A nonlinear fluctuation – dissipation test for Markovian systems

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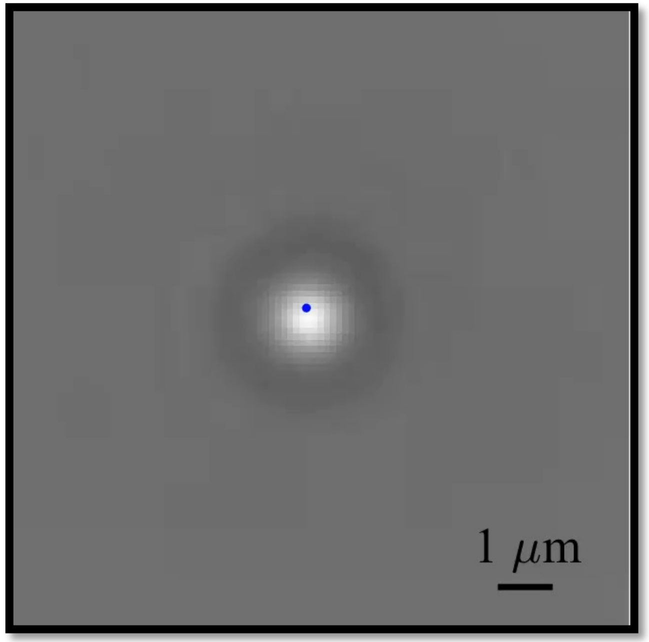
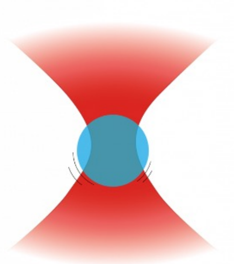
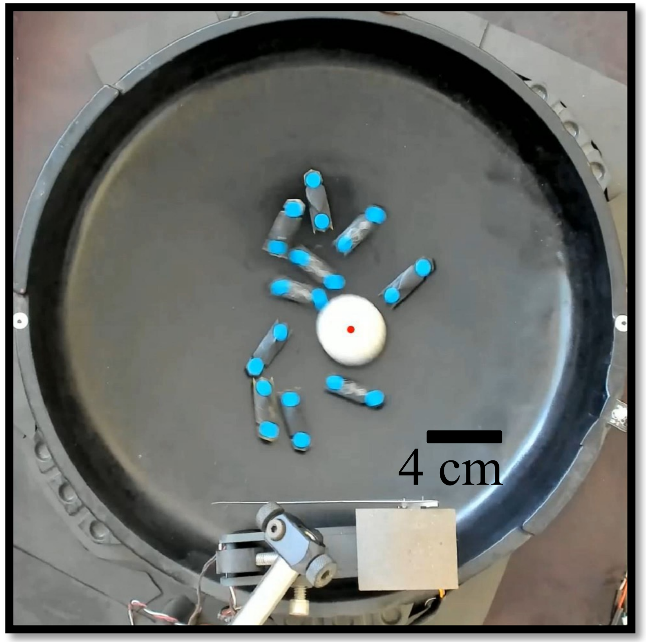
Thermal or active forces?

<u>Thermal fluctuations:</u>



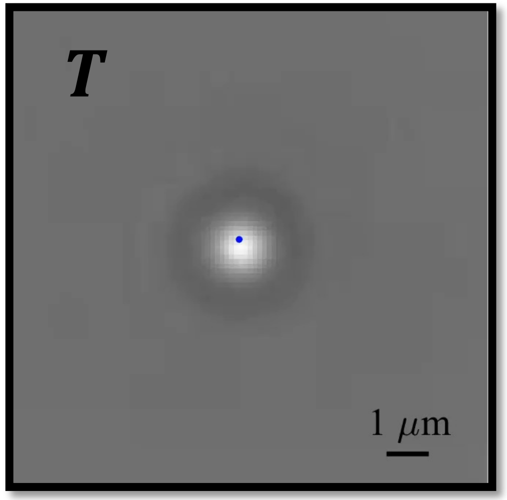


<u>Active fluctuations:</u>



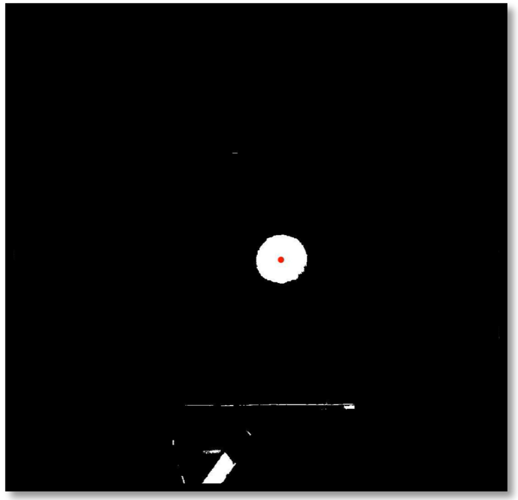
Thermal or active forces?

<u>Thermal fluctuations:</u>



- ❑ Markovian dynamics.
- ❑ Equilibrium FDT: temperature.

<u>Active fluctuations:</u>



- ❑ Non-Markovian dynamics?
- ❑ Generalized FDR?
- ❑ Effective temperature?

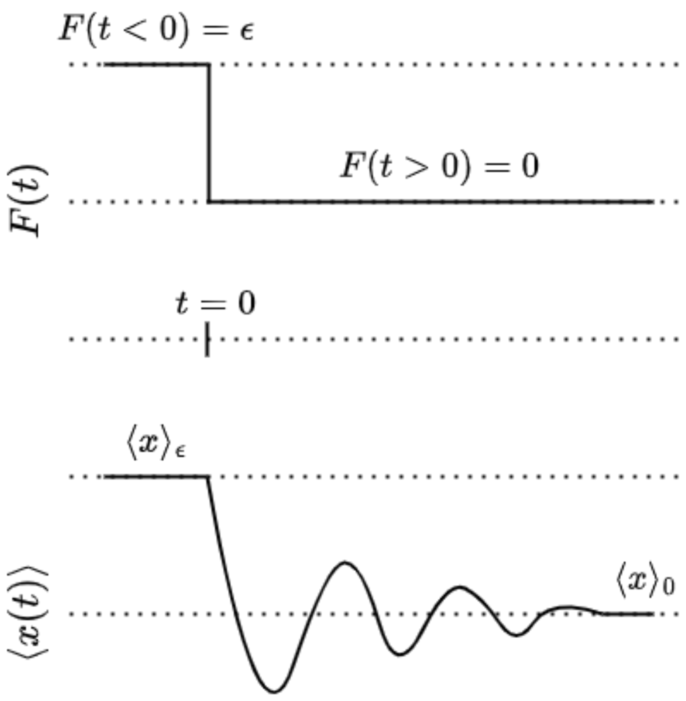
Fluctuation – Dissipation Theorem: a test for equilibrium

- A step-stimulus: $F(t) = \epsilon \cdot \Theta(-t)$

- The Fluctuation – Dissipation Relation (FDR):

$$\langle x(t>0)\rangle = \frac{\epsilon}{k_B T}\langle x(t')x(t'+t)\rangle_0 \equiv \frac{\epsilon}{k_B T} C_{xx}(t)$$

- **If violated: the system operates out-of-equilibrium.**



Generalized (linear) FDR: a test for Markovianity

- A step – stimulus: $F(t) = \epsilon \cdot \Theta(-t)$

- **Generalized (linear) FDR:**

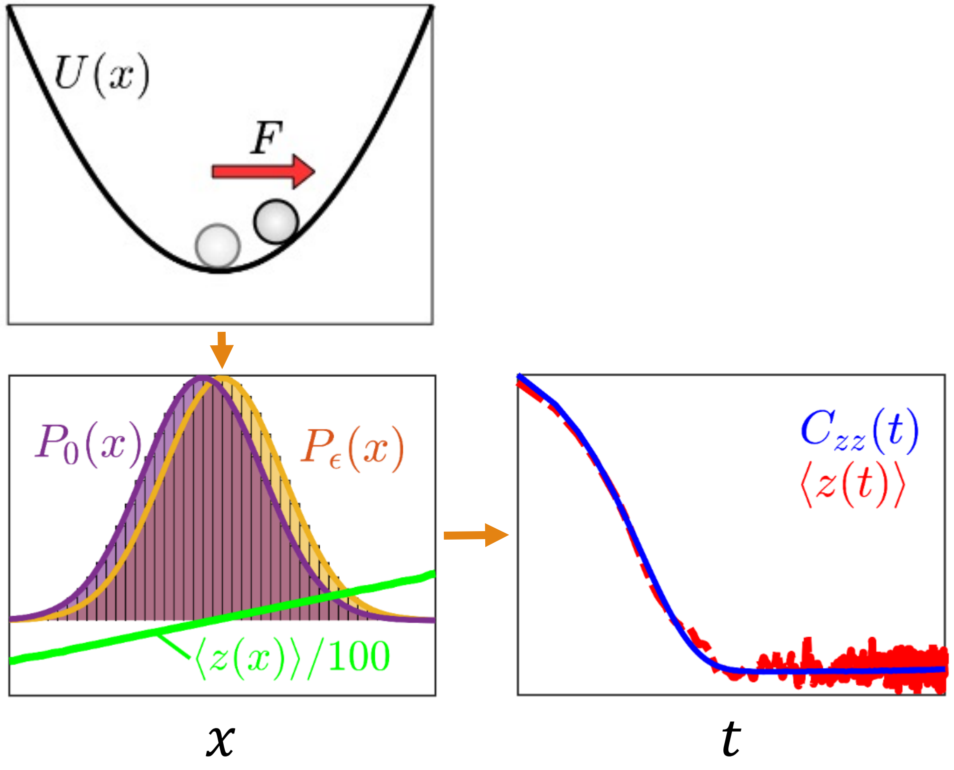
$$z_L\big(x(t)\big) \equiv \epsilon \cdot \frac{\partial}{\partial \epsilon} \ln P_\epsilon(x) \Big|_{\epsilon=0}$$

$$\langle z_L(t > 0)\rangle = C_{zz}(t)$$

- **If violated: the coordinate $\{x\}$ is non-Markovian.**

- Restricted to linear-response: extensive amount of trials.

$$\left[\dot{x} = -\mu k x + f(t) + \sqrt{2\mu k_B T}\,\xi_{th}(t)\right]$$



Nonlinear FDR
an efficient test for Markovianity

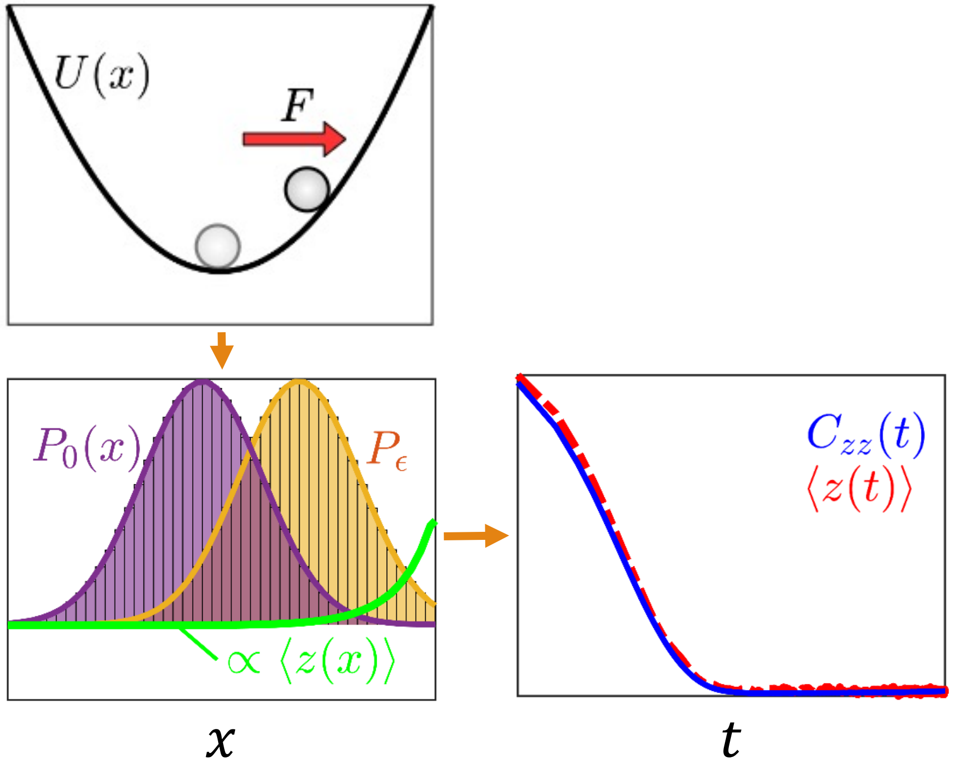
- ❑ A step – stimulus: $F(t) = \epsilon \cdot \Theta(-t)$
- ❑ **Nonlinear FDR:**

$$z_{NL}\big(x(t)\big) \equiv \frac{P_\epsilon(x)}{P_0(x)} - 1$$

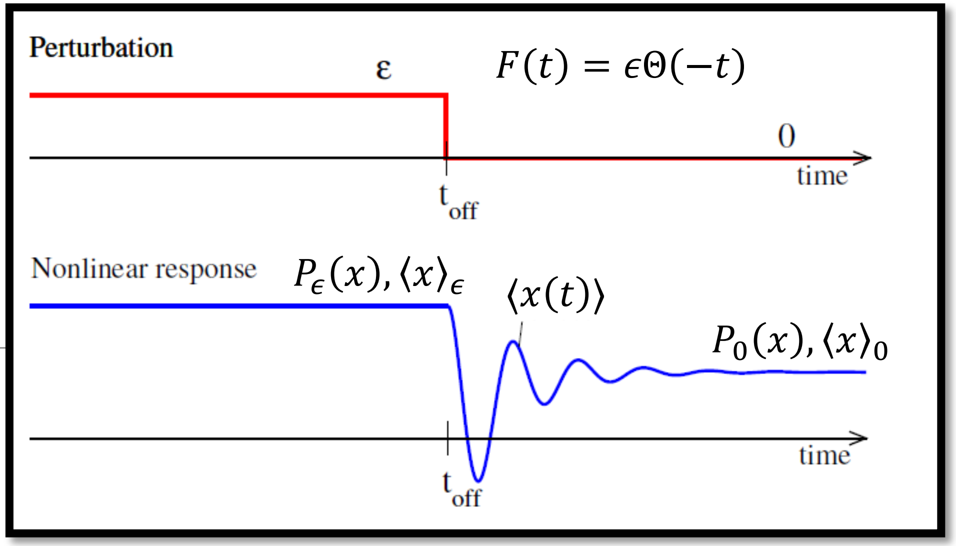
$$\langle z_{NL}(t > 0)\rangle = C_{zz}(t)$$

- ❑ **If violated: the coordinate $\{x\}$ is non-Markovian.**
- ❑ **Not restricted to weak perturbations: requires less averaging.**

$$\left[\dot{x} = -\mu k x + f(t) + \sqrt{2\mu k_B T}\,\xi_{th}(t)\right]$$



NL-FDR: step-like response derivation

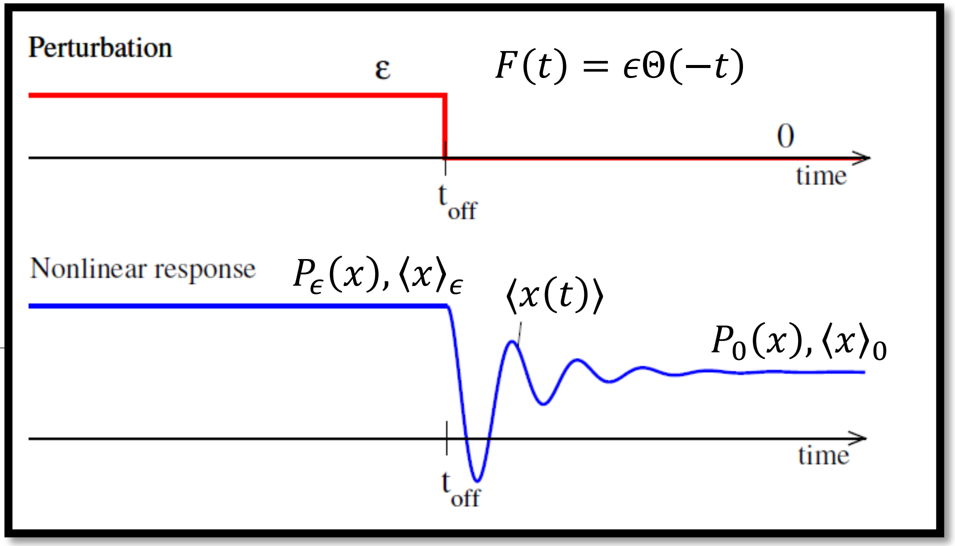


$z(x) \equiv \left(\frac{P_\epsilon(x)}{P_0(x)} - 1\right)$ - Conjugated variable.

$\langle z(x(t))\rangle = \int dx_1 \int dx_2 \cdot z(x_2) \cdot P_0(x_2, t|x_1, t_{\text{off}}) \cdot P_\epsilon(x_1)$ - mean value of $z(x(t > t_{\text{off}}))$.

$\langle \delta z(x(t))\rangle \equiv \langle z(x(t))\rangle - \langle z(x(t))\rangle_0 = \int dx_1 \int dx_2 \cdot z(x_2) \cdot (P_\epsilon(x_1) - P_0(x_1)) \cdot P_0(x_2, t|x_1, t_{off})$

NL-FDR: step-like response derivation



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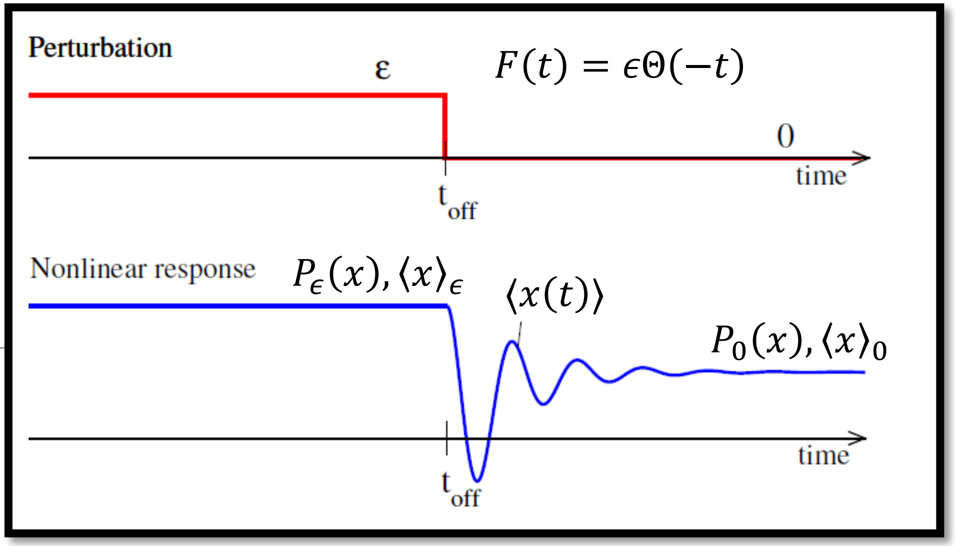

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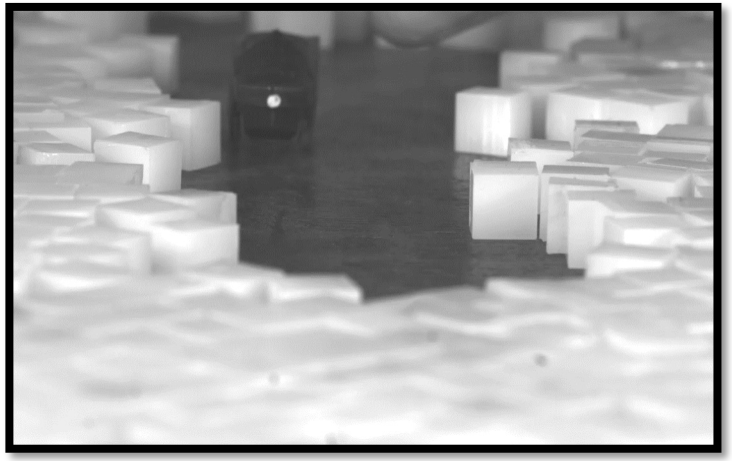
$\langle z(\tau)\rangle = \langle z(\tau) z(0)\rangle_0 \equiv C_{zz}(\tau)$ - **Nonlinear fluctuation-response relation**.

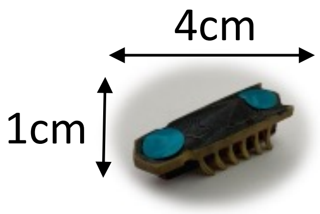
The system: mechanical perturbation

- The amplitude of the perturbation is controlled by the fan voltage.
- A shutter is added to simulate a step – stimulus.

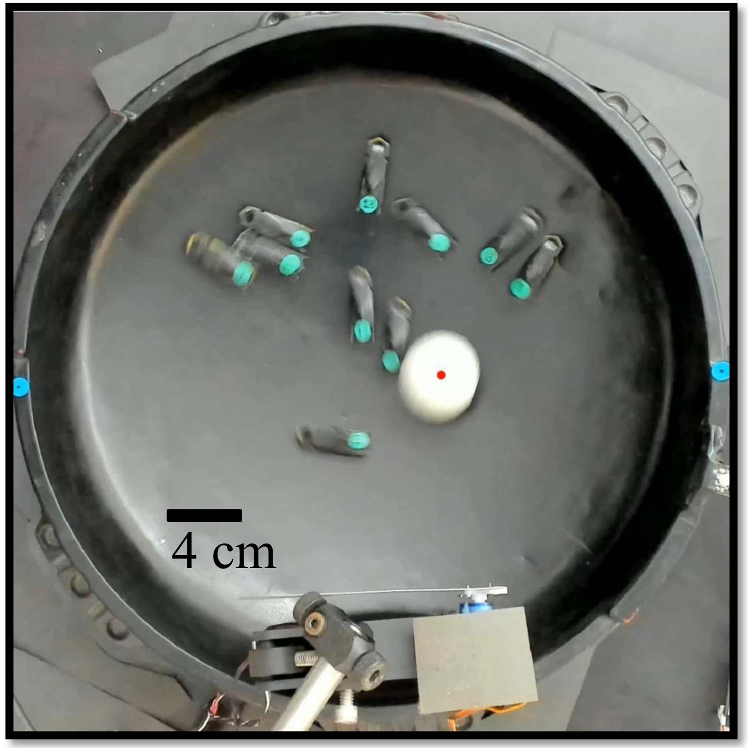
The system: bristlebots

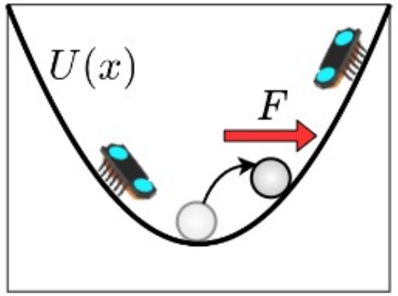
<u>Self-propelled bristlebots:</u>





<u>Bristlebots in a harmonic trap</u>



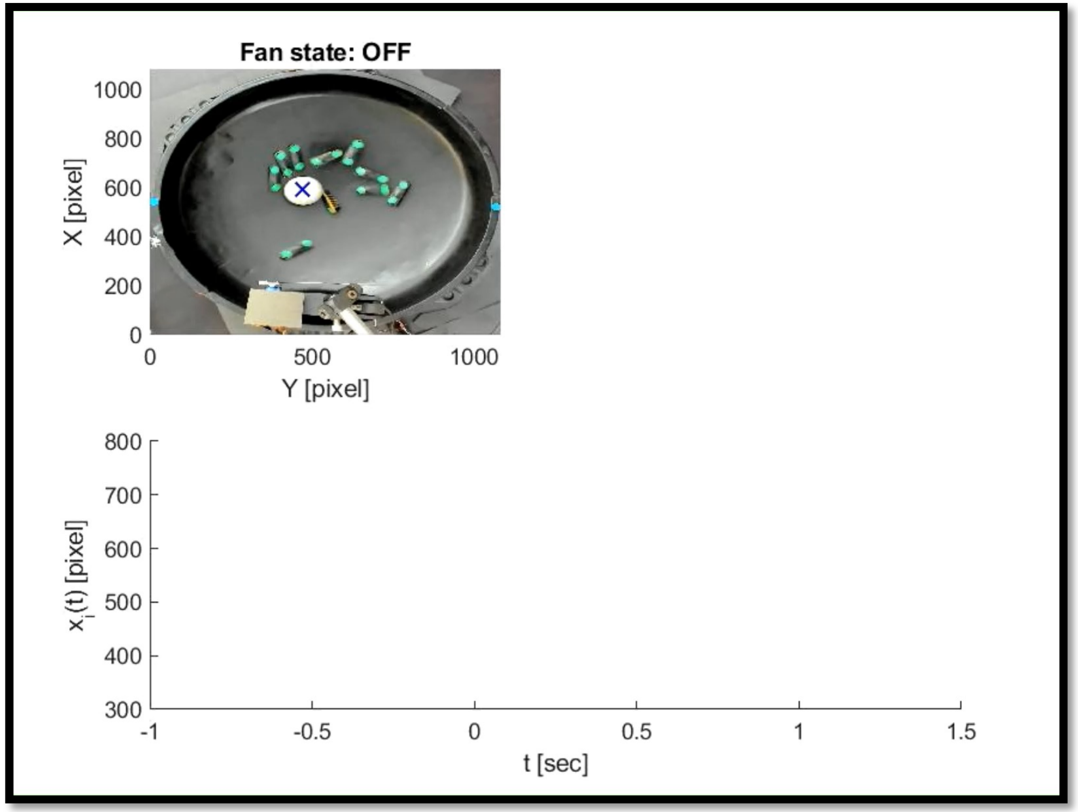


fluctuation-response protocol: passive tracer in a bristle-bot bath

- Time sequences $x_i(t)$ are recorded during a time window $T = 2$ min (1 min force on/off).

- $M = 375$ trials are averaged.

- The steady-state densities and the nonlinear conjugated variable $z\big(x(t)\big)$ are obtained.



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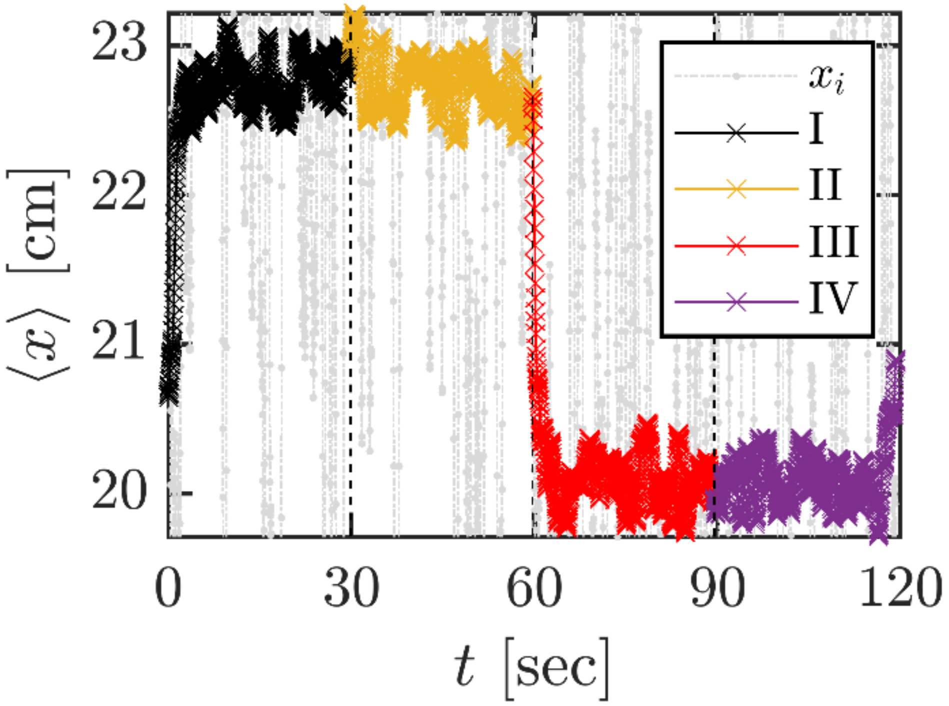

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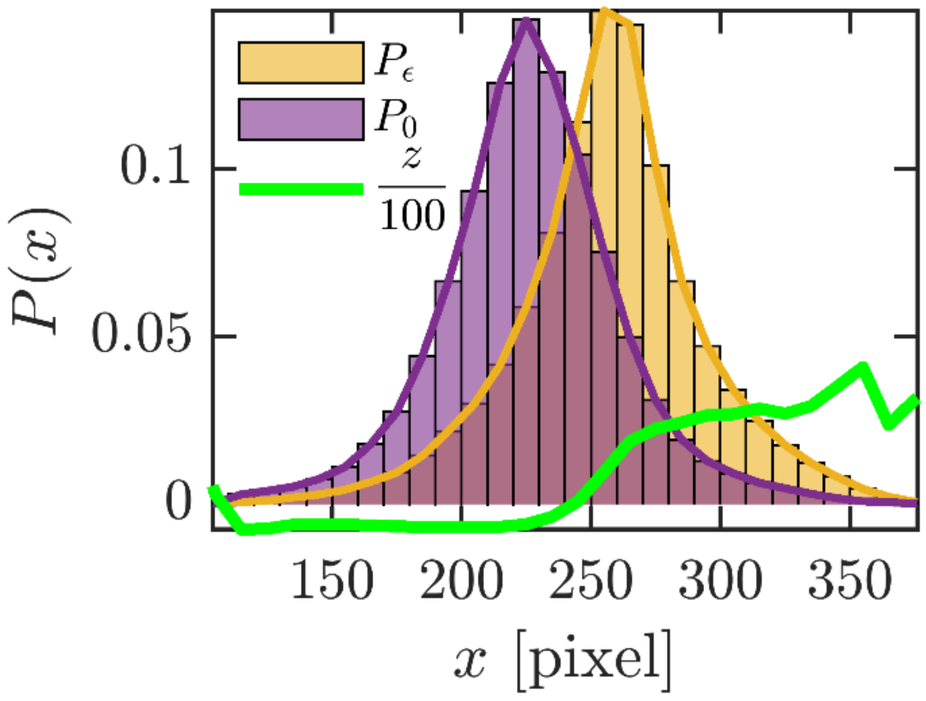
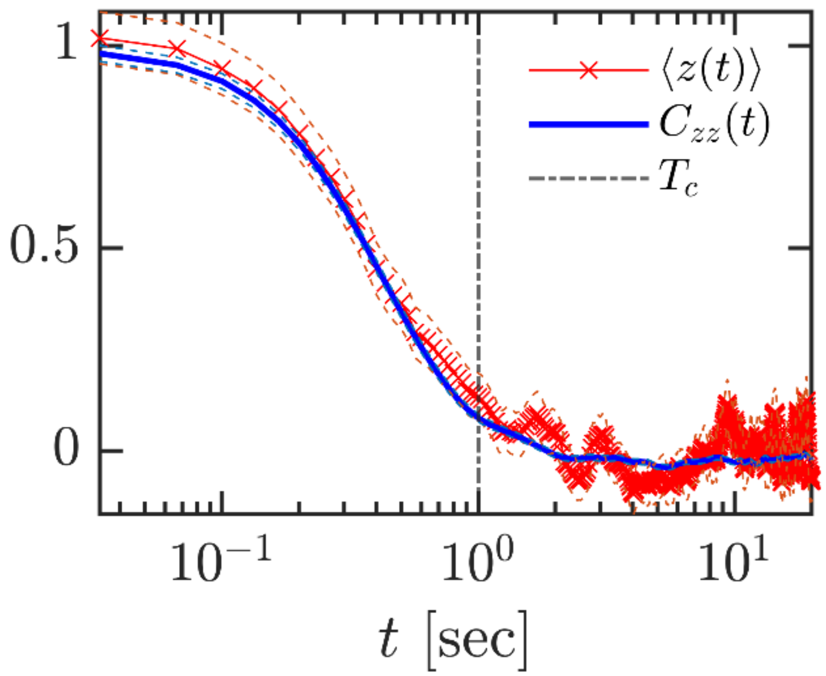
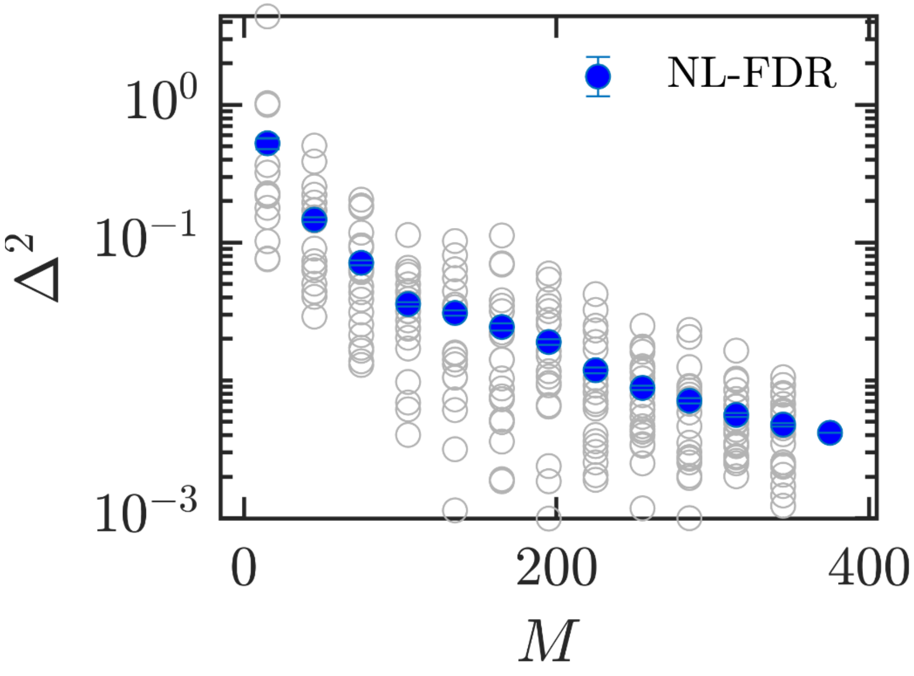
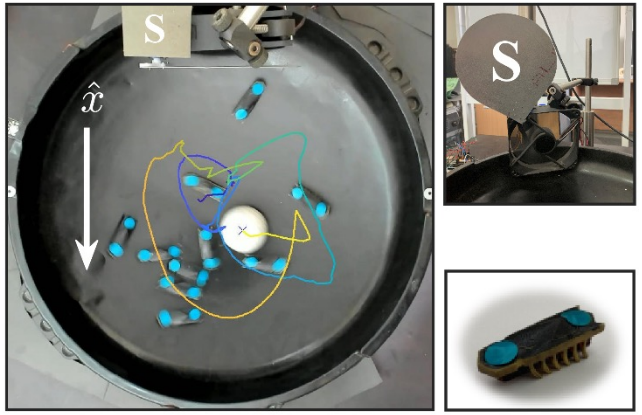


Results: verification of the NL-FDR





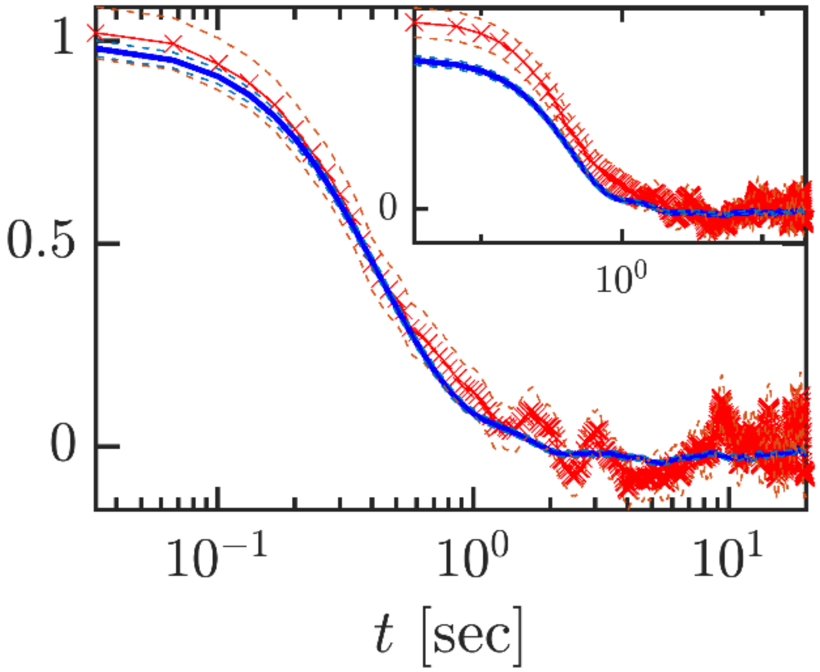


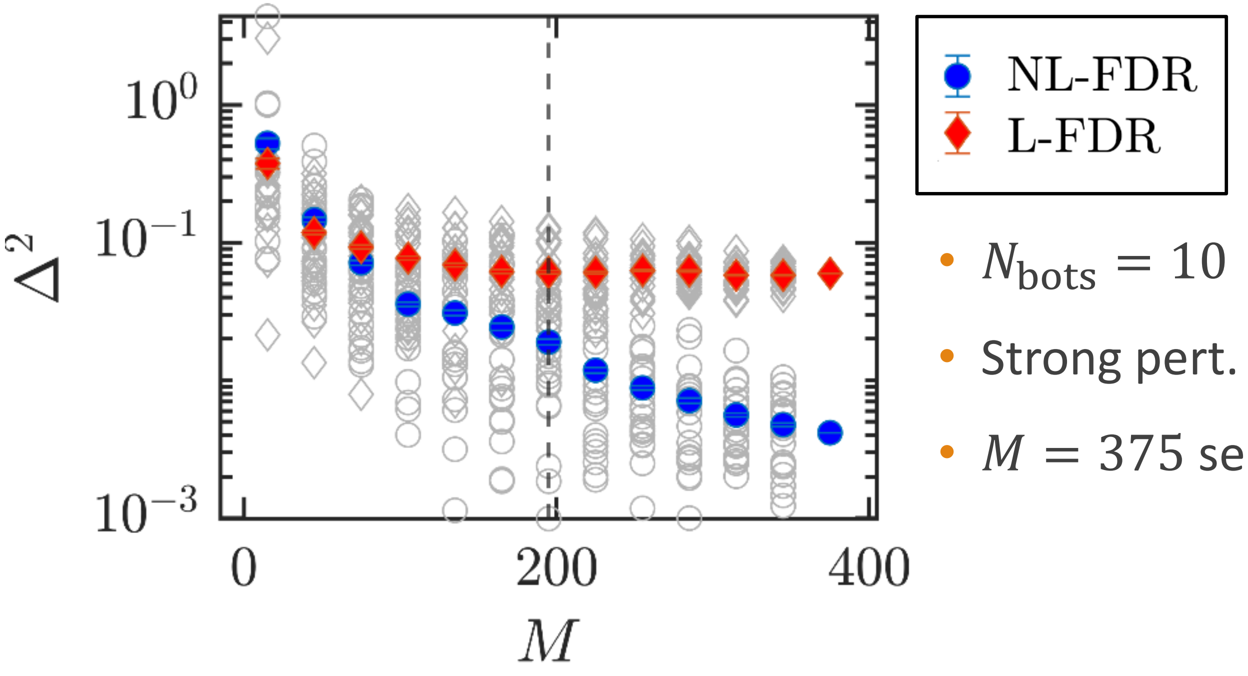
- $N_{\text{bots}} = 10$
- Strong pert. ($12V$ fan)
- $M = 375$ sequences.

$$\langle z(t) \rangle = \langle z(t+\tau) z(t) \rangle_0$$

Δ^2 - relative squared deviations

Results:
convergence in the nonlinear regime

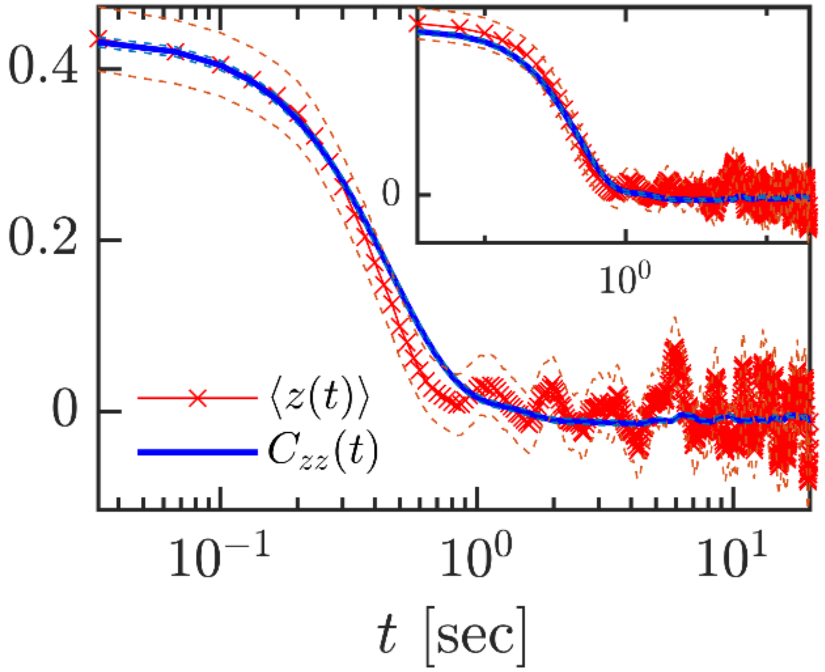


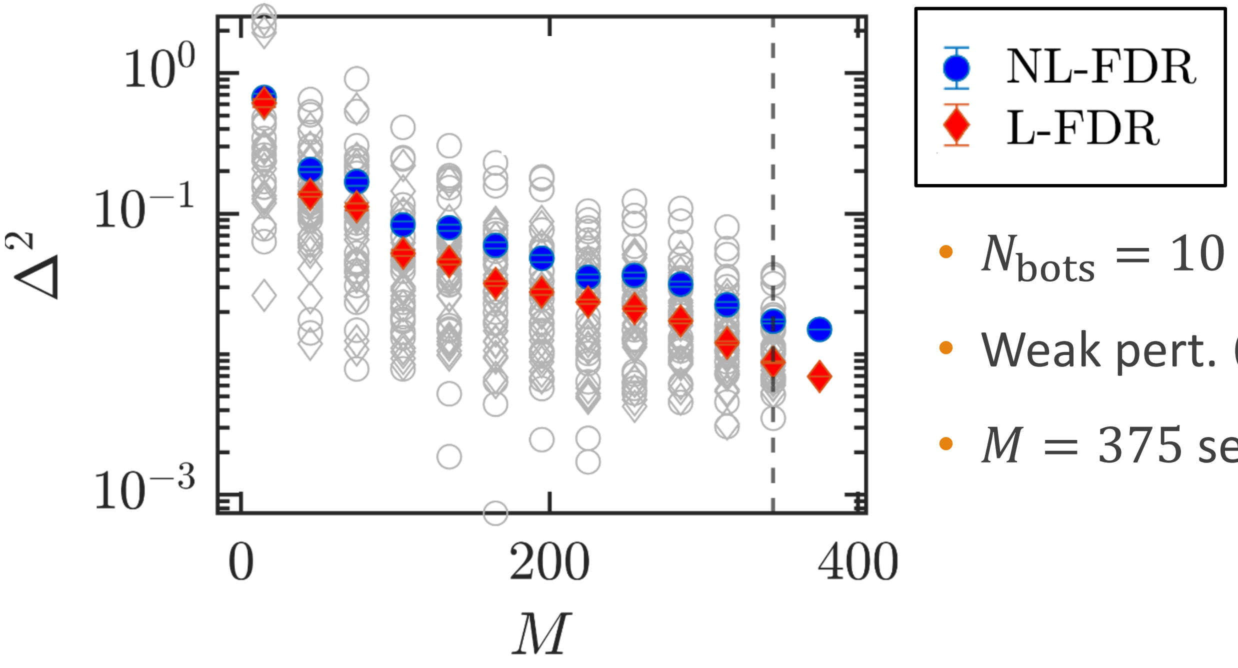


- $N_{\text{bots}} = 10$
- Strong pert. ($12V$ fan)
- $M = 375$ sequences.

Inset: linear FDR violation for a strong perturbation.

Results:
both relations verified in the linear regime

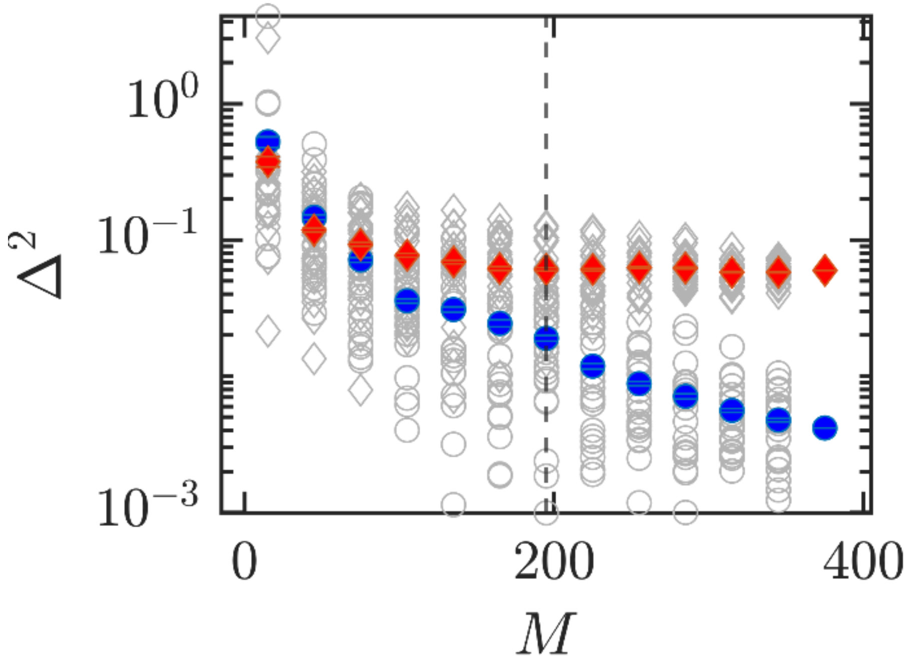




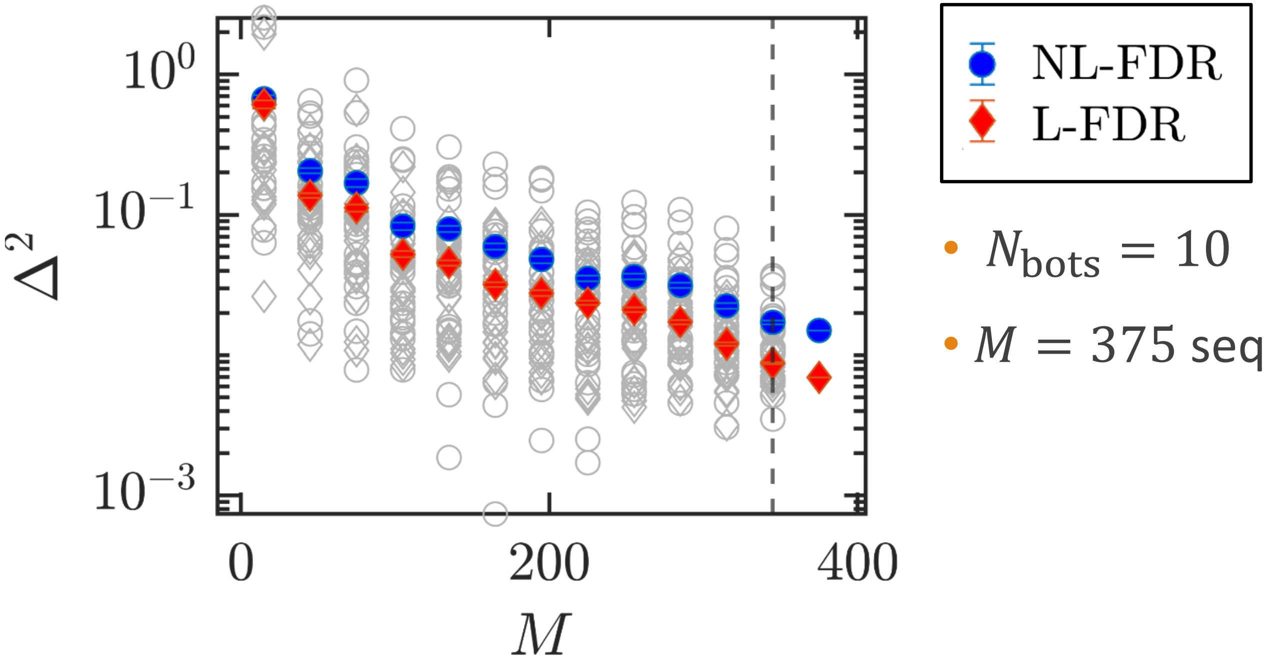
- $N_{\text{bots}} = 10$
- Weak pert. ($9V$ fan)
- $M = 375$ sequences.

Inset: linear FDR verified for a weak perturbation.

Results: relative-squared-deviation



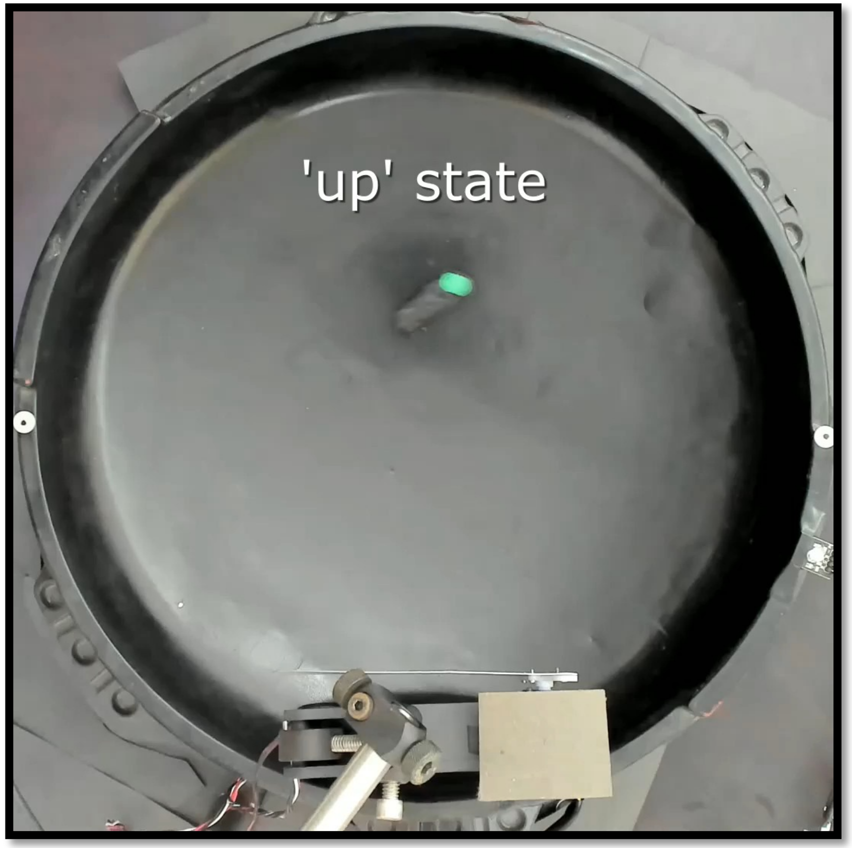
Strong perturbation (12V)



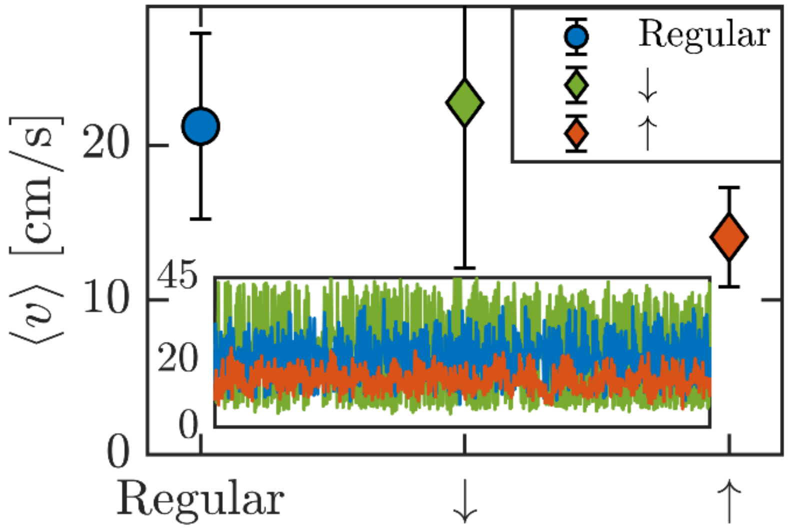
Weak perturbation (9V)

- $N_{\text{bots}} = 10$
- $M = 375$ sequences.

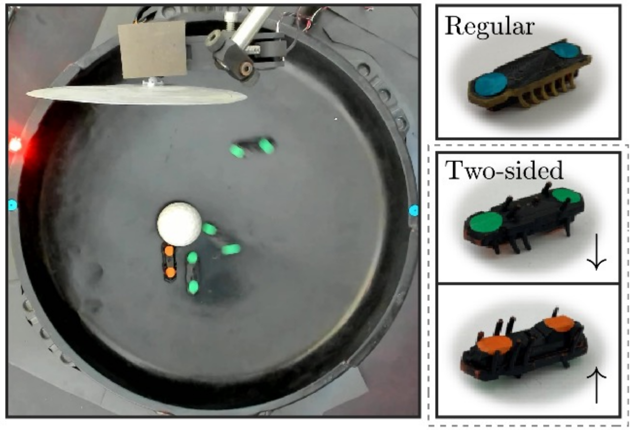
Non-Markovian setup: violation of the NL-FDR

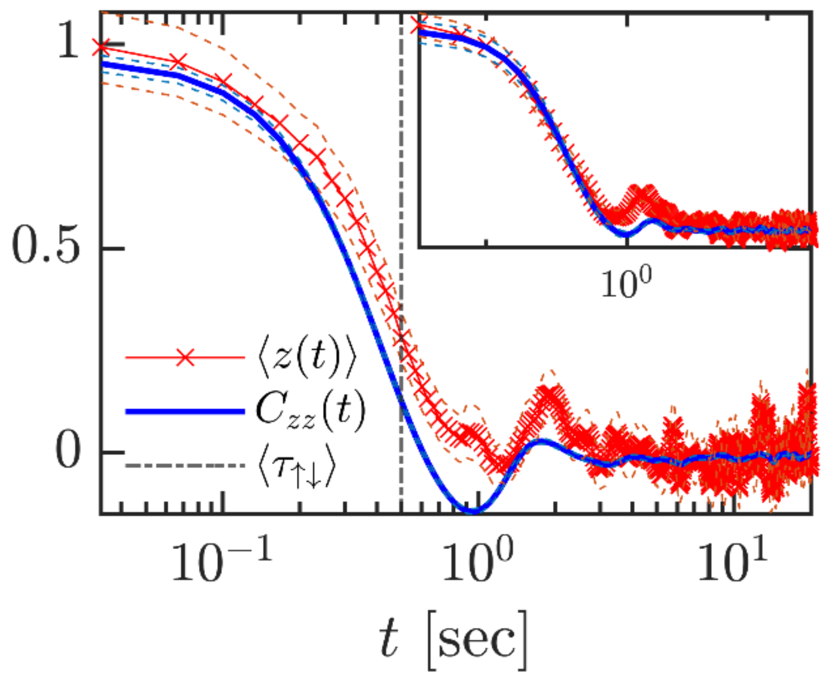


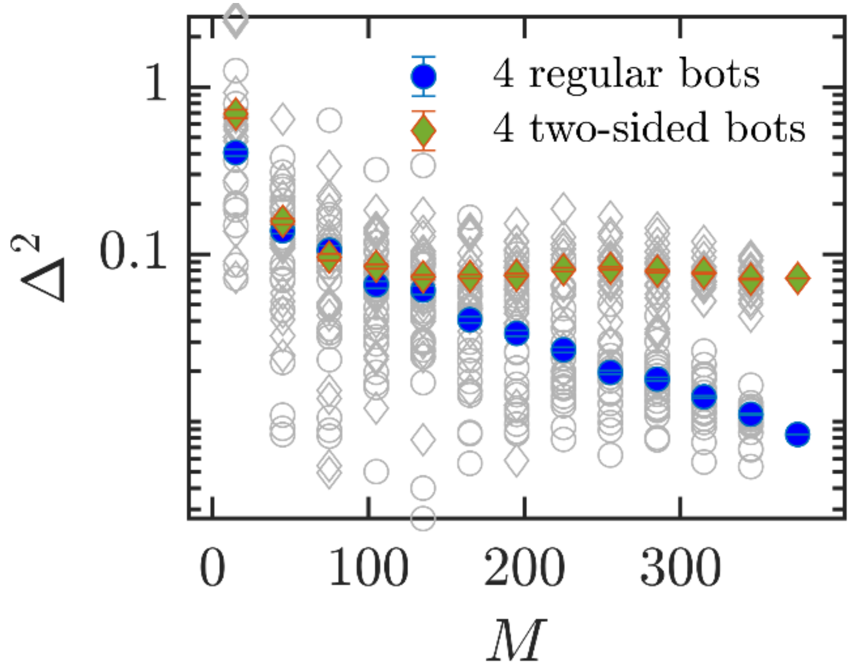
- All green ‘states’ → ‘uniform’ system state.
- Otherwise (green & orange) → ‘mixed’ system state.



Non-Markovian setup: violation of the NL-FDR







- $N_{\text{bots}} = 4$ two-sided.
- Strong pert. ($12V$ fan)
- $M = 375$ sequences.

Inset: 4 'regular' bots NL-FDR.

Conclusions: non-linear FDR for Markovian systems

- We presented the NL-FDR as a model-free tool to exclude Markovianity.

- The NL-FDR was experimentally verified in the nonlinear force regime.
Requiring significantly less data then the linear FDR.

- We witnessed a violation of the NL-FDR in a non-Markovian system.

- **The NL-FDR is an experimentally accessible tool to exclude Markovianity.**

A nonlinear fluctuations – dissipation test for Markovian systems

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[3] Physics Department of Humboldt University Berlin, Germany.

[4] Bernstein Center for Computational Neuroscience Berlin, Germany.

הפקולטה למדעים מדויקים ע"ש ריימונד ובברלי סאקלר אוניברסיטת תל אביב

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