Relativistic Equations of Motion for a Spinning Particle in a Background Yang-Mills Field

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QCD Theory Seminar

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based on arXiv: 2511.20083 and work in progress

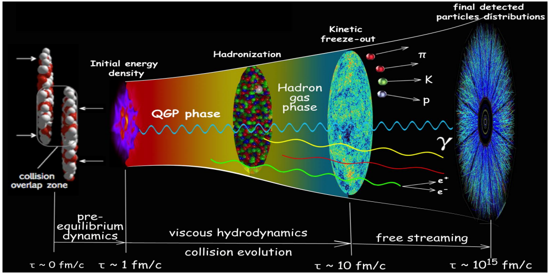
Outline

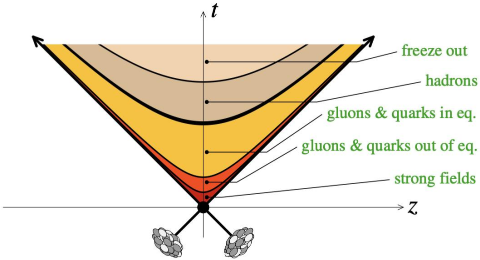
01 Introductions: Glasma, Hard Probes & Constrained Hamiltonian Formalism

02 Covariant Equations of Motion of Spinning and Colored Particles in a Background Yang-Mills Field

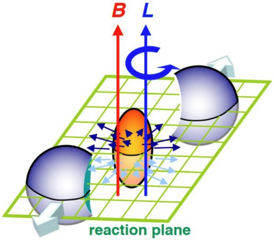
03 Massless Limit and Chiral Kinetic Equations in E.M. Field

04 Summary





Credit: Chun Shen



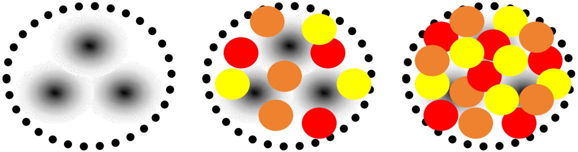
Electromagnetic Field: eB ~ 1-100 m_π^2 (Classical Abelian guage field)

Phenomena

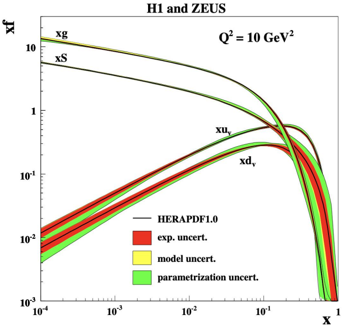
- CME related effects
- Phase diagram
- QGP transport coefficient, Heavy quarkonium
- Spin polarization & alignment......

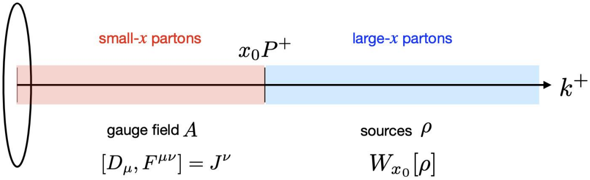
Color Glass Condensate (CGC)

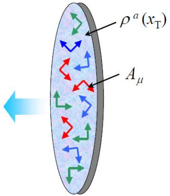
Higher energies (smaller x → 0)

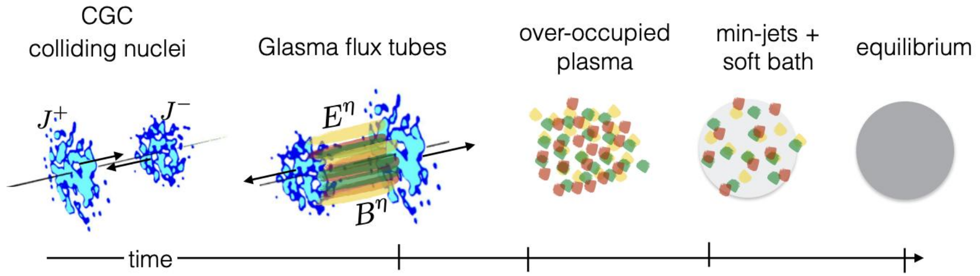


L. D. McLerran and R. Venugopalan, PRD 49, 3352 (1994)
L. D. McLerran and R. Venugopalan, PRD 50, 2225 (1994)









Glasma: Chromo-Electromagnetic Field (i.e., Classical Non-Abelian Yang-Mills Field)

$$A^i(\tau,\vec{x}_\perp)\big|_{\tau=0} = \alpha^i_A(\vec{x}_\perp) + \alpha^i_B(\vec{x}_\perp)$$

$$A^\eta(\tau,\vec{x}_\perp)\big|_{\tau=0} = \frac{ig}{2}\left[\alpha^i_A(\vec{x}_\perp), \alpha^i_B(\vec{x}_\perp)\right]$$

$$\partial_\tau A^i(\tau,\vec{x}_\perp)\big|_{\tau=0} = \partial_\tau A^\eta(\tau,\vec{x}_\perp)\big|_{\tau=0} = 0$$

$$\partial_\tau P^i = \tau \mathcal{D}_j F_{ji} - \frac{ig}{\tau}\left[A_\eta,\, \mathcal{D}_i A_\eta\right]$$

$$\partial_\tau P^\eta = \frac{1}{\tau}\mathcal{D}_i(\mathcal{D}_i A_\eta)$$

$$P^i = \tau\,\partial_\tau A_i, \qquad P^\eta = \frac{1}{\tau}\partial_\tau A_\eta$$

Field strength: $gB^a \sim 100 m_\pi^2$

Lifetime: $\tau \lesssim 1$ fm/c

A. Kovner, L. D. McLerran, and H. Weigert, PRD 52, 6231 (1995)

Gluon saturation

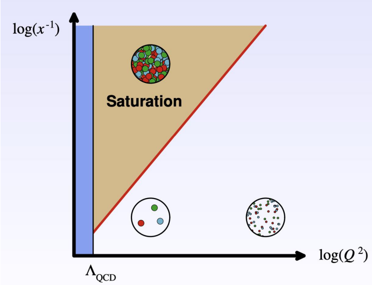
- Forward rapidity hadron production
- Inclusive and diffractive dijet production
- Vector meson photoproduction in UPC...

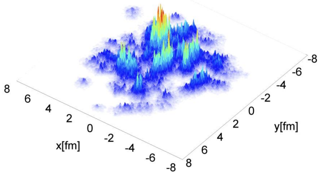
H. Mäntysaari et al., PRL 124, 112301 (2020)
Y. Shi et al., PRL 128, 202302 (2022)

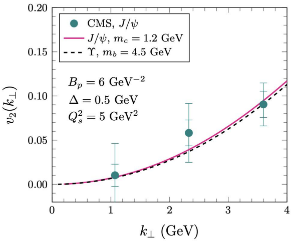
Initial conditions

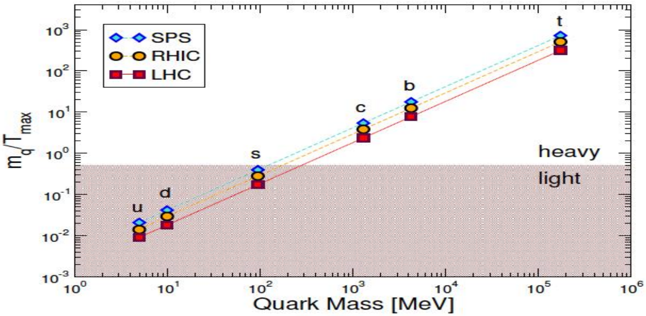
- Initial density, fluctuations, and correlations
- Collective flow in small systems...

B. Schenke, P. Tribedy, and R. Venugopalan, PRL 108, 252301 (2012)
C. Zhang et al., PRL 122, 172302 (2019)





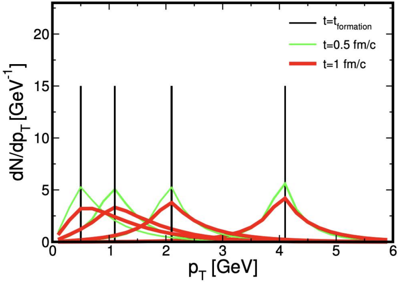




Heavy quarks and jets as effective probes

- Clean production: $m_{c,b} >> \Lambda_{QCD}$, $m_{c,b} >> T_{RHIC,LHC}$ production by pQCD hard processes and negligible thermal production
- Early Formation: $t_{form} \approx 0.08$ fm/c
- Relatively weak interaction with the QGP

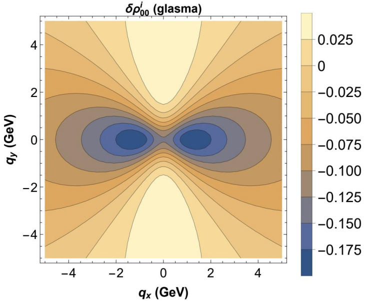
H.F. and Jet momentum diffusion



M. Ruggieri et al., PRD 98, 094024 (2018)
Y. Sun et al., PLB 798, 134933 (2019)
A. Ipp et al., PRD 102, 074001 (2020)
K. Boguslavski et al., JHEP 09 (2020) 077
C. Andres et al., PLB 803, 135318 (2020)
K. Boguslavski et al., NPA 1005, 121970 (2021)
J.-H. Liu et al., PRD 103, 034029 (2021)
P. Khowal et al., EPJP 137, 307 (2022)
M. Ruggieri et al., PRD 106, 034032 (2022)
M. E. Carrington et al., PLB 834, 137464 (2022)
C. Andres et al., JHEP 03 (2023) 189
D. Avramescu et al., PRL 134, 172301 (2025)

Spin polarization & alignment

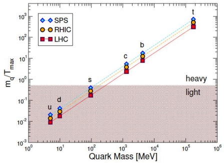
$$\mathrm{Tr}_c\langle\hat{\mathcal{P}}^i_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle \approx \frac{\int d\Sigma_X\cdot q\big(\langle\tilde{a}^{si}_q(\boldsymbol{q}/2,X)\tilde{a}^{si}_{\bar{q}}(\boldsymbol{q}/2,X)\rangle + \langle\tilde{a}^{ai}_q(\boldsymbol{q}/2,X)\tilde{a}^{ai}_{\bar{q}}(\boldsymbol{q}/2,X)\rangle/(2N_c^2)\big)}{m_q m_{\bar{q}}\int d\Sigma_X\cdot q\big(f^{\mathrm{s}}_{\mathrm{V}q}(\boldsymbol{q}/2,X)f^{\mathrm{s}}_{\mathrm{V}\bar{q}}(\boldsymbol{q}/2,X)\big)}$$



A. Kumar, B. Müller and D.L. Yang, PRD 108, 016020 (2023)
D.L. Yang, PRD 111, 056005 (2025)

Hard Probes in the Glasma

Wong Equations: **Classical point** colored particle in a background Yang-Mills field



$$p^\mu \partial_\mu f + F^\mu \frac{\partial f}{\partial p^\mu} = C[f]$$

Dilute and out of equilibrium

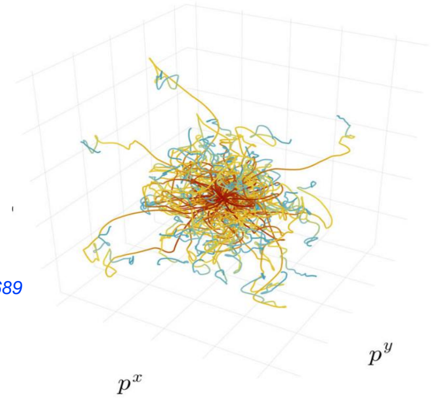
Solved by **test particle method**

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \frac{p^\mu}{m}$$

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = gQ^a F^{\mu\nu,a}\frac{p_\nu}{m}$$

$$\frac{\mathrm{d}Q^a}{\mathrm{d}\tau} = -gf^{abc}A^b_\mu Q^c \frac{p^\mu}{m}$$

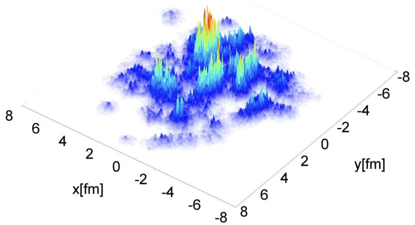
S.K.Wong, Il Nuovo Cimento 65A, 689 (1970)



Spin Degrees of freedom are missing for quarks and gluons

Why spin matters

- Stern-Gerlach force in highly non-uniform Glasma field $\vec{\nabla}(\vec{\mu}\cdot\vec{B})$



EOMs of spinning Particles in the Background Yang-Mills Field is required!

- Spin polarization & alignment (heavy flavor hadrons)

Non-Relativistic Treatment

Issue: Not applicable for RHICs

$$\frac{dx^i}{dt} = \frac{P^i}{m}$$

$$\frac{dP^i}{dt} = -\frac{g}{m}Q_a\left(mF_a^{i0} - P^j F_a^{ij}\right) - \frac{g}{m}Q_a S^j(\mathcal{D}^i B_a^j)$$

$$\frac{dQ_a}{dt} = \frac{g}{m}f_{abc}Q_c\left(P^i A_b^i + S^i B_b^i\right)$$

$$\frac{dS^i}{dt} = -\frac{g}{m}Q_a F_a^{ij} S^j$$

N. Linden, A. J. Macfarlane, and J. W. van Holten, CJP 46, 209 (1996)
Pooja, S. K. Das, V. Greco, and M. Ruggieri, EPJP 138, 313 (2023)

Grassmann Variables

Issue: Numerically impractical

$$\dot{x}^\mu = \epsilon P^\mu$$

$$\dot{P}^\mu = \epsilon g F^{a\,\mu\nu} Q^a P_\nu - \frac{i\epsilon g}{2}\psi^\alpha (D^\mu F_{\alpha\beta})^a Q^a \psi^\beta$$

$$\dot{\psi}^\mu = \epsilon g F^{a\,\mu\nu} Q^a \psi_\nu$$

$$\dot{\lambda}_a^\dagger = -igv^\mu t_{ab}^c A_\mu^c \lambda_b^\dagger - \frac{\epsilon g}{2}\psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c^\dagger \psi^\nu$$

$$\dot{\lambda}_a = igv^\mu t_{ab}^c A_\mu^c \lambda_b + \frac{\epsilon g}{2}\psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c \psi^\nu$$

$$Q^a \equiv \lambda_c^\dagger t_{cd}^a \lambda_d \qquad S_{\mu\nu} = -i\psi_\mu \psi_\nu$$

A. P. Balachandran, P. Salomonson, B.-S. Skagerstam, and J.-O. Winnberg, PRD 15, 2308 (1977)
N. Mueller and R. Venugopalan, PRD 99, 056003 (2019)

Dirac-Equation Based and **Lorentz Covariance**

$$\mathcal{H}\,\psi \equiv -\frac{1}{2m}\left\{\left[\pi^\mu + \frac{g}{c}\hat{A}^\mu\right]^2 + \frac{g\hbar}{2c}\,\sigma^{\mu\nu}\hat{F}_{\mu\nu} - m^2c^2\right\}\psi = 0$$

Heisenberg EOM

$$\frac{dq}{d\tau} = \frac{1}{i\hbar}[q,\mathcal{H}]$$

$$S^\mu = -\frac{1}{2}\,\epsilon^{\mu\nu\lambda\rho}\,u_\nu\,S_{\lambda\rho}$$

U. W. Heinz, PLB 144, 228 (1984)

$$\frac{dx^\mu}{d\tau} = \frac{p^\mu}{m}$$

$$\frac{dp^\mu}{d\tau} = g\,Q^a F^{a\,\mu\nu}\,u_\nu + \frac{g}{2mc}\,Q_a\,(D^\mu F_{\lambda\nu})^a S^{\lambda\nu}$$

$$\frac{dQ^a}{d\tau} = -g f_{abc}\left[u^\mu A^b_\mu + \frac{1}{2mc}S^{\mu\nu}F^b_{\mu\nu}\right]Q^c$$

$$\frac{dS^{\mu\nu}}{d\tau} = \frac{g}{mc}\,Q^a\left(F^{\mu}_{a\,\lambda}\,S^{\lambda\nu} - F^{\nu}_{a\,\lambda}\,S^{\lambda\mu}\right)$$

$$\frac{dx^\mu}{d\tau} = c\,u^\mu$$

$$\frac{dp^\mu}{d\tau} = g\,Q^a F^{\mu\nu}_a u_\nu - \frac{g}{mc}\,Q^a(D_\mu\tilde{F}_{\alpha\beta})_a\,u^\alpha S^\beta$$

$$\frac{dQ^a}{d\tau} = -g f_{abc}\left[u^\mu A^b_\mu - \frac{1}{mc}\,u^\alpha\tilde{F}^b_{\alpha\beta}S^\beta\right]Q^c$$

$$\frac{dS^\mu}{d\tau} = \frac{g}{mc}\,Q^a\left[F^{\mu\nu}_a S_\nu + \frac{1}{mc}(u^\mu S^\nu - u^\nu S^\mu)(D_\nu\tilde{F}_{\alpha\beta})_a\,u^\alpha S^\beta\right]$$

Degree of Freedom (DOF)

Position: $\vec{x}(t)$; Momentum: $\vec{p}(t)$

Spin vector in rest frame: $\vec{s}(t)$; Color Charge: $Q^a(t)$

	DOF	Physical DOF
$\frac{dx^\mu}{d\tau} = \frac{p^\mu}{m}$	4	3
$\frac{dp^\mu}{d\tau} = g\, Q^a F^{a\,\mu\nu}\, u_\nu + \frac{g}{2mc}\, Q_a\, (D^\mu F_{\lambda\nu})^a S^{\lambda\nu}$	4	3
$\frac{dQ^a}{d\tau} = -g f_{abc} \left[u^\mu A^b_\mu + \frac{1}{2mc} S^{\mu\nu} F^b_{\mu\nu} \right] Q^c$	$N_c^2 - 1$	$N_c^2 - 1 - (N_c - 1)$
$\frac{dS^{\mu\nu}}{d\tau} = \frac{g}{mc}\, Q^a \left(F^{\mu}_{a\,\lambda}\, S^{\lambda\nu} - F^{\nu}_{a\,\lambda}\, S^{\lambda\mu} \right)$	6	2
	$Q_n = d^{abc\cdots} q^a q^b q^c \ldots$	$N_c - 1$ Classical Casimirs

The particle evolves within a **subspace** of whole phase space.

Mass-Shell Constraint

$$u_\mu = \frac{p_\mu}{mc}$$

$$u^\mu u_\mu = 1$$

Spin Supplementary Condition (e.g., Tulczyjew–Dixon Condition)

$$P^\mu S_{\mu\nu} = 0$$

$$S^{\mu\nu} S_{\mu\nu} = 8 s^2$$

Classical Casimirs

$$Q_n = d^{abc\ldots} q^a q^b q^c \ldots .$$

$$\frac{dx^\mu}{d\tau} = \frac{p^\mu}{m}$$

$$\frac{dp^\mu}{d\tau} = g\, Q^a F^{a\,\mu\nu}\, u_\nu + \frac{g}{2mc}\, Q_a\, (D^\mu F_{\lambda\nu})^a S^{\lambda\nu}$$

$$\frac{dQ^a}{d\tau} = -g f_{abc} \left[u^\mu A^b_\mu + \frac{1}{2mc} S^{\mu\nu} F^b_{\mu\nu} \right] Q^c$$

$$\frac{dS^{\mu\nu}}{d\tau} = \frac{g}{mc}\, Q^a \left(F^\mu_{a\,\lambda}\, S^{\lambda\nu} - F^\nu_{a\,\lambda}\, S^{\lambda\mu} \right)$$

Issue: Even if the constraint is satisfied initially, the EOMs do not preserve it at later times.
The EOMs should ensure that the particle trajectory **lies on the constraint surface** in phase space.

$$
\begin{aligned}
0 =&\delta(\xi \mp \hbar \Sigma_S^{\alpha\beta} F_{\alpha\beta}) \\
&\times\Big[\Big(p\cdot\Delta \pm \frac{\hbar}{2}\Sigma_S^{\mu\nu}\left(\nabla_\rho F_{\mu\nu} - p_\lambda R^\lambda{}_{\rho\mu\nu}\right)\partial_p^\rho\Big) f_\pm \\
&+ \frac{\hbar}{2}(f_+ - f_-)\Big(\left(\nabla_\rho F_{\mu\nu} - p_\lambda R^\lambda{}_{\rho\mu\nu}\right)\partial_p^\rho \Sigma_S^{\mu\nu} \\
&- \frac{1}{2m}\tilde{F}^{\nu\sigma}\partial_\nu^p\left(p\cdot\Delta\theta_\sigma - F_{\sigma\lambda}\theta^\lambda\right)\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
0 =&\delta(\xi)\Big[f_A p\cdot\Delta\theta^\mu - f_A F^{\mu\nu}\theta_\nu + \theta^\mu p\cdot\Delta f_A \\
&- \frac{\hbar}{4m}\epsilon^{\mu\nu\rho\alpha}p_\alpha\left(\nabla_\sigma F_{\nu\rho} - p_\lambda R^\lambda{}_{\sigma\nu\rho}\right)\partial_p^\sigma f \\
&- \frac{\hbar}{2m}\tilde{F}^{\mu\nu}\partial_\nu^p(p\cdot\Delta f)\Big],
\end{aligned}
$$

$\Rightarrow$

$$
\begin{aligned}
\frac{dx^\mu}{d\tau} &= \frac{p^\mu}{m} \\
\frac{dp^\mu}{d\tau} &= F^{\mu\lambda}\frac{p_\lambda}{m} \pm \frac{\hbar}{2m}\Sigma_S^{\alpha\beta}\,\partial^\mu F_{\alpha\beta} \\
\frac{d\,\hbar\Sigma_S^{\mu\nu}}{d\tau} &= \frac{2}{m}F_\sigma^{[\mu}\,\hbar\Sigma_S^{\nu]\sigma}
\end{aligned}
$$

Y.C. Liu and X.G. Huang, CPC 44, 094101 (2020)

EOMs do not preserve the constraints. $p^2 - m^2 \pm \hbar\,\Sigma_S^{\alpha\beta}F_{\alpha\beta} = 0$ $\quad P^\mu \Sigma_{S\,\mu\nu} = 0$, $\quad \Sigma_S^{\mu\nu}\Sigma_{S\,\mu\nu} = 8s^2$

Wigner-function approaches **may not** serve this purpose either: they rely on $\hbar$ expansion

EOM Requirements

- Lorentz covariance
- Constraint preservation

$$P^2 - \frac{1}{2}\mu g\, q^a\, F^a_{\mu\nu}\, S^{\mu\nu} - m^2 = 0$$ Mass-Shell Condition

$$P^\mu S_{\mu\nu} = 0$$
$$S^{\mu\nu} S_{\mu\nu} = 8s^2$$ Tulczyjew-Dixon Spin Supplementary Condition

$$\dot{Q}_n = 0 \qquad Q_n = d^{abc\ldots} q^a q^b q^c \ldots.$$ Classical Casimirs Conservation

- Arbitrary chromo-magnetic moment (Landé g-factor ≠ 2) $\mu = g_S/2$,

Classical Mechanics of Point Particles

$$S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^i} - \frac{\partial L}{\partial q^i} = 0$$

$$p_i = \frac{\partial L}{\partial \dot{q}^i}$$

$$H(q, p, t) = p_i \dot{q}^i - L$$

$$\dot{q}^i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q^i}$$

Constraints arise when the Hessian of the Lagrangian is singular

$$\det\left(\frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j}\right) = 0$$

EOMs do not uniquely fix the time evolution

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu(\partial_\mu A^\mu) = J^\nu$$

$$H_P \equiv H_C + \lambda_a \phi_a$$

$$\dot{F} = \{F, H_P\}_{\mathrm{PB}} = \{F, H_C\}_{\mathrm{PB}} + \lambda_a \{F, \phi_a\}_{\mathrm{PB}} + \phi_a \{F, \lambda_a\}_{\mathrm{PB}} \approx \{F, H_C\}_{\mathrm{PB}} + \lambda_a \{F, \phi_a\}_{\mathrm{PB}}$$

ϕ_a stand for constraints

Constrained Hamiltonian Formalism

World-line parametrization

$$L = \frac{1}{2\det\tilde{G}}\left[g_3(\dot{x}N\dot{x}) - 2g_7(\dot{x}ND\omega) + g_1(D\omega ND\omega)\right]$$
$$+ \frac{e}{c}A_\mu\dot{x}^\mu - \frac{g_4}{2}(\omega^2 - a_4) - \frac{g_1}{2}m^2c^2 + \frac{g_3}{2}a_3,$$

$$D\omega^\mu = \dot{\omega}^\mu - g_1\frac{e\mu}{c}(F\omega)^\mu$$

$$N^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\omega^\mu\omega^\nu}{\omega^2}$$

$$H = \frac{g_1}{2}\left(\mathcal{P}^2 - \frac{\mu e}{2c}(JF) + m^2c^2\right) + \frac{g_3}{2}(\pi^2 - a_3) + \frac{g_4}{2}(\omega^2 - a_4)$$
$$+ \lambda_5(\omega\pi) + \lambda_6(\mathcal{P}\omega) + g_7(\mathcal{P}\pi) + \lambda_{g_i}\pi_{g_i},$$

A.A. Deriglazov and A.M. Pupasov-Maksimov, NPB 885, 1-24 (2014)

Constrained Hamiltonian

- Systematic treatment on singular Lagrangians
- Classical dynamics: Hamiltonian formulism easier than Lagrangian with constraints, **constraints on phase space**
- Gauge theories, general relativity
- Quantum Mechanics: QCD (Faddeev–Popov determinant), general relativity

The Dirac-Bergmann Algorithm

- Primary Constraints: ϕ_a=0 $$p_i = \frac{\partial L}{\partial \dot{q}_i} \implies \dot{q}_i \cancel{=} \mathcal{V}_i(q,p)$$
- Primary Hamiltonian: $H_C = \sum_i p_i \dot{q}_i - L$ $\quad H_P \equiv H_C + \lambda_a \phi_a$
- Consistency Conditions: $\dot{\phi}_a = \{\phi_a, H_P\}_{\mathrm{PB}} \approx 0$ impose restrictions on the Lagrange multipliers or generate new constraints
- Secondary/Higher Constraints: iterate until no further constraints arise. Collect all constraints as ϕ_a
- If L is not known, one just specify two ingredients on the phase space: **1. The phase space + Poisson bracket 2. The Hamiltonian function H**

Classification of Constraints

- Compute $\mathcal{C}_{ab} = \{\phi_a, \phi_b\}_{\mathrm{PB}}$
- First-Clsss Constraints (FCC) χ: generate gauge transformations (combinations of constraints) $\{\chi_i, \phi_b\}_{\mathrm{PB}} \approx 0,$
  - ✓ Null vectors of matrix C
  - ✓ extended Hamiltonian $H_E = H_T + \lambda_i \chi_i \quad \dot{F} = \{F, H_E\}_{\mathrm{PB}}$
- Second-Clsss Constraints (SCC) ξ: non-vanishing Poisson with at least one other constraint; Used to construct **Dirac bracket**

$$\{F,G\}_{\mathrm{DB}} \equiv \{F,G\}_{\mathrm{PB}} - \{F,\xi_\alpha\}_{\mathrm{PB}}\,\Delta^{-1}_{\alpha\beta}\,\{\xi_\beta,G\}_{\mathrm{PB}} \qquad \Delta_{\alpha\beta} = \{\xi_\alpha,\xi_\beta\}_{\mathrm{PB}}$$

$$H_E = H_T + \lambda_i \chi_i$$

EOMs: $\dot{F} \approx \{F, H_{FR}\}_{\mathrm{DB}}$ H_{FR}: fully reduced Hamiltonian

Phase space measure: $d\Gamma_{\mathrm{phys}} = d\Gamma \prod_a \delta(\phi_a)\,\sqrt{\det\{\phi_a,\phi_b\}}$

$$(q,p) \;\rightarrow\; (Q^A, P_A) \qquad d\Gamma_{\mathrm{phys}} = \sqrt{\det \omega_D}\,\prod_A dZ^A \qquad \omega_D^{AB} = \{Z^A, Z^B\}_D$$

$$A = 1,\ldots,N-m$$

P. A. M. Dirac, Lectures on Quantum Mechanics (Dover Publications, 1964)
A. W. Wipf, arXiv:hepth/9312078
J. D. Brown, Universe 8, 171 (2022)

World-line parametrization

$$H = \frac{g_1}{2}\left(\mathcal{P}^2 - \frac{\mu e}{2c}(JF) + m^2c^2\right) + \frac{g_3}{2}(\pi^2 - a_3) + \frac{g_4}{2}(\omega^2 - a_4)$$
$$+ \lambda_5(\omega\pi) + \lambda_6(\mathcal{P}\omega) + g_7(\mathcal{P}\pi) + \lambda_{g_i}\pi_{g_i},$$

$$H_P = \frac{1}{2}g_1\left(P^2 - 2\mu g q^a F^a_{\mu\nu}\,\omega^\mu\pi^\nu - m^2\right) + \frac{1}{2}g_2(\pi^2 - a_2)$$
$$+\frac{1}{2}g_3(\omega^2 - a_3) + g_5(P\pi) + \lambda_4(P\omega) + \lambda_6(\omega\pi)$$
$$+\lambda_{g_i}\pi_{g_i}$$

$$T_1 : -\frac{1}{2}(P^2 - 2\mu g q^a F^a_{\mu\nu}\omega^\mu\pi^\nu - m^2),$$
$$T_2 : -\frac{1}{2}(\pi^2 - a_2), \quad T_3 : -\frac{1}{2}(\omega^2 - a_3),$$
$$T_4 : -P\omega, \quad T_5 : -P\pi, \quad T_6 : \omega\pi,$$

$$P_\mu = p_\mu - g q^a A^a_\mu$$

$$[\hat{x}^\mu, \hat{p}^\nu] = i\hbar g^{\mu\nu}$$
$$[\hat{S}^{\mu\nu}, \hat{S}^{\rho\sigma}] = 2i\hbar\left(g^{\nu\rho}\hat{S}^{\mu\sigma} - g^{\mu\rho}\hat{S}^{\nu\sigma} + g^{\mu\sigma}\hat{S}^{\nu\rho} - g^{\nu\sigma}\hat{S}^{\mu\rho}\right)$$
$$S^{\mu\nu} = 2(\omega^\mu\pi^\nu - \omega^\nu\pi^\mu)$$
$$[\hat{\omega}^\mu, \hat{\pi}^\nu] = i\hbar g^{\mu\nu}$$
$$[\hat{q}^a, \hat{q}^b] = i\hbar f^{abc}\hat{q}^c$$

$$\{x^\mu, p^\nu\}_{\mathrm{PB}} = g^{\mu\nu}$$
$$\{S^{\mu\nu}, S^{\rho\sigma}\}_{\mathrm{PB}} = 2(g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} + g^{\mu\sigma}S^{\nu\rho} - g^{\nu\sigma}S^{\mu\rho})$$
$$\{\omega^\mu, \pi^\nu\}_{\mathrm{PB}} = g^{\mu\nu}$$
$$\{q^a, q^b\}_{\mathrm{PB}} = f^{abc}q^c$$

J. Zhou, Y. S. Zhao and Y. Sun, arXiv: 2511.20083

$$\dot{\phi}_a = \{\phi_a, H_P\}_{\mathrm{PB}} \approx 0$$

$$\mathcal{C}_{ab} = \{\phi_a, \phi_b\}_{\mathrm{PB}}$$

$$\mathcal{C}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & (1-\mu)c_2+\mu b_2 & (1-\mu)c_1+\mu b_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & 0 & 0 & -a_3 \\ (\mu-1)c_2-\mu b_2 & 0 & 0 & 0 & (2\mu+1)c_3+m^2 & 0 \\ (\mu-1)c_1-\mu b_1 & 0 & 0 & -(2\mu+1)c_3-m^2 & 0 & 0 \\ 0 & -a_2 & a_3 & 0 & 0 & 0 \end{pmatrix}$$

FCC: $$\chi_1 = \frac{m^2+(2\mu+1)c_3}{\mu b_2+(1-\mu)c_2}T_1 - \frac{\mu b_1+(1-\mu)c_1}{\mu b_2+(1-\mu)c_2}T_4 + T_5 \qquad \chi_2 = \frac{a_3}{a_2}T_2 + T_3$$

SCC: $$\xi_i = \{T_3, T_4, T_5, T_6\}$$

$$\{x^\mu, x^\nu\}_{\mathrm{DB}} = \frac{\pi^\mu\omega^\nu - \omega^\mu\pi^\nu}{\Delta},$$

$$\{P^\mu, P^\nu\}_{\mathrm{DB}} = gq^a g^{\mu\rho}g^{\nu\sigma}F^a_{\rho\sigma} + \frac{1}{\Delta}g^2 q^a q^b F^a_{\rho\sigma}F^b_{\alpha\beta}(g^{\mu\rho}g^{\nu\beta}\pi^\alpha\omega^\sigma - g^{\mu\alpha}g^{\nu\sigma}\pi^\beta\omega^\rho),$$

$$\{x^\mu, P^\nu\}_{\mathrm{DB}} = g^{\mu\nu} + \frac{1}{\Delta}gq^a F^a_{\rho\sigma}(\pi^\rho g^{\nu\sigma}\omega^\mu - \pi^\mu g^{\nu\sigma}\omega^\rho),$$

$$\{x^\mu, F^a_{\rho\sigma}\}_{\mathrm{DB}} = \frac{1}{\Delta}(\partial_\nu F^a_{\rho\sigma})(\pi^\mu\omega^\nu - \pi^\nu\omega^\mu),$$

$$\{P^\mu, F^a_{\rho\sigma}\}_{\mathrm{DB}} = -\partial^\mu F^a_{\rho\sigma} + \frac{g}{\Delta}(\partial_\nu F^a_{\rho\sigma})F^b_{\alpha\beta}g^{\mu\alpha}(\pi^\beta\omega^\nu - \pi^\nu\omega^\beta),$$

$$\dot{x}^\mu = \lambda_1 P^\mu + \lambda_1 \frac{1}{\Delta} g(\mu-1) q^a F^a_{\rho\sigma} P^\sigma (\omega^\rho \pi^\mu - \omega^\mu \pi^\rho) - \lambda_1 \frac{1}{\Delta} \mu g g_{\alpha\beta} q^a (D^\alpha F^a_{\rho\sigma}) \omega^\rho \pi^\sigma (\omega^\beta \pi^\mu - \pi^\beta \omega^\mu),$$

$$\dot{P}^\mu = g q^a g^{\mu\rho} F^a_{\rho\sigma} \dot{x}^\sigma + \lambda_1 \mu g q^a (D^\mu F^a_{\rho\sigma}) \omega^\rho \pi^\sigma,$$

$$\dot{\omega}^\mu = \lambda_2 \pi^\mu - \frac{1}{\Delta}(\mu-1)\lambda_1 g q^a F^a_{\rho\sigma} P^\rho \omega^\sigma P^\mu - \frac{1}{\Delta} \mu \lambda_1 g g_{\alpha\beta} q^a (D^\alpha F^a_{\rho\sigma}) P^\mu \pi^\sigma \omega^\rho \omega^\beta - \mu \lambda_1 g q^a F^a_{\rho\sigma} g^{\mu\sigma} \omega^\rho$$

$$\dot{\pi}^\mu = -\frac{a_2 \lambda_2 \omega^\mu}{a_3} - \frac{1}{\Delta}(\mu-1)\lambda_1 g q^a F^a_{\rho\sigma} P^\rho \pi^\sigma P^\mu - \frac{1}{\Delta} \mu \lambda_1 g g_{\alpha\beta} q^a (D^\alpha F^a_{\rho\sigma}) P^\mu \pi^\sigma \pi^\beta \omega^\rho + \mu \lambda_1 g q^a F^a_{\rho\sigma} g^{\mu\rho} \pi^\sigma$$

$$\dot{q}^a = -\lambda_1 g f^{abc} q^c P^\mu A_\mu - \mu g f^{abc} q^c \lambda_1 F^b_{\mu\nu} \omega^\mu \pi^\nu + \frac{1}{\Delta}(1-\mu)\lambda_1 g^2 q^c q^d f^{abc} F^d_{\rho\sigma} A^b_\mu P^\rho (\pi^\sigma \omega^\mu - \omega^\sigma \pi^\mu)$$
$$- \frac{1}{\Delta} \mu \lambda_1 g^2 q^c q^d f^{abc} (D_\nu F^d_{\rho\sigma}) \omega^\rho \pi^\sigma A^b_\mu (\omega^\mu \pi^\nu - \omega^\nu \pi^\mu),$$

Two unknown λ due to two FCC

λ_1: reparametrization invariance of the worldline; this invariance is inevitable in any consistent relativistic formulation

$$\dot{x}^{\mu} = \lambda_1 P^{\mu} + \frac{g}{\Delta}\lambda_1 S^{\mu\nu}\left\{\frac{\mu}{4}\,\mathrm{Tr}[qD_{\nu}F_{\rho\sigma}]\,S^{\rho\sigma} - (\mu-1)\,\mathrm{Tr}[qF_{\nu\rho}]\,P^{\rho}\right\}$$

$$\dot{P}^{\mu} = 2gg^{\mu\nu}\,\mathrm{Tr}[qF_{\nu\rho}]\,\dot{x}^{\rho} + \frac{1}{2}\mu\lambda_1 g\,\mathrm{Tr}[qD^{\mu}F_{\rho\sigma}]\,S^{\rho\sigma}$$

$$\dot{S}^{\mu\nu} = 2\mu\lambda_1 g\,\mathrm{Tr}[qF_{\rho\sigma}]\,g^{\mu[\rho}S^{\sigma]\nu} + \frac{2g}{\Delta}\lambda_1 P^{[\mu}S^{\nu]\alpha}\left\{\frac{\mu}{4}\,\mathrm{Tr}[qD_{\alpha}F_{\rho\sigma}]\,S^{\rho\sigma} - (\mu-1)\,\mathrm{Tr}[qF_{\alpha\rho}]\,P^{\rho}\right\}$$

$$\dot{q} = ig[A_{\mu}q]\,\dot{x}^{\mu} + i\frac{g}{4}\mu\lambda_1[F_{\mu\nu}q]\,S^{\mu\nu}$$

$$D_{\mu}F_{\rho\sigma} = \partial_{\mu}F_{\rho\sigma} - ig[A_{\mu}, F_{\rho\sigma}] \qquad \Delta = m^2 + q^a(2\mu+1)F^a_{\mu\nu}\omega^{\mu}\pi^{\nu} = P^2 + \tfrac{1}{4}q^a F^a_{\mu\nu}S^{\mu\nu}$$

All constraints are satisfied

$$\dot{S}^{\mu\nu} = 2\mu\lambda_1 g\,\mathrm{Tr}[qF_{\rho\sigma}]\,g^{[\mu\rho}S^{\sigma\nu]} + 2\left(p^{\mu}\dot{x}^{\nu} - p^{\nu}\dot{x}^{\mu}\right)$$

Spin–Orbital Angular Momentum Conversion

$$S^\mu = \frac{1}{2\sqrt{P^2}}\,\epsilon^{\mu\nu\alpha\beta}P_\nu S_{\alpha\beta}$$

not

$$S^\mu = \frac{1}{2m}\,\epsilon^{\mu\nu\alpha\beta}\,P_\nu\,S_{\alpha\beta}$$

$$P^\mu S_\mu = 0,\qquad S^\mu S_\mu = -4s^2,$$

$$\begin{aligned}
\frac{dx^\mu}{d\tau} &= c\,u^\mu \\
\frac{dp^\mu}{d\tau} &= g\,Q^a F_a^{\mu\nu}u_\nu - \frac{g}{mc}\,Q^a(D_\mu\tilde{F}_{\alpha\beta})_a\,u^\alpha S^\beta \\
\frac{dQ^a}{d\tau} &= -g\,f_{abc}\left[u^\mu A_\mu^b - \frac{1}{mc}\,u^\alpha\tilde{F}_{\alpha\beta}^b S^\beta\right]Q^c \\
\frac{dS^\mu}{d\tau} &= \frac{g}{mc}\,Q^a\left[F_a^{\mu\nu}S_\nu + \frac{1}{mc}(u^\mu S^\nu - u^\nu S^\mu)(D_\nu\tilde{F}_{\alpha\beta})_a\,u^\alpha S^\beta\right]
\end{aligned}$$

U. W. Heinz, PLB 144, 228 (1984)

$$\Longleftrightarrow$$

$$\begin{aligned}
\dot{x}^\mu &= \lambda_1 P^\mu + \frac{g}{\Delta\sqrt{P^2}}\lambda_1\epsilon^{\mu\nu\alpha\beta}P_\alpha S_\beta\left\{\frac{\mu}{4}\mathrm{Tr}\left[qD_\nu F_{\rho\sigma}\right]\epsilon^{\rho\sigma\alpha'\beta'}P_{\alpha'}S_{\beta'} - (\mu-1)\mathrm{Tr}\left[qF_{\nu\rho}\right]P^\rho\right\} \\
\dot{P}^\mu &= 2gg^{\mu\nu}\mathrm{Tr}\left[qF_{\nu\rho}\right]\dot{x}^\rho + \frac{g}{2\sqrt{P^2}}\mu\lambda_1\epsilon^{\rho\sigma\alpha\beta}P_\alpha S_\beta\mathrm{Tr}\left[qD^\mu F_{\rho\sigma}\right] \\
\dot{S}^\mu &= 2\mu\lambda_1 g\mathrm{Tr}\left[qF^{\nu\mu}\right]S_\nu + \frac{1}{P^2}P^\mu\left\{2\mu\lambda_1 g\mathrm{Tr}\left[qF^{\rho\sigma}\right]P_\rho S_\sigma - S^\nu\dot{P}_\nu\right\} \\
\dot{q} &= ig[A_\mu, q]\dot{x}^\mu + i\frac{g}{4\sqrt{P^2}}\mu\lambda_1[F_{\mu\nu}, q]\epsilon^{\mu\nu\alpha\beta}P_\alpha S_\beta
\end{aligned}$$

J. Zhou, Y. S. Zhao and Y. Sun, arXiv: 2511.20083

Implications

- Spin-dependent velocity (mass shell); velocity not parallel to momentum (**also appears in chiral kinetic theory**)
- Uniform Abelien gauge fields → reduces to TBMT-type equations but **with modifications**
- Uniform Non-Abelian → correction on spin dynamics

$$\dot{x}^\mu = \lambda_1 P^\mu + \frac{g}{\Delta\sqrt{P^2}}\lambda_1 \epsilon^{\mu\nu\alpha\beta} P_\alpha S_\beta \left\{\frac{\mu}{4}\mathrm{Tr}\left[qD_\nu F_{\rho\sigma}\right]\epsilon^{\rho\sigma\alpha'\beta'}P_{\alpha'}S_{\beta'} - (\mu-1)\mathrm{Tr}\left[qF_{\nu\rho}\right]P^\rho\right\}$$

$$\dot{P}^\mu = 2gg^{\mu\nu}\mathrm{Tr}\left[qF_{\nu\rho}\right]\dot{x}^\rho + \frac{g}{2\sqrt{P^2}}\mu\lambda_1\epsilon^{\rho\sigma\alpha\beta}P_\alpha S_\beta \mathrm{Tr}\left[qD^\mu F_{\rho\sigma}\right]$$

$$\dot{S}^\mu = 2\mu\lambda_1 g\mathrm{Tr}\left[qF^{\nu\mu}\right]S_\nu + \frac{1}{P^2}P^\mu\left\{2\mu\lambda_1 g\mathrm{Tr}\left[qF^{\rho\sigma}\right]P_\rho S_\sigma - S^\nu\dot{P}_\nu\right\}$$

$$\dot{q} = ig[A_\mu, q]\dot{x}^\mu + i\frac{g}{4\sqrt{P^2}}\mu\lambda_1[F_{\mu\nu}, q]\epsilon^{\mu\nu\alpha\beta}P_\alpha S_\beta$$

$$A'_\mu = UA_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger,$$
$$F'_{\mu\nu} = UF_{\mu\nu}U^\dagger,$$
$$(D_\mu F_{\rho\sigma})' = UD_\mu F_{\rho\sigma}U^\dagger,$$
$$q' = UqU^\dagger,$$
$$\dot{q} - ig[A_\mu, q]\dot{x}^\mu - i\frac{g}{4}\mu\lambda_1[F_{\mu\nu}, q]S^{\mu\nu}$$
$$(\cdot)' = U(\cdot)U^\dagger = 0$$

lightcone time λ = x^+ or coordinate time λ = x^0

For λ = x^0 $\quad dx^0/d\lambda = 1$

$$\lambda_1 = \frac{1}{P^0 + \frac{g}{\Delta}S^{0i}\left\{\frac{\mu}{4}\mathrm{Tr}[qD_i(FS)] - (\mu-1)\mathrm{Tr}[qF_{i\nu}P^\nu]\right\}}$$

Formal method: Gauge fixing

$$T_g : x^0 - \lambda = 0$$

$$\{T_g, H_P\}_{\mathrm{PB}} + \frac{\partial T_g}{\partial\lambda} = 0$$

Outline

$P^\mu S_{\mu\nu} = 0$ Spin is not defined in the particle rest frame for massless particle

N=(1,0,0,0)

$$S^{0i} = 0$$
$$S^{ij} = h\,\epsilon_{ijk}\,\hat{p}^k$$

Spin locking

$$S^{\mu\nu} = \frac{\lambda\epsilon^{\mu\nu\rho\sigma}P_\rho N_\sigma}{\sqrt{(P\cdot N)^2 - \frac{\mu e}{2}F_{\mu\nu}S^{\mu\nu}N^2}} = \frac{\lambda\epsilon^{\mu\nu\rho\sigma}P_\rho N_\sigma}{|\vec{p}|}$$

$$N^\mu S_{\mu\nu} = 0$$
$$P^\mu S_{\mu\nu} = 0$$
$$S^{\mu\nu}S_{\mu\nu} = 8s^2$$
$$P^2 - \frac{\mu}{2}\,q\,e\,F^{\mu\nu}\,S_{\mu\nu} = 0$$

E.M. Field

$$H_p = \frac{1}{2}g_1(P^2 - \frac{\mu e}{2}F^{\mu\nu}J_{\mu\nu}) + \frac{1}{2}g_2(\pi^2 - a_2) + \frac{1}{2}g_3(\omega^2 - a_3) + g_5(P\pi) + \lambda_4(P\omega) + \lambda_6(\omega\pi)$$
$$+ g_7(N\omega) + g_8(N\pi) + \lambda_{g_i}\pi_{g_i} \qquad (21)$$

$$\dot{x}^\mu = \lambda_1\left(P^\mu + \frac{e(\mu-1)}{2\Delta_1}F_{\rho\sigma}P^\sigma S^{\mu\rho} + \frac{e\mu}{8\Delta_1}(\partial_\rho F_{\alpha\beta})S^{\alpha\beta}S^{\rho\mu} + \frac{e\mu\,F_{\rho\sigma}N^\rho S^{\mu\sigma}}{2\Delta_2}\right)$$

$$\dot{P}^\mu = eF^{\mu\nu}\dot{x}_\nu + \lambda_1\frac{e\mu}{4}\partial^\mu F_{\rho\sigma}S^{\rho\sigma}$$

$$\frac{1}{\Delta_1} = \frac{N^2}{(NP)^2 - \frac{e(2\mu+1)}{4}N^2F_{\mu\nu}S^{\mu\nu}}$$

$$\frac{1}{\Delta_2} = \frac{P\cdot N}{(PN)^2 - \frac{(2\mu+1)}{4}S_{\mu\nu}F^{\mu\nu}N^2}$$

Relativistic EOMs

$$\sqrt{G}\,\dot{x}_i = \frac{P_i}{P_0}\left(1+\mu e\,\vec{B}\cdot\vec{b}\right) + B_i e(1-\mu)\frac{\vec{p}\cdot\vec{b}}{P_0} + e(\vec{E}\times\vec{b})_i + e\mu\frac{|\vec{p}|^2}{P_0}\left(\vec{\nabla}(\vec{B}\cdot\vec{b})\times\vec{b}\right)_i$$

$$\sqrt{G} = 1 + e\,\mathbf{b}\cdot\mathbf{B}$$

$$\sqrt{G}\,\dot{P}_i = e\left(\vec{E}+\frac{\mu|\vec{p}|^2}{P_0}\vec{\nabla}(\vec{B}\cdot\vec{b})\right)_i + e\,\frac{(\vec{p}\times\vec{B})_i}{P_0}\left(1+\mu e\,\vec{B}\cdot\vec{b}\right) + e^2\left(\left(\vec{E}+\frac{\mu|\vec{p}|^2}{P_0}\vec{\nabla}(\vec{B}\cdot\vec{b})\right)\cdot\vec{B}\right)b_i$$

$$\mathbf{b} = \lambda\,\frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

$$\sqrt{G}\,\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \boldsymbol{E}\times\boldsymbol{b} + \boldsymbol{B}(\hat{\boldsymbol{p}}\cdot\boldsymbol{b})\,;$$
$$\sqrt{G}\dot{\boldsymbol{p}} = \boldsymbol{E} + \hat{\boldsymbol{p}}\times\boldsymbol{B} + \boldsymbol{b}(\boldsymbol{E}\cdot\boldsymbol{B})\,.$$

Path integral approach

$$G = (1+\boldsymbol{b}\cdot\boldsymbol{B})^2$$

M. A. Stephanov and Y. Yin, PRL 109, 162001 (2012) → $\boldsymbol{\mu = 0}$

$$\left\{\partial_t + \frac{1}{\sqrt{G}}(\tilde{\mathbf{v}} + \hbar\epsilon Q(\tilde{\mathbf{v}}\cdot\mathbf{b}_h)\mathbf{B} + \hbar\epsilon Q\tilde{\mathbf{E}}\times\mathbf{b}_h)\cdot\nabla_{\mathbf{x}} + \frac{\epsilon Q}{\sqrt{G}}(\tilde{\mathbf{E}} + \tilde{\mathbf{v}}\times\mathbf{B} + \hbar\epsilon Q(\tilde{\mathbf{E}}\cdot\mathbf{B})\mathbf{b}_h)\cdot\nabla_{\mathbf{p}}\right\} f_h^{\epsilon}(x,\mathbf{p}) = 0$$

$$\sqrt{G} = (1+\hbar\epsilon Q\mathbf{b}_h\cdot\mathbf{B}), \qquad \tilde{\mathbf{E}} = \mathbf{E} - \frac{1}{\epsilon Q}\nabla_{\mathbf{x}}E_{\mathbf{p}},$$

Wigner function approach → $\boldsymbol{\mu = 1}$, $+\hbar$ ***order***

$$E_{\mathbf{p}} = |\mathbf{p}|(1-\hbar\epsilon Q\mathbf{B}\cdot\mathbf{b}_h), \qquad \tilde{\mathbf{v}} = \frac{\partial E_{\mathbf{p}}}{\partial\mathbf{p}} = \hat{\mathbf{p}}(1+2\hbar\epsilon Q\mathbf{B}\cdot\mathbf{b}_h) - \hbar\epsilon Q b_h\mathbf{B}$$

Y. Hidaka, S. Pu and D.L. Yang, PRD 95, 091901 (2017)
A. Huang, S. Shi, Y. Jiang, J. Liao, and P. Zhuang, PRD 98, 036010 (2018)

Applications for Chiral Particles

$$\{\xi^a, \xi^b\} = (\omega^{-1})^{ab} \equiv \omega^{ab}$$

$$\dot{\xi}^a = \{H, \xi^a\} = -\{\xi^a, \xi^b\}\partial_b H$$

$$\mathrm{d}\Gamma = \sqrt{\omega}\,\mathrm{d}\xi = (1 + \mathbf{B}\cdot\boldsymbol{\Omega})\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{x}}{(2\pi)^3}$$

$$\{p_i, p_j\} = -\frac{\epsilon_{ijk}B_k}{1+\mathbf{B}\cdot\boldsymbol{\Omega}},$$

$$\{x_i, x_j\} = \frac{\epsilon_{ijk}\Omega_k}{1+\mathbf{B}\cdot\boldsymbol{\Omega}},$$

$$\{p_i, x_j\} = \frac{\delta_{ij}+\Omega_i B_j}{1+\mathbf{B}\cdot\boldsymbol{\Omega}},$$

D.T. Son and N. Yamamoto, PRL 109, 181602 (2012)

Dirac Bracket

$$\{F, G\}_{\mathrm{DB}} \equiv \{F, G\}_{\mathrm{PB}} - \{F, \xi_\alpha\}_{\mathrm{PB}}\,\Delta^{-1}_{\alpha\beta}\,\{\xi_\beta, G\}_{\mathrm{PB}}$$

$$\Delta_{\alpha\beta} = \{\xi_\alpha, \xi_\beta\}_{\mathrm{PB}}$$

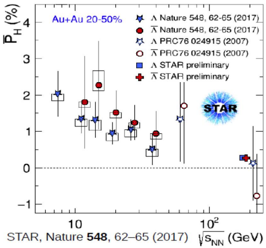
$$\{p_i, p_j\}_{\mathrm{DB}} = -\frac{\epsilon_{ijk}B_k}{1+\mathbf{B}\cdot\boldsymbol{\Omega}}$$

$$\{x_i, x_j\}_{\mathrm{DB}} = \frac{\epsilon_{ijk}\Omega_k}{1+\mathbf{B}\cdot\boldsymbol{\Omega}}$$

$$\{x_i, p_j\}_{\mathrm{DB}} = \frac{\delta_{ij}+\Omega_i B_j}{1+\mathbf{B}\cdot\boldsymbol{\Omega}}$$

Summary

- We use the constrained Hamiltonian formalism to derive equations of motion that respect the constraints; Applying the constraints to the massless case, we recover the chiral kinetic equations
- How it modifies the HQ momentum diffusion
- Remaining questions:
  - ➢ Derivation of kinetic theory in collisionless limit
  - ➢ Gradient-expansion effects on spin polarization
  - ➢ Incorporation of collision
  - ➢ Extension to gluon jets



STAR, Nature **548**, 62–65 (2017)

Thank you!

$$\frac{d\mathbf{x}}{dt} = \frac{c\boldsymbol{\beta}[1+2a(\mu-1)\gamma((\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s}))] - \frac{2ac(\mu-1)}{\gamma}\mathbf{B}(\boldsymbol{\beta}\mathbf{s})}{[1+2a(\mu-1)\gamma(\boldsymbol{\beta}^2(\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s}))]},$$

$$\frac{d\boldsymbol{\mathcal{P}}}{dt} = \frac{e(1+2a(\mu-1)\gamma((\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s})))[\boldsymbol{\beta},\mathbf{B}]}{c[1+2a(\mu-1)\gamma(\boldsymbol{\beta}^2(\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s}))]}, \qquad \frac{d\mathcal{P}^0}{dt}=0,$$

$$\frac{d\mathbf{s}}{dt} = \frac{e\mu[\mathbf{s},\mathbf{B}]}{\mathcal{P}^0[1+2a(\mu-1)\gamma(\boldsymbol{\beta}^2(\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s}))]} + \frac{e(\mu-1)(\mathbf{s},\boldsymbol{\mathcal{P}},\mathbf{B})\boldsymbol{\mathcal{P}}[1-2a\gamma((\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s}))]}{\mathcal{P}^2\mathcal{P}^0[1+2a(\mu-1)\gamma(\boldsymbol{\beta}^2(\mathbf{B}\mathbf{s})-(\mathbf{B}\boldsymbol{\beta})(\boldsymbol{\beta}\mathbf{s}))]},$$

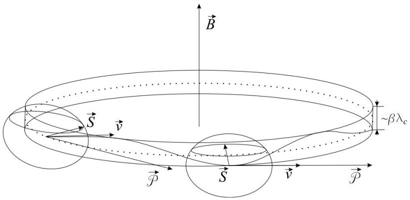
$$\boldsymbol{\mathcal{P}}(t) = |\boldsymbol{\mathcal{P}}(0)|(\mathbf{e}_x\cos(\Omega_p t)+\mathbf{e}_y\sin(\Omega_p t)),$$

$$\mathbf{s}(t) = \mathbf{e}_x s^{(0)}\left[\cos\left(\frac{\Omega_s'}{\gamma}t+\phi\right)\cos(\Omega_p t) - \frac{1}{\gamma}\sin\left(\frac{\Omega_s'}{\gamma}t+\phi\right)\sin(\Omega_p t)\right] + \mathbf{e}_y s^{(0)}\left[\cos\left(\frac{\Omega_s'}{\gamma}t+\phi\right)\sin(\Omega_p t) + \frac{1}{\gamma}\sin\left(\frac{\Omega_s'}{\gamma}t+\phi\right)\cos(\Omega_p t)\right] + \mathbf{e}_z s_z(0),$$

$$\mathbf{x}(t) = \mathbf{x}_c + \frac{c\boldsymbol{\mathcal{P}}(t)}{eB} - \frac{2ac|\boldsymbol{\mathcal{P}}(0)|s^{(0)}}{e\gamma^2(1-2a\gamma(\mathbf{B}\mathbf{s}))}\mathbf{e}_z\sin\left(\frac{\Omega_s'}{\gamma}t+\phi\right),$$

$$\Omega_p = \frac{eB(1+2a(\mu-1)\gamma(\mathbf{B}\mathbf{s}))}{\mathcal{P}^0(1+2a(\mu-1)\gamma\beta^2(\mathbf{B}\mathbf{s}))}$$

$$\Omega_s' = \frac{eB(\mu-1)\mathcal{P}^0(1-2a\gamma(\mathbf{B}\mathbf{s}))}{\mathcal{P}^2(1+2a(\mu-1)\gamma\beta^2(\mathbf{B}\mathbf{s}))}$$



Trajectory

Orbital motion frequency

helicity variation frequency

A.A. Deriglazov and A.M. Pupasov-Maksimov, NPB 885, 1-24 (2014)

Examples: charged particle in e.m. field

Lagrangian Formalism

$$S = -m \int d\tau - e \int A_\mu dx^\mu$$

$$\frac{d}{dt}(\gamma m \boldsymbol{v}) = e\boldsymbol{E} + e(\boldsymbol{v} \times \boldsymbol{B})$$

Hamiltonian Formalism

$$H = \sqrt{(p - eA)^2 + m^2} + e\Phi$$

$$\frac{d\boldsymbol{x}}{dt} = \frac{\boldsymbol{P}}{\sqrt{\boldsymbol{P}^2 + m^2}}$$

$$\frac{d\boldsymbol{P}}{dt} = e\boldsymbol{E} + e\left(\frac{d\boldsymbol{x}}{dt} \times \boldsymbol{B}\right)$$

m=0 limit? Lorentz Covariance?

$$L = \frac{1}{2g}(\dot{x}^\mu \dot{x}_\mu) + \frac{1}{2} g m^2 + e A_\mu \dot{x}^\mu$$

$$H_P = \frac{g}{2}\left[(p - eA)^2 - m^2\right] + \lambda p_g$$