Can migration and disorder rescue metacommunities from extinctions?

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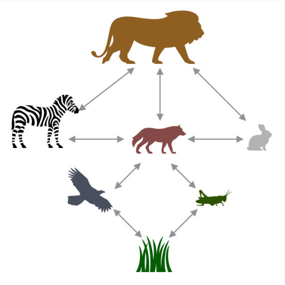
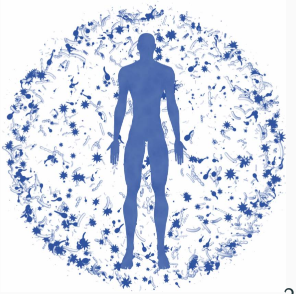
Theoretical Community Ecology

- Ecosystems of many species
  $\rightarrow$ Statistical physics
- Collective properties shared by different ecosystems
  $\rightarrow$ **Universality**
- Heterogeneous interactions
  $\rightarrow$ sample **randomly**





May, Nature (1972).

Theoretical Community Ecology

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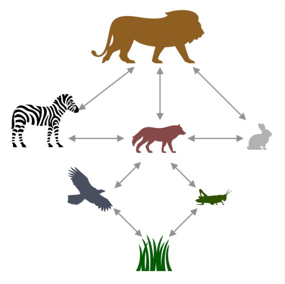
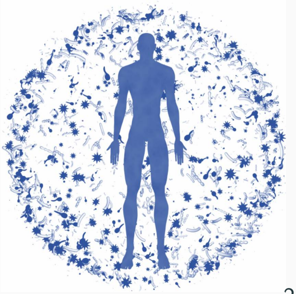
- Diversity? Stability? Chaotic dynamics?





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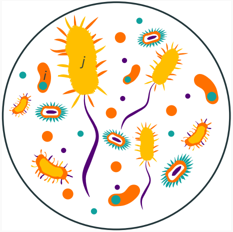
Lotka-Volterra (meta)community

$$\dot{N}_i = N_i \left(k - N_i - \sum_j \alpha_{ij} N_j \right)$$

$S \to \infty$ species



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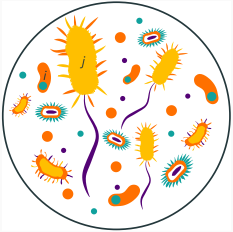

- Self-regulation

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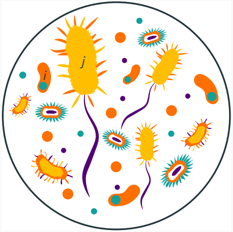

- Self-regulation
- **Random** inter-species interactions

$$\langle \alpha_{ij} \rangle = \mu / S$$
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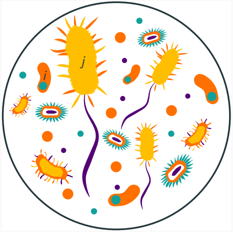

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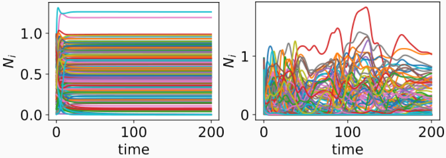
$\longrightarrow$ Different phases:

Stability, multistability, chaos



Bunin (2017), Biroli et al. (2018), Altieri et al. (2021).

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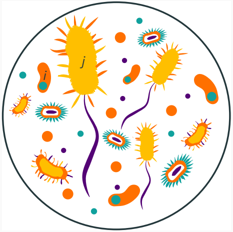

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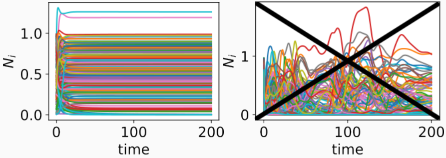

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Lotka-Volterra (meta)community

$$\dot{N}_i = N_i \left(k - N_i - \sum_j \alpha_{ij} N_j\right) + \eta_i \sqrt{N_i}$$

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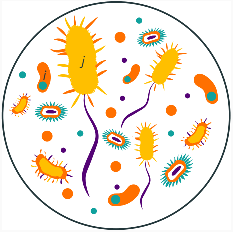

- Self-regulation
- **Random** inter-species interactions
- Demographic fluctuations

$$\boxed{\langle \eta_i(t)\eta_j(t')\rangle = 2T\delta_{ij}\delta(t-t')}$$

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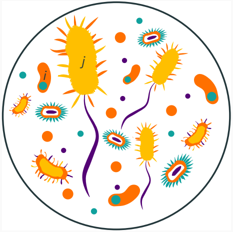

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- **Random** inter-species interactions
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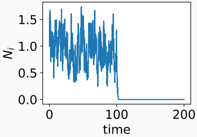
  $\longrightarrow$ **extinctions**



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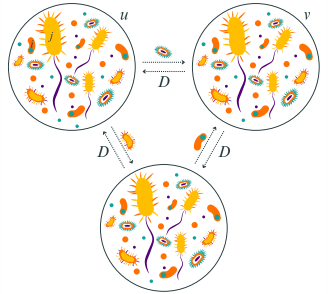

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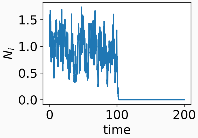
- Diffusion between communities



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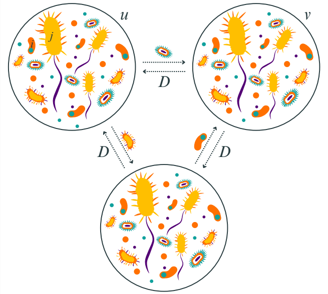
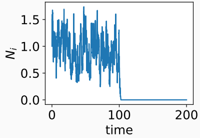
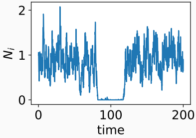
$S \to \infty$ species

$L \to \infty$ sites



- Self-regulation
- **Random** inter-species interactions
- Demographic fluctuations
  $\longrightarrow$ **extinctions**
- Diffusion between communities
  $\longrightarrow$ **insurance effect**





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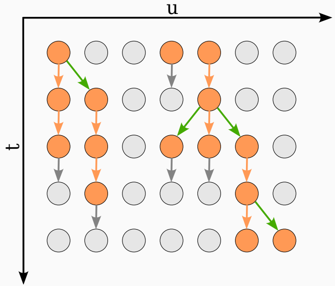
Coupled Directed Percolation processes

$$\dot{N}_i{}^u = N_i{}^u\left(k - N_i{}^u - \sum_j \alpha_{ij} N_j^u\right) + \eta_i{}^u(t)\sqrt{N_i{}^u} + \frac{D}{L}\sum_v \left(N_i{}^v - N_i{}^u\right)$$



Denk, Hallatschek (2022)

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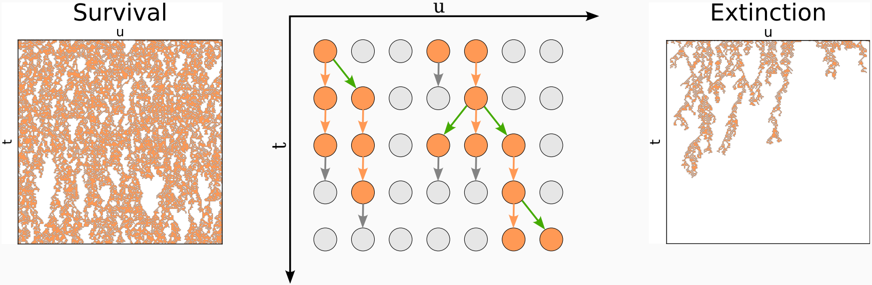

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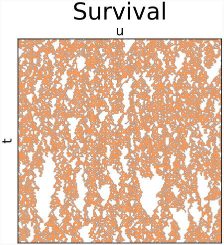

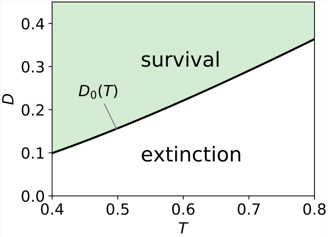

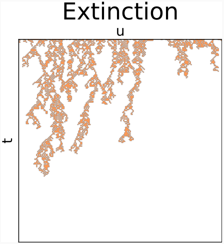

Continuous phase transition

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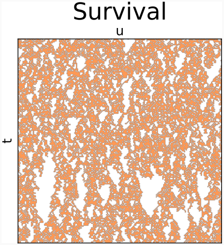

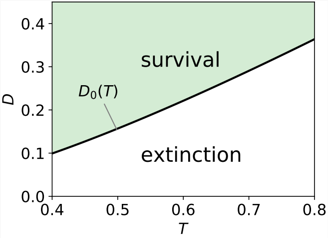

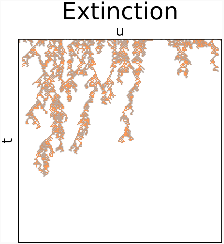

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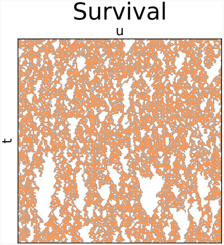

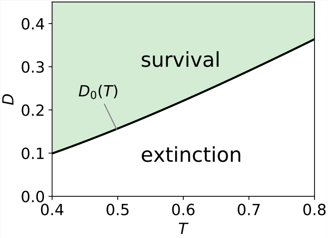

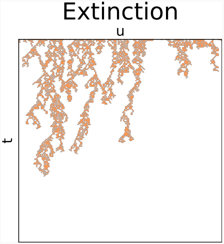

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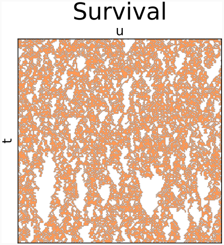

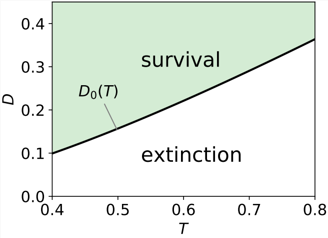

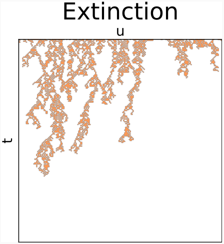

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Denk, Hallatschek (2022)

Double Dynamical Mean Field Theory

$$S \to \infty$$

$$\sum_j \alpha_{ij} N_{j,u} \longrightarrow \mu h_u + \sigma \xi_u(t) - \sigma^2 \gamma \int_0^t \sum_v R_{uv}(t,t') N_v(t') dt'$$

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Effective dynamics for single species in single site + self-consistency

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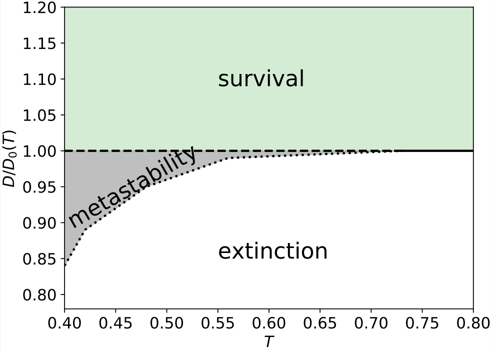
Effective dynamics for single species in single site + self-consistency

Symmetric interactions $\longrightarrow$ FDT $\longrightarrow$ $P_{eq} \propto e^{-\beta H_{eff}}$

Phase diagram



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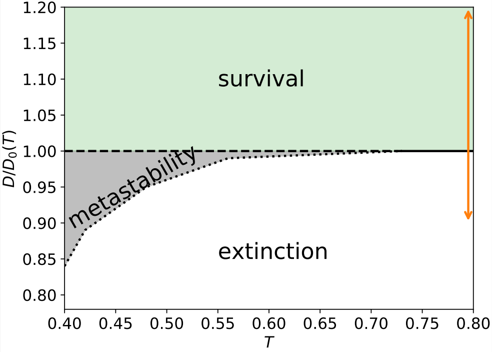

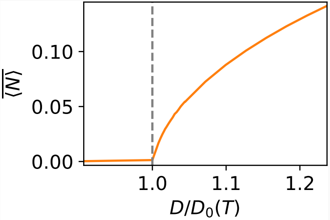

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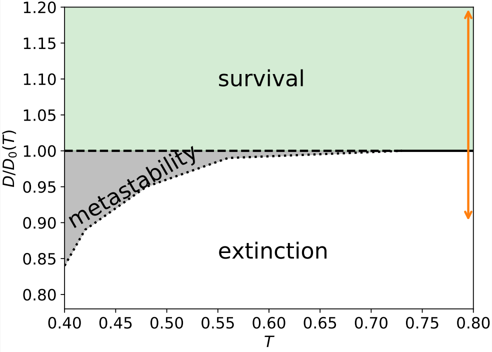
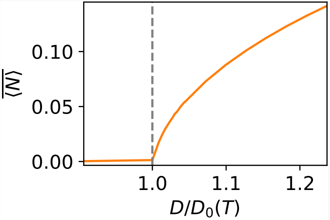
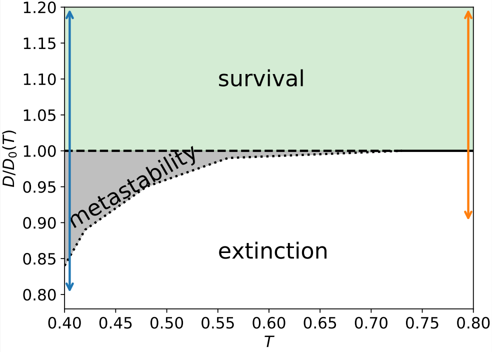
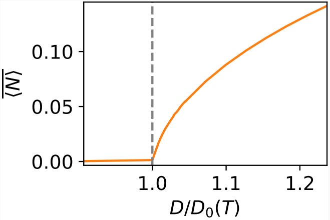




DP universality class

Phase diagram





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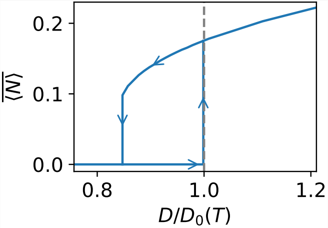

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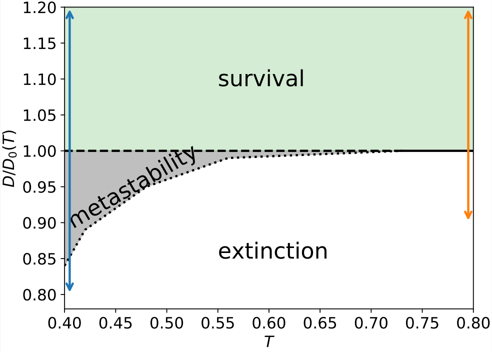
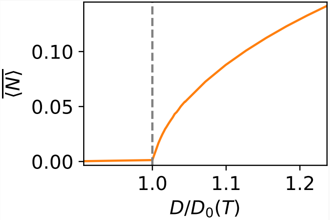
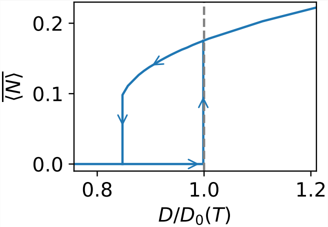


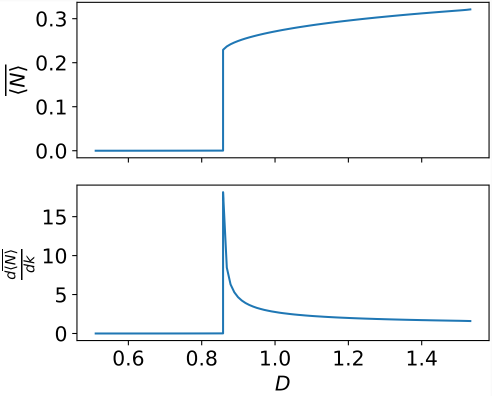


DP universality class



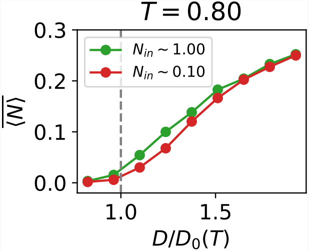
New universality class

Fragility of the metastable state

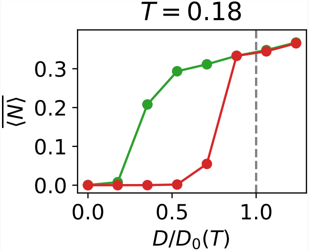


Approaching the tipping point (*spinodal*) the response of the abundance to perturbations diverges

Continuous transition



Discontinuous transition



Mutualism

- Similar discontinuous transition with **mutualistic** interactions: collaboration → survival in harsh conditions → collapse

Denk, Hallatschek (2023)

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Denk, Hallatschek (2023)

Mutualism

- Similar discontinuous transition with **mutualistic** interactions: collaboration $\rightarrow$ survival in harsh conditions $\rightarrow$ collapse
- In our case mutualism naturally **emerging** in the surviving community close to extinction
- Enables survival, but leads to **fragility**

Denk, Hallatschek (2023)

Can migration and disorder rescue metacommunities from extinction?

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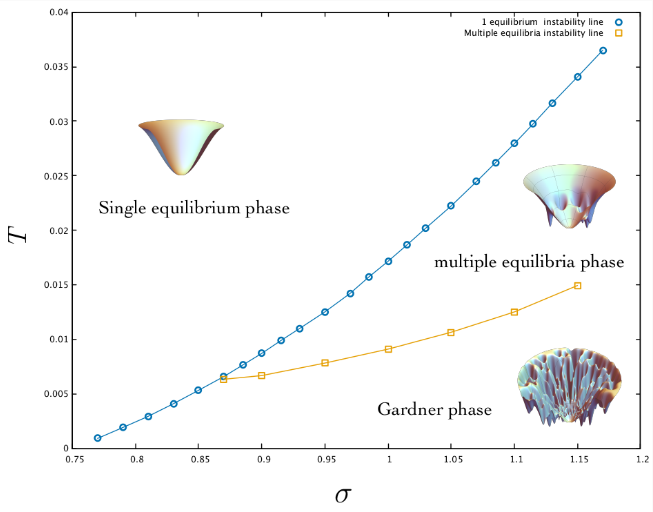
Perspectives:

- Finite dimensional spatial networks
- Strong asymmetry in the interactions
- Strong heterogeneity: multiple equilibria, chaotic dynamics

Thank you

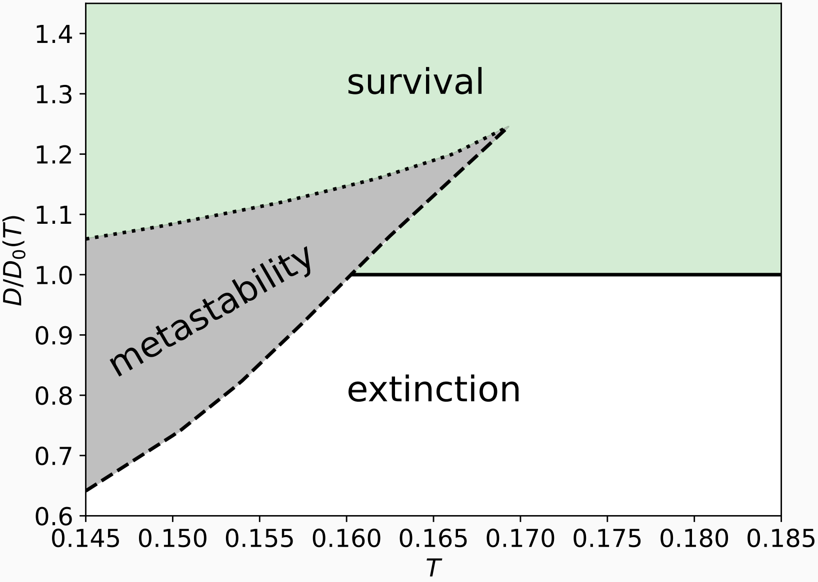
for your attention!

Phase diagram



Altieri et al. (2021)

Phase diagram $\rho = 0$



Self-consistent conditions

$$N^*(z) = \langle N \rangle = \frac{\int_0^\infty dN N e^{-\beta H_{eff}}}{\int_0^\infty dN e^{-\beta H_{eff}}}$$

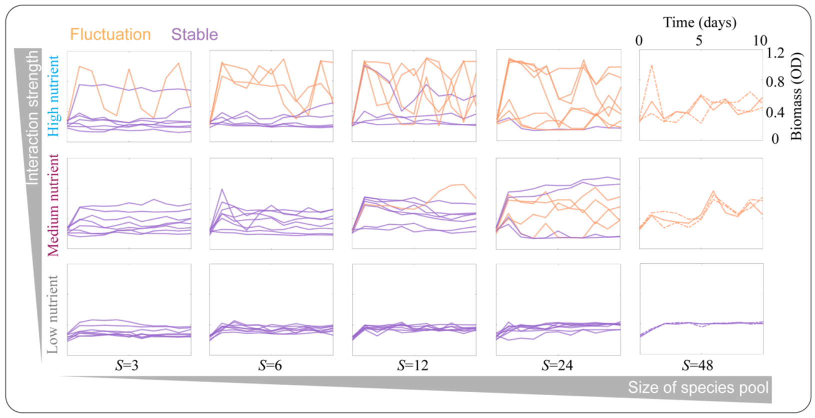
$$h = \overline{\langle N \rangle} = \int \mathcal{D}z \frac{\int_0^\infty dN N e^{-\beta H_{eff}}}{\int_0^\infty dN e^{-\beta H_{eff}}}$$

$$C_d^0 = \overline{\langle N^2 \rangle} = \int \mathcal{D}z \frac{\int_0^\infty dN N^2 e^{-\beta H_{eff}}}{\int_0^\infty dN e^{-\beta H_{eff}}}$$

$$C_d^\infty = \overline{\langle N \rangle^2} = \int \mathcal{D}z \left(\frac{\int_0^\infty dN N e^{-\beta H_{eff}}}{\int_0^\infty dN e^{-\beta H_{eff}}} \right)^2$$

$$H_{eff} = \left(1 - \frac{\sigma^2}{T}\left(C_d^0 - C_d^\infty\right)\right)\frac{N^2}{2} - \left(k - \mu h - D + \right.$$
$$\left. + D\sigma^2 N^* R_0^{int} + z\sqrt{C_d^\infty}\sigma\right) N + (T - DN^*) \ln N$$

Experimental validation



Hu et al., Science (2022).

Extensions

- Spatial variation of the interactions $\langle \alpha_{ij}^{u} \alpha_{ij}^{v} \rangle_c = \rho\sigma^2/S$
- (Small) asymmetry in the interactions $\langle \alpha_{ij}^{u} \alpha_{ji}^{u} \rangle_c = \gamma\sigma^2/S$

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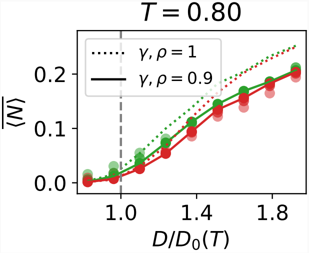
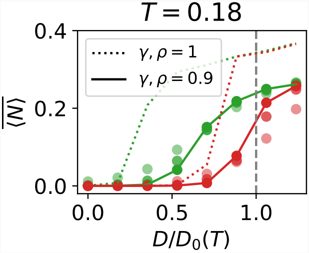
$\longrightarrow$ Same qualitative behavior

Continuous transition



Discontinuous transition



Numerical dynamics