Hadron Spin Decomposition via QCD Sum Rules

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Refs.

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Outline

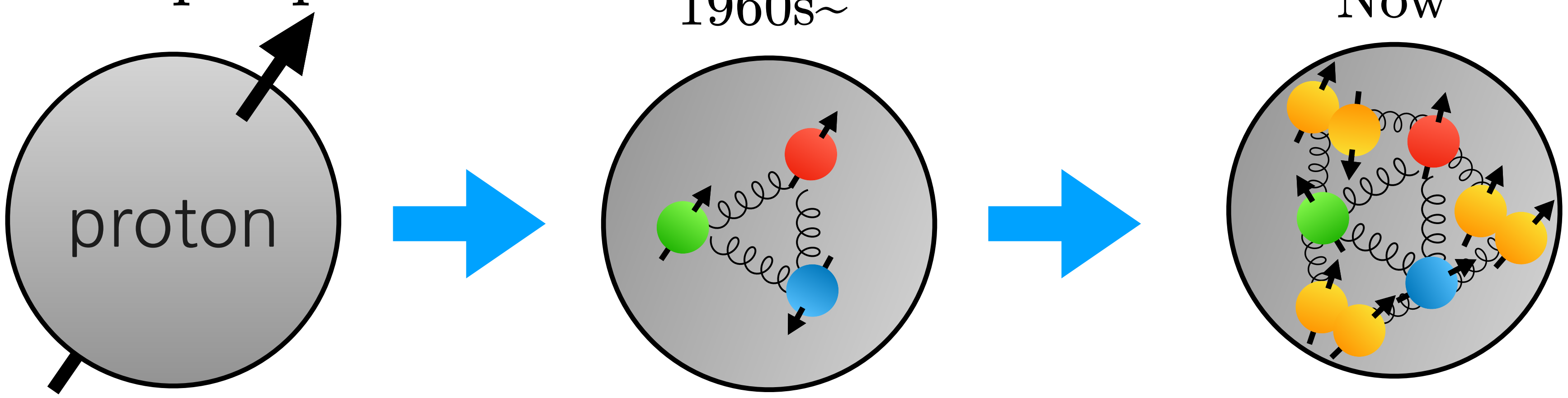
- Two-point function and its spin structure
- Basic idea of QCD Sum Rules (QCDSR)
- Hadron Spin Decomposition via QCDSR
- Application 1: Spin-1 quarkonium
- Application 2: Spin-1/2 proton
- Summary

Motivation

- Hadron is composed of quarks and gluons
  - *meson=quark+anti-quark, baryon=3 quarks*

- Naive quark model is successful, but insufficient

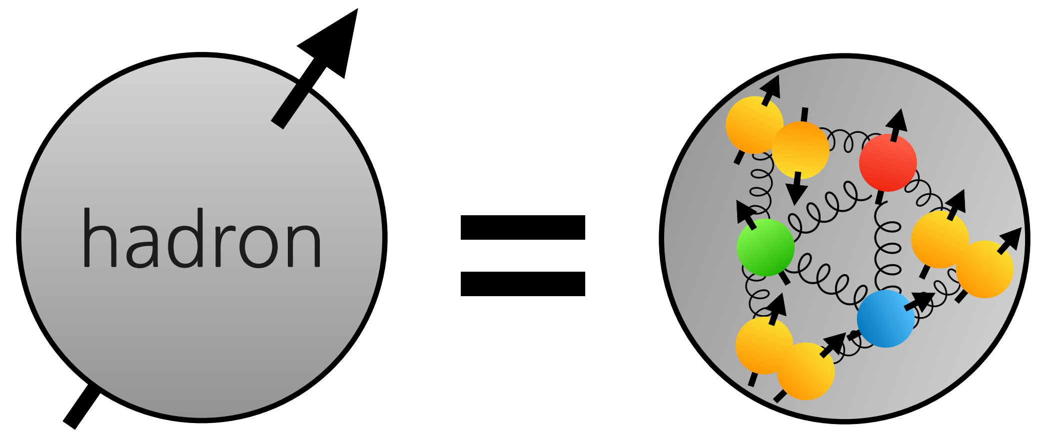
- ex) Proton spin puzzle



$$\frac{1}{2} = \underbrace{\boxed{\text{Quark Spin}}}_{\sim 30\%\ \text{(COMPASS)}} + \underbrace{\boxed{\text{Quark Orbit}}}_{?\ \%} + \underbrace{\boxed{\text{Gluon AM}}}_{?\ \%}$$

=> new approach for studying spin structures of hadrons

Two-point function (2PF)

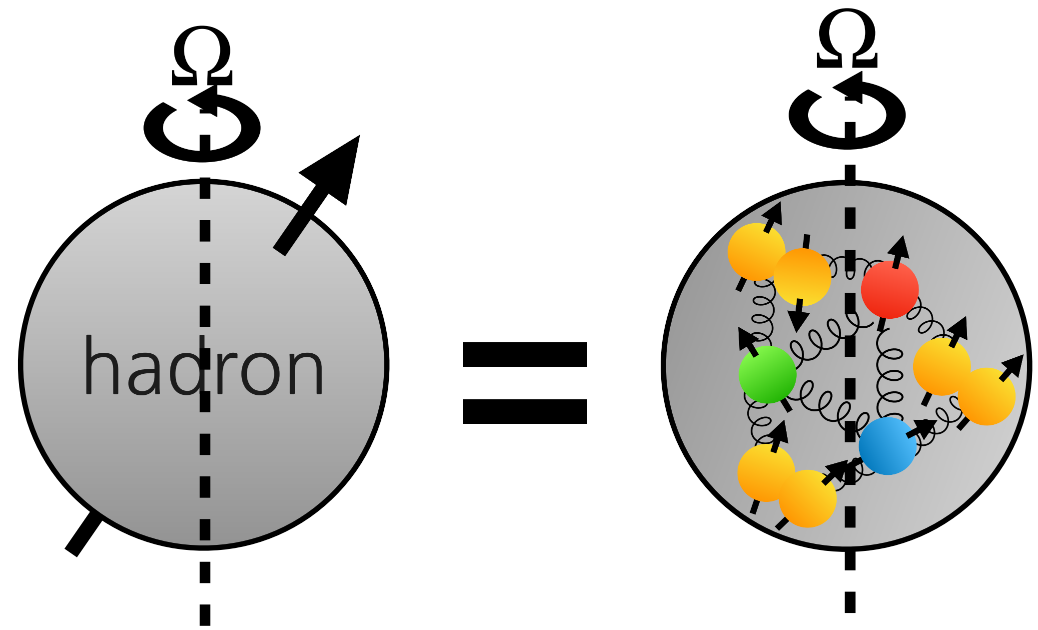


$$\Pi(q) = i\int d^4x e^{iqx}\langle 0|\mathrm{T}\{\phi(x)\phi^\dagger(0)\}|0\rangle$$

$$= \frac{|\langle 0|\phi|h\rangle|^2}{q^2 - m_h^2 + i\epsilon} + \cdots$$

- Basic tool for studying properties of a particle created by $\phi(x)$
- In QCD, a composite field can create hadronic states
- Study hadron mass m_h in terms of quarks and gluons

In a rotating frame



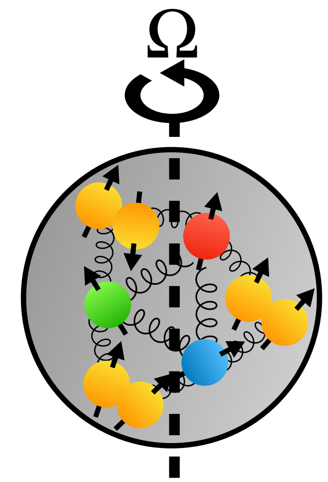
$$\Pi(q,\Omega) = i\int d^4x e^{iqx}\langle 0|\mathrm{T}\{\phi(x)\phi^\dagger(0)\}|0\rangle_\Omega$$

$$= \Pi(q) + \underline{\Pi^{\mathrm{rot}}(q)}\cdot\Omega + \mathcal{O}(\Omega^2)$$

- Uniform rotation with angular velocity Ω along the z-axis
- Two independent ways to derive the $\Pi^{\mathrm{rot}}(q)$
  - Method A : quarks and gluons
  - Method B : total system

$\Rightarrow$ spin structure of 2PF

Method A



For $\Omega \ll 1$, ψ and A^a_μ introduce their total AM operators.

Insert "QCD total AM" into the two-point function

(Ji's decomposition)

$$J_{\rm QCD}(x) = \underbrace{\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma^5\psi}_{S_q\,:\,\text{quark spin}} + \underbrace{\psi^\dagger(\vec{x}\times(-i\overrightarrow{D}))\psi}_{L_q\,:\,\text{quark orbit}} + \underbrace{\vec{x}\times(\overrightarrow{E}\times\overrightarrow{B})}_{J_g\,:\,\text{gluon AM}}$$

insert

$$\Pi^{\rm rot}_{\rm A}(q) = i\int d^4x\, d^4y\, e^{iqx}\langle 0\,|\,T\{\phi(x)\, \overset{\text{insert}}{J_{\rm QCD}(y)}\, \phi^\dagger(0)\}\,|\,0\rangle$$

$$= \langle S_q\rangle + \langle L_q\rangle + \langle J_g\rangle$$

: AM decomposition of two-point function

Method B

For $\Omega \ll 1$, ϕ field introduce its total AM operator.

$$\phi'(x) \to (1 + iJ_z\Omega x^0)\phi(x) + \mathcal{O}(\Omega^2)$$

$$\Pi_B^{\mathrm{rot}}(q) = \left[S_z + (\vec{q} \times (i\vec{\partial}_q))_3\right]\partial_q^0 \Pi(q)$$

total spin

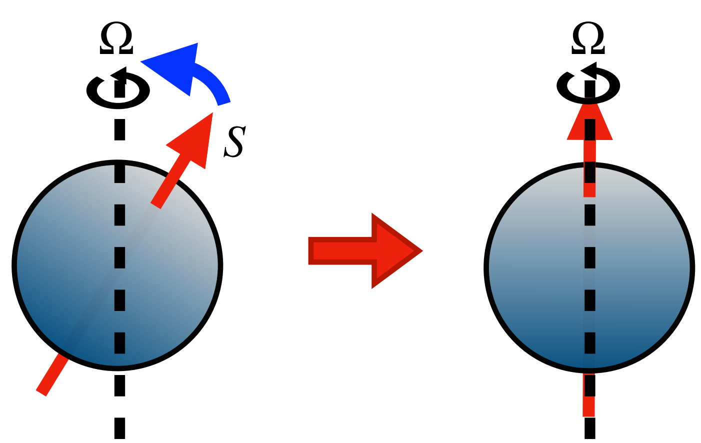
total orbital AM

In the rest frame,

$$\Pi_B^{\mathrm{rot}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

s_z: eigenvalue of S_z

Spin-Rotating Coupling (SRC)



$$H_{\mathrm{SRC}} = -S \cdot \Omega$$

$$\Pi(\omega) \to \Pi(\omega + s_z\Omega) = \Pi(\omega) + s_z \frac{\partial \Pi(\omega)}{\partial \omega} \cdot \Omega + \mathcal{O}(\Omega^2)$$

$$\Pi_B^{\mathrm{rot}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

A = B

- In the rest frame,

$$\Pi_{\mathrm{A}}^{\mathrm{rot}}(\omega) = \lim_{\vec{q}\to 0} \langle S_q \rangle + \langle L_q \rangle + \langle J_g \rangle$$

: quantifies how quarks and gluons contribute to the total spin of the system

$$\Pi_{\mathrm{B}}^{\mathrm{rot}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

: depends on total spin & inertial frame 2PF

- Both methods are describing the same quantity

$$\Rightarrow \Pi_{\mathrm{A}}^{\mathrm{rot}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

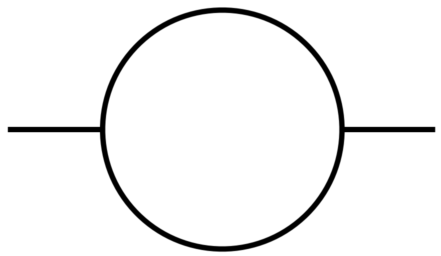
: very useful insight into the spin structure of 2PF

Implications

$$\Pi_{\mathrm{A}}^{\mathrm{rot}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

(1) Spin decomposition of individual Feynman diagram

$$\Pi(\omega) = \frac{1}{s_z} \int^{\omega} d\omega' \, \Pi_{\mathrm{A}}^{\mathrm{rot}}(\omega')$$



$$= \langle S_q(\omega) \rangle + \langle L_q(\omega) \rangle + \langle J_g(\omega) \rangle$$

(2) Verify computation of $\Pi_{\mathrm{A}}^{\mathrm{rot}}$

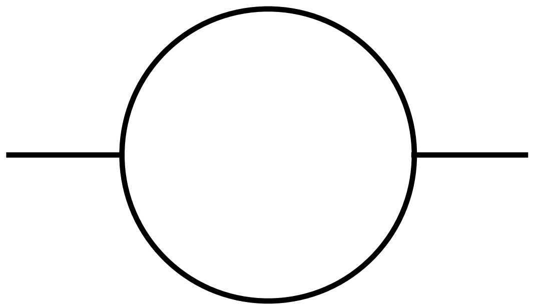
$\Pi(\omega)$ is much easier to compute or is already given.

(3) Constraint on the spectral density in a rotating frame

$$\rho = \mathrm{Im}\Pi \implies \rho^{\mathrm{rot}} = \mathrm{Im}\Pi^{\mathrm{rot}} = s_z \frac{\partial \rho(\omega)}{\partial \omega}$$

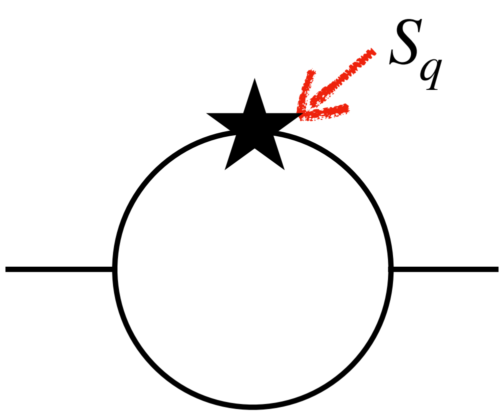
Example

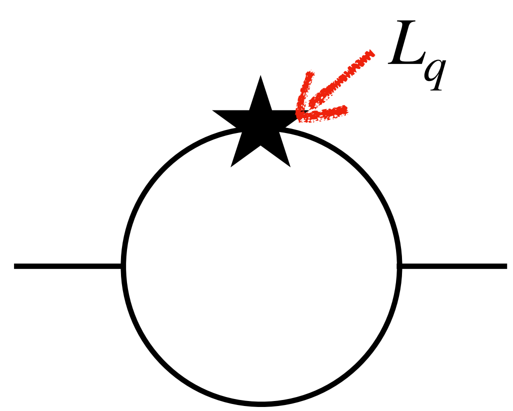
Vacuum polarization. $\phi(x) = \bar{\psi}(x)\gamma^{\mu}\psi(x)$



$$\Pi(q) = i\int d^4x e^{iqx}\langle 0 | \mathrm{T}\{\phi(x)\phi^{\dagger}(0)\} | 0\rangle$$

$$\mathrm{Im}\Pi(\omega) = \frac{-3\omega^2}{4\pi}\sqrt{1-\frac{4m^2}{\omega^2}}\left(1+\frac{2m^2}{\omega^2}\right)$$



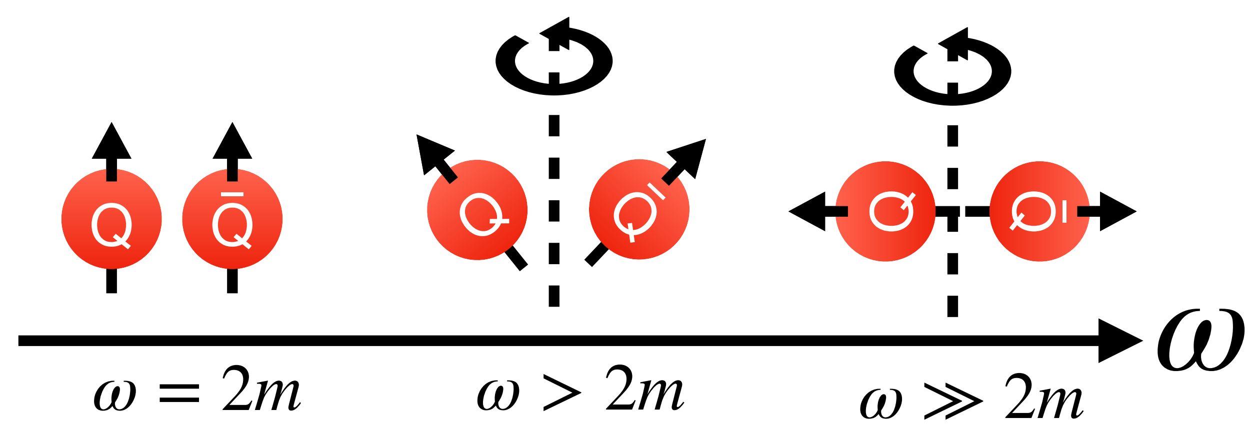


$$\mathrm{Im}\langle S_q(\omega)\rangle = \frac{3m^2}{2\pi\sqrt{\omega^2}\sqrt{\omega^2-4m^2}}$$

$$\mathrm{Im}\langle L_q(\omega)\rangle = \frac{(\omega^2-m^2)\sqrt{\omega^2-4m^2}}{2\pi(\omega^2)^{3/2}}$$

$$\mathrm{Im}\Pi^{\mathrm{rot}}_{\mathrm{A}}(\omega) = \mathrm{Im}\left\{\langle S_q(\omega)\rangle + \langle L_q(\omega)\rangle\right\}$$

$$\mathrm{Im}\Pi^{\mathrm{rot}}_{\mathrm{A}}(\omega) = s_z\frac{\partial \mathrm{Im}\Pi(\omega)}{\partial\omega} \text{ with } s_z = 1$$

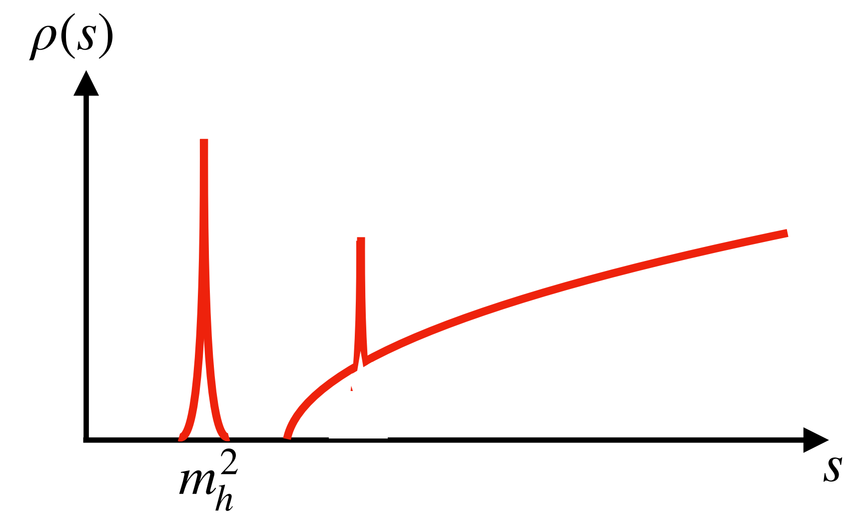


spin-1 $Q\bar{Q}$ system at rest

Two-point function - revisit

$$\Pi(q) = i\int d^4x e^{iqx}\langle 0|\mathrm{T}\{\phi(x)\phi^\dagger(0)\}|0\rangle$$
$$= \int_0^\infty ds \frac{\rho(s)}{s - q^2 - i\epsilon}$$
$$= \frac{|\langle 0|\phi|h\rangle|^2}{q^2 - m_h^2 + i\epsilon} + \big(\text{excited+multi-particles}\big)$$

$\rho(s)$: spectral density



- Not only a single hadron, but also excited & multi-particle states
- Extracting a single state is challenging at low energy scale
- QCD Sum Rule, a technique to extract the ground state from 2PF

QCD Sum Rules (QCDSR)

$$\boxed{\Pi(q) = i\int d^4x e^{iqx}\langle 0|\,\mathrm{T}\{\phi(x)\phi^\dagger(0)\}\,|0\rangle}$$

OPE $(Q^2 = -q^2 \gg 0)$

$$\Pi^{\mathrm{OPE}}(Q^2) = \sum_n C_n(Q^2)\langle \mathcal{O}_n\rangle$$

"Wilson coefficients"

"Vacuum condensate"

Spectral rep. $(q^2 > 0)$

"Spectral density"

$$\Pi^{\mathrm{phen}}(q^2) = \int_0^\infty ds \frac{\rho(s)}{s - q^2 - i\epsilon}$$

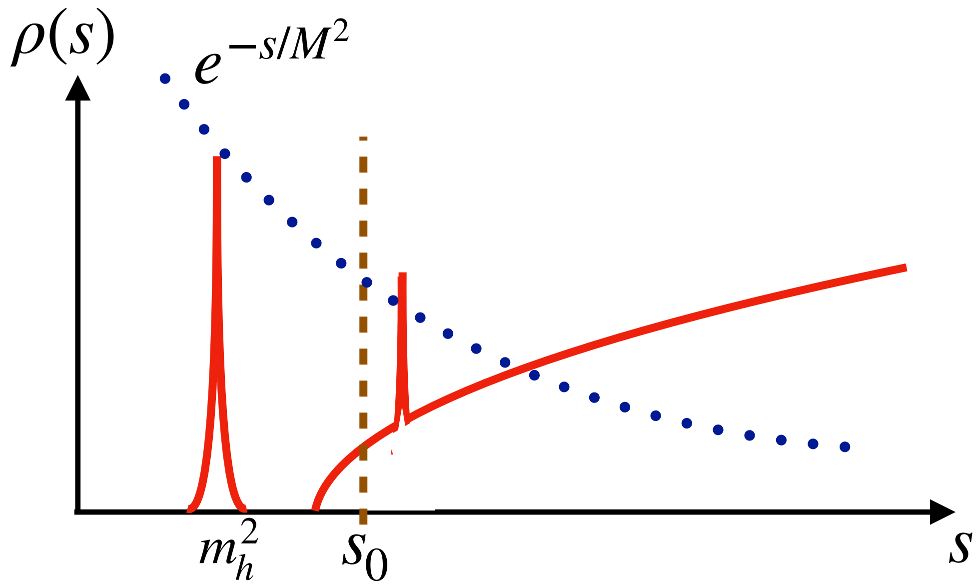
(Borel transformed)
Sum rule

$$\boxed{\int_0^{s_0} ds e^{-s/M^2} \mathrm{Im}\Pi^{\mathrm{OPE}}(s) \approx \int_0^{s_0} ds\, e^{-s/M^2}\rho(s)}$$

- Approximate relation, but useful
- Spectral density is constrained by OPE series
- s_0 is a threshold parameter that suppresses the continuum contribution
- M is a parameter that controls "OPE convergence"+"continuum suppression"

Example

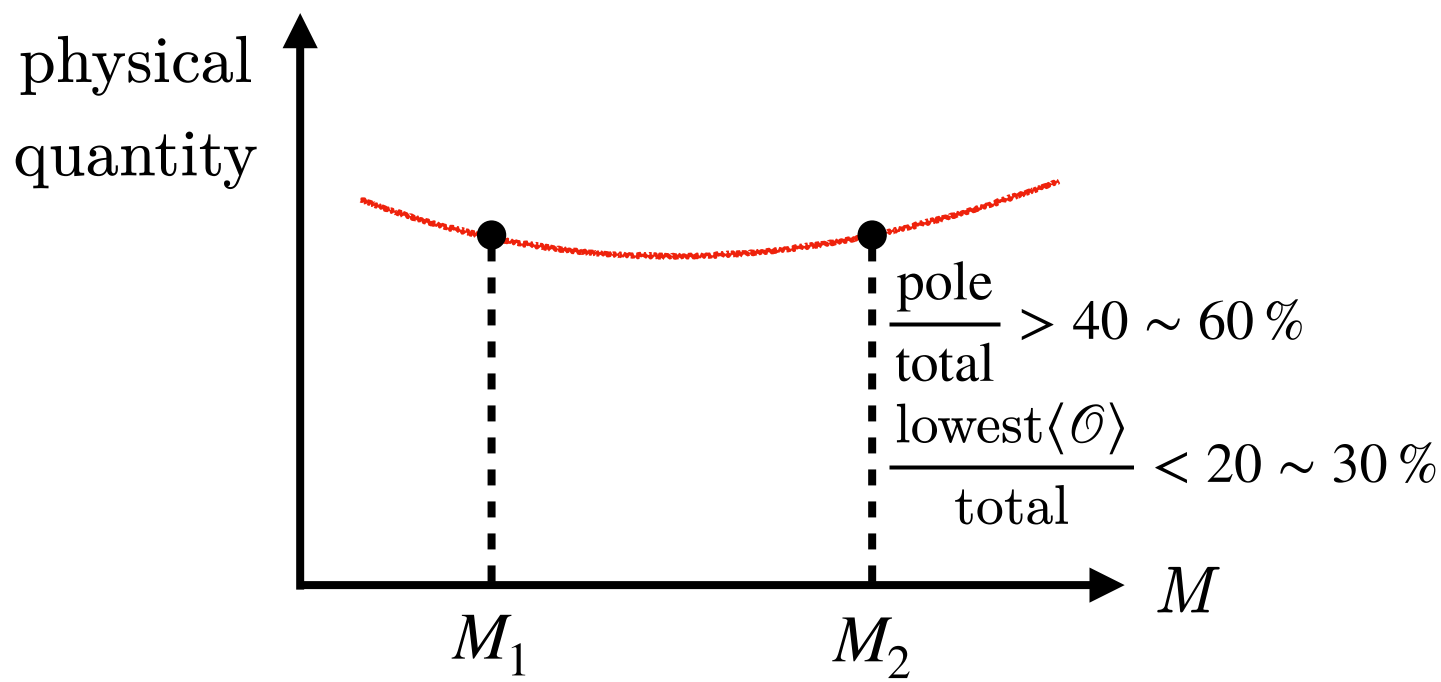
$$\boxed{\int_0^{s_0} ds e^{-s/M^2} \mathrm{Im}\Pi^{\mathrm{OPE}}(s) \approx \int_0^{s_0} ds\, e^{-s/M^2} \rho(s)}$$



$$\rho(s) \approx f_h \delta(s - m_h^2) + \theta(s - s_0) \cdot \text{continuum}$$

$$\bar{\Pi}(M) \equiv \int_0^{s_0} ds e^{-s/M^2} \mathrm{Im}\Pi^{\mathrm{OPE}}(s) \propto e^{-m_h^2/M^2}$$

$$m_h^2 = -\frac{\partial \bar{\Pi}(M^2)/\partial(1/M^2)}{\bar{\Pi}(M^2)}$$



Approximate, but reliable within limited range of M.

So far, widely used to predict and understand masses of various hadrons

Hadron Spin Decomposition via QCDSR

Inertial frame

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | \mathrm{T}\{\phi(x)\phi^\dagger(0)\} | 0 \rangle$$

Rotating frame

$$\Pi^{\mathrm{rot}}_{\mathrm{B}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

$$\Pi^{\mathrm{rot}}_{\mathrm{A}}(\omega) = \langle S_q(\omega) \rangle + \langle L_q(\omega) \rangle + \langle J_g(\omega) \rangle$$

QCDSR

$$\int_0^{s_0} ds \, \mathrm{Im}\Pi^{\mathrm{rot}}_{\mathrm{B}}(s) e^{-s/M^2} \propto s_z \times \text{spin indep.}$$

$$\Pi^{\mathrm{rot}}_{\mathrm{B}} = \Pi^{\mathrm{rot}}_{\mathrm{A}}$$

$$\int_0^{s_0} ds \, \mathrm{Im}\Pi^{\mathrm{rot}}_{\mathrm{A}}(s) e^{-s/M^2} \propto s_q + l_q + j_g$$

s_q: quark spin
l_q: quark orbital
j_g: gluon total

Equivalence

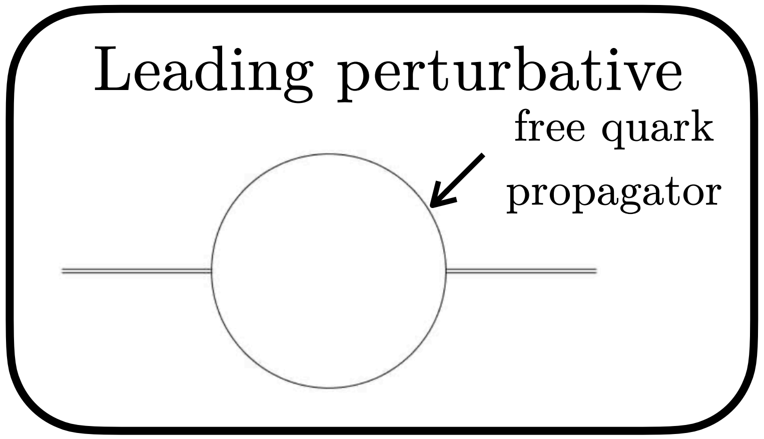
$$\frac{\int_0^{s_0} ds \, \mathrm{Im}\Pi^{\mathrm{rot}}_{\mathrm{A}}(s) e^{-s/M^2}}{\int_0^{s_0} ds \, \mathrm{Im}\Pi^{\mathrm{rot}}_{\mathrm{B}}(s) e^{-s/M^2}} = \frac{s_q + l_q + j_g}{s_z} = 1$$

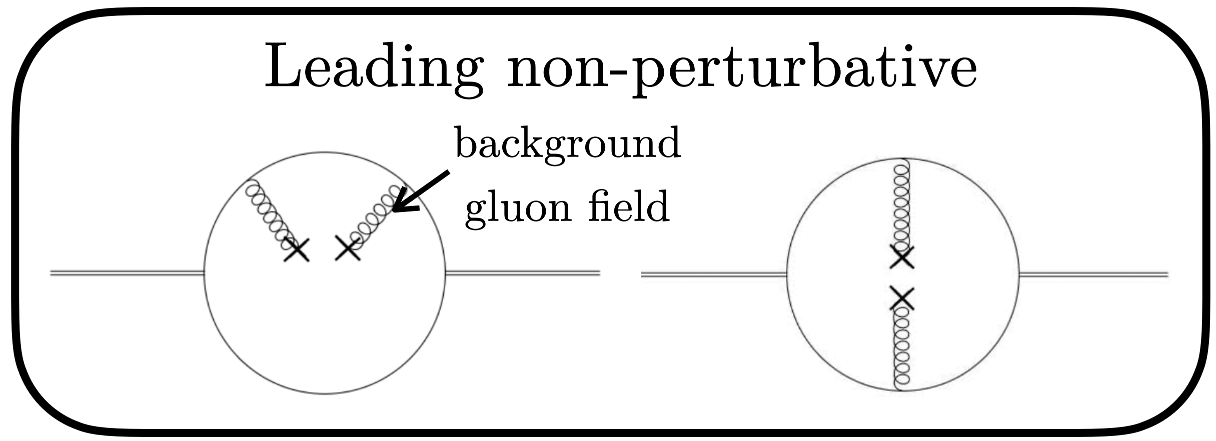
spin independent terms cancel out,
spin decomposition of the ground state

Spin-1 Quarkonium

- $\phi(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$ or $\bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$ $\quad (\psi = c$ or $b)$

- Feynman diagrams (truncated OPE upto D=4)



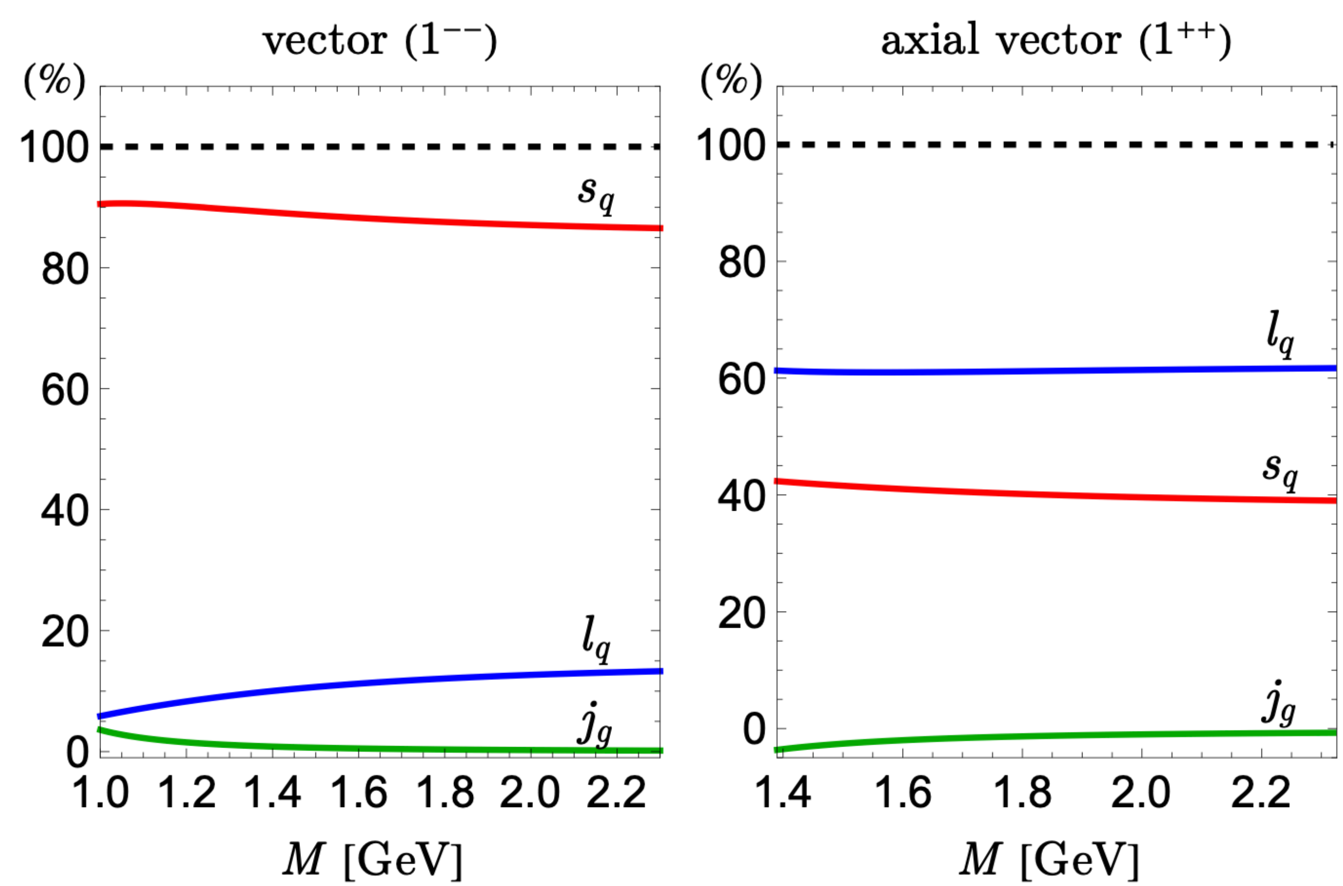


$\propto \langle \frac{\alpha_s}{\pi} G^2 \rangle$

- QCDSR analysis

  ex) charmonium

$$\frac{s_q + l_q + j_g}{s_z} = 1$$



Take average over a reliable range of M

Spin-1 Quarkonium

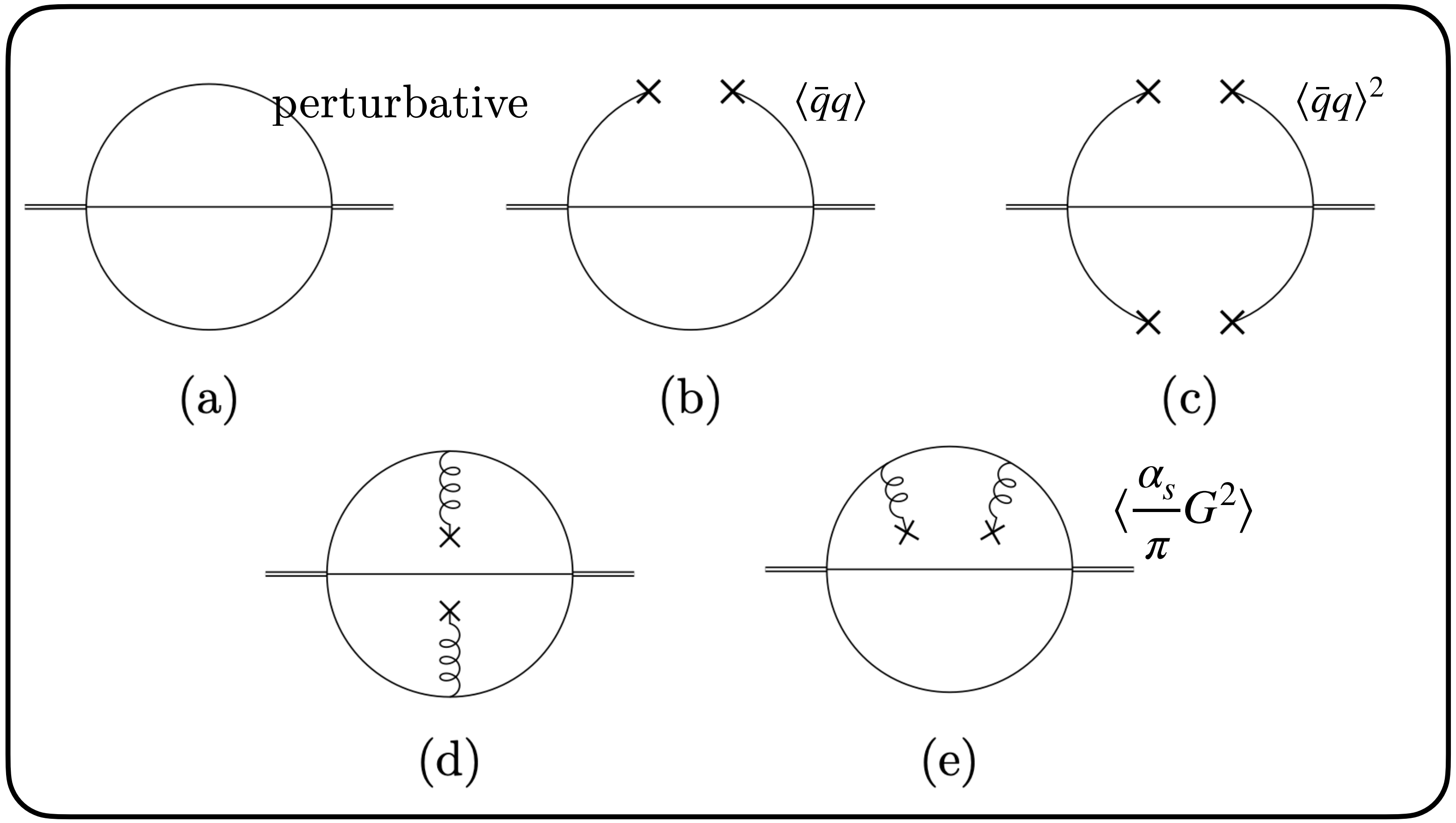
	Vector (%)			Axial (%)		
	S-wave	$\Upsilon(1S)$	J/ψ	P-wave	χ_{b1}	χ_{c1}
s_q	**100**	92	87.6	**50**	43	40
l_q	0	7.6	11.8	**50**	57	61
j_g	0	0.015	0.6	0	-0.005	-1.5

- Total sum $=$ 100 %
- Classical picture from the naive Q.M.

  S-wave: quark spin(100%) , P-wave: quark spin(50%) quark oam(50%)
- Spin structures are slightly different from the classical picture.
- As quark mass becomes lighter, spin content deviates further from classical picture

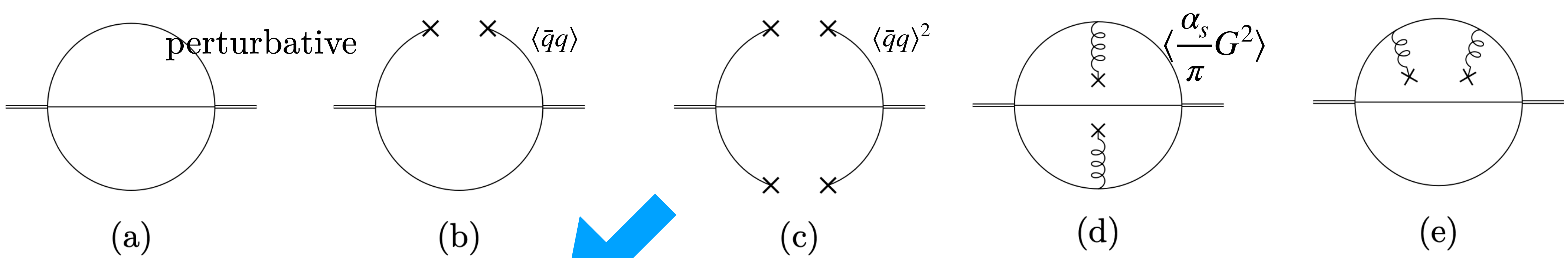
Spin-1/2 Proton

- $\phi(x) = \epsilon_{abc}[u^{a\mathrm{T}}(x)C\gamma^5 d^b(x)]u^c(x)$
- massless quark limit: $m_{u,d} \to 0$
- Feynman diagrams (truncated OPE upto D=4)



$\langle\bar{q}q\rangle$: quark condensate, chiral symmetry breaking in QCD

OPE calculations



$$\bar{\Pi}(\omega)/\omega = -\frac{5\omega^4 \log(-\omega^2)}{512\pi^4} + \frac{7\omega \log(-\omega^2)}{16\pi^2}\langle \bar{q}q \rangle - \frac{7}{6\omega^2}\langle \bar{q}q \rangle^2 - \frac{5\log(-\omega^2)}{256\pi^2}\langle G_0 \rangle,$$

$$\begin{aligned}\bar{\Pi}_{\mathrm{B}}^{\mathrm{rot}}(\omega) &= s_z \frac{\partial \bar{\Pi}(\omega)}{\partial \omega} \\ &= -\frac{25\omega^4 \log(-\omega^2)}{1024\pi^4} + \frac{7\omega \log(-\omega^2)}{16\pi^2}\langle \bar{q}q \rangle \\ &\quad + \frac{7}{12\omega^2}\langle \bar{q}q \rangle^2 - \frac{5\log(-\omega^2)}{512\pi^2}\langle G_0 \rangle,\end{aligned}$$

$$\begin{array}{lccccccccccl}
\Pi_{\mathrm{A}}^{\mathrm{rot}}(\omega) = & & \langle S_u \rangle & : & \langle S_d \rangle & : & \langle L_u \rangle & : & \langle L_d \rangle & : & \langle J_g \rangle & \\
\text{diagram (a)}: & (& -\frac{6}{25} & + & -\frac{1}{25} & + & \frac{36}{25} & + & -\frac{4}{25} & + & 0 &) \times \left[\frac{-25\omega^4 \log(-\omega^2)}{1024\pi^4}\right], \\
\text{diagram (b)}: & (& \frac{6}{7} & + & -\frac{1}{21} & + & \frac{8}{21} & + & -\frac{4}{21} & + & 0 &) \times \left[\frac{7\omega \log(-\omega^2)}{16\pi^2}\langle \bar{q}q \rangle\right], \\
\text{diagram (c)}: & (& \frac{10}{7} & + & -\frac{3}{7} & + & 0 & + & 0 & + & 0 &) \times \left[\frac{7}{12\omega^2}\langle \bar{q}q \rangle^2\right], \\
\text{diagram (d-e)}: & (& -\frac{2}{5} & + & -\frac{11}{15} & + & \frac{40}{9} & + & \frac{56}{45} & + & -\frac{32}{9} &) \times \left[\frac{-5\log(-\omega^2)}{512\pi^2}\langle G_0 \rangle\right].
\end{array}$$

Indeed, we confirm that A=B

Spin-1/2 Proton

$$\frac{s_q + l_q + j_g}{s_z} = 1$$

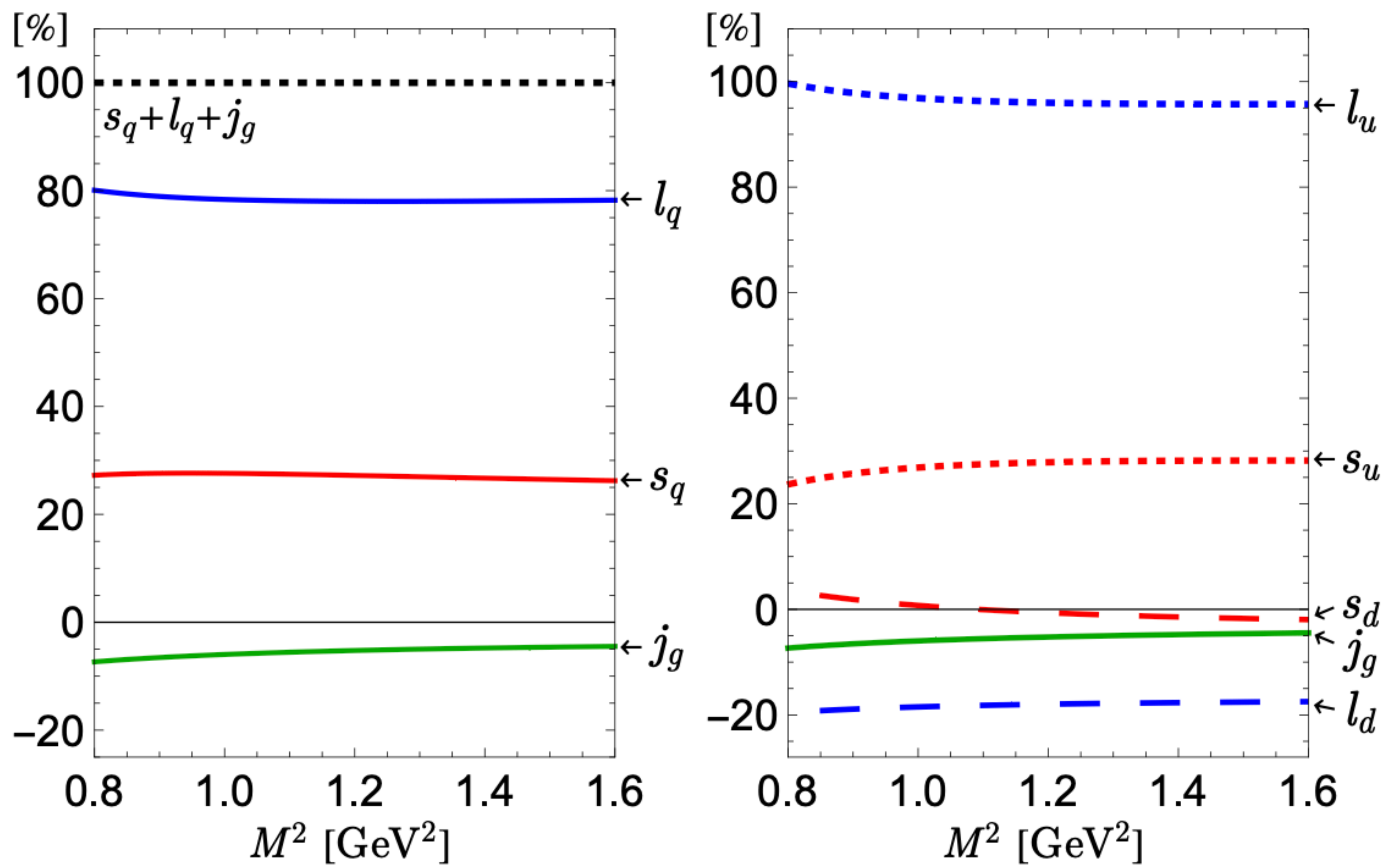


TABLE I: Spin decomposition of the proton at rest

Component	Overall (%)	Up-quark (%)	Down-quark (%)
s_q	27.1 ± 0.4	27.3 ± 1.1	-0.2 ± 1.5
l_q	78.3 ± 0.5	96.4 ± 0.6	-18.2 ± 0.6
j_g	-5.4 ± 0.8	—	—

$\langle \bar{q} q \rangle$ contribution, i.e. (b)+(c), $\sim 50\%$

significantly affects not only proton mass but also its spin structure

Summary

- For a given 2PF,

$$\Pi_{\mathrm{A}}^{\mathrm{rot}}(\omega) = s_z \frac{\partial \Pi(\omega)}{\partial \omega}$$

- 2PF in an inertial frame + QCDSR => hadron mass

- 2PF in a rotating frame + QCDSR => hadron spin structure

Thank you for listening !