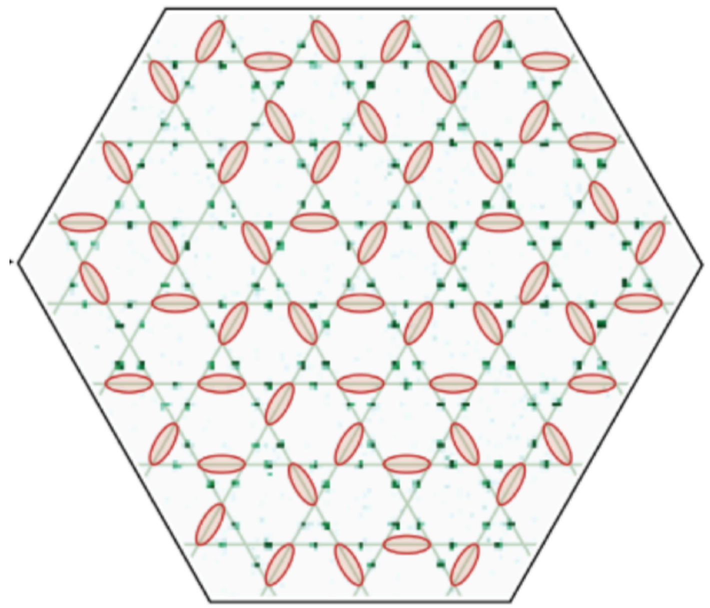
Motivation

Quantum devices / NISQ (noisy intermediate scale quantum) platforms



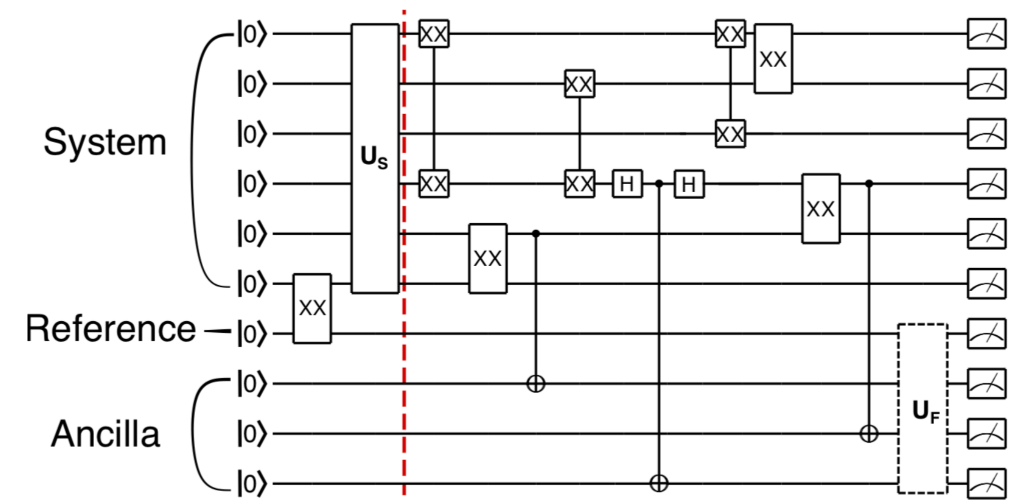
superconducting circuits

K. Satzinger et al.,
Science (2021)



Rydberg tweezers

G. Semeghini et al.,
Science (2021)



trapped ions

C. Noel et al.,
Nature Physics (2022)

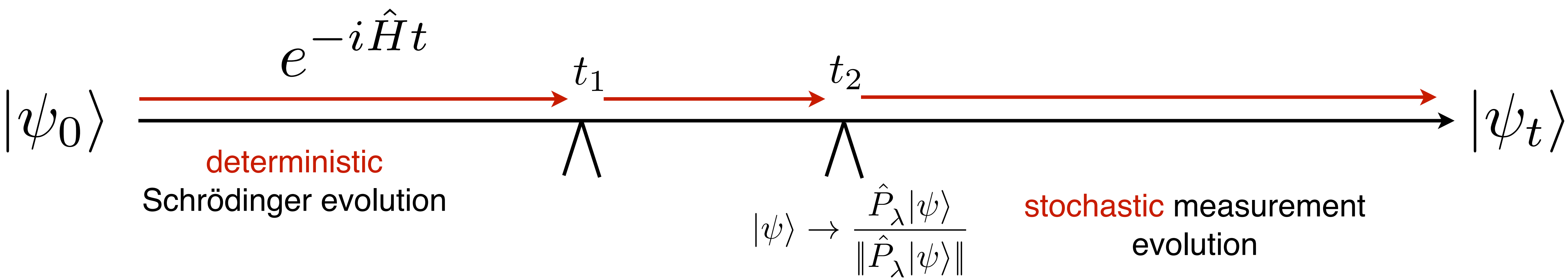
➡ realize many-body driven open quantum dynamics: interplay of unitary evolution, decoherence/dissipation, measurements

➡ require concepts and tools from (quantum) non-equilibrium statistical mechanics

Measurements in many-body systems

Small quantum systems: Measurements

- two types of quantum dynamics



for measurement observable $\hat{M} = \sum_\lambda m_\lambda |\lambda\rangle\langle\lambda| \equiv \sum_\lambda m_\lambda \hat{P}_\lambda$

- dynamics non-trivial (eigenstates not shared) once $[\hat{H}, \hat{M}] \neq 0$

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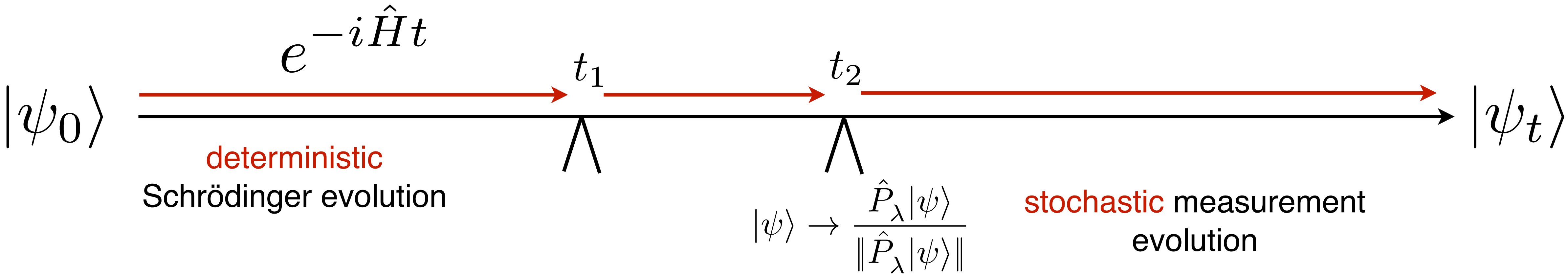

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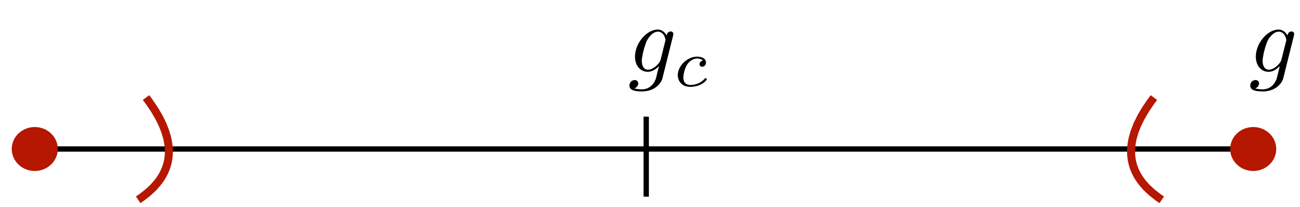
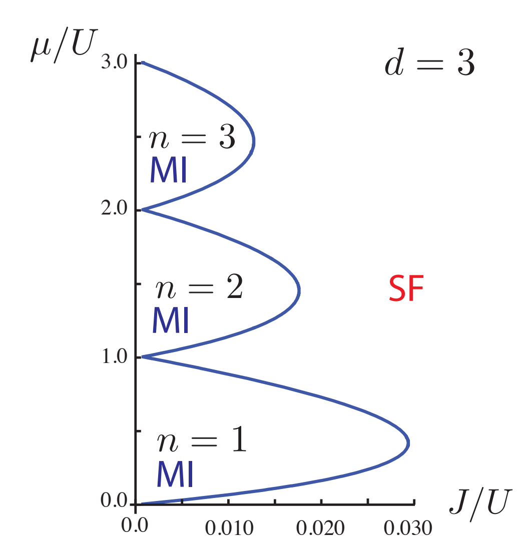
Many-body systems: Phase transitions

- non-commuting operators lead to (quantum) phase transitions

$$\hat{H} = \hat{H}_1 + g\hat{H}_2 \qquad [\hat{H}_1, \hat{H}_2] \neq 0$$





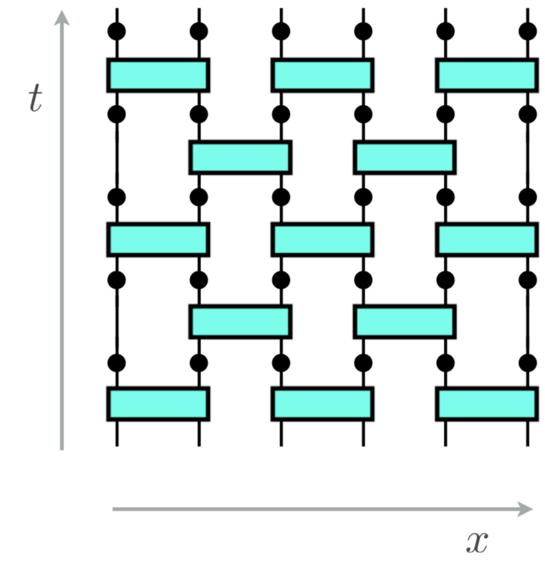
e.g. Mott-insulator to superfluid transition in cold atoms

➡ combine measurement and many particles: similar scenario?

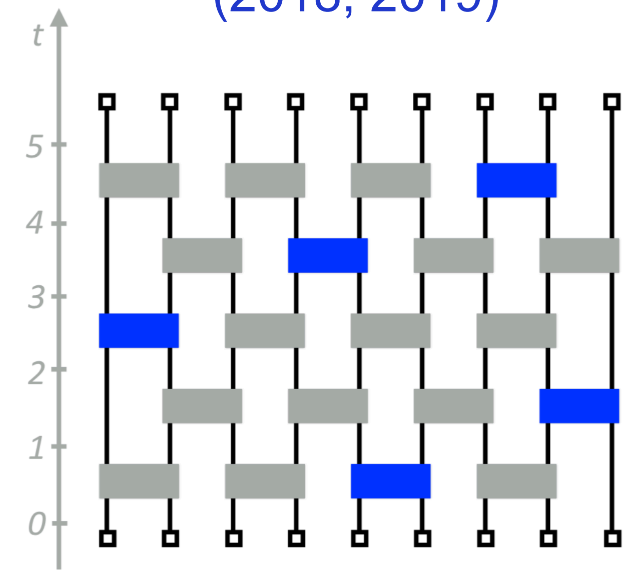
Entanglement Phase Transitions in Random Circuits

- model and key ingredients:
  - randomly chosen local entangling unitary gates (random circuit)
  - projective local measurement of non-commuting observables

Skinner, Ruhman, Nahum PRX (2019)



Li, Chen, Fisher, PRB (2018, 2019)



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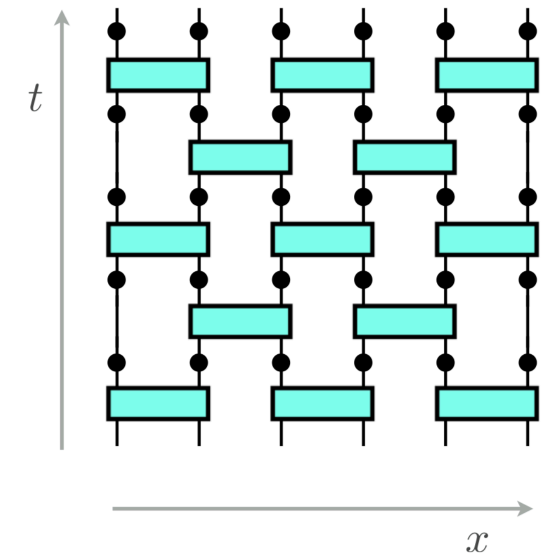

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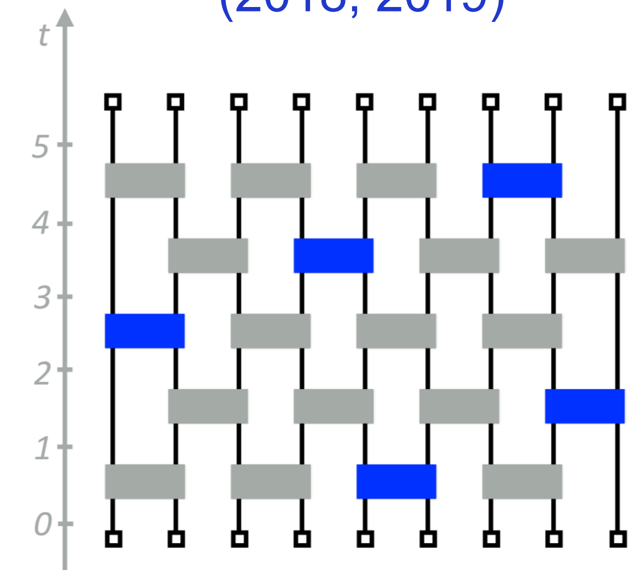

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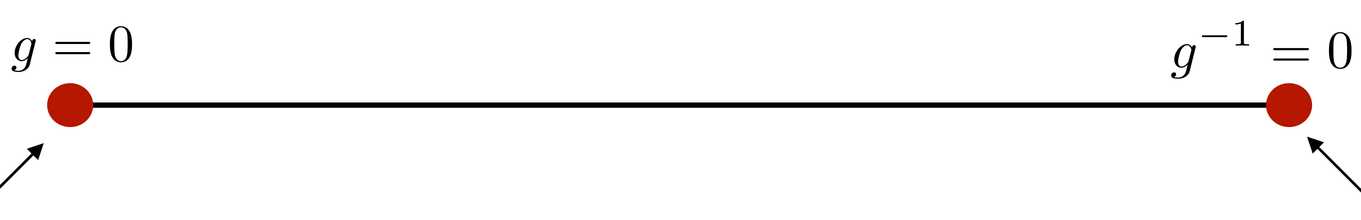
- basic picture: competition in many-body context (measure σ_i^z)



$$g = \frac{\text{\# measurements/time}}{\text{\# unitaries/time}}$$

chaotic dynamics

➡ volume law entanglement growth

product state $\prod_i |\sigma_i\rangle \quad \sigma_i = \uparrow, \downarrow$

➡ area law entanglement growth

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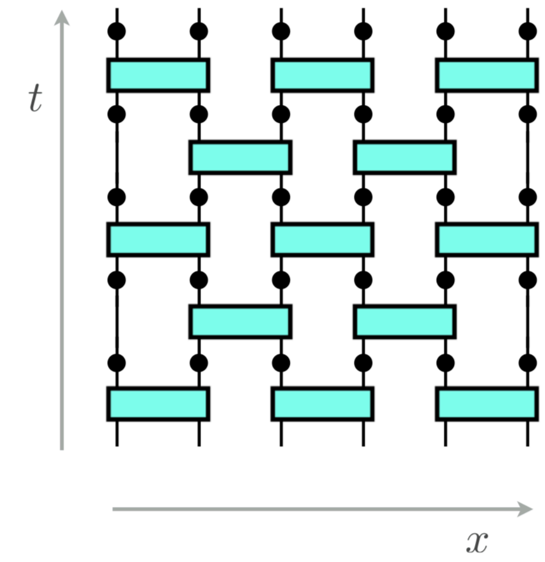

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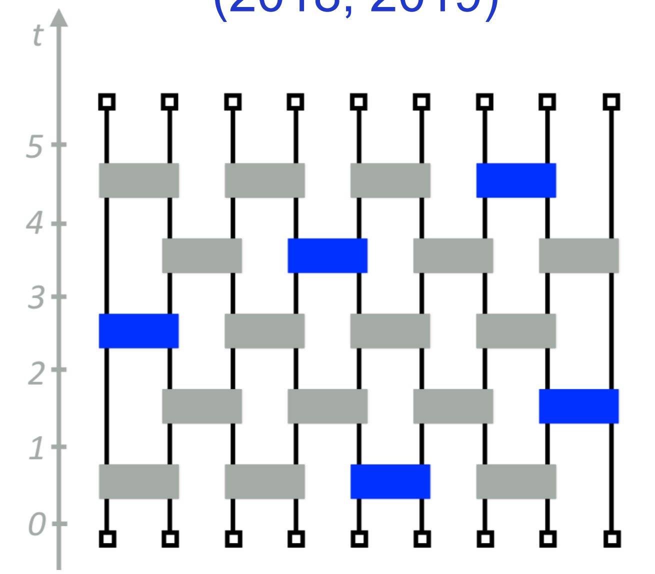

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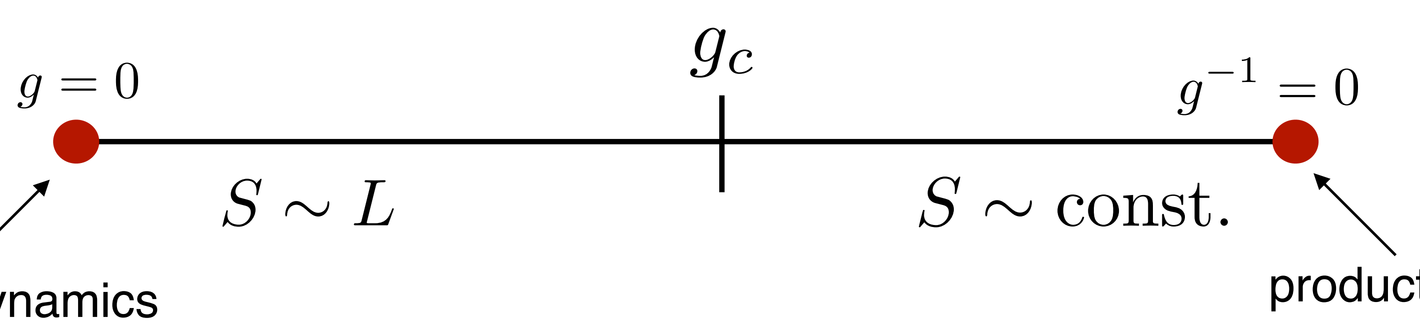

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➡ volume law entanglement growth

➡ area law entanglement growth

➡ Phase transition in entanglement growth at finite competition ratio g

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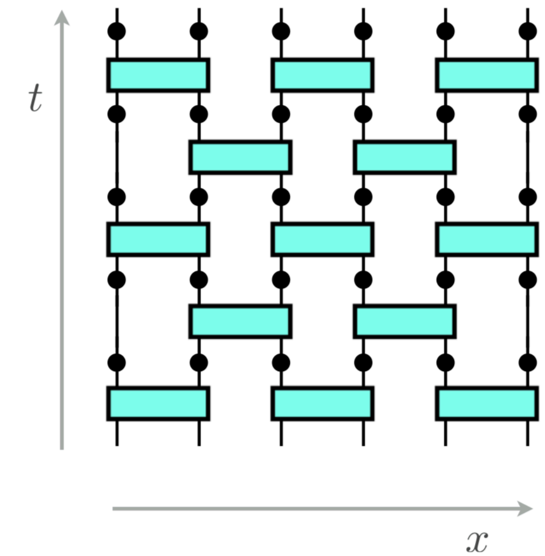

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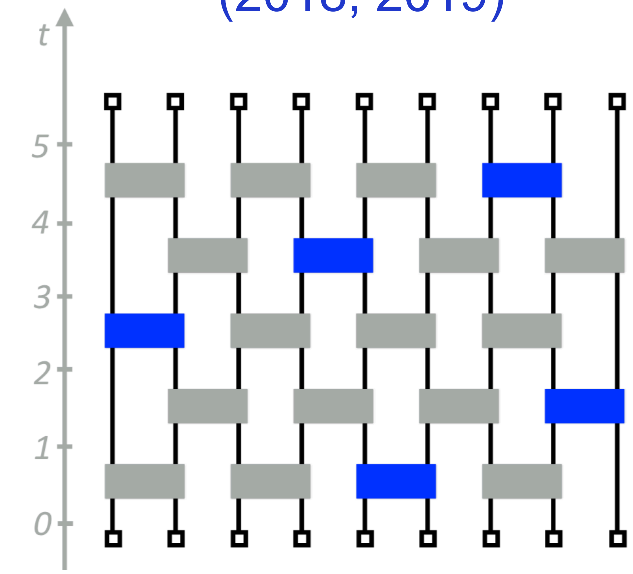

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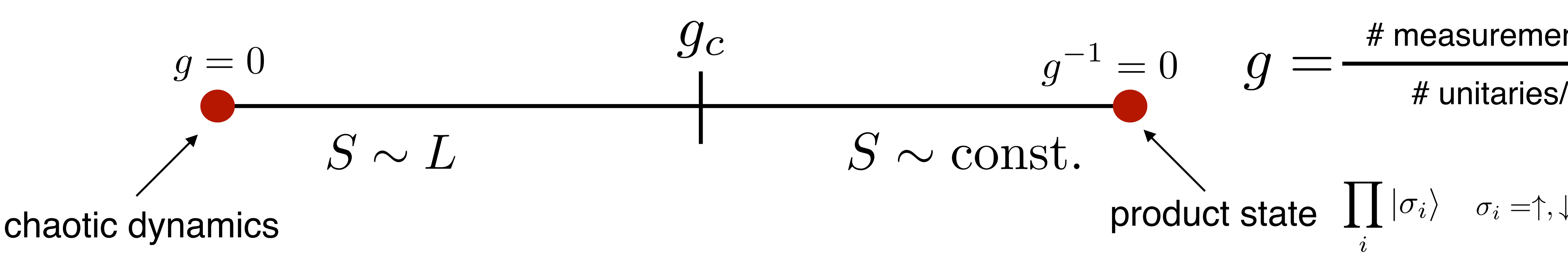

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➡ volume law entanglement growth

➡ area law entanglement growth

- Physical pictures
  - quantum error correction — Choi, Bao, Qi, Altman, PRL (2020); Fan et al. PRB (2021); Li Fisher PRB (2021)
  - purification transition — Gullans, Huse, PRX (2020); Gopalakrishnan, Gullans PRL (2021)
  - mapping to stat mech models — Jian, You, Vasseur, Ludwig, PRB (2020), Nahum et al., PRX Quantum (2021)

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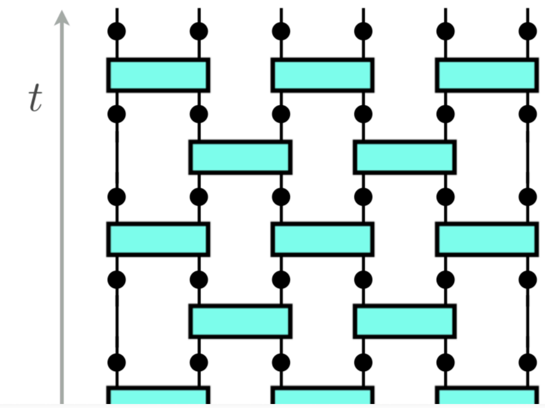

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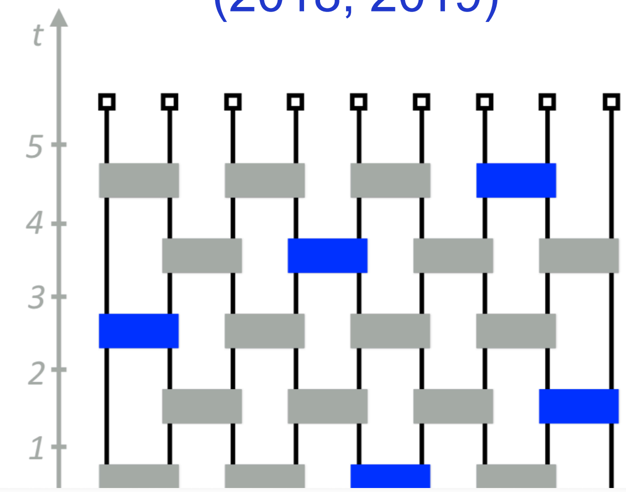

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Outline

- Entanglement transition: from critical phase to area law
- Physical understanding: mapping to field theory
- Observability: Pre-selection & connection to absorbing states

chaotic dynamics

product state $\prod_i |\sigma_i\rangle$ $\sigma_i = \uparrow, \downarrow$

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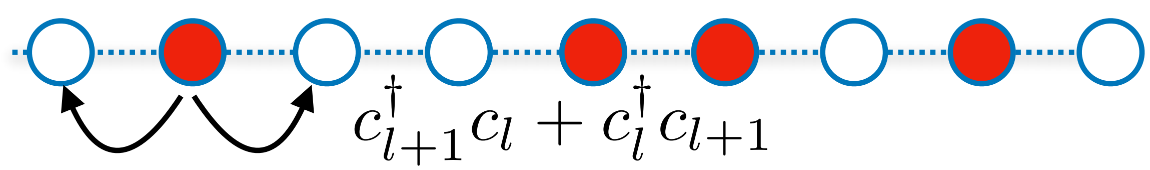
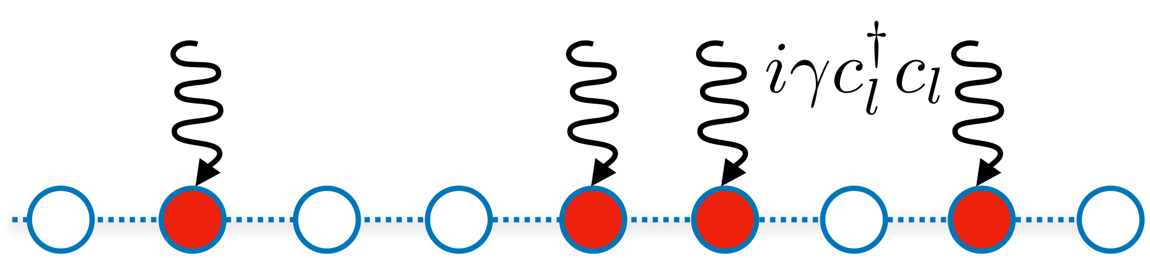
Entanglement Phase Transition in a Monitored Free Fermion Chain

Hamiltonian:

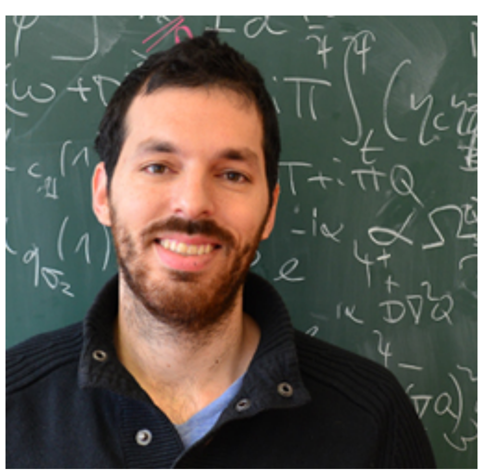


entanglement growth



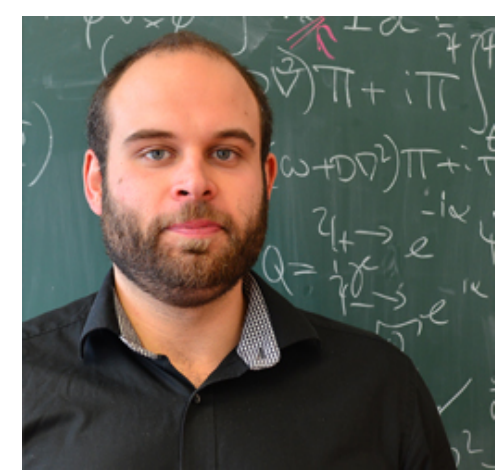
Monitoring:

entanglement saturation



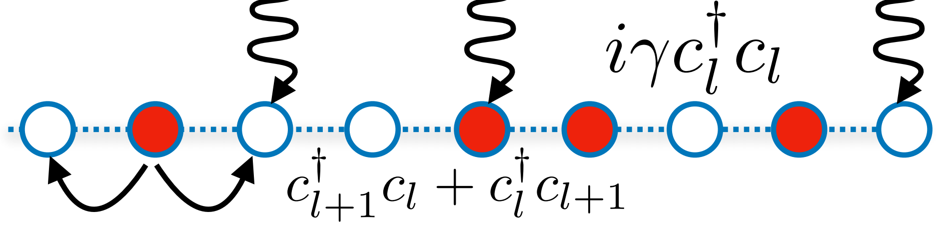
Ori Alberton

O. Alberton, M. Buchhold, SD,
PRL 126, 170602 (2021)



Michael Buchhold

Monitored Fermion Dynamics



- Weak continuous measurements: Stochastic Schrödinger Eq.

Belavkin, Phys. Lett A (1989); Gisin, Percival, JPA (1993)

$$d|\psi_t\rangle = dt(-i\hat{H} - \frac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l dW_l \hat{M}_l|\psi_t\rangle$$

(dW_l: Gaussian white noise)

related formulations:
Dalibard, Castin, Mølmer, PRL (1992)
Carmichael, PRL (1993)

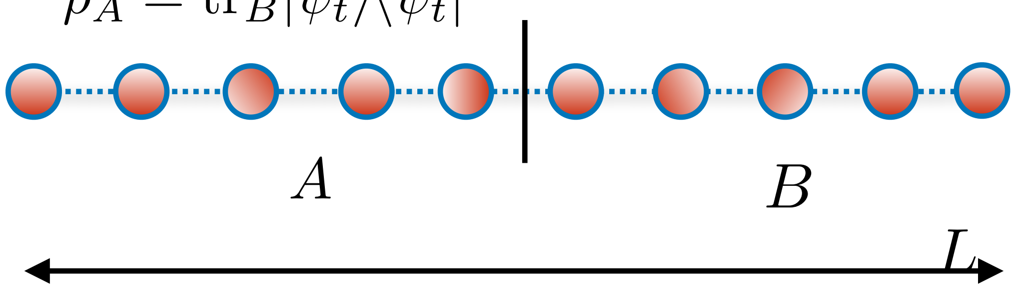
- unitary dynamics: hopping

$$H = -J\sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l\right)$$

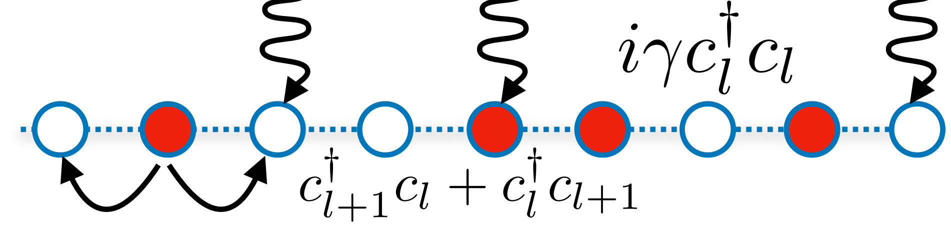
➡ volume law entanglement entropy

$$S_{vN}(L/2, L) = \mathrm{tr}\rho_A \log(\rho_A) \overset{t\to\infty}{\sim} L$$

$$\rho_A = \mathrm{tr}_B|\psi_t\rangle\langle\psi_t|$$



Monitored Fermion Dynamics



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- H = 0: continuous collapse into dark state

$$\hat{M}_l|\psi_t\rangle = 0 \quad \text{for} \quad \hat{n}_l|\psi_t\rangle = n_l|\psi_t\rangle$$

eigenstate of measurement operator

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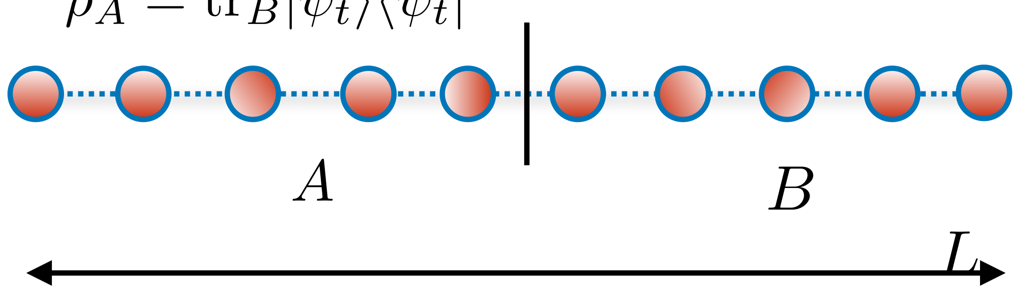
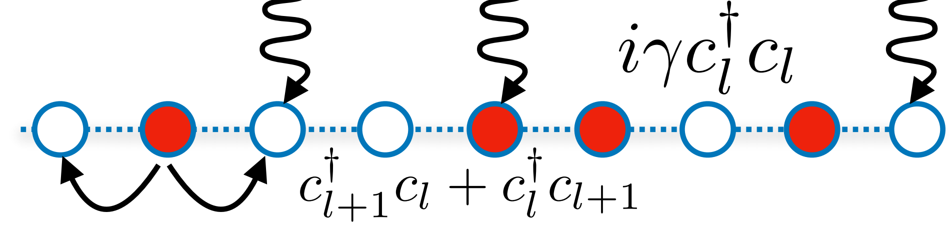


➡ area law entanglement entropy

$$S_{vN}(L/2, L) = s_0$$

Monitored Fermion Dynamics



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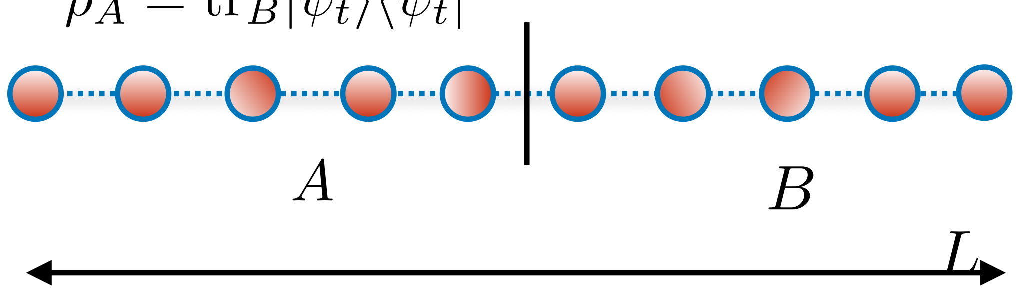

➡ area law entanglement entropy

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- caveat: $|\psi_{t\to\infty}\rangle$ is a random variable

➡ consider trajectory ensemble to extract information

Monitored Fermion Dynamics: Extracting Information

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- each wave function is a random object —> ensemble average trivial in stationary state

$$\hat{\rho}_t = \overline{|\psi_t\rangle\langle\psi_t|} = \sum_{\text{trajectories}} |\psi_t\rangle\langle\psi_t| \sim \mathbf{1} \qquad \longleftrightarrow \qquad \partial_t\hat{\rho}_t = -i[\hat{H},\hat{\rho}_t] - \gamma\sum_l[\hat{n}_l,[\hat{n}_l,\hat{\rho}_t]]$$

state projector

maximally mixed / infinite temperature (configuration entropy) — Lindblad quantum master equation

- standard quantum mechanical observable do not yield information

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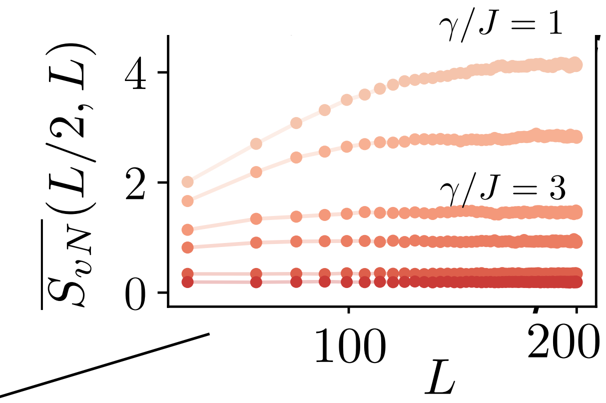
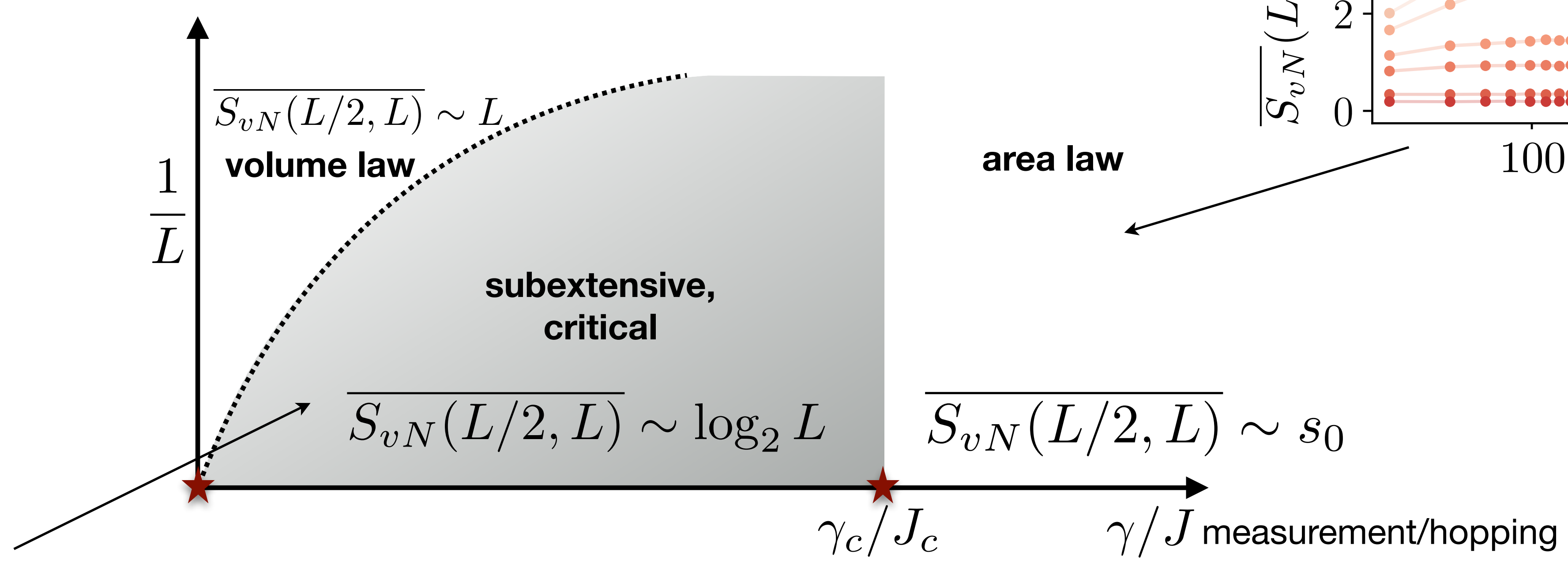
- use state dependent observables instead:

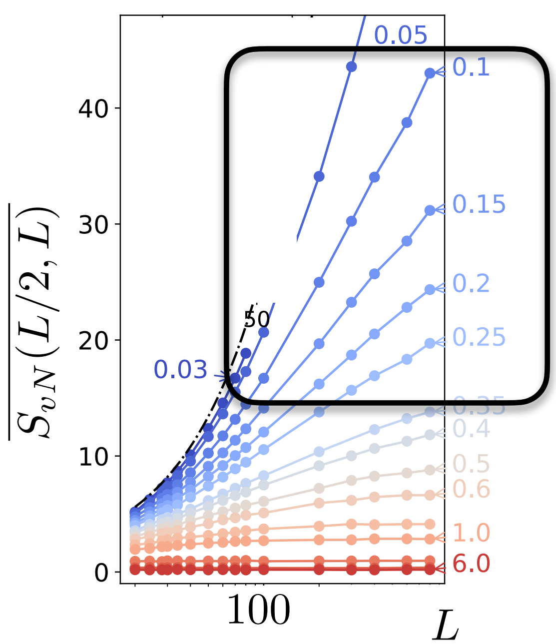
$$F(\overline{\hat{\rho}}) \neq \overline{F[\hat{\rho}]}$$

- examples:
  - von Neumann entropy $\overline{S_{vN}(l,L)} = \overline{\langle\log(\rho_A)\rangle}$ — arbitrarily high power of state projector
  - correlation function $\overline{\langle\hat{n}_i\rangle\langle\hat{n}_j\rangle}$ — quadratic in state projector

Trajectory Ensemble Phase Diagram







➡ extended critical phase at intermediate monitoring

critical phase but no transition for non-unitary circuits: Chen, Li, Fisher, Lucas PRR (2020)

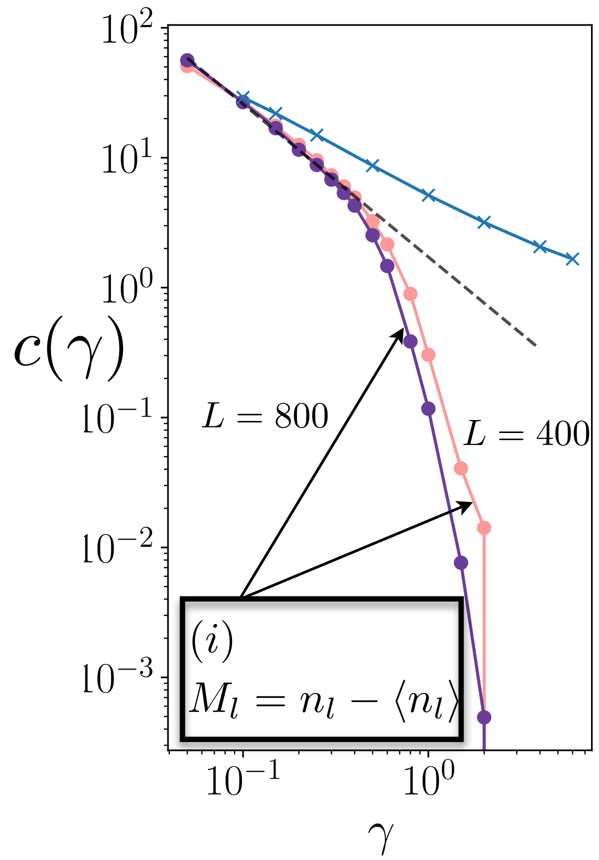
Characterizing the Weak Monitoring Phase & Phase Transition

- effective central charge $c(\gamma)$

$$\overline{S_{vN}(l,L)} = \frac{c(\gamma)}{3}\log_2\left[\frac{L}{\pi}\sin\left(\frac{\pi l}{L}\right)\right] + s(\gamma)$$

parameter dependent c

random systems: Cardy Jacobsen PRL (1997);
Refael, Moore PRL (2004)



➡ sudden jump reminiscent of BKT

measurement-induced BKT:
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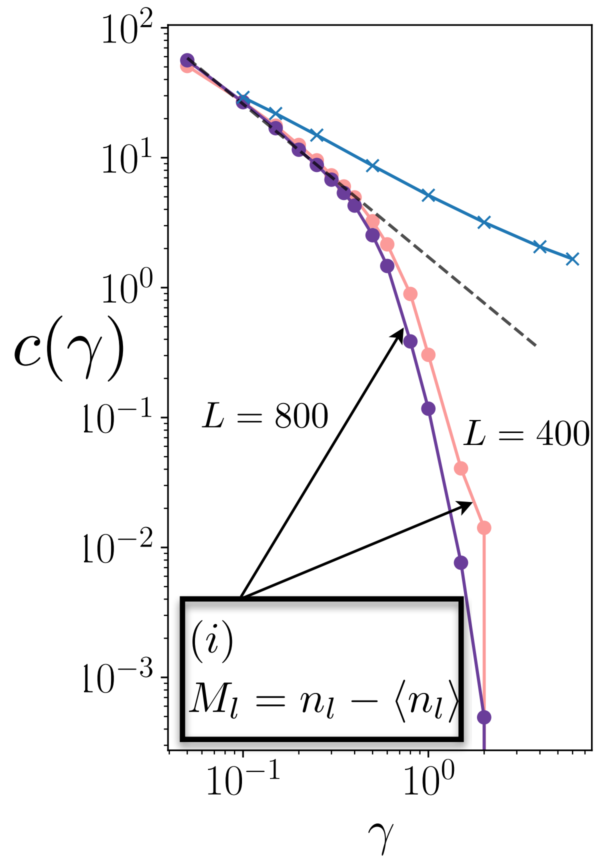

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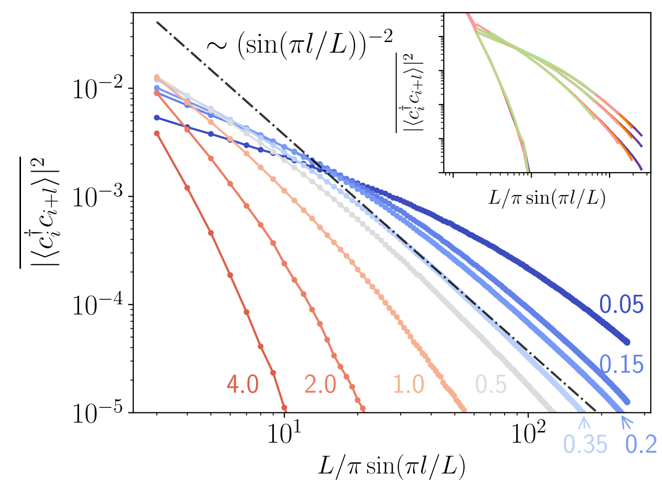
measurement-induced BKT:

Bao, Choi, Altman, Annals of Physics (2021)

- extended criticality: Connected correlation function

$$C_{i,i+l} = \overline{\langle \hat{n}_i \rangle \langle \hat{n}_{i+l} \rangle} - \overline{\langle \hat{n}_i \hat{n}_{i+l} \rangle}$$

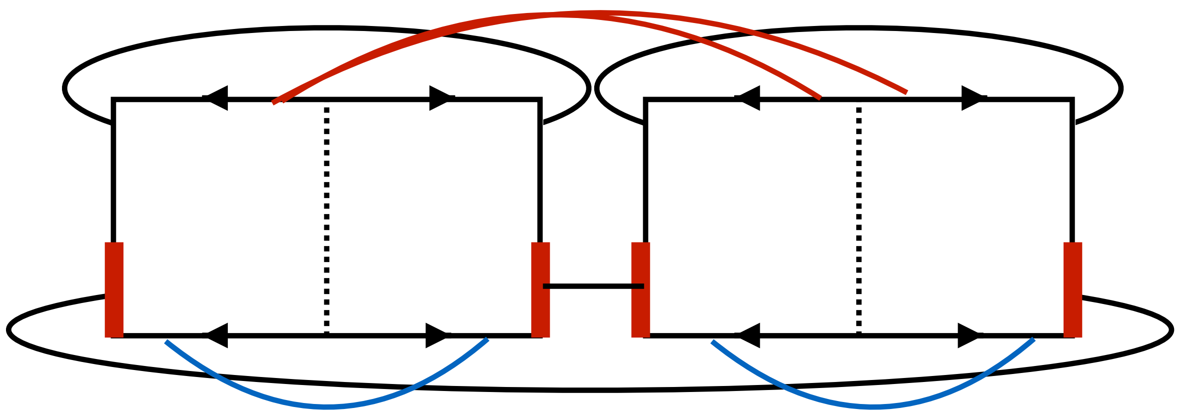
$$\sim l^{-2}$$

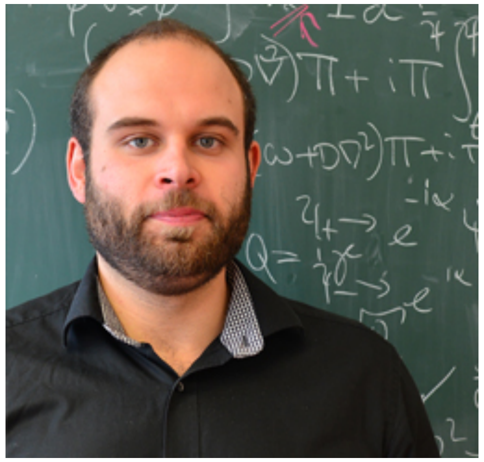


$$C_{i,i+l} \sim \begin{cases} 0 & \text{for } H = 0 \\ \exp(-l/\xi) & \text{for } \gamma \gg J \\ l^{-2} & \text{for } \gamma \ll J \\ l^{-1} & \text{for } \gamma = 0 \end{cases}$$

➡ correlation functions equally characterize the transition

Effective Replica Field Theory
for
Measurement Induced Phase Transitions



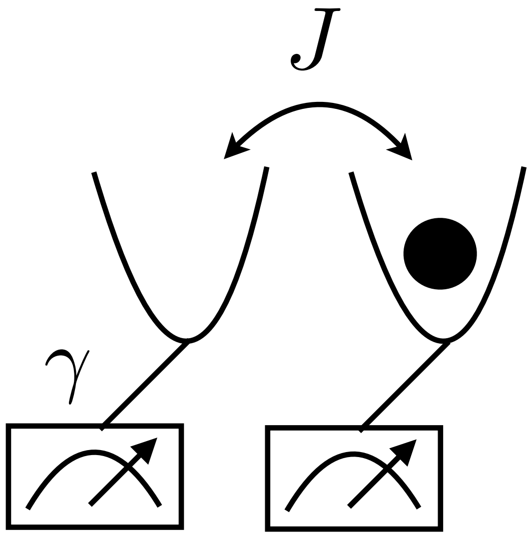


Michael Buchhold

M. Buchhold, Y. Minoguchi, A. Altland, SD, PRX 11, 041004 (2021)

microphysics → macrophysics

Pinning picture: Toy model



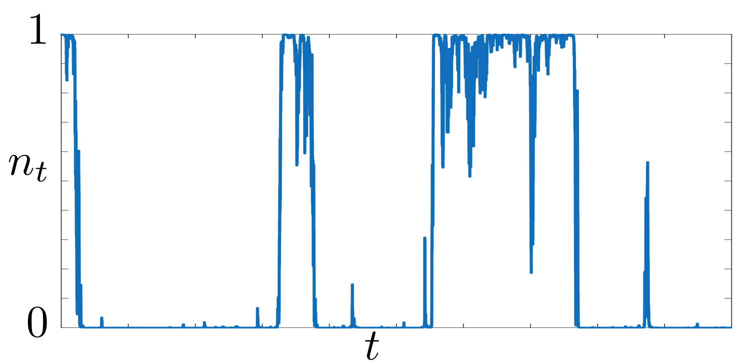
- toy model: trajectory evolution of single fermion on two sites

$$|\psi_{t+dt}\rangle = |\psi_t\rangle - idt\hat{H}_{\text{eff}}|\psi_t\rangle + \sum_{l=1}^{2} dW_l \left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)|\psi_t\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hat{K} \qquad \hat{H} = -J\left(c_1^\dagger c_2 + h.c.\right) \qquad \hat{K} = \frac{\gamma}{2}\sum_{l=1}^{2}\left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)^2$$

➡ H=0: collapse into dark state at long times $\hat{n}_l|\psi_t\rangle = \langle \hat{n}_l \rangle |\psi_t\rangle \Longrightarrow n_l = 0, 1$

- strong monitoring $J/\gamma \ll 1$



➡ pinning to measurement eigenstate

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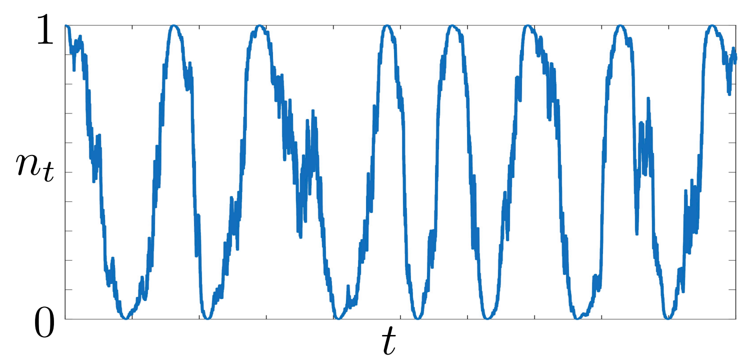

➡ vanishing time spent in eigenstate

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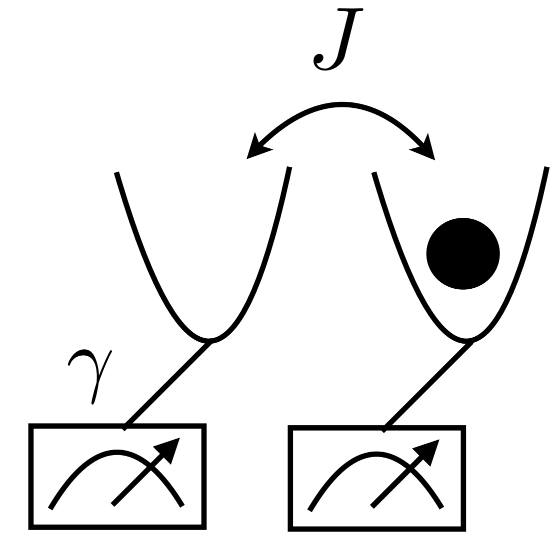

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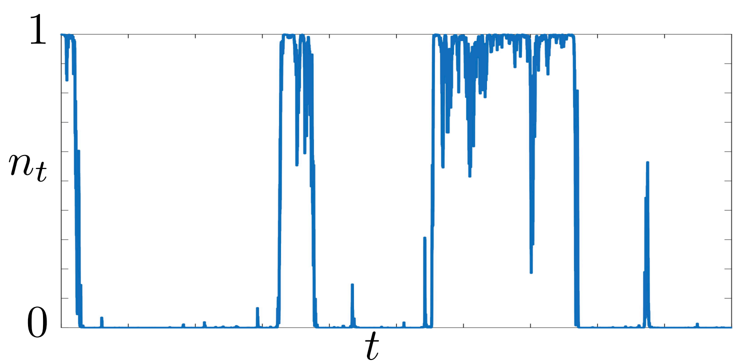

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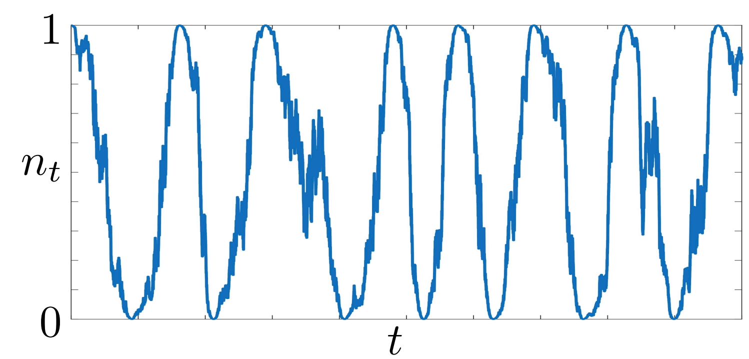

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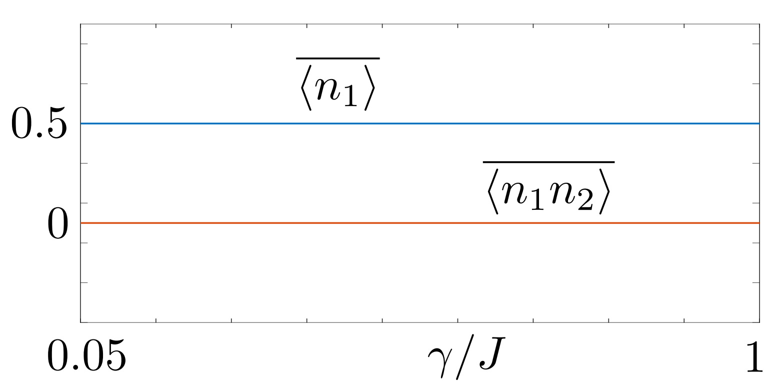
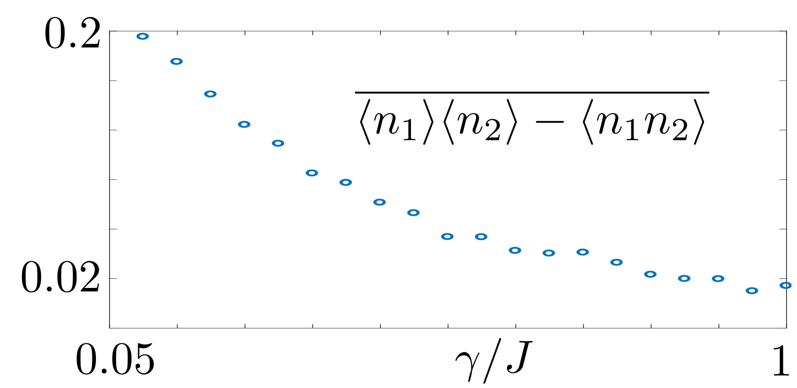
- invisible in linear averages



- seen in averaged trajectory covariance matrix



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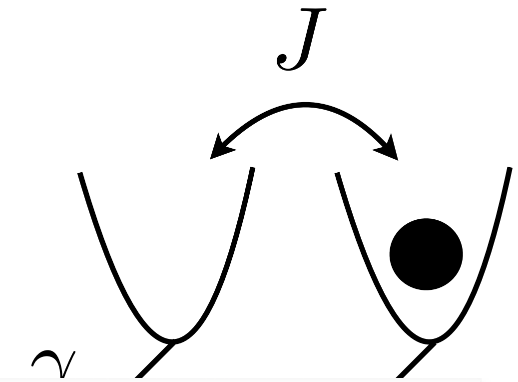

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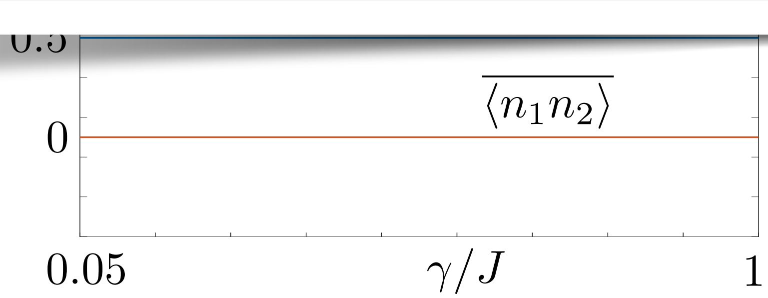
2

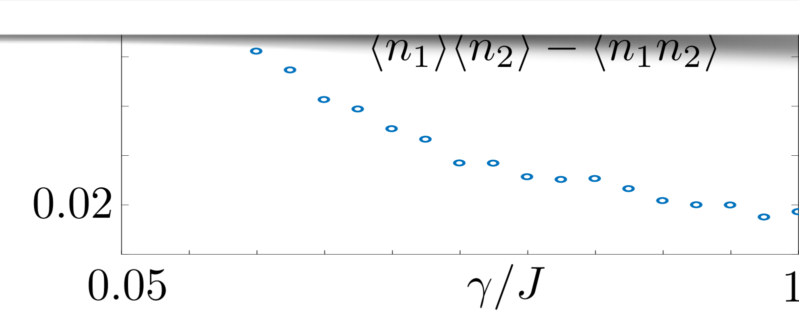
guiding picture and practical approach:

- thermodynamic limit: pinning quantum phase transition at sharply defined point
  - ➡ minimal continuum model in (1+1) dimensions
- signalled in state dependent ‘observables’, like the covariance matrix
  - ➡ replica construction

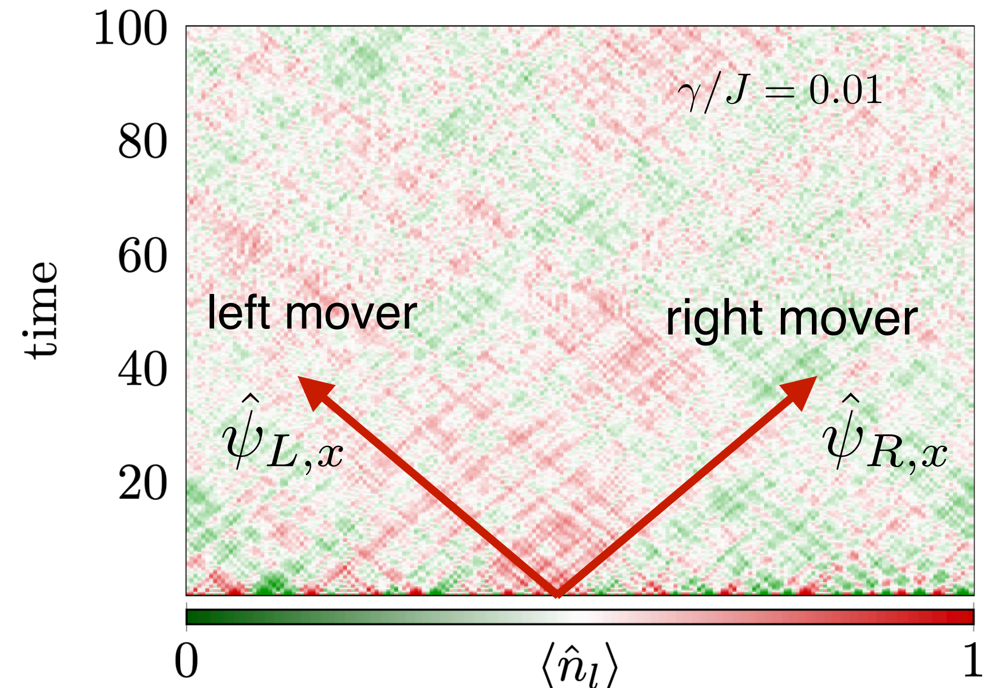
main insight:

➡ pinning transition in replica degrees of freedom in BKT universality class





Continuum (1+1) dimensional Model



Luttinger liquid

- model: phenomenologically motivated continuum limit for lattice fermions (weak measurement regime)

- Hamiltonian: massless Dirac fermions $\hat{\Psi}_x = (\hat{\psi}_{R,x}, \hat{\psi}_{L,x})^T$

$$\hat{H} = iv \int_x \hat{\Psi}_x^\dagger \sigma_z \partial_x \hat{\Psi}_x \longrightarrow \hat{H} = \frac{v}{2\pi} \int_x [(\partial_x \hat{\theta}_x)^2 + (\partial_x \hat{\phi}_x)^2]$$

phase density

fermionic variant

bosonized variant

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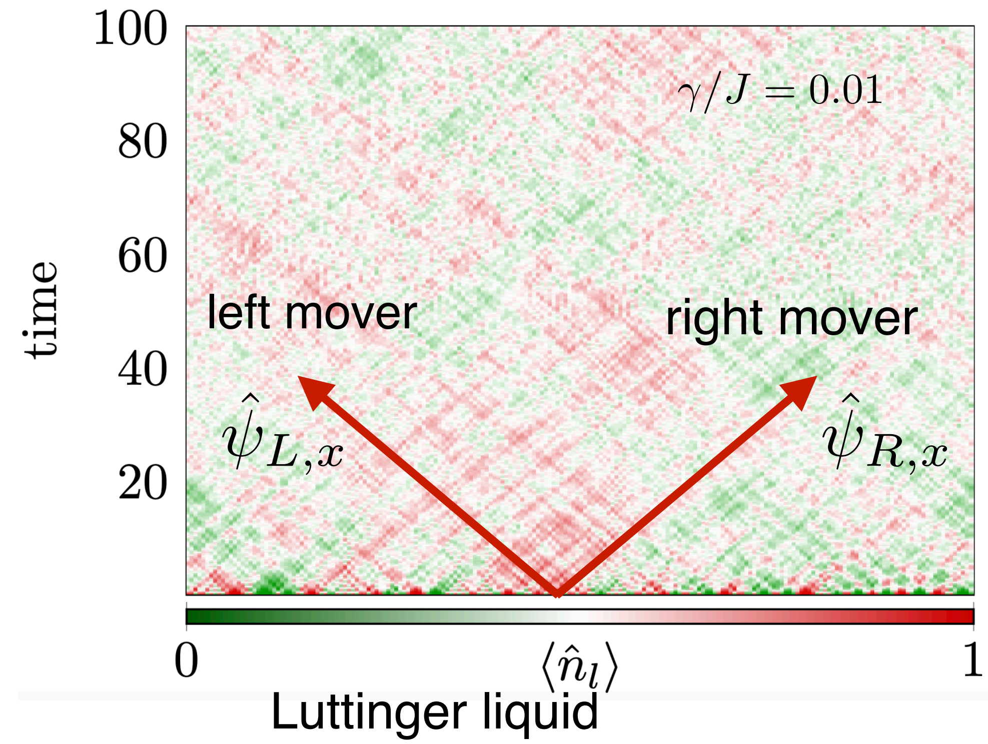

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phase density

fermionic variant bosonized variant

- measurement operators: current and vertex operators

rate γ_1 : $\hat O_{1,x} = \Psi^\dagger_x \Psi_x = \hat J^{(0)}_x \longrightarrow \hat O_{1,x} = -\frac{1}{\pi}\partial_x\hat\phi_x$ linear gapless

rate γ_2 : $\hat O_{2,x} = \Psi^\dagger_x \sigma_x \Psi_x \longrightarrow \hat O_{2,x} = m\cos(2\hat\phi_x)$ nonlinear

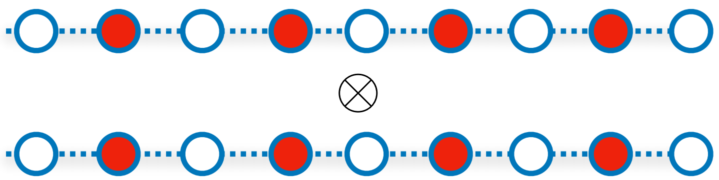
m: $\mathcal{O}(1)$

common eigenstates: $\hat\phi_x|\Psi_D\rangle = \phi_x|\Psi_D\rangle$

- stabilize product dark states: exactly local
- realize competition: do not commute with H (phase fluctuations)

Replica Approach (n=2)

- Access state-dependent observables, e.g. covariance matrix $C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$
- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$



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$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

➡ linear statistical average of replica density matrix

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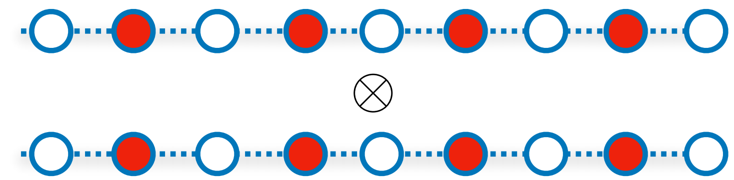

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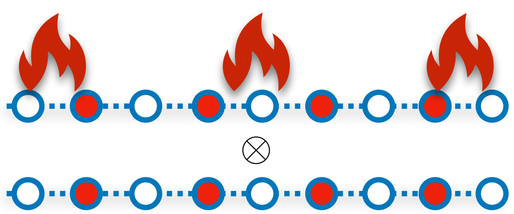

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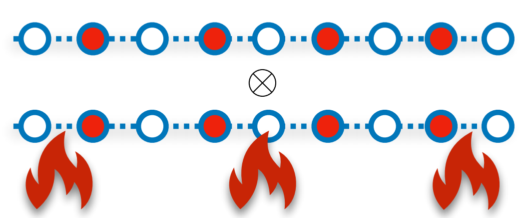

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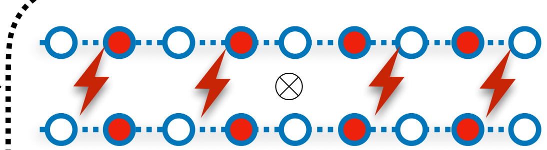

$$i[\rho^{2R}, H^{(\alpha)}] - \frac{\gamma}{2}[\hat{M}_x^{(\alpha)}, [\hat{M}_x^{(\alpha)}, \rho^{2R}]]$$

individual heating Lindbladians

$$\gamma\{\hat{M}_x^{(1)}, \{\hat{M}_x^{(2)}, \rho^{2R}\}\}$$

replica coupling

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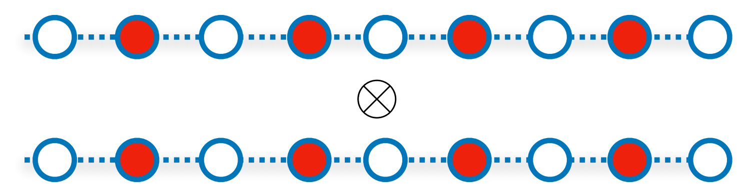

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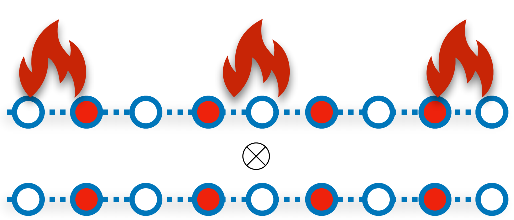

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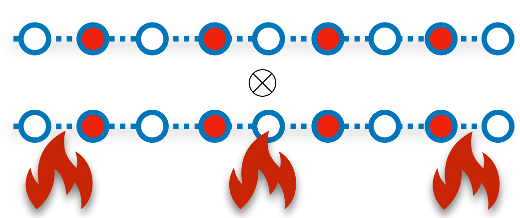

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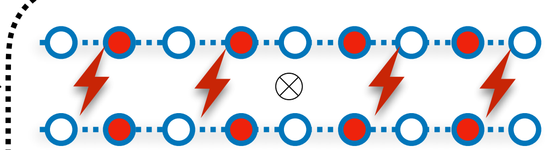

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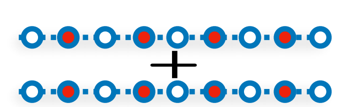
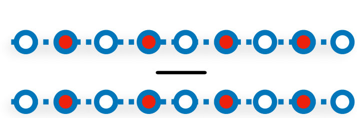
- New degrees of freedom



$: \hat{\phi}^{(a)} = \hat{\phi}^{(1)} + \hat{\phi}^{(2)}$ average coordinate



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Boson Replica Quantum Master Equation

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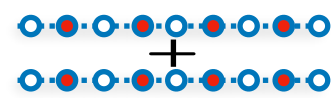

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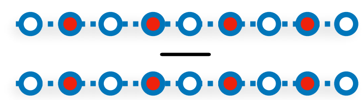

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➡ Master equation becomes separable (exact for Gaussian dynamics, useful more generally)

- Average coordinate: heating to infinite temperature

$$\partial_t \rho^{(a)} = i[\rho^{(a)}, H^{(a)}] + \frac{2\gamma}{\pi} \sum_l \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle}\right) \rho^{(a)} \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle}\right)$$

⟵ only jump term!

- Relative coordinate: cooling/damping into dark state

$$\partial_t \rho^{(r)} = i[\rho^{(r)}, H^{(r)}] - \frac{\gamma}{\pi} \sum_l \left\{ (\partial_x \hat{\phi}^{(r)})^2, \rho^{(r)} \right\}$$

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Boson Replica Quantum Master Equation

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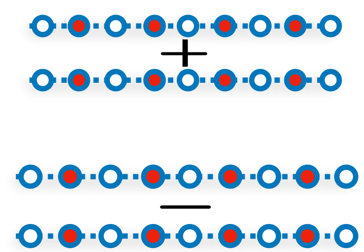

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- Present model: non-Hermitian Schrödinger equation for relative coordinate

$$\partial_t |\psi_t^{(r)}\rangle = -i H_{\text{eff}} |\psi_t^{(r)}\rangle$$

➡ cooling into dark state

$$H_{\text{eff}} = \frac{\nu}{2\pi} \int_x (\partial_x \hat{\theta})^2 + (1 - i\eta^2)(\partial_x \hat{\phi})^2 \; - i \frac{\gamma m}{\pi} \int_x [1 - \cos(\sqrt{8} \hat{\phi}_x)]$$

effect of non-linearity

➡ non-Hermitian Sine-Gordon: pinning via cos term, depinning via theta term

➡ extract physics in path integral approach

Phase diagram

➡ Sine-Gordon action with complex coefficients

Fendley, Saleur, Zamolodchikov,
International Journal of Modern Physics (1993)

$$S = \int_{t,x} \left\{ \frac{K}{16\pi} \left[\frac{1}{\eta} (\partial_t \phi)^2 - \eta (\partial_x \phi)^2 \right] - i\lambda \cos(\phi) \right\}$$

➡ RG flow: standard KT flow with complex K, λ

- flow modified at short distance

➡ shift of phase border

- standard BKT flow at long distance

➡ same long wavelength properties

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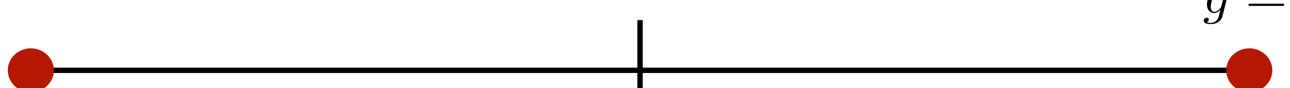
- resulting phase diagram:

$g = \gamma / J$



g_c

$C_{xy} = \frac{C(\gamma)}{|x-y|^2}$

➡ algebraic decay (generalized CFT)

➡ depinned phase

$C_{xy} \sim e^{-\frac{|x-y|}{\xi}}$ $\quad \xi^{-1} = \log(2\pi\gamma/v)$

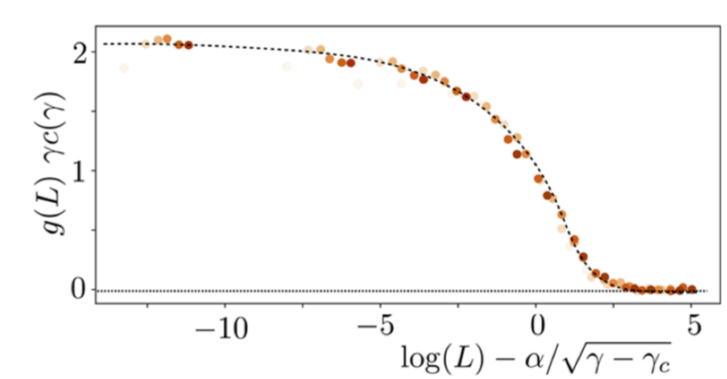
➡ exponential decay

➡ pinned phase

➡ phase transition in the BKT universality class

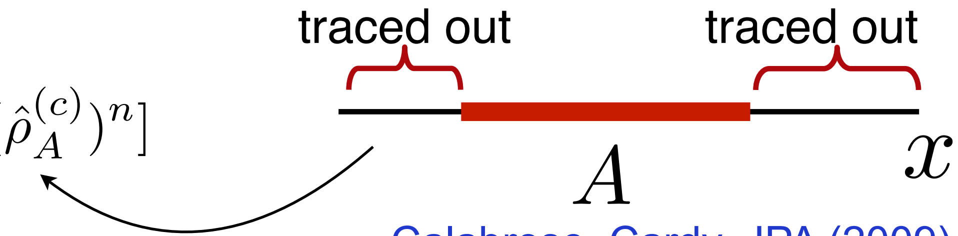
➡ all in line with numerics for lattice fermions

scaling collapse of c



Entanglement Entropies: n-Replica Keldysh approach

- Rényi entropy $S_n(L) = \frac{1}{1-n}\overline{\log Z_A(n,\{dW\})}, \; Z_A(n,\{dW\}) \equiv \mathrm{tr}[(\hat{\rho}_A^{(c)})^n]$
- von Neumann entropy: $n \to 1$



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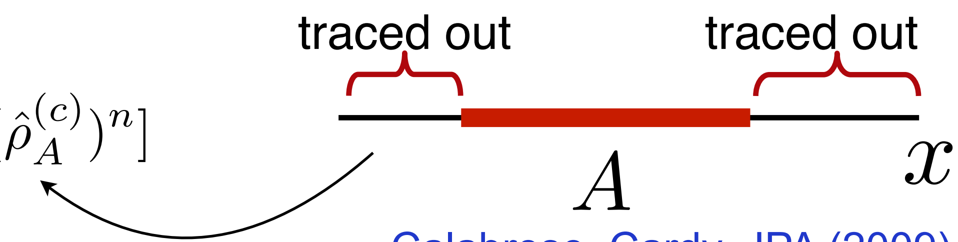


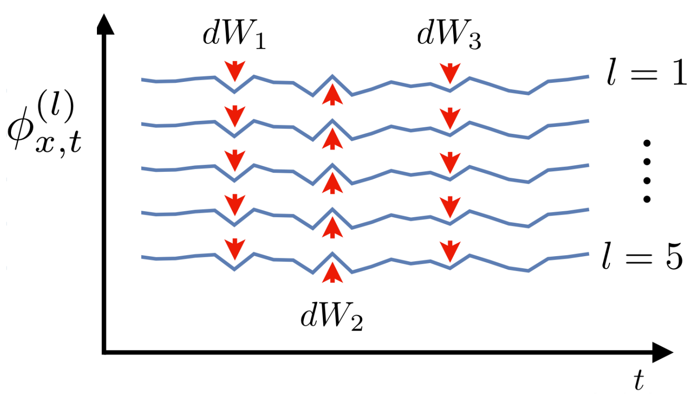
Calabrese, Cardy, JPA (2009)

- approach: Keldysh replica field theory for n replicas (entropies via modified boundary conditions)

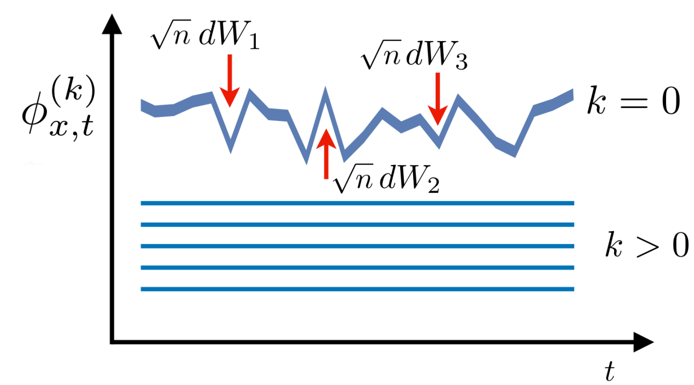
ground states: Casini, Fosco, Huerta, J. Stat. Mech. (2005)

- decoupling of center-of-mass and relative modes (exact for Gaussian states -> good away from transition)

replica space picture



reciprocal replica space picture



- 1 mode heats up (noisy)
- n-1 modes cool down (noiseless)

→

- noisy contribution A independent
- all A dependence in noiseless modes!

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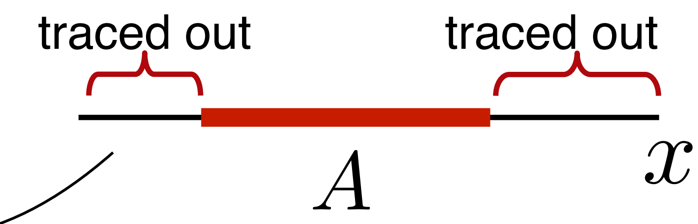

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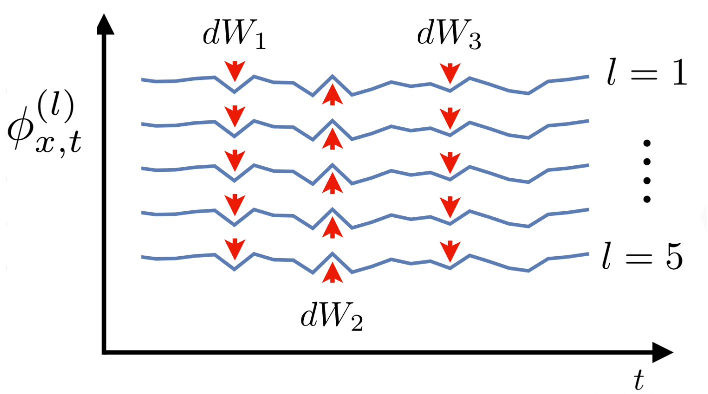

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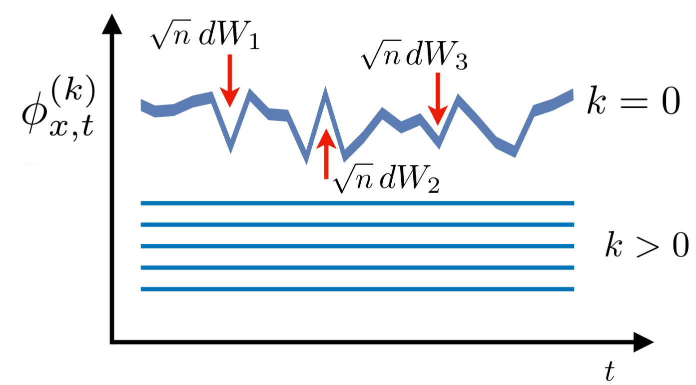

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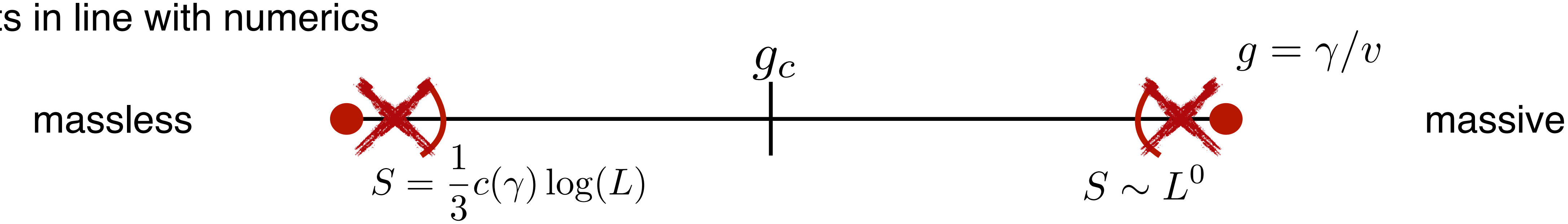
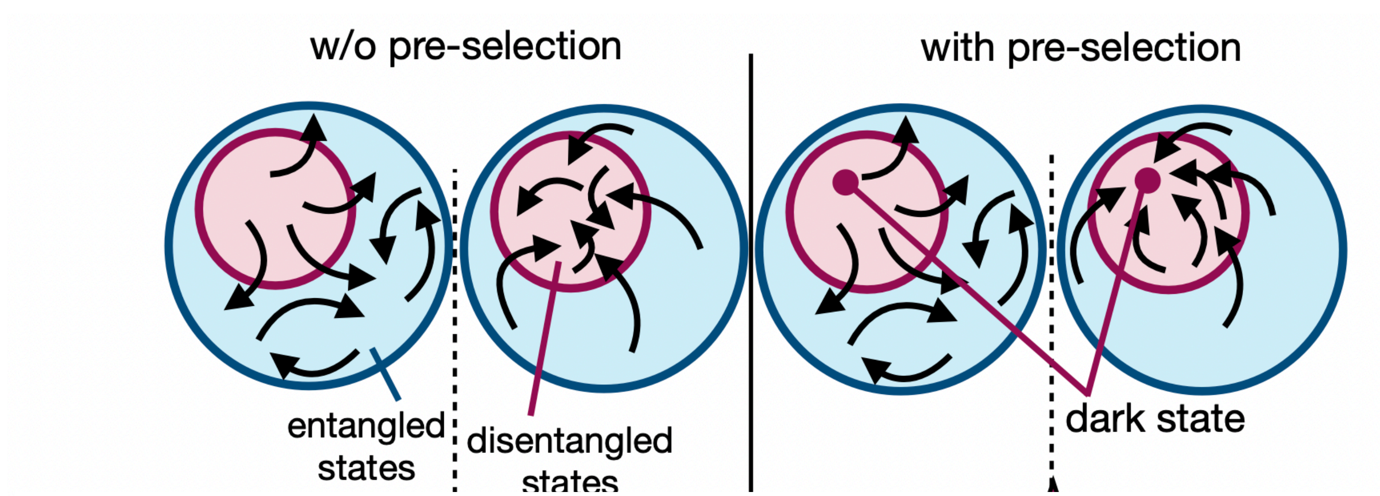
➡ Rényi entropy calculation as for ground states

➡ results in line with numerics

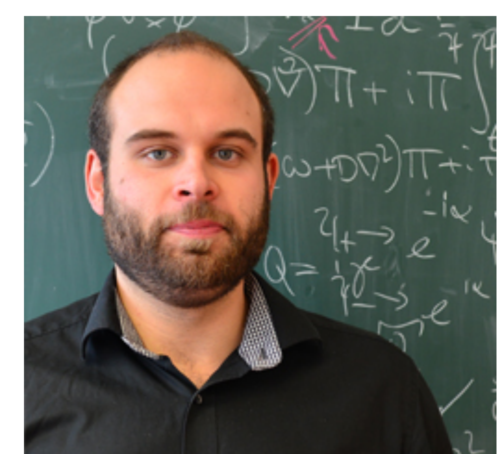


Revealing measurement-induced phase transitions by pre-selection




Thomas Müller

M. Buchhold, T. Müller, SD,
arxiv:2208.10506 (2022)


Michael Buchhold

Observability of the transition?

- Standard quantum mechanical observables: need to reproduce identical state
- Randomness of measurements outcomes masks transition in qm observables

time

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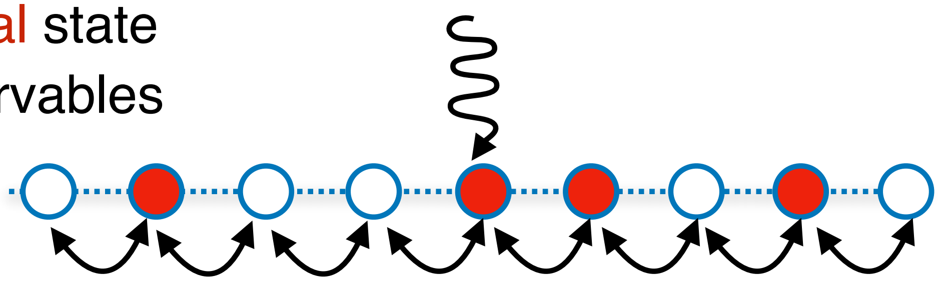
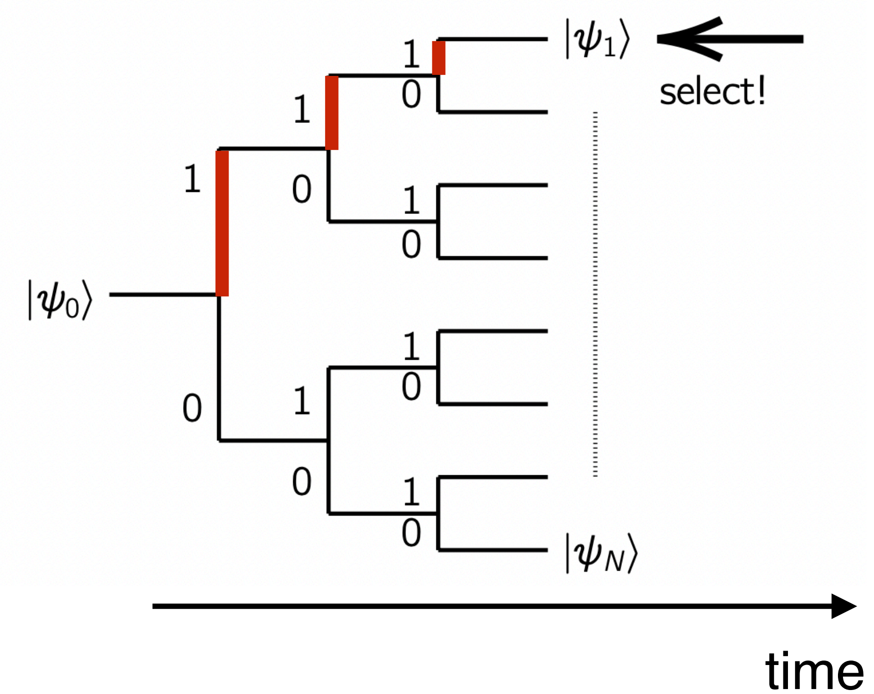


- Post-selection problem: way out for Cliffords: Gullans, Huse, PRL (2020); exp: C. Noel et al. Nat Phys. (2021)
    - Select one string of measurement outcomes (trajectory)
    - build an ensemble of identical states with same trajectory

➡ number of outcomes (single site)

$$2^{\#\text{ of measurements}}$$

➡ post-selection exponentially costly



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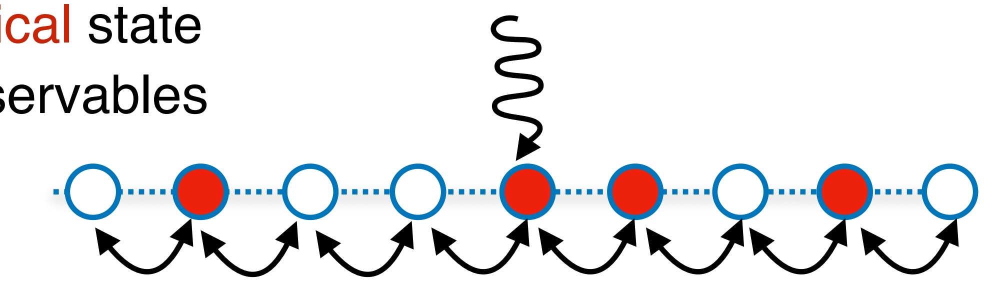

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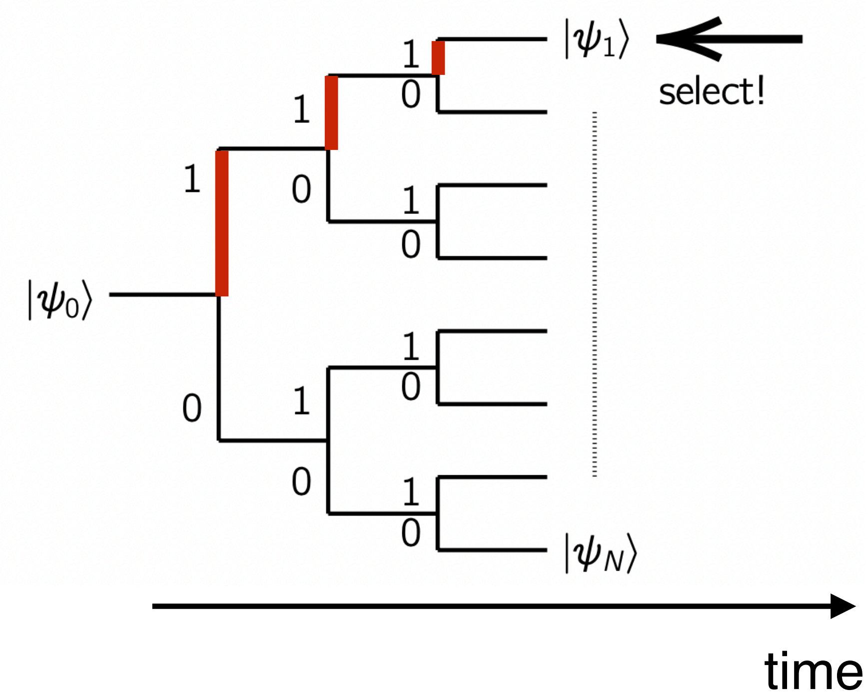

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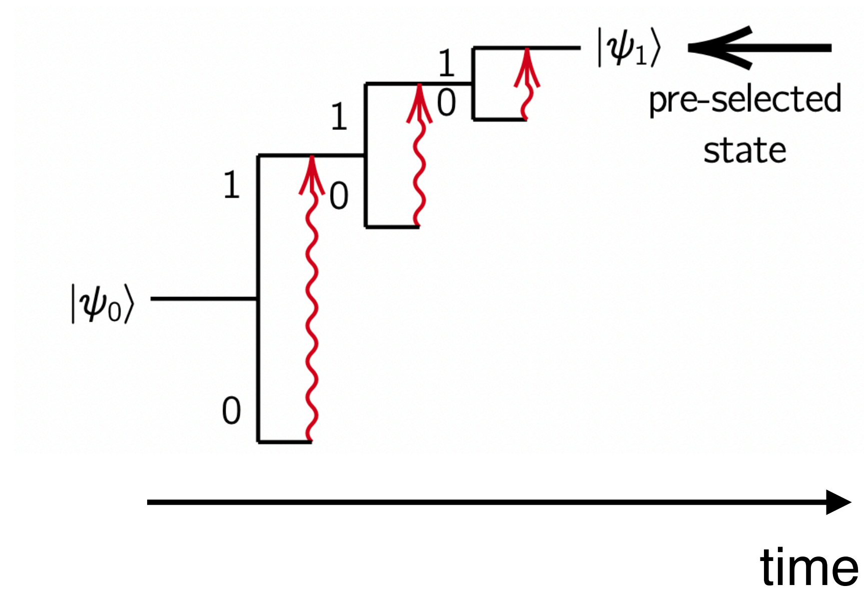
- Pre-selection solution: M. Buchhold, T. Müller, SD, arxiv:2208.10506 (2022)

  - break randomness in measurement outcomes
  - steer system into unique representative state in Hilbert space (dark state)

➡ can be done gently: no modification of entanglement structure & universal properties (as post-selection)

➡ no exponential overhead: observable by standard means in NISQ platforms



Pre-selection concept

- pull MIPT to observable level <—> study measurement averaged density matrix

$$\hat{\rho}_{t+\delta t} = \hat{\mathcal{X}}[\hat{\rho}_t]\delta t$$

e.g. temporal continuum limt

$$\hat{\mathcal{X}}[\hat{\rho}] = -i[\hat{H}, \hat{\rho}] + \sum_l \hat{L}_l \hat{\rho} \hat{L}_l^\dagger - \frac{1}{2}\{\hat{L}_l^\dagger \hat{L}_l, \hat{\rho}\}$$

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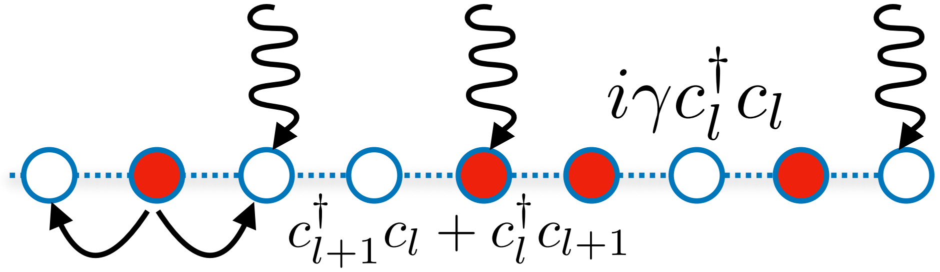
- unmodified dynamics w/o preselection:

$\hat{\mathcal{X}}^{(0)}$ with $\hat{H}^{(0)}, \quad \hat{L}_l^{(0)} = (\hat{L}_l^{(0)})^\dagger$ hermitian

$\Longrightarrow \hat{\mathcal{X}}^{(0)}[\hat{\mathbf{1}}] = 0$ fully mixed stationary state



ex.: monitored fermions

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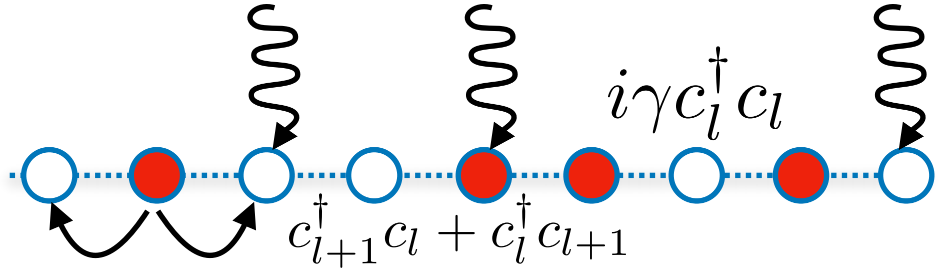

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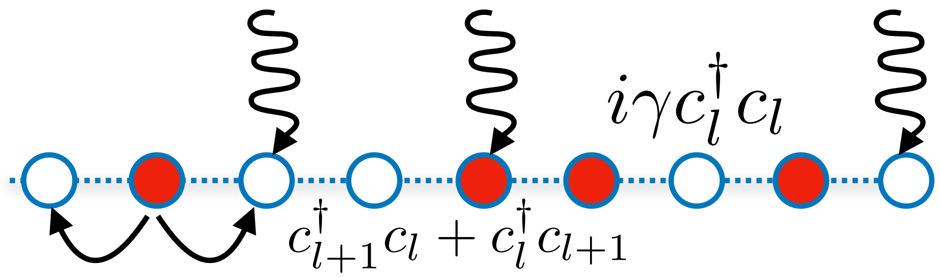

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1. There exists a pure dark state representing one possible measurement outcome,

$\hat{\rho}_D = |D\rangle\langle D|$ s.t. $\hat{\mathcal{X}}[\hat{\rho}_D] = 0$

➡ pre-select representative state (e.g. with order parameter)

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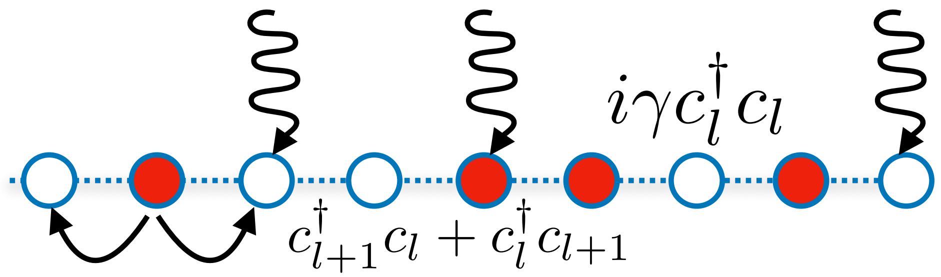

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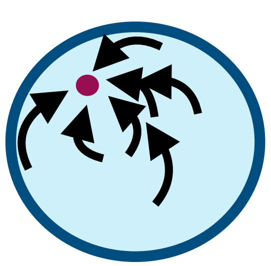
2. The dark state is unique

➡ directionality in Hilbert space



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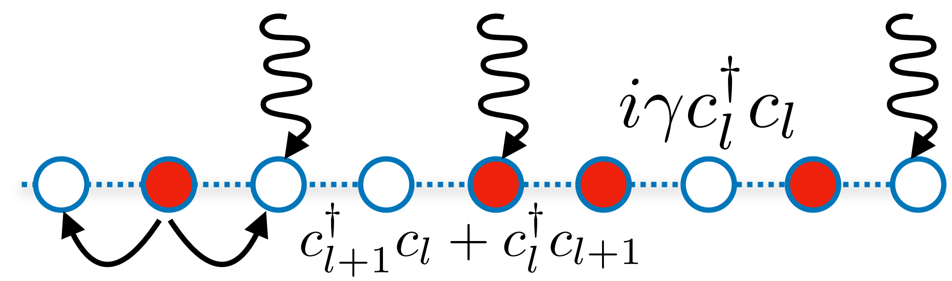

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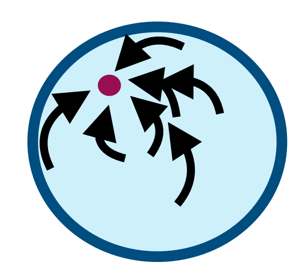


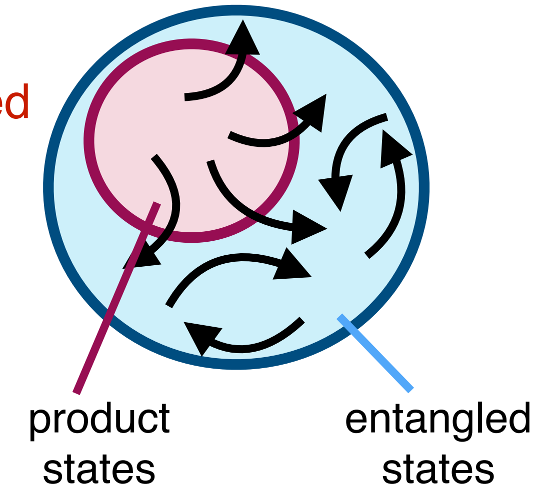
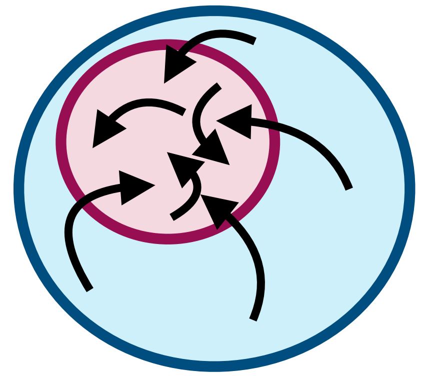
3. Modifications preserve symmetries and are irrelevant in the RG sense

➡ phases and phase transitions unmodified

Pre-selection: connection to absorbing state transitions

- dynamics $\partial_t \hat{\rho} = \mathcal{X}(\hat{\rho}) = \mathcal{X}_1(\hat{\rho}) + \gamma \mathcal{X}_2(\hat{\rho})$

unitary (for $\mathcal{X}_1(\hat{\rho})$) non-unitary (for $\gamma \mathcal{X}_2(\hat{\rho})$)

	$\mathcal{X}_1(\hat{\rho})$:		$\mathcal{X}_2(\hat{\rho})$:	
unmodified measurement dynamics:	prefers entangled		prefers product	

trajectory entanglement entropy:

$$\gamma \geq \gamma_c : \quad S(\hat{\rho}_A) \sim |A|^0$$

$$\gamma < \gamma_c : \quad S(\hat{\rho}_A) \sim |A|, \log(|A|)$$

but: extensive configuration entropy

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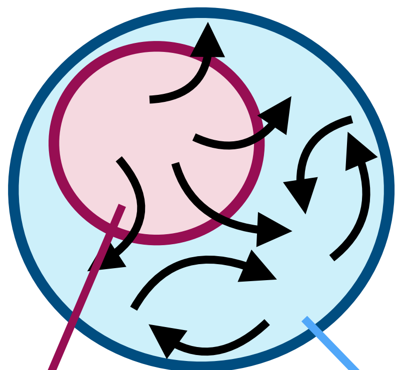
unitary non-unitary

	$\mathcal{X}_1(\hat{\rho}):$	$\mathcal{X}_2(\hat{\rho}):$	
• unmodified measurement dynamics:	prefers entangled	prefers product	trajectory entanglement entropy:
• pre-selected dynamics w/ dark state:	dynamically unstable	dynamically stable	

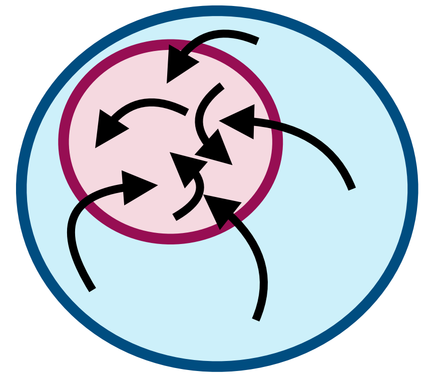
prefers entangled



product states

entangled states

prefers product



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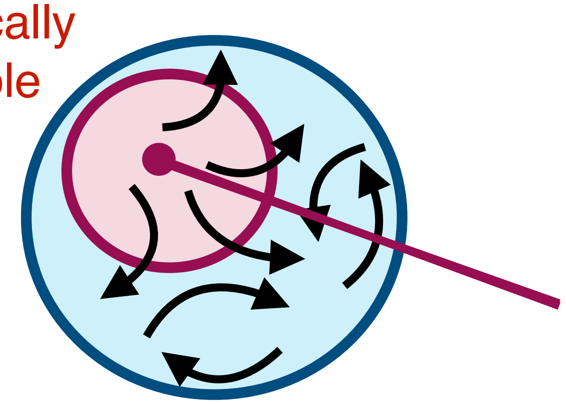
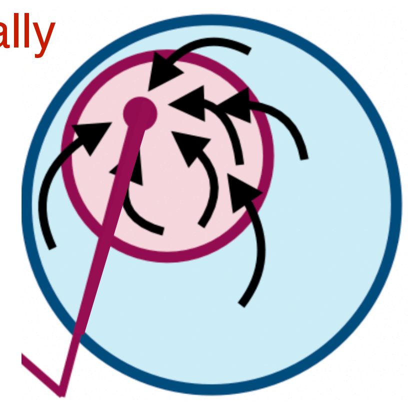
dynamically unstable



$\hat{\rho}_D$

dynamically stable



phase transition:

pre-selection fails

$$\gamma < \gamma_c : \quad \hat{\rho} \not\to \hat{\rho}_D$$

pre-selection succeeds

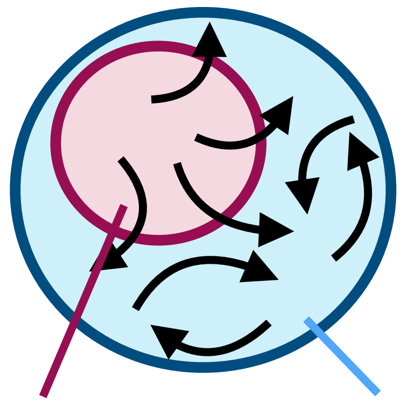
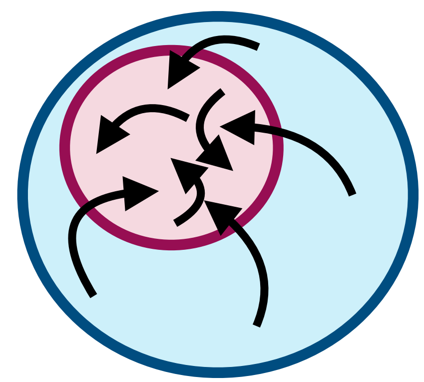
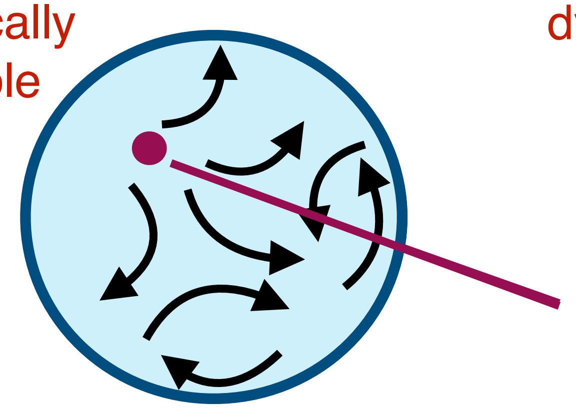
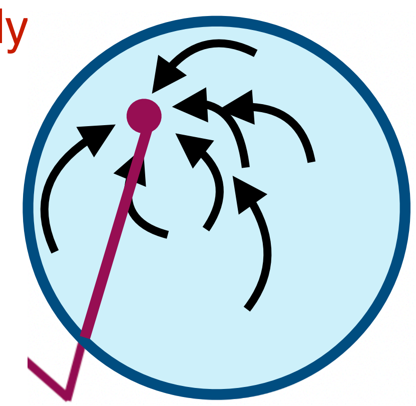
$$\gamma \geq \gamma_c : \quad \hat{\rho} \to \hat{\rho}_D$$

configuration entropy removed!

Pre-selection: connection to (quantum) absorbing state transitions

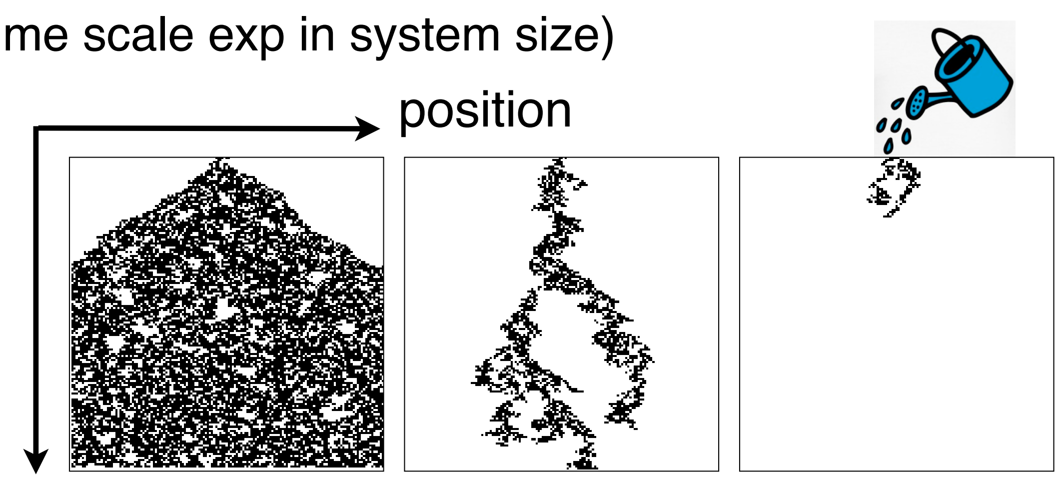
- dynamics $\partial_t \hat\rho = \mathcal{X}(\hat\rho) = \mathcal{X}_1(\hat\rho) + \gamma \mathcal{X}_2(\hat\rho)$

unitary — non-unitary

	$\mathcal{X}_1(\hat\rho)$:	$\mathcal{X}_2(\hat\rho)$:	
• unmodified measurement dynamics:	prefers entangled 	prefers product 	trajectory entanglement entropy: $\gamma \geq \gamma_c : \quad S(\hat\rho_A) \sim \lvert A\rvert^0$ $\gamma < \gamma_c : \quad S(\hat\rho_A) \sim \lvert A\rvert, \log(\lvert A\rvert)$ but: extensive configuration entropy
• pre-selected dynamics w/ dark state:	dynamically unstable  $\hat\rho_D$	dynamically stable 	$\gamma \geq \gamma_c : \quad \hat\rho \to \hat\rho_D$ $\gamma < \gamma_c : \quad \hat\rho \not\to \hat\rho_D$

(up to time scale exp in system size)

wetting transition



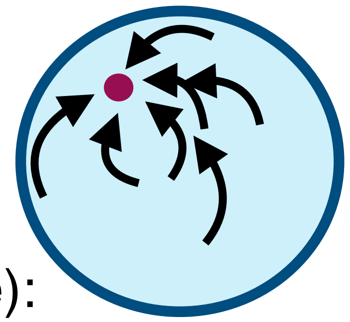
➡ transition in the dynamics

➡ structure of absorbing state transition in Hilbert space!

H. Hinrichsen, Adv. Phys. (2000);
G. Odor, RMP (2004)

Example: ’Classical’ preselection dynamics in monitored fermions

- choice of representative dark state: $\hat{\rho}_D = |101010...\rangle\langle 101010...|$



- modify generator of dynamics: condition hopping on state (non-linear in state):

$$\hat{H}^{(0)} \to \hat{H} = \hat{H}[\hat{\rho}] \quad \text{with} \quad \hat{H}[\hat{\rho}_D] = 0$$

Wiseman, Milburn, PRL (1993)

$$J \to J_l[\hat{\rho}] = 2 - (\lfloor\langle \hat{n}_{l-1}\rangle\rfloor - \lfloor\langle \hat{n}_l\rangle\rfloor)^2 - (\lfloor\langle \hat{n}_{l+1}\rangle\rfloor - \lfloor\langle \hat{n}_{l+2}\rangle\rfloor)^2$$

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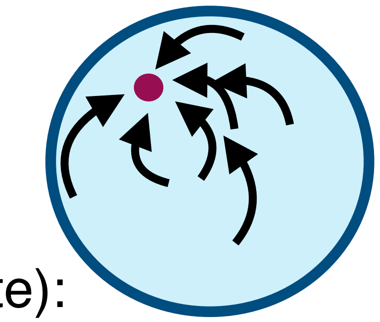

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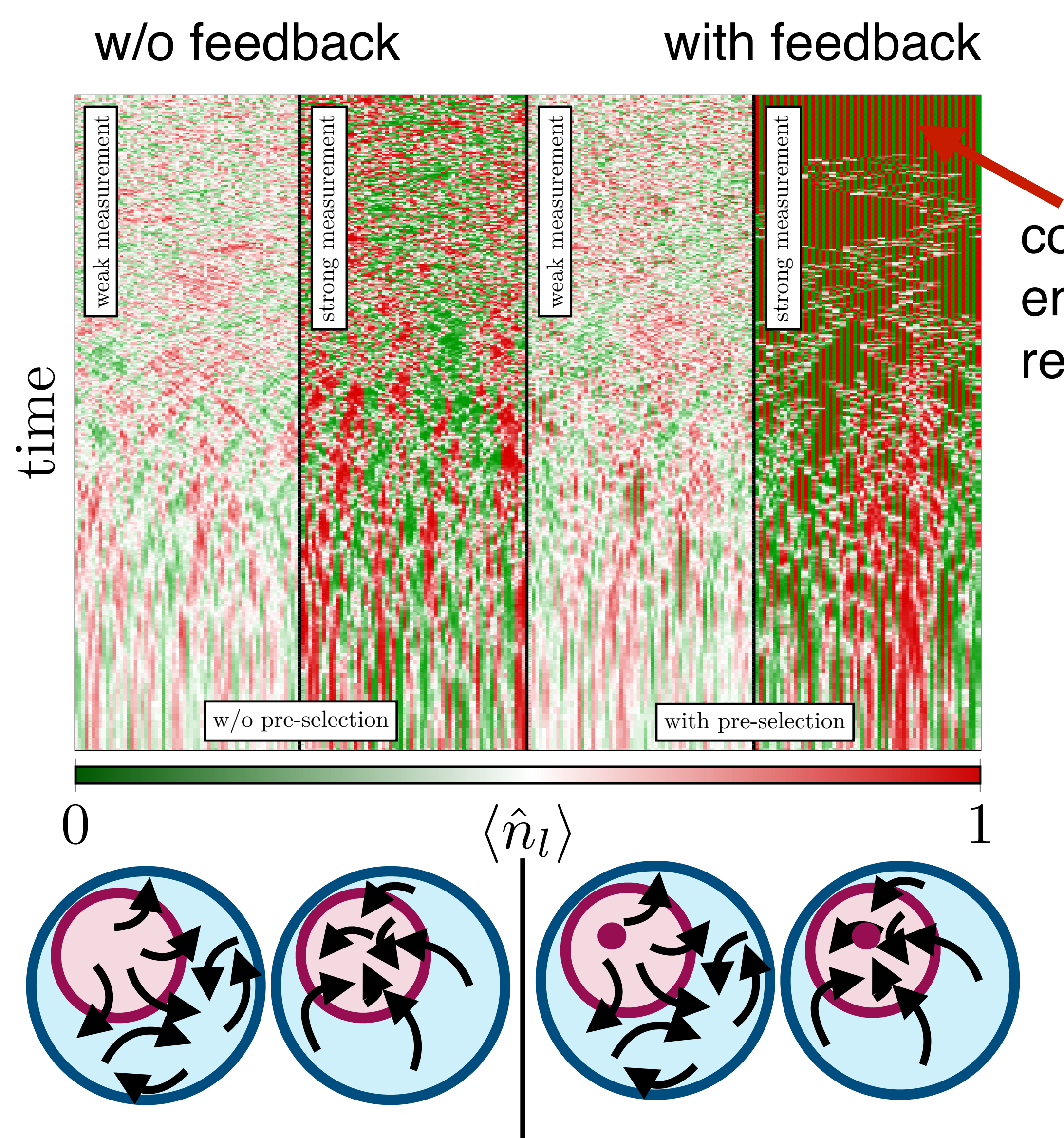
- distinguishes phases via Néel order



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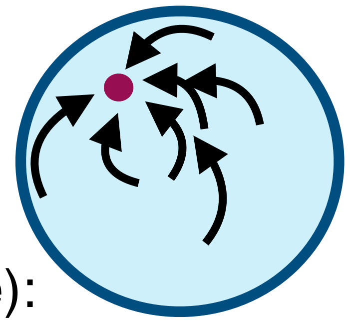

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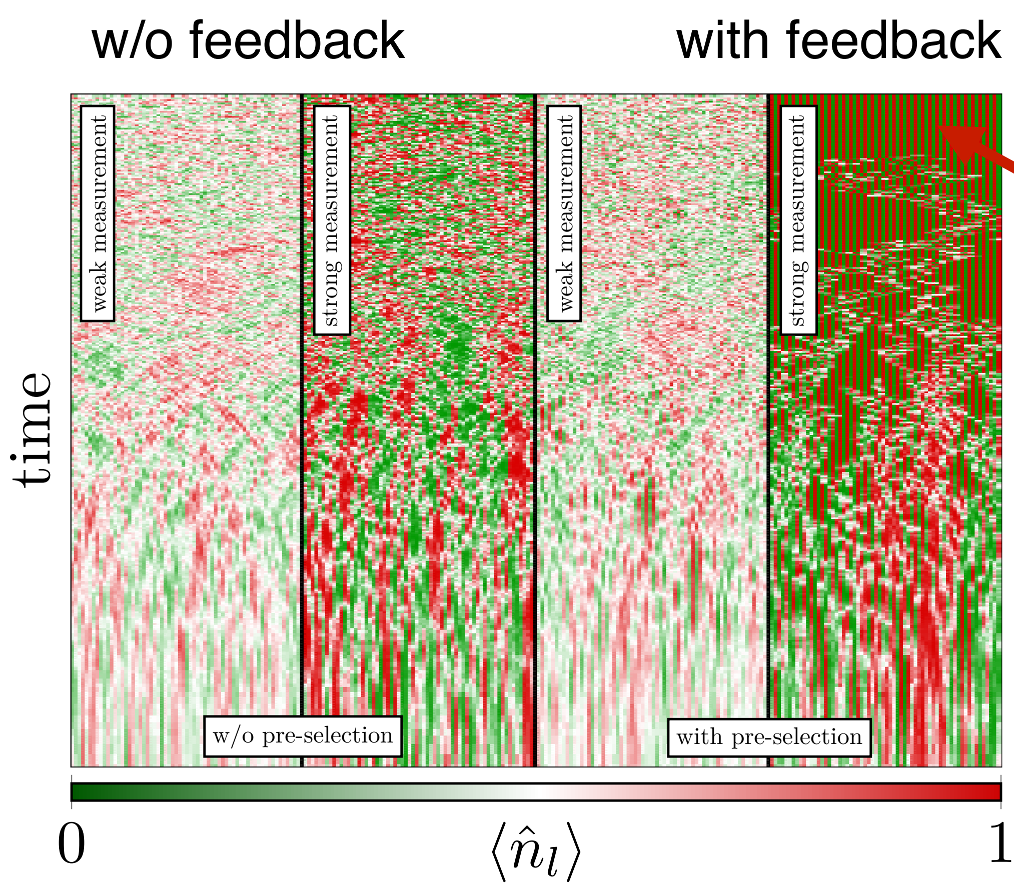
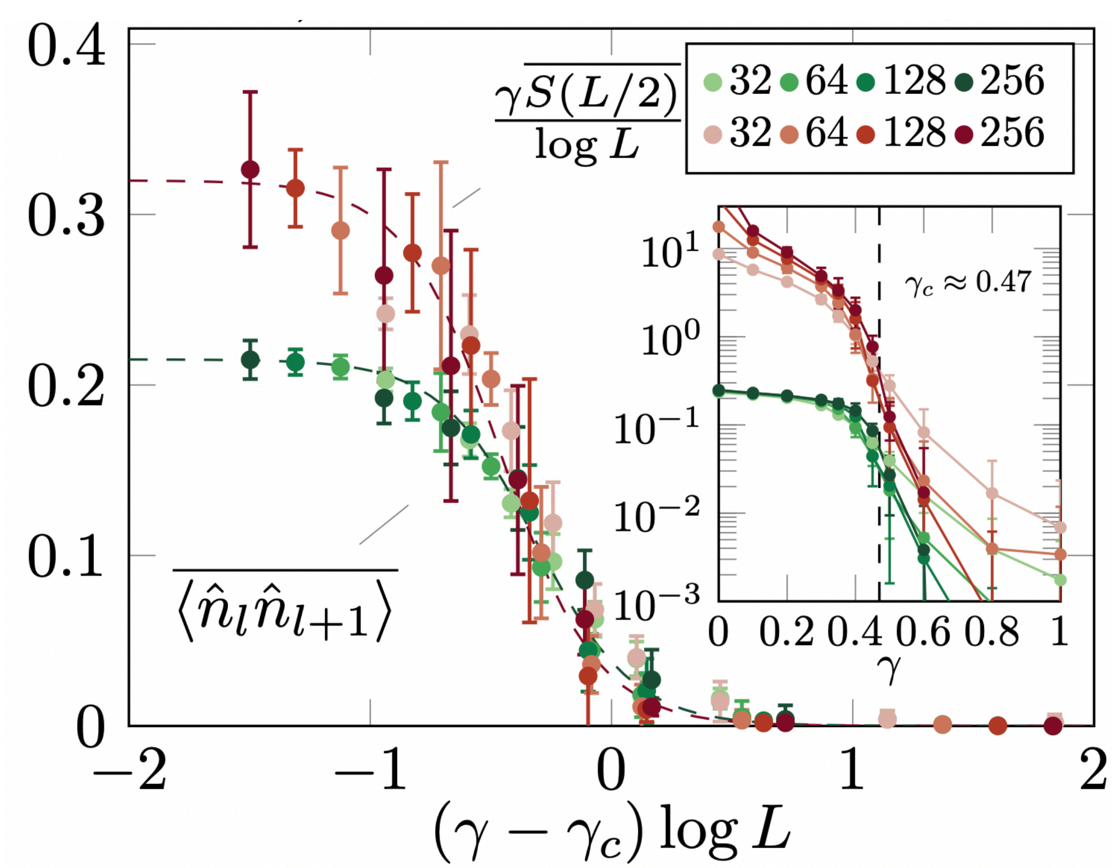


config. entropy removed!

- dark state phase transition: BKT



➡ entanglement entropies coincide: phases / phase transition unmodified

➡ but there exist standard qm observables witnessing it

Example: 'quantum' preselection

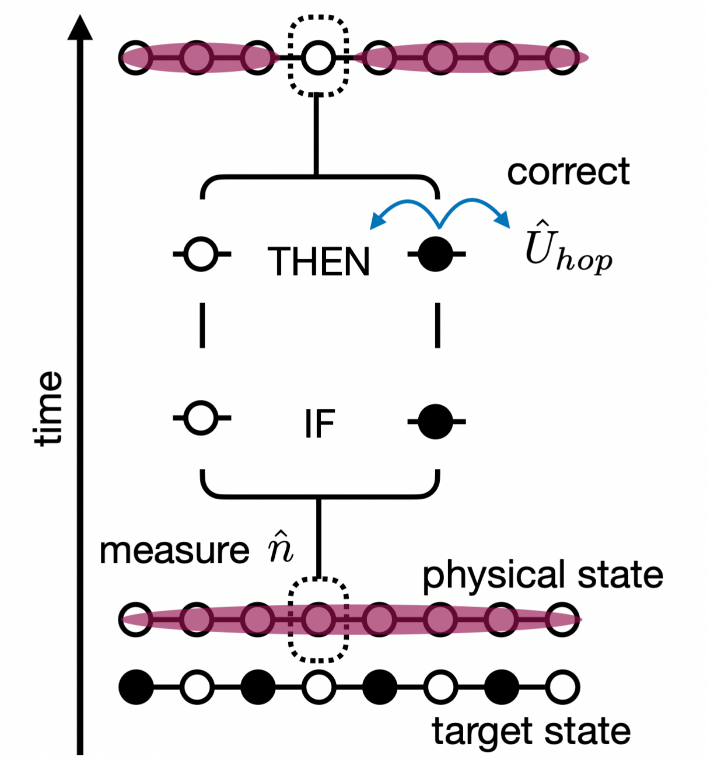
- modify operators: linear in state

modified measurement operators

$$\partial_t \rho = i[\rho, H] - \gamma \sum_l \{L_l^\dagger L_l, \rho\} - 2L_l \rho L_l^\dagger$$

$$[\hat{H}, \hat{\rho}_D] = 0$$

$$\hat{L}_l \hat{\rho}_D = 0$$



- measurement preselection:

$$\hat{L}_l = \hat{U}_l \hat{n}_l$$

$$[\hat{U}_l, \hat{n}_l] \neq 0$$

- non-Gaussian evolution

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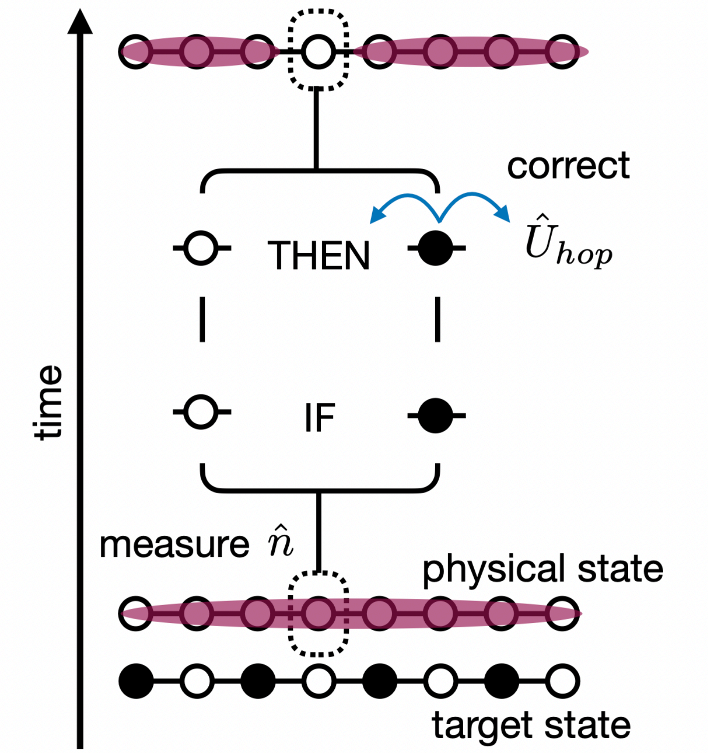

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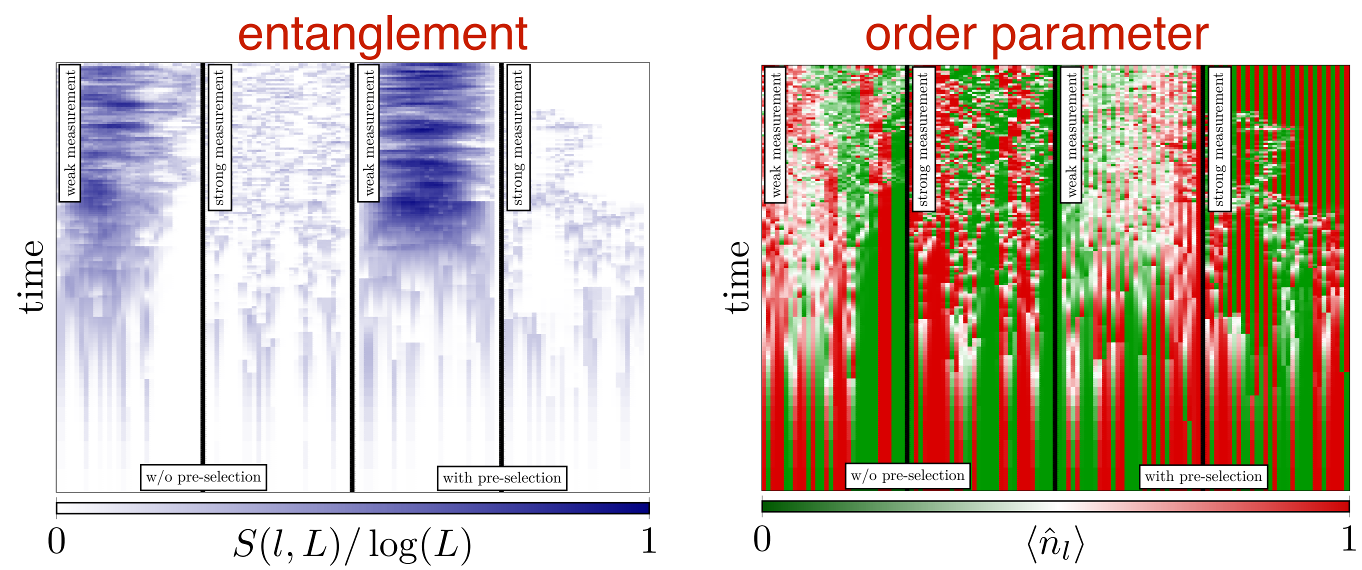

DMRG simulations, M. Buchhold, T. Müller, SD, in progress

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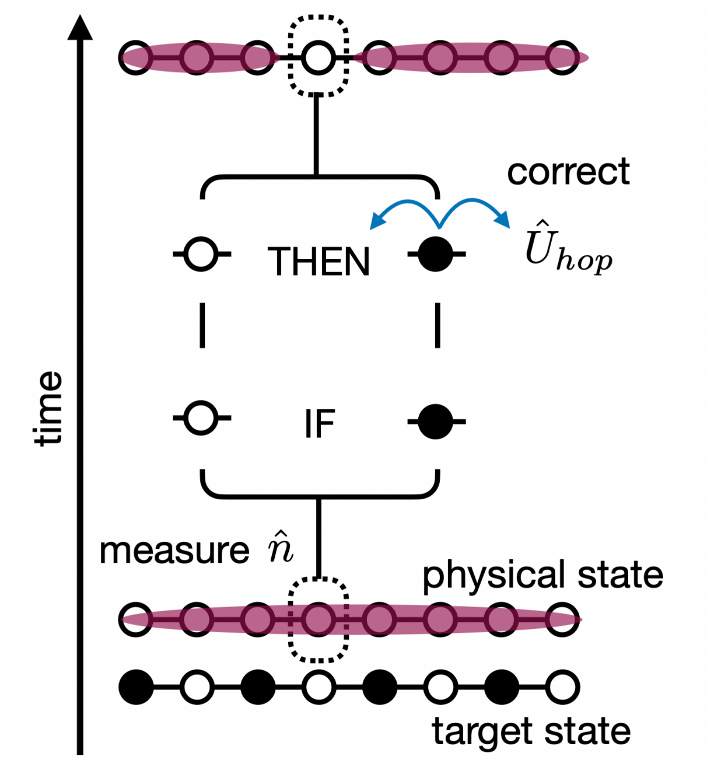

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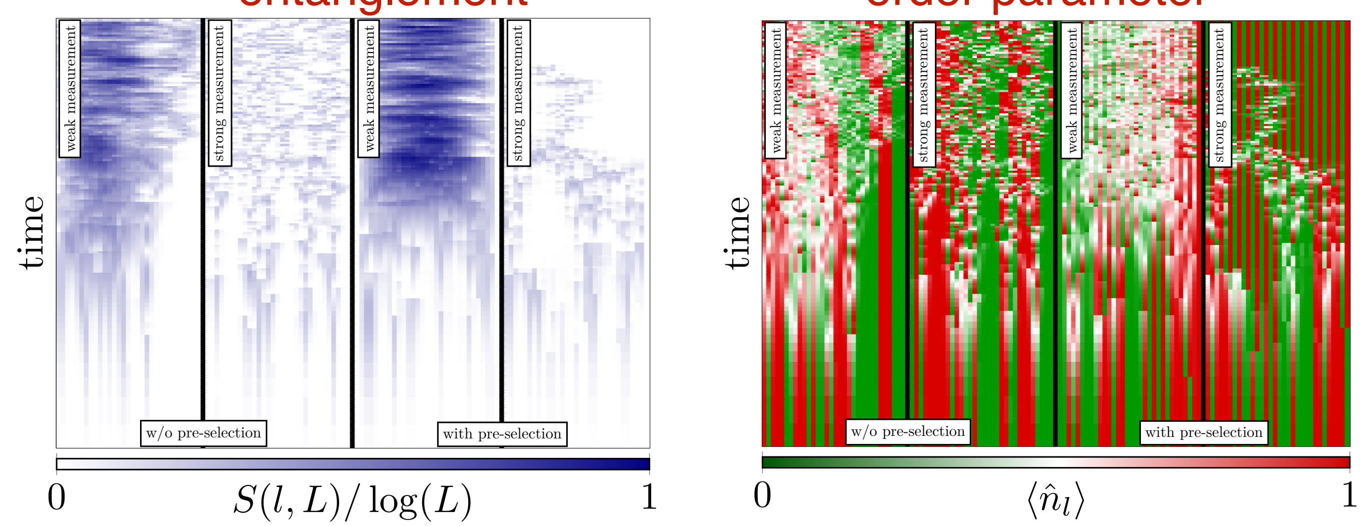
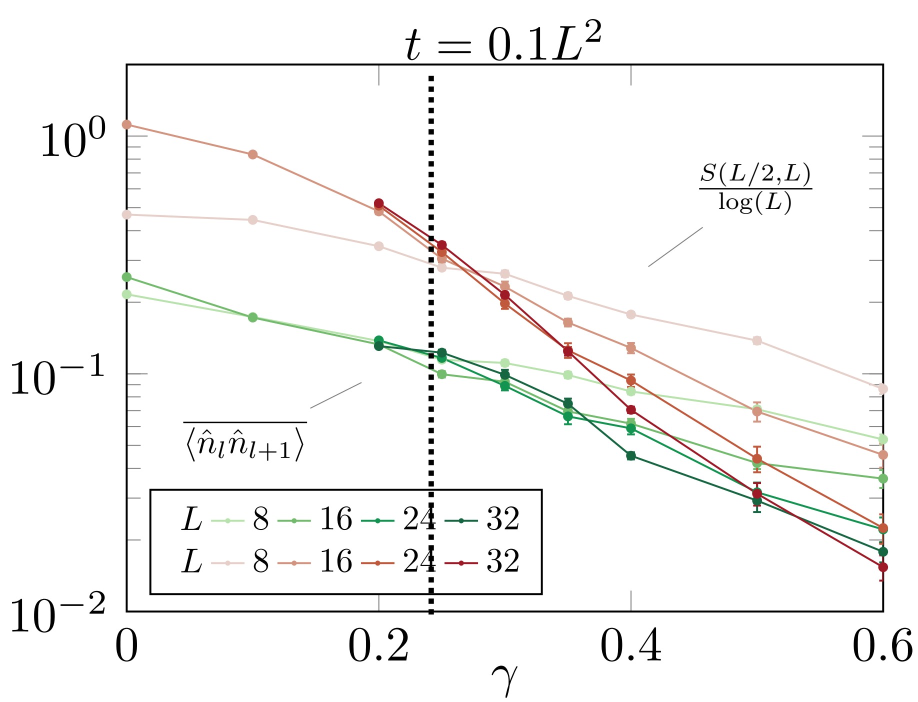


DMRG simulations, M. Buchhold, T. Müller, SD, in progress

- detect transition



Example: ‘quantum’ preselection

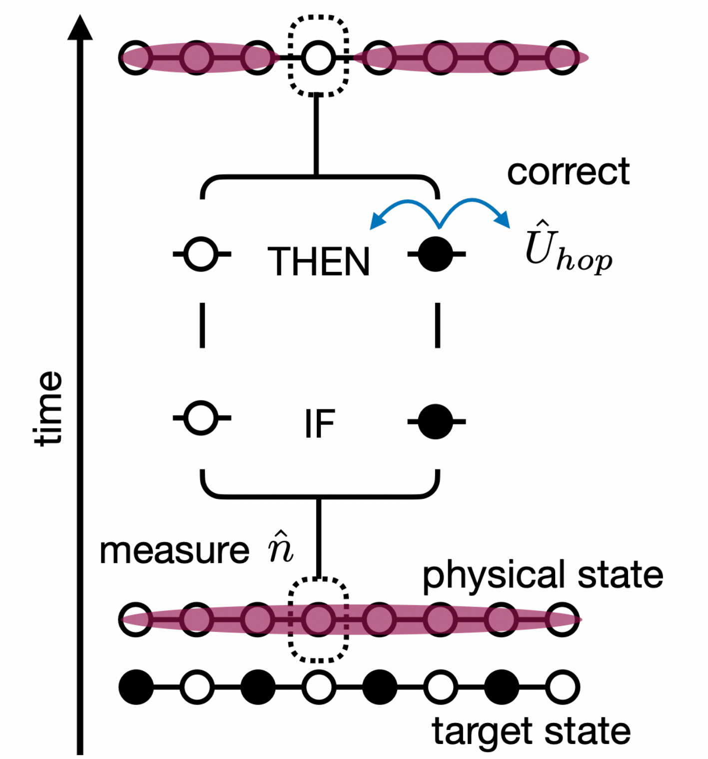
- modify operators: linear in state

$$\partial_t \rho = i[\rho, H] - \gamma \sum_l \{L_l^\dagger L_l, \rho\} - 2 L_l \rho L_l^\dagger$$

modified measurement operators

$$[\hat{H}, \hat{\rho}_D] = 0$$

$$\hat{L}_l \hat{\rho}_D = 0$$

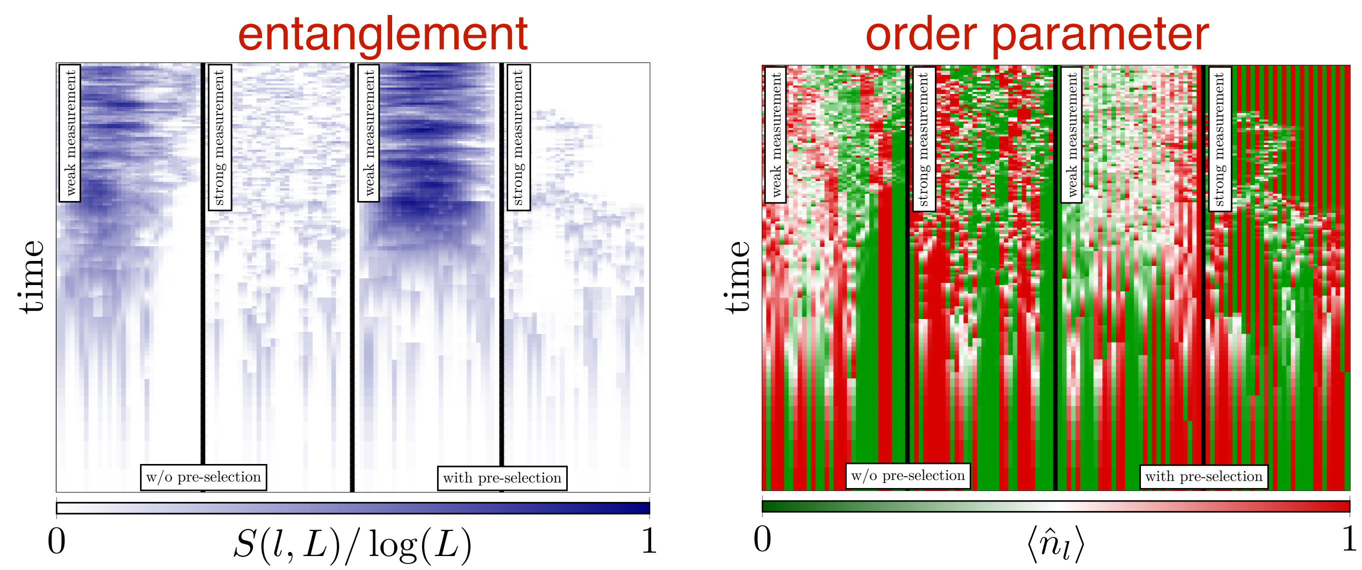


- measurement preselection:

$$\hat{L}_l = \hat{U}_l \hat{n}_l$$

$$[\hat{U}_l, \hat{n}_l] \neq 0$$

- non-Gaussian evolution



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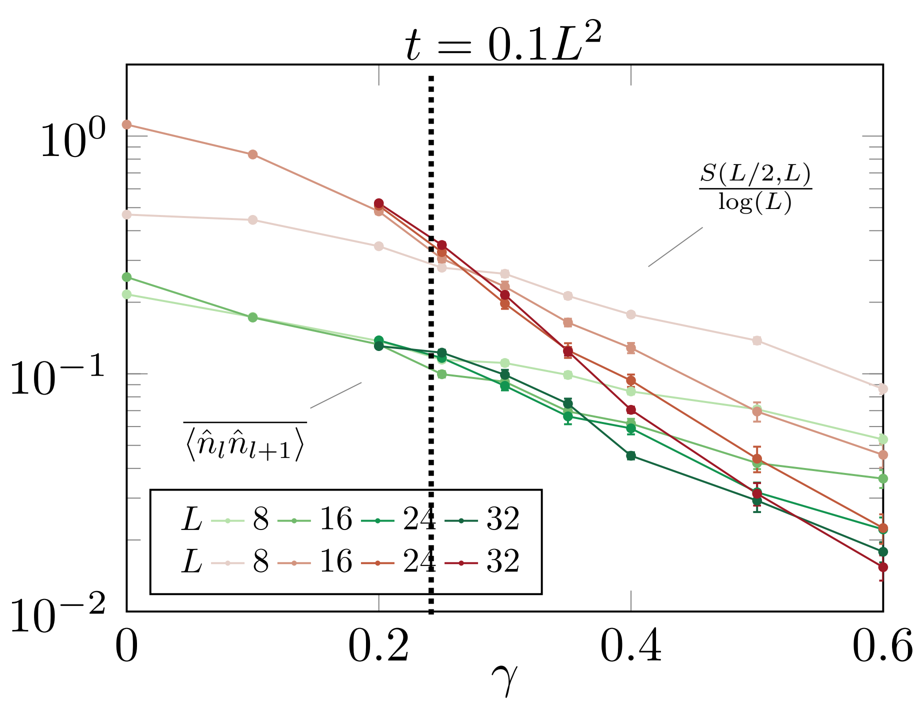


- field theory

$$\hat{L}_l = \hat{n}_l \to \exp[i\pi(c_l^\dagger c_m + c_m^\dagger c_l)]\ \hat{n}_l$$

$$\partial_x \hat{\phi}_x + \cos(\hat{\phi}_s) \to \partial_x \hat{\phi}_x + \cos(\hat{\phi}_s) + i\partial_x \hat{\theta}_x$$

- Heisenberg-Langevin theory (or equivalent Keldysh)

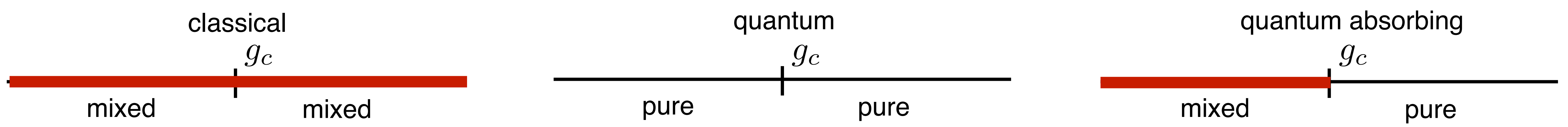
$$(\partial_t^2 - \partial_x^2)\hat{\phi}_x + \gamma \cos\hat{\phi}_x + \gamma \sin\hat{\phi}_x(\partial_t \hat{\phi}_x + \sin\hat{\phi}_x) = \hat{\xi}_x$$

➡ irrelevant in weak measurement regime: infinite T state

➡ becomes relevant at transition: pinning

Outlook: Quantum absorbing transitions beyond directed percolation?

- Grassberger-Janssen conjecture: No! Janssen, Z. Phys. B (1981); Grassberger, Z. Phys. B (1982)
- Qualitative overview:



Outlook: Quantum absorbing transitions beyond directed percolation?

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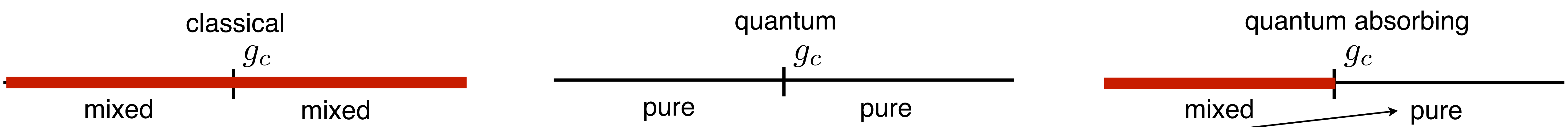


- possible pure state scenarios:

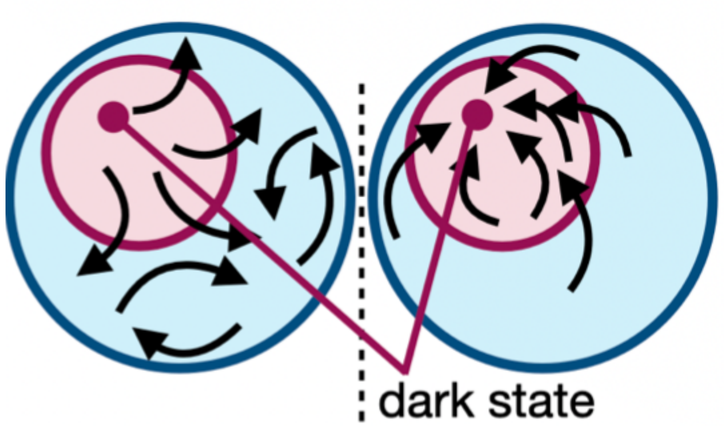
Sieberer, Buchhold, SD, ROPP (2016)

  - general quantum dynamics w/ dark/absorbing state (or equivalent Keldysh)

$$\partial_t \hat\rho = -i[\hat H, \hat\rho] + \gamma \sum_i \hat L_i \hat\rho \hat L_i^\dagger - \tfrac{1}{2}\{\hat L_i^\dagger \hat L_i, \hat\rho\} \qquad \hat H = \sum_i \hat h_i \to \hat H = \sum_i \hat h_i \hat L_{i-1}^\dagger \hat L_{i-1}$$

repulsive fixed point

attractive fixed point



dark/absorbing state

$$\hat L_i |D\rangle = 0 \ \forall i$$

Outlook: Quantum absorbing transitions beyond directed percolation?

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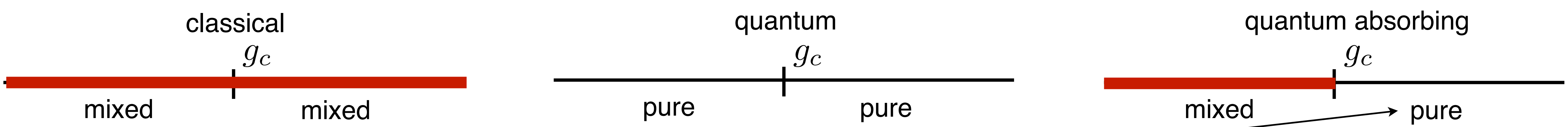


- possible pure state scenarios:

Sieberer, Buchhold, Diehl ROPP (2016)

  - general quantum dynamics w/ dark/absorbing state (or equivalent Lindblad-Keldysh functional integral)

$$\partial_t \hat\rho = -i[\hat H, \hat\rho] + \gamma \sum_i \hat L_i \hat\rho \hat L_i^\dagger - \tfrac{1}{2}\{\hat L_i^\dagger \hat L_i, \hat\rho\} \qquad \hat H = \sum_i \hat h_i \to \hat H = \sum_i \hat h_i \hat L_{i-1}^\dagger \hat L_{i-1}$$

$\hat L_i |D\rangle = 0 \ \forall i$ dark/absorbing state

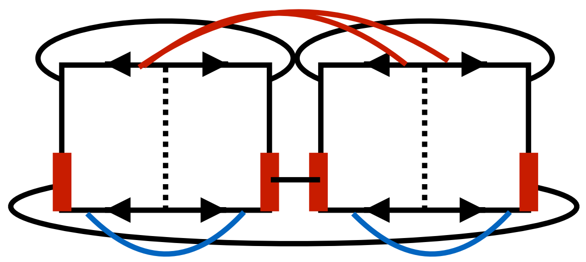
State

Dynamics	product (w/o entanglement)	entangled ('non-trivial vacua')
w/o conservation law	directed percolation	$\lvert D\rangle =$ (BCS superfluid)
w/ conservation law	1+1: present problem, BKT d+1, d > 1: tbd	$\lvert D\rangle =$ (topological insulator)

Summary

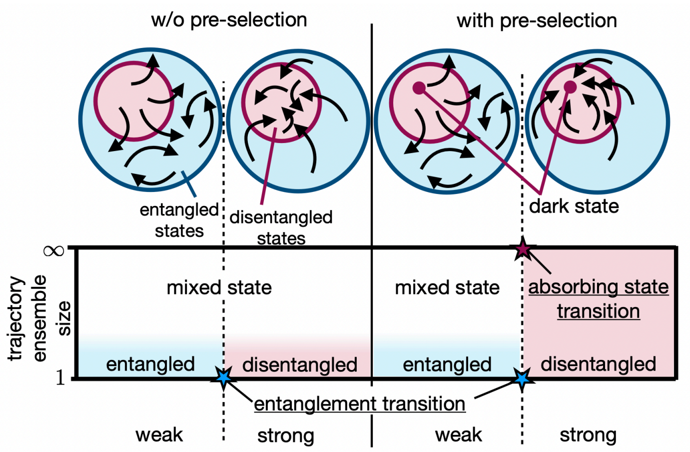
General non-unitary quantum dynamics hosts new types of phases and phase transitions

- here: critical to area law entanglement phase transition
- BKT transition revealed by Keldysh replica field theory



Observability via pre-selection

- like post-selection, but no exponential overhead
- various strategies for NISQ platforms
- link to quantum absorbing state transitions



Outlook

- general principles for observability?
- higher dimensions: novel quantum absorbing state transitions beyond directed percolation?
- relation of measurement vs. disorder problems? role of conservation laws? symmetry classification? Poboiko, Pöpperl, Gornyi, Mirlin, arXiv:2304.03138 (2023)
- emergent CFT behavior?