香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

CLQCD

QCD theory seminars

Total Gluon Helicity Contribution to the Proton Spin from Lattice QCD

2026.06.09 JST/PM 2:00-3:00

Speaker: Dian-Jun Zhao

DJZ et al., arXiv: 2512.24315

In Collaboration with Long Chen, Hongxin Dong, Xiangdong Ji, Liuming Liu, Zhuoyi Pang, Andreas Schäfer, Peng Sun, Yi-Bo Yang, Jianhui Zhang, Shiyi Zhong

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    1. Jaffe & Manohar decomposition, quark and gluon helicity in Lattice
    2. $(E \times A)_i$ and K_μ strategy for extraction of gluon helicity
2. Lattice Setup
3. BME extraction, renormalization and continuum extrapolation

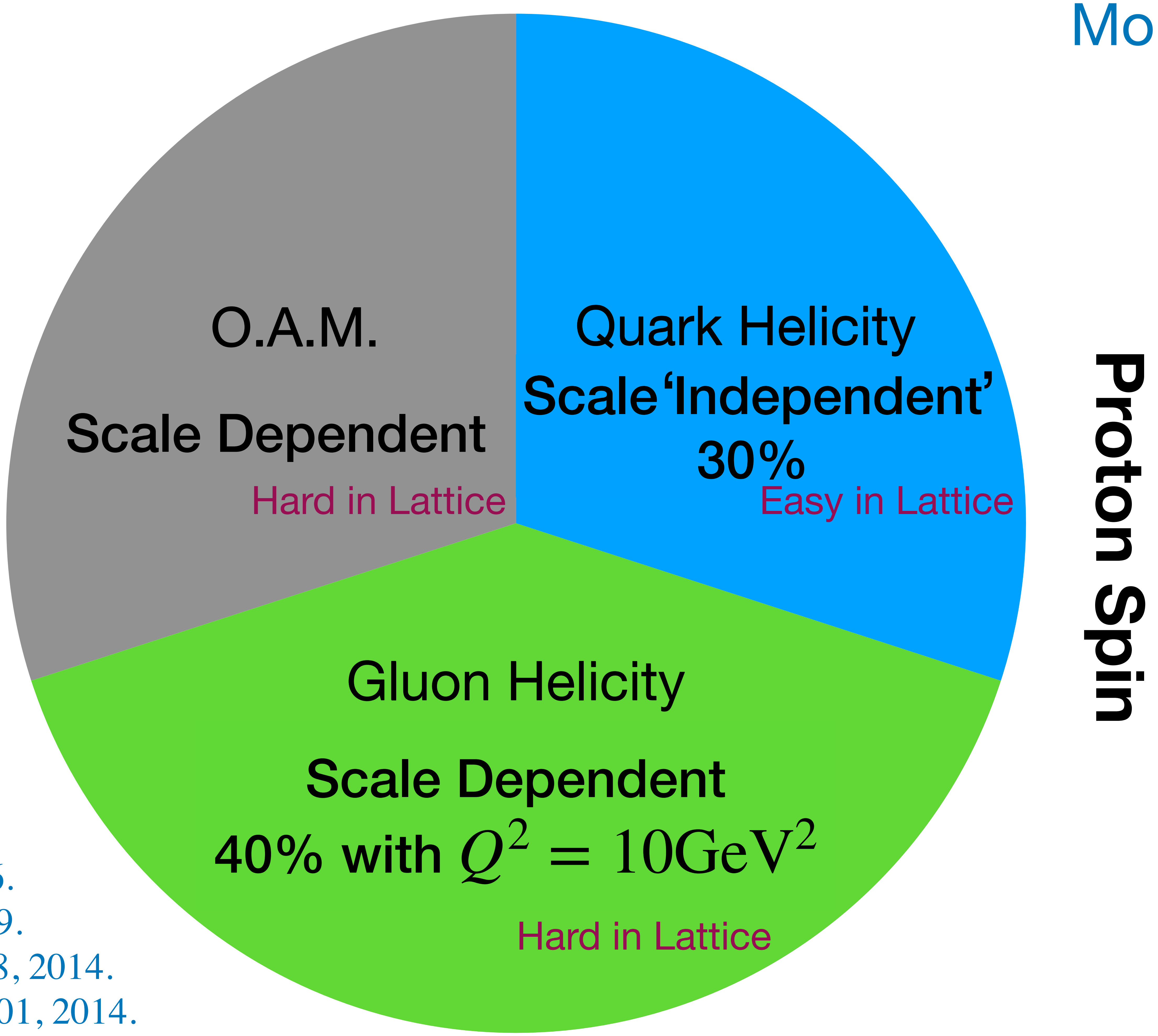
Introduction on spin structure

Moment of twist-3 GTMD, WD

Jaffe—Manohar spin decomposition: $\frac{1}{2} = \underline{L_q^{\mathrm{JM}} + L_g^{\mathrm{JM}}} + \frac{1}{2}\Delta\Sigma + \Delta G$

R. L. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990).

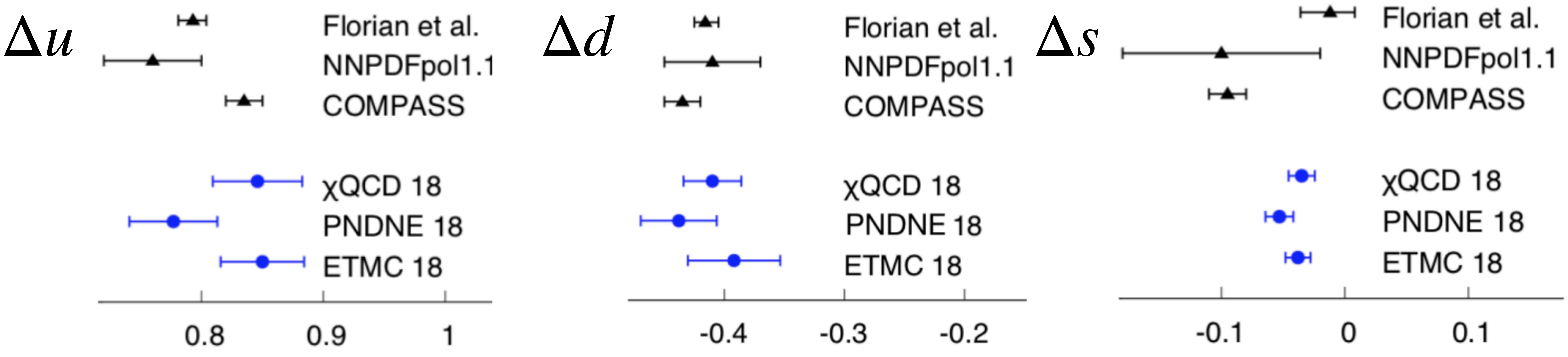
Moment of polarized PDF

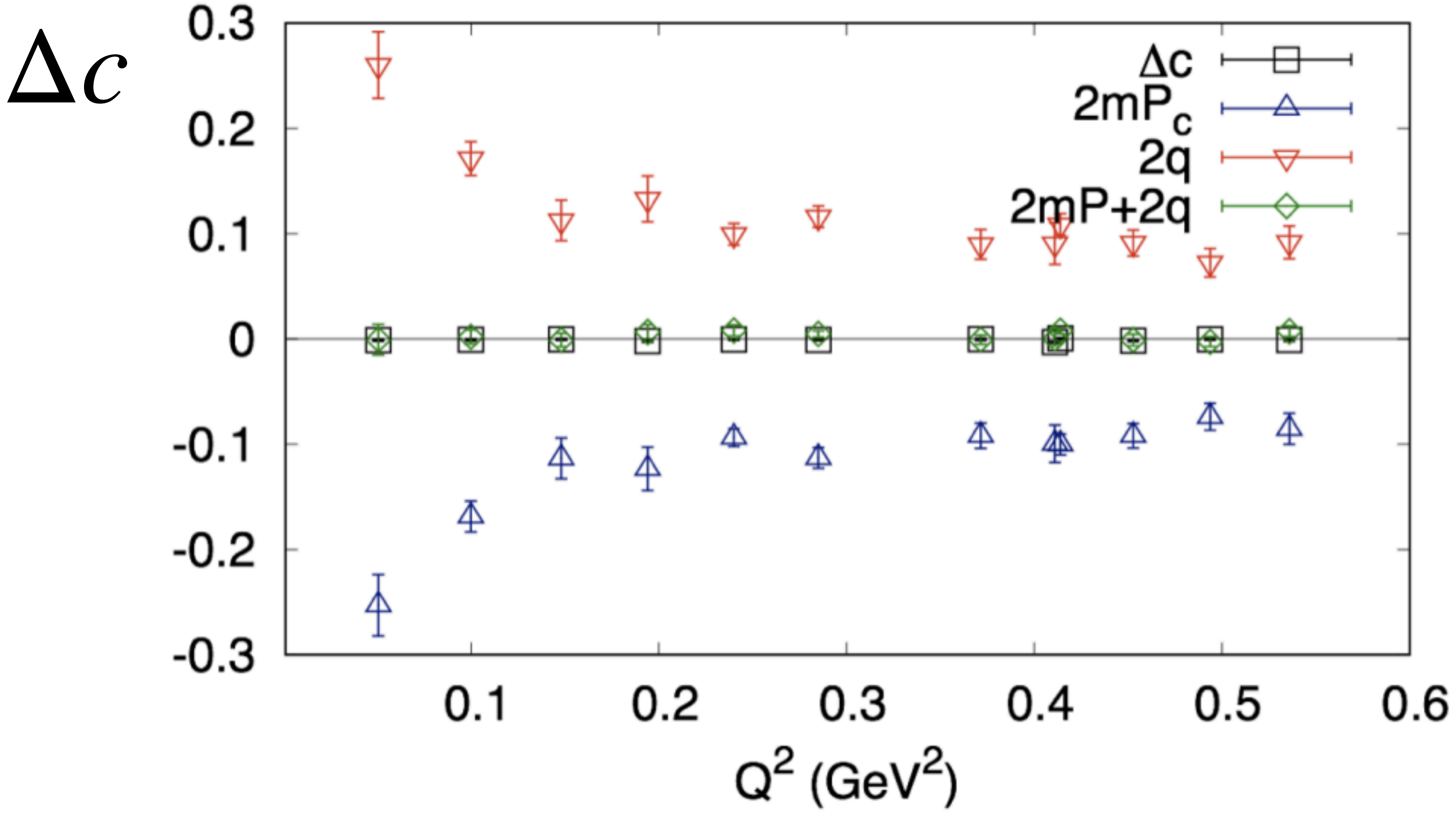


C. Adolph et al. Phys. Lett., B753:18–28, 2016.
D. Florian et al. Phys. Rev. D, 80:034030, 2009.
E. R. Nocera, et al. Nucl. Phys., B887:276–308, 2014.
D. Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

Lattice interpretation of quark helicity

$$\Delta q = \langle \mathrm{PS} \,|\, \bar{q}\gamma_5 \vec{\gamma} \cdot \vec{S} q \,|\, \mathrm{PS} \rangle \qquad \int d^3x (\bar{q}\gamma_\mu \gamma_5 q)(x) = 2m_f \int d^3x \vec{x} P(x) - 2i \int d^3x \vec{x} q(x)$$





$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum \Delta q = 0.155(25)$$

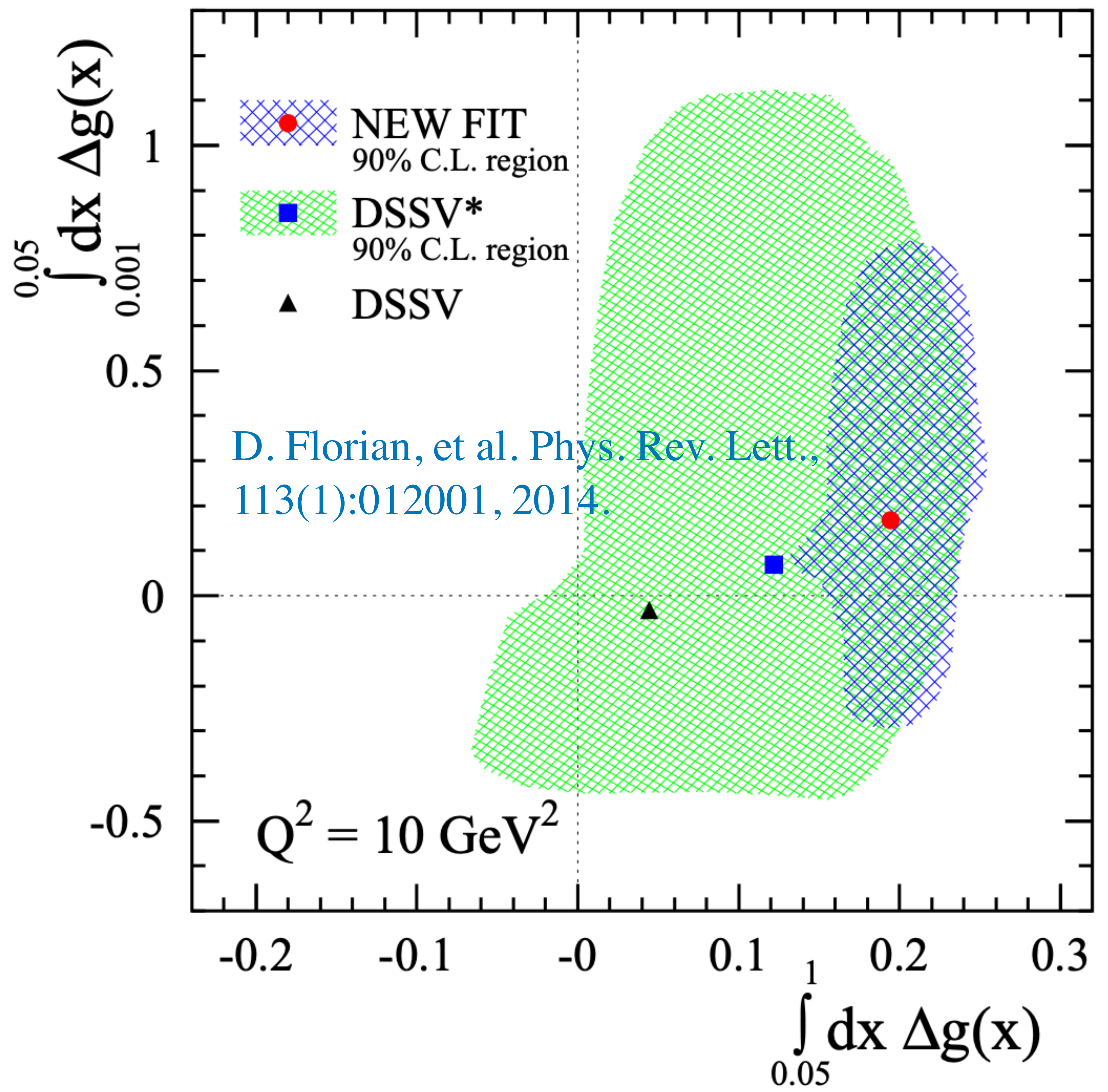
J. Liang, el al. Phys. Rev. D, 98(7):074505, 2018.
H.-W. Lin, et al. Phys. Rev. D, 98:094512, 2018.
C. Alexandrou, et al. Phys. Rev. Lett., 119(14):142002, 2017.
M. Gong, et al. Phys. Rev., D95(11):114509, 2017.

Lattice interpretation of gluon helicity

$$\Delta G = \int dx \Delta g(x) = \int dx \frac{i}{2xP^+} \int \frac{d\varepsilon^-}{2\pi} e^{-ix\varepsilon^- P^+}$$

$$\langle \mathrm{PS} \,|\, F_a^{+\mu}(\varepsilon^-) \mathscr{L}_{ab}(\varepsilon^-, 0) \tilde{F}^+_{b,\mu}(0) \,|\, \mathrm{PS} \rangle$$

Integrating LCPDF



ΔG is difficult to calculate in LQCD because light-cone gauge (L.G.) $\not\leftrightarrow$ Euclidean space!

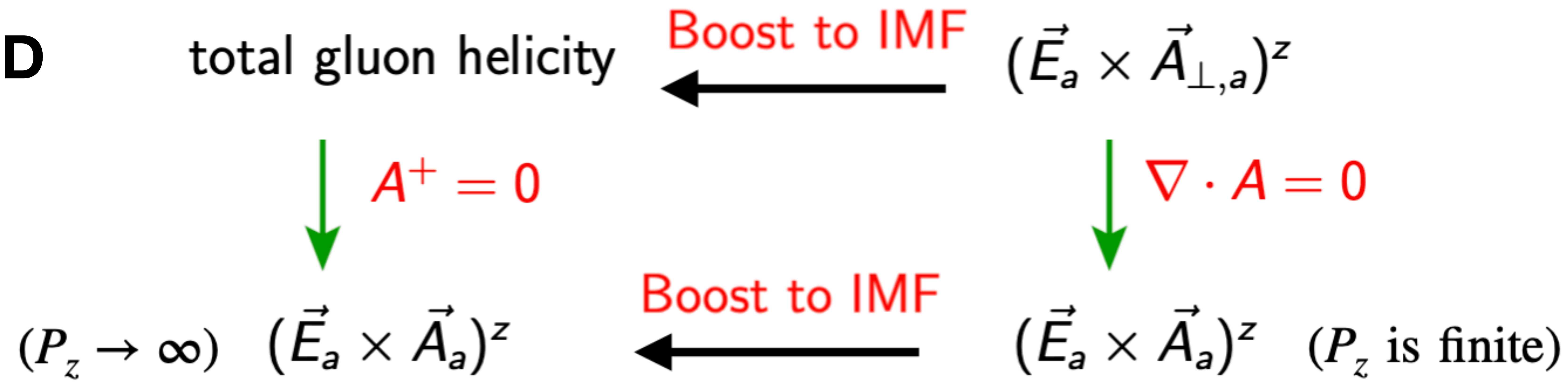
Gauge invariant non-local $\longrightarrow$ Coulomb local

LaMET suggests that Coulomb gauge fixing (C.G.) condition become to L.G. when nucleon to IMF, then $\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}}$ with matching for their difference in UV behavior.

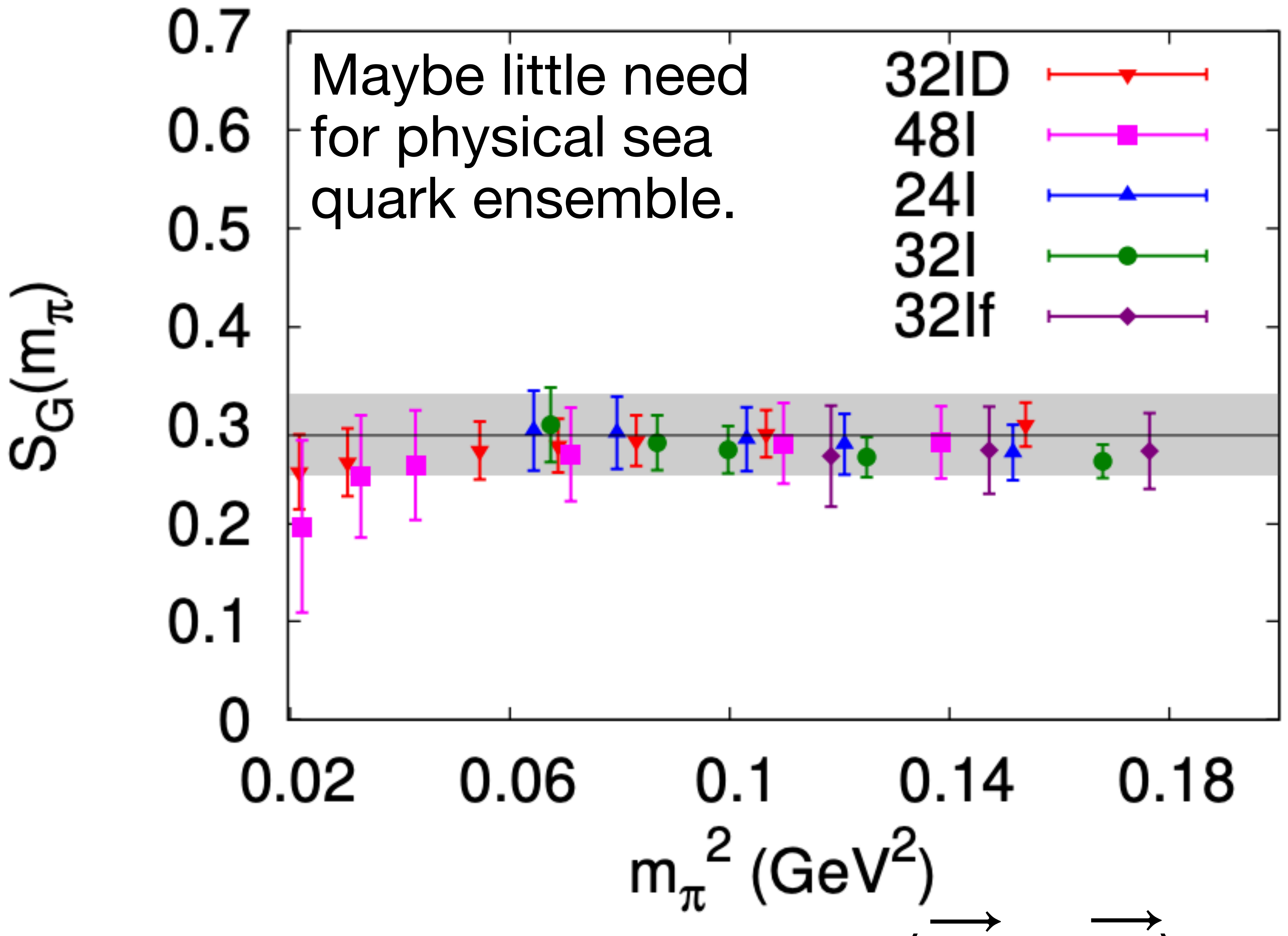
X.-D. Ji et al. 10.1103/PhysRevLett.111.112002

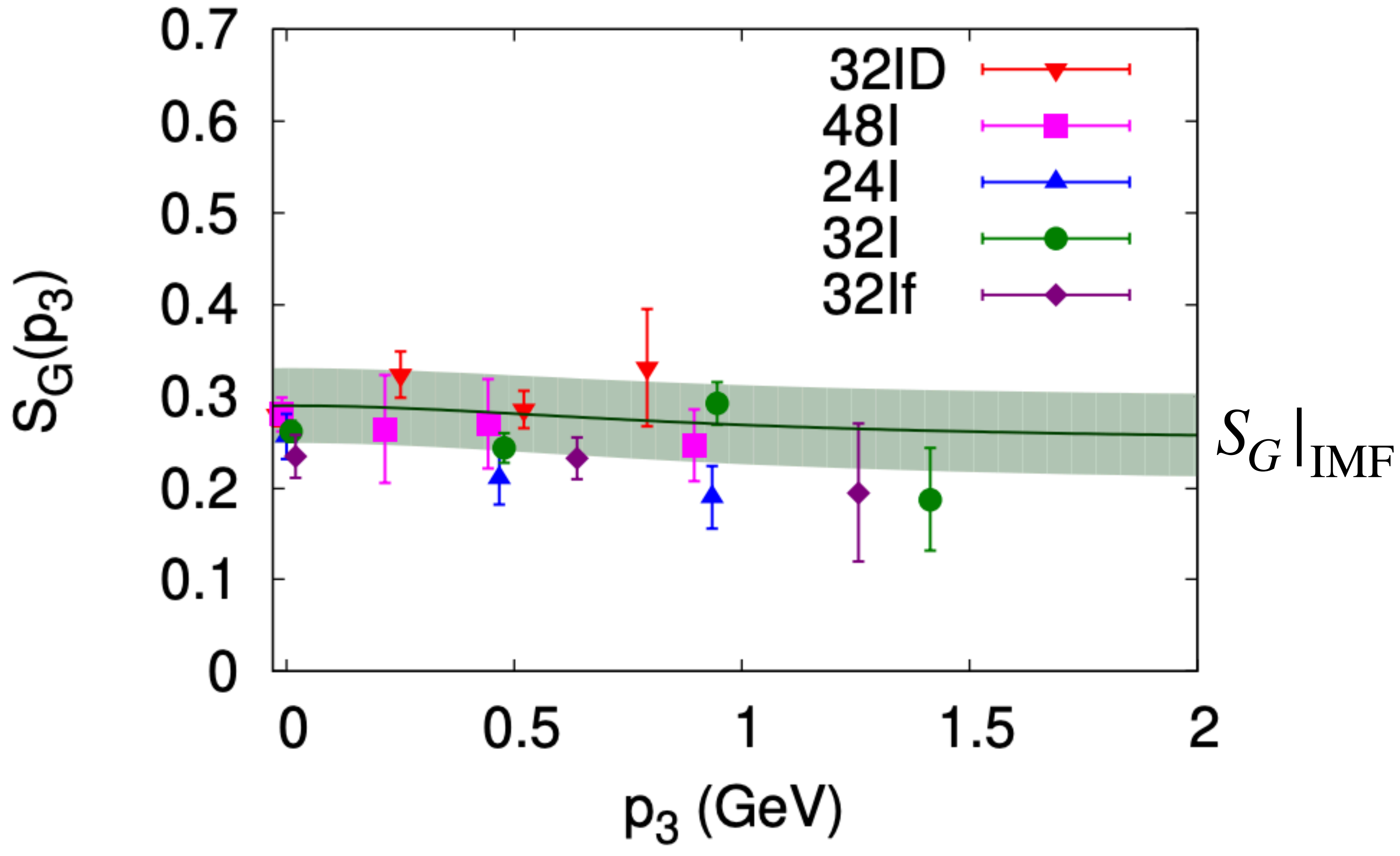
Lattice interpretation of gluon helicity

Gluon helicity in lattice QCD



Y.-B. Yang et al., 10.1103/PhysRevLett.118.102001





$$\Delta G \propto \langle \overrightarrow{E} \times \overrightarrow{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}} = 0.251(47)^{\text{sts.}}(16)^{\text{sym.}}$$

Lattice interpretation of gluon helicity

Y.-B. Yang et al., 10.1103/PhysRevLett.118.102001

Potential problems with ΔG extraction using $\langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}}$

1. Large momentum external state is not large enough;
2. Should be non-perturbative renormalization + perturbative matching;
3. Inconsistency of $\int \frac{dy}{|y|} C'_{gg}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) \Delta g(y) + \cdots \neq C_{gg} \Delta G + \cdots.$

Intrinsic momentum scale in matching shall be parton momentum, not proton momentum.

Lattice interpretation of gluon helicity

Z.-Y. Pang et al., 10.1007/JHEP07(2024)222

Gluon helicity ΔG from topological current

We propose a scheme that relates ΔG to **local topological current** $K^{\mu}_{\text{C.G.}}$.
This scheme solves all of these problems and doesn't require lattice perturbative theory.

$$\boxed{\text{Topological Current } K^{\mu}(x) = \epsilon^{\mu\nu\rho\sigma}\text{Tr}[A_{\nu}F_{\rho\sigma} - 2ig_s A_{\nu}A_{\rho}A_{\sigma}/3](x)}$$

$$\text{Target Three-PT } \langle \text{PS}_{\text{Proj.i}} | K^{t/i} | \text{PS}_{\text{Proj.i}} \rangle_{\text{C.G.}} \propto S^{t/i}\Delta G + \text{h.t.}$$

Lattice Setup

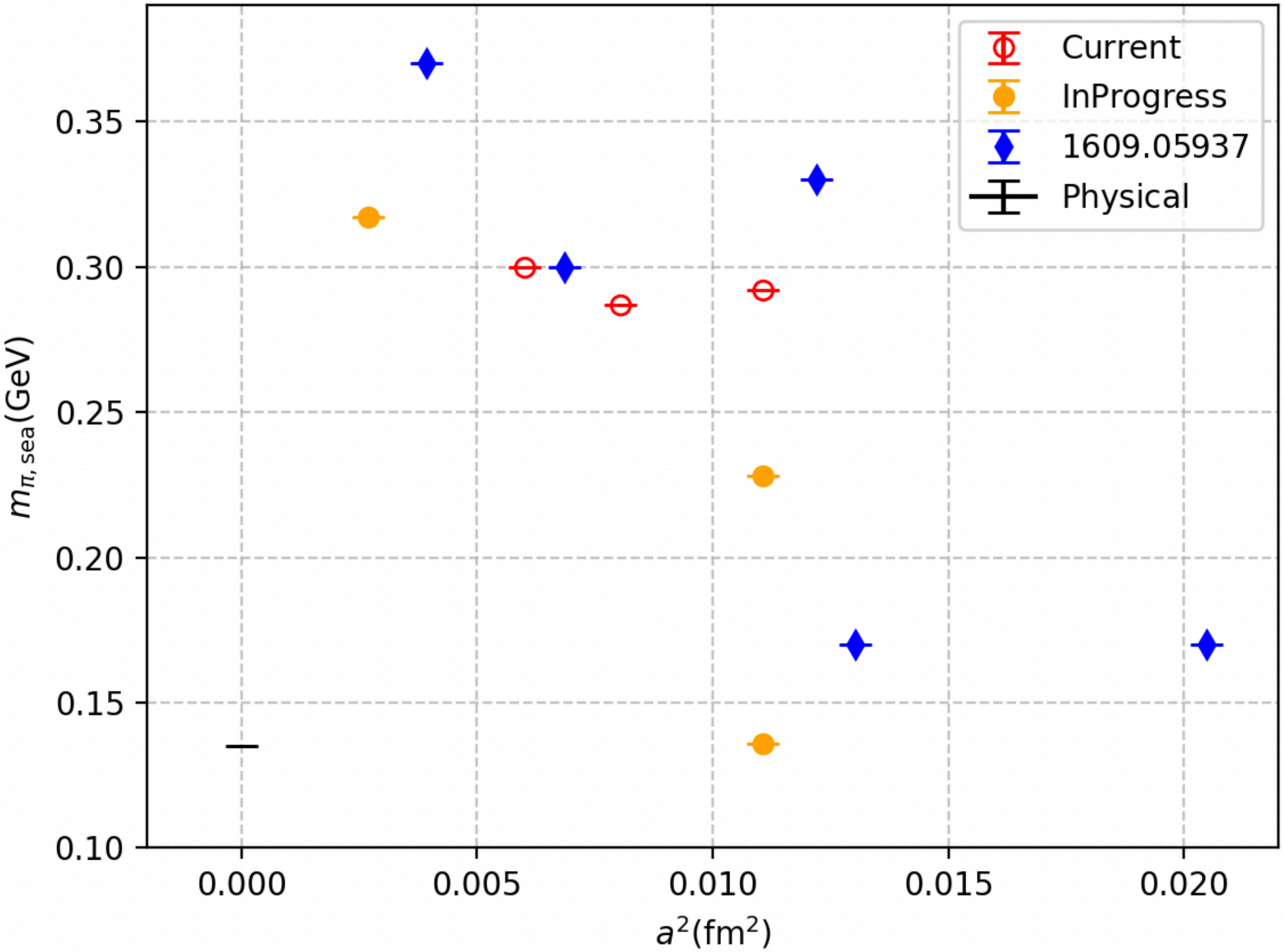
Z.-C. Hu et al. 10.1103/PhysRevD.109.054507

CLQCD	Sea Type	$N_L^3 \times N_T$	Lattice Spacing(fm)	$m_\pi^{(s)}$(MeV)	Valence Type	Ncfg	Nsource
C24P29	Clover (2+1) + TITLS	$24^3 \times 72$	0.1052	292.3(1.0)	Clover	**880**	72
C48P23		$48^3 \times 96$		224.1(1.2)		**380**	Volume
E32P29		$32^3 \times 64$	0.0897	287.3(2.5)		**890**	64
F32P30		$32^3 \times 96$	0.0775	300.4(1.2)		**780**	96

Ensemble used for renormalization has same UV properties as ensemble used for bare matrix element calculations.

Lattice Setup

Z.-C. Hu et al. 10.1103/PhysRevD.109.054507.
Y.-B. Yang et al., 10.1103/PhysRevLett.118.102001



Calculation of quark propagator

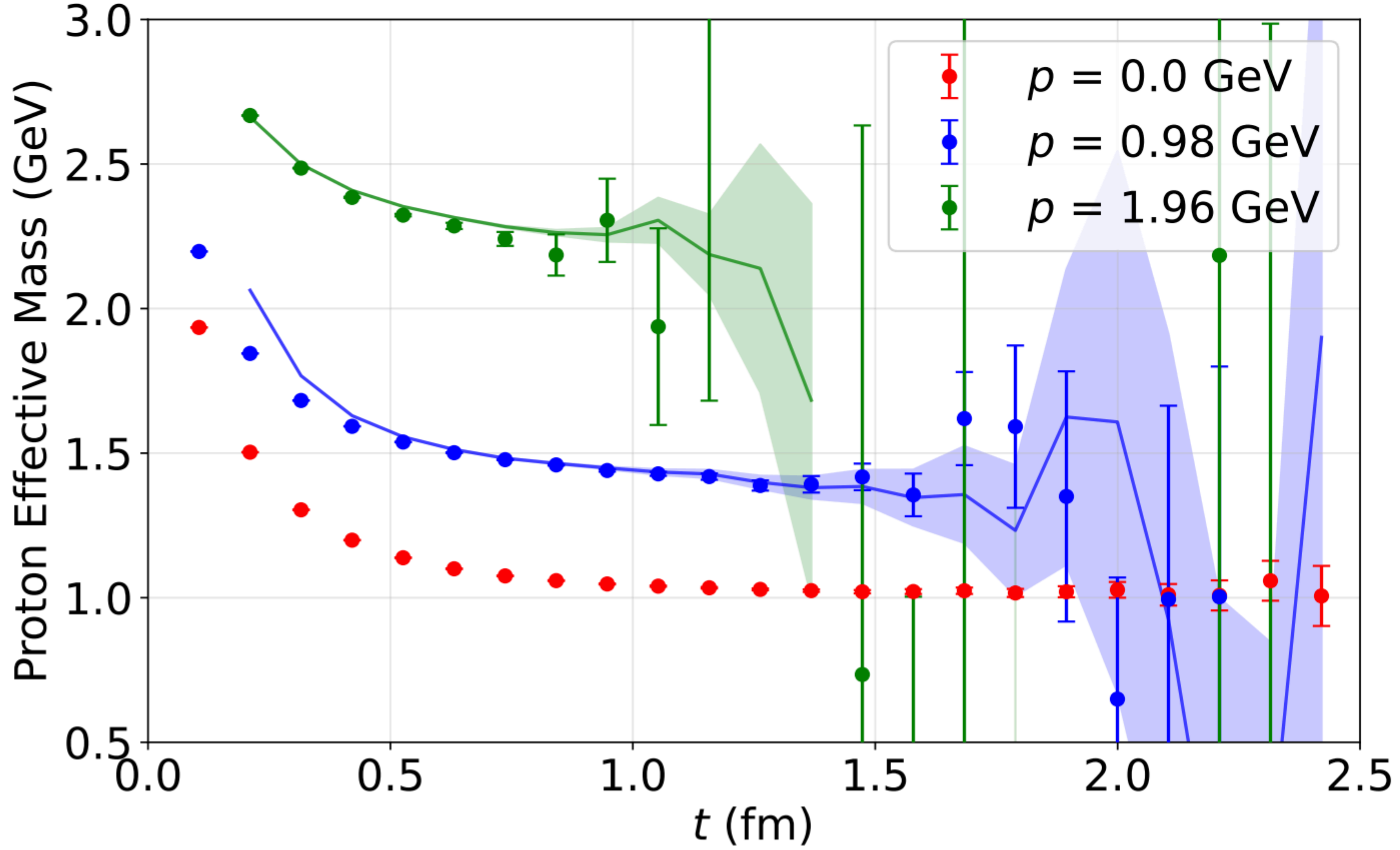
2.2G/Fixed source prop in $24^3 \times 72$

The dimension of the propagator is $(N_t^2, N_z^2, N_y^2, N_x^2, 4, 4, 3, 3, 2)$. Then we have $Dx =$ source, where x is propagator (fixed source), which is a system of linear equations. We need to use CG/BiCGstab/MG.

All-to-all propagator: Distillation allows us to solve the eigenvalue problem by changing the dimensionality of spatial direction from $N_z^2 N_y^2 N_x^2$ to N_{ev}^2.

M. Peardon et al., *Phys.Rev.D* 80 (2009) 054506

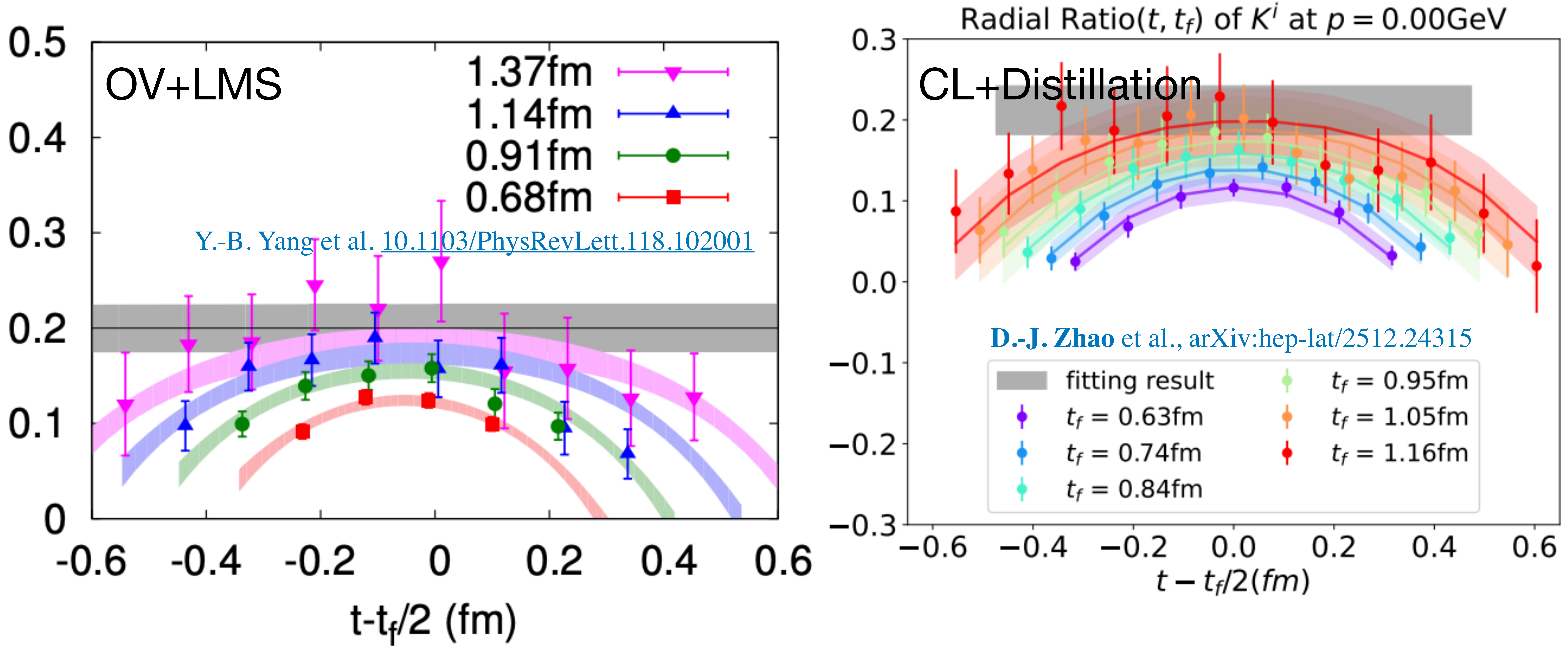
Equivalent to increasing statistics of a single point source propagator by $N_z^2 N_y^2 N_x^2$ times.



a≈0.1fm

BME with optimization smear methods

Distillation for small p_z



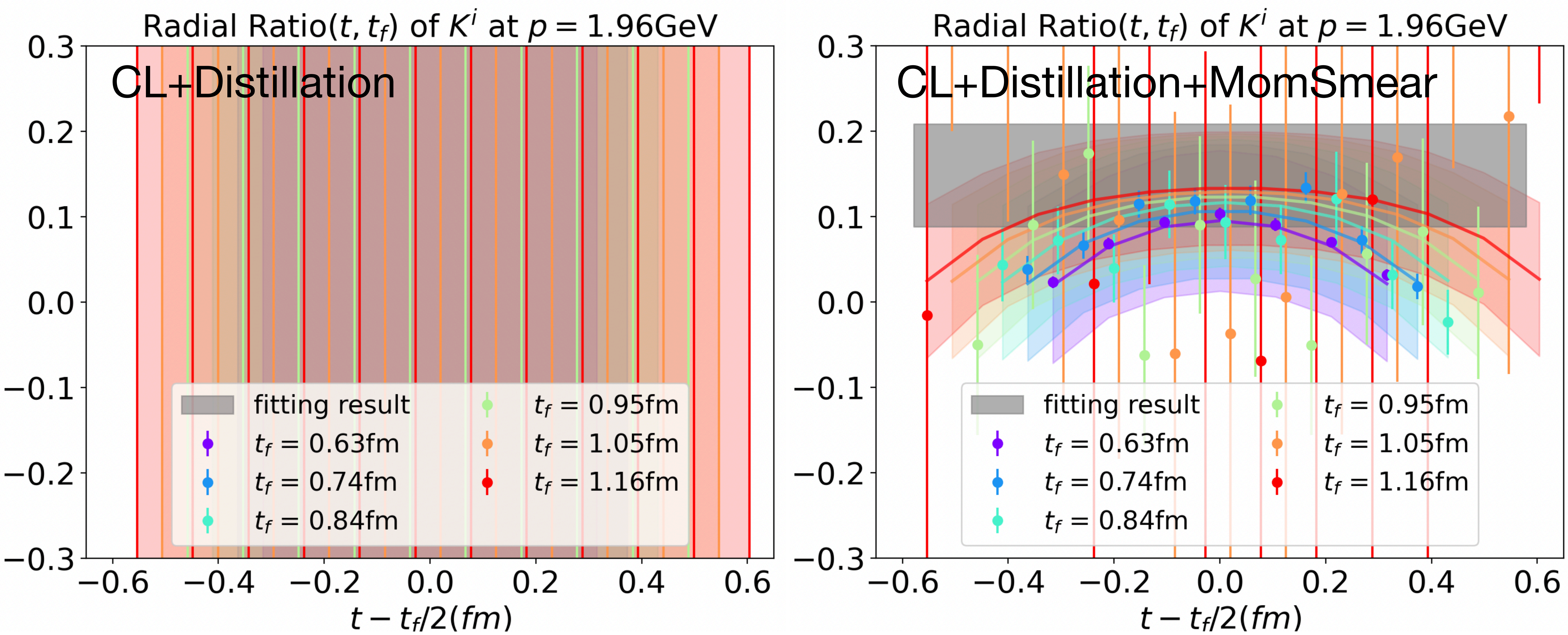
Fitting Form: $R(p; t_f, t_i) = \langle\{K^\mu, E \times A\}\rangle_N^{\mathrm{B.}}(p) + c_1 e^{-\Delta E(t_f - t_i)} + c_2 e^{-\Delta E t_i}$

a≈0.1fm

BME with optimization smear methods

D.-J. Zhao et al., arXiv:hep-lat/2512.24315
C. Egerer et al., *Phys.Rev.D* 103 (2021) 3, 034502

Distillation + Momentum Smear for large p_z

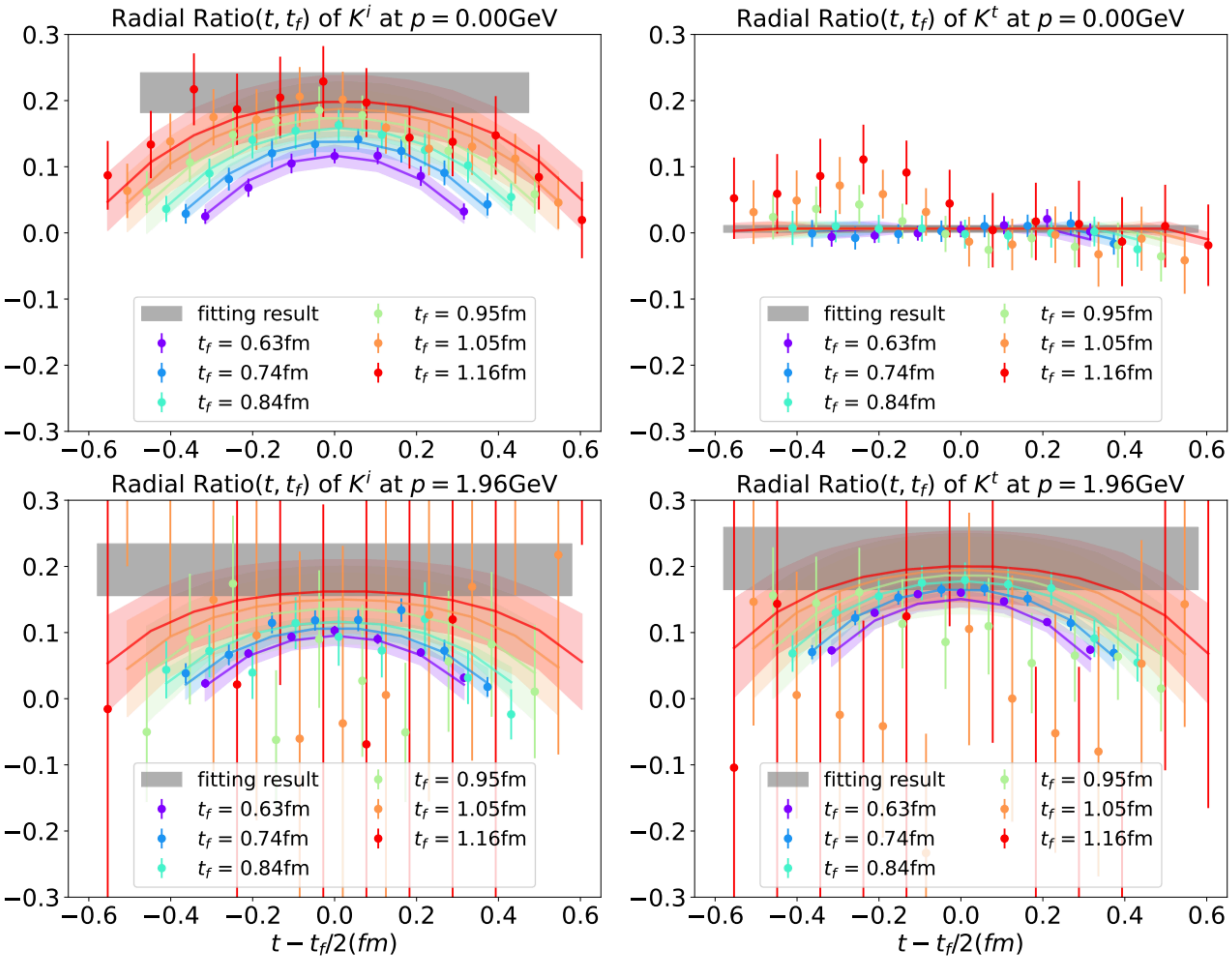


Fitting Form: $R(p; t_f, t_i) = \langle\{K^\mu, E \times A\}\rangle_N^{\text{B.}}(p) + c_1 e^{-\Delta E(t_f - t_i)} + c_2 e^{-\Delta E t_i}$

BME Performance under different momentum

a≈0.1fm

D.-J. Zhao et al., arXiv:hep-lat/2512.24315



Fitting Form:

$$R_{K^\mu}(p; t_f, t_i) = \langle K^\mu \rangle_N^{\mathrm{B.}}(p)$$
$$+c_1 e^{-\Delta E(t_f - t_i)} + c_2 e^{-\Delta E t_i}$$

Almost all of $\langle K^\mu \rangle$'s signal comes from $\epsilon^{\mu\nu\rho\sigma}Tr[A_\nu F_{\rho\sigma}]$.

Renormalization of topological current K^μ

RI/MOM is non-perturbative renormalization scheme on LQCD.

Z.-Y. Pang et al. 10.1007/JHEP07(2024)222
D.-J. Zhao et al., arXiv:hep-lat/2512.24315

Lattice to RI/MOM

$$\langle PS \,|\, \{\overbrace{K^\rho, J^{5\rho}}\} \,|\, PS\rangle^{\text{tree.}} = Z^{\text{RI}}_{\{1,2\}1}\langle PS \,|\, K^\rho \,|\, PS\rangle^{\text{lat.}} + Z^{\text{RI}}_{\{1,2\}2}\langle PS \,|\, J^{5\rho} \,|\, PS\rangle^{\text{lat.}}$$

Coupled

$$\langle K^\mu\rangle^{\text{RI}}_N = Z^{\text{RI}}_{11}\left(\langle K^\mu\rangle^{\text{B.}}_N - \frac{K^{\text{lat.,q}}}{J_5^{\text{lat.,q}}}\langle J^{5\rho}\rangle^{\text{B.}}_N\right)$$

RG Running

$$R^{\text{3loop}}_{ij}(\mu^{\text{RI}}, \mu = e^{-\frac{5}{6}}\mu^{\text{RI}}) \to R^{\text{3loop}}_{ij}(\mu^{\text{RI}}, \mu = \sqrt{10}\text{GeV})$$

S. J. Brodsky, et al., Phys. Rev. D 28, 228 (1983).

Improve the first few terms' convergence of perturbative series.

RI/MOM to $\overline{\text{MS}}$

$$\langle K^\mu\rangle^{\overline{\text{MS}}}_N = R^{\text{3loop}}_{11}\langle K^\mu\rangle^{\text{RI}}_N + R^{\text{3loop}}_{12}\langle J^{5\rho}\rangle^{\text{RI}}_N) \approx \bar{Z}^{\text{lat}}_{11}\langle K^\mu\rangle^{\text{B.}}_N$$

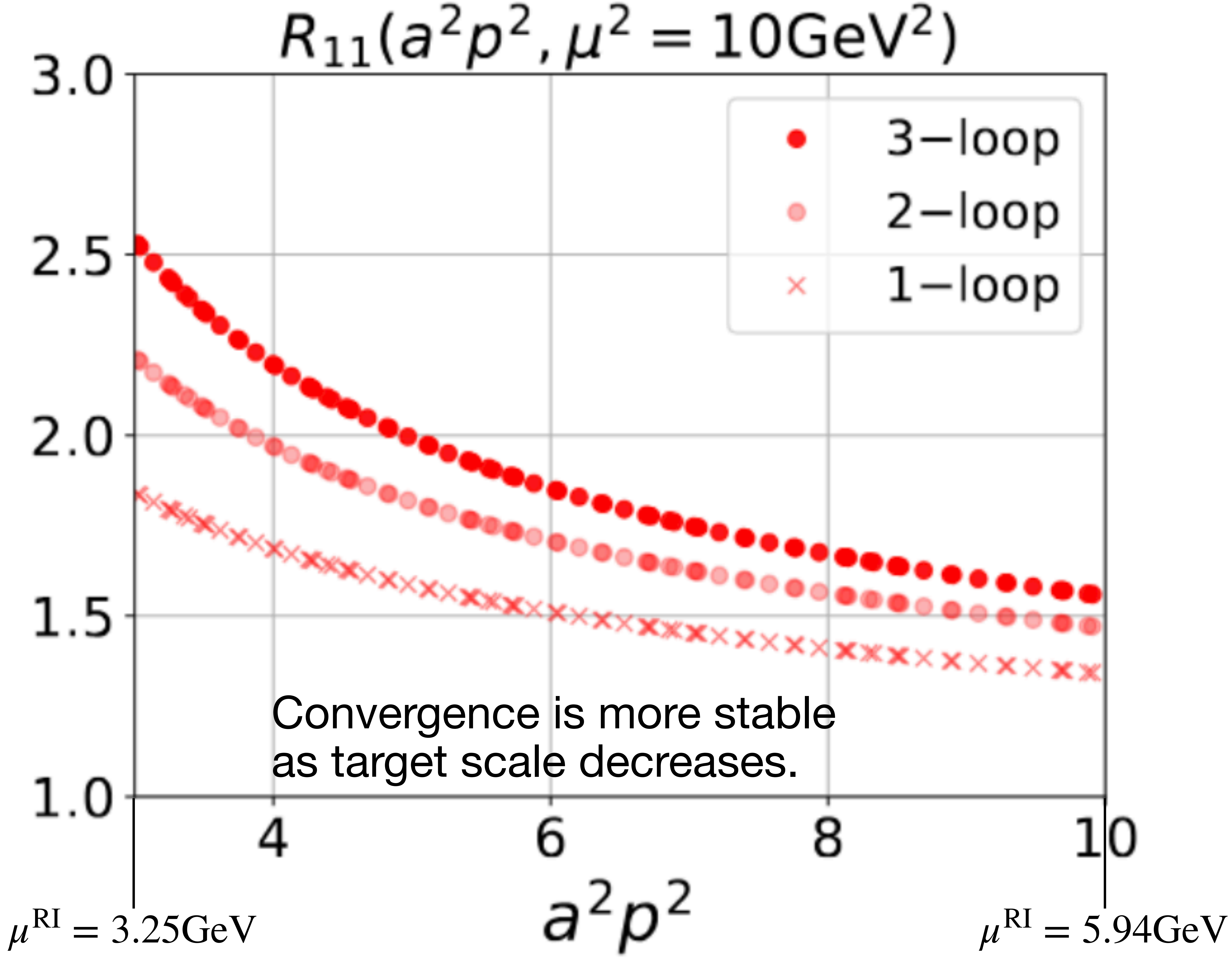
$$\langle J^{5\rho}\rangle^{\overline{\text{MS}}}_N \approx \bar{Z}^{\text{lat}}_{22}\langle J^{5\rho}\rangle^{\text{B.}}_N$$

where $\underbrace{\bar{Z}^{\text{lat}}_{11}}_{3} \equiv \underbrace{R^{\text{3loop}}_{11}}_{1}\underbrace{Z^{\text{RI}}_{11}}_{2}$

Convergence of perturbation matching

$\bar{Z}_{11}^{\mathrm{lat}} \equiv R_{11}^{\mathrm{3loop}} Z_{11}^{\mathrm{RI}}$

$$R_{ij}(\mu^{\mathrm{RI}}, \mu = e^{-\frac{5}{6}}\mu^{\mathrm{RI}}) \to R_{ij}(\mu^{\mathrm{RI}}, \mu = \sqrt{10}\mathrm{GeV})$$



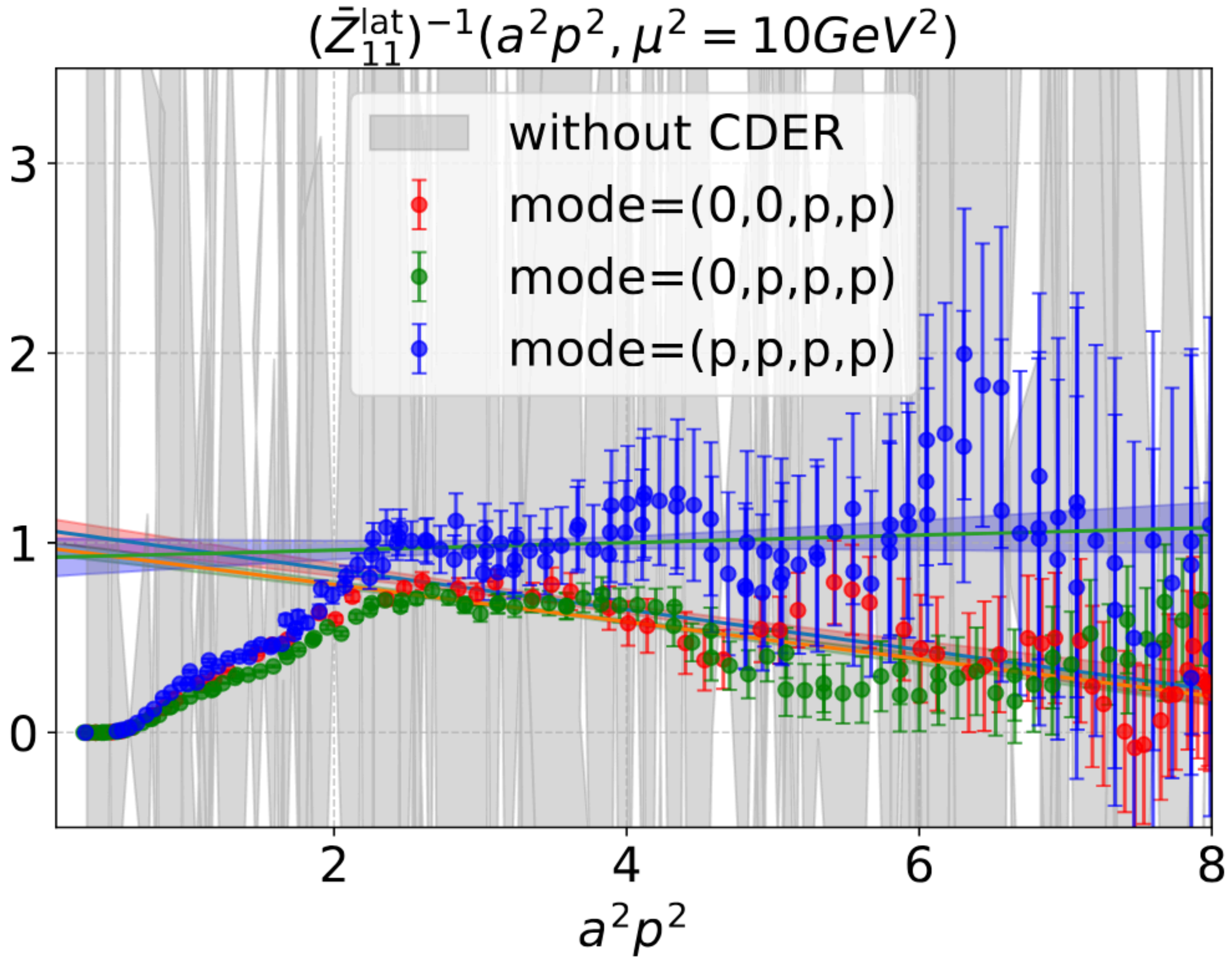
Cluster Decomposition Error reduction

$\bar{Z}_{11}^{\text{lat}} \equiv R_{11}^{\text{3loop}} Z_{11}^{\text{RI}}$

$$Z_{11}^{\text{RI}}(\mu^{\text{RI}}) = \frac{Z_g(\mu^{\text{RI}})}{\text{Tr}[S_g^{-1}(p_1)G_K(p_1,p_2)S_g^{-1}(p_2)(K^{\text{tree,g}})^{-1}]}$$

Z_{11}^{RI} contributed by gluon Green functions is difficult to see signal, so we use **CDER** scheme to enhance SNR.

Y.-B. Yang et al., 10.1103/PhysRevD.98.074506



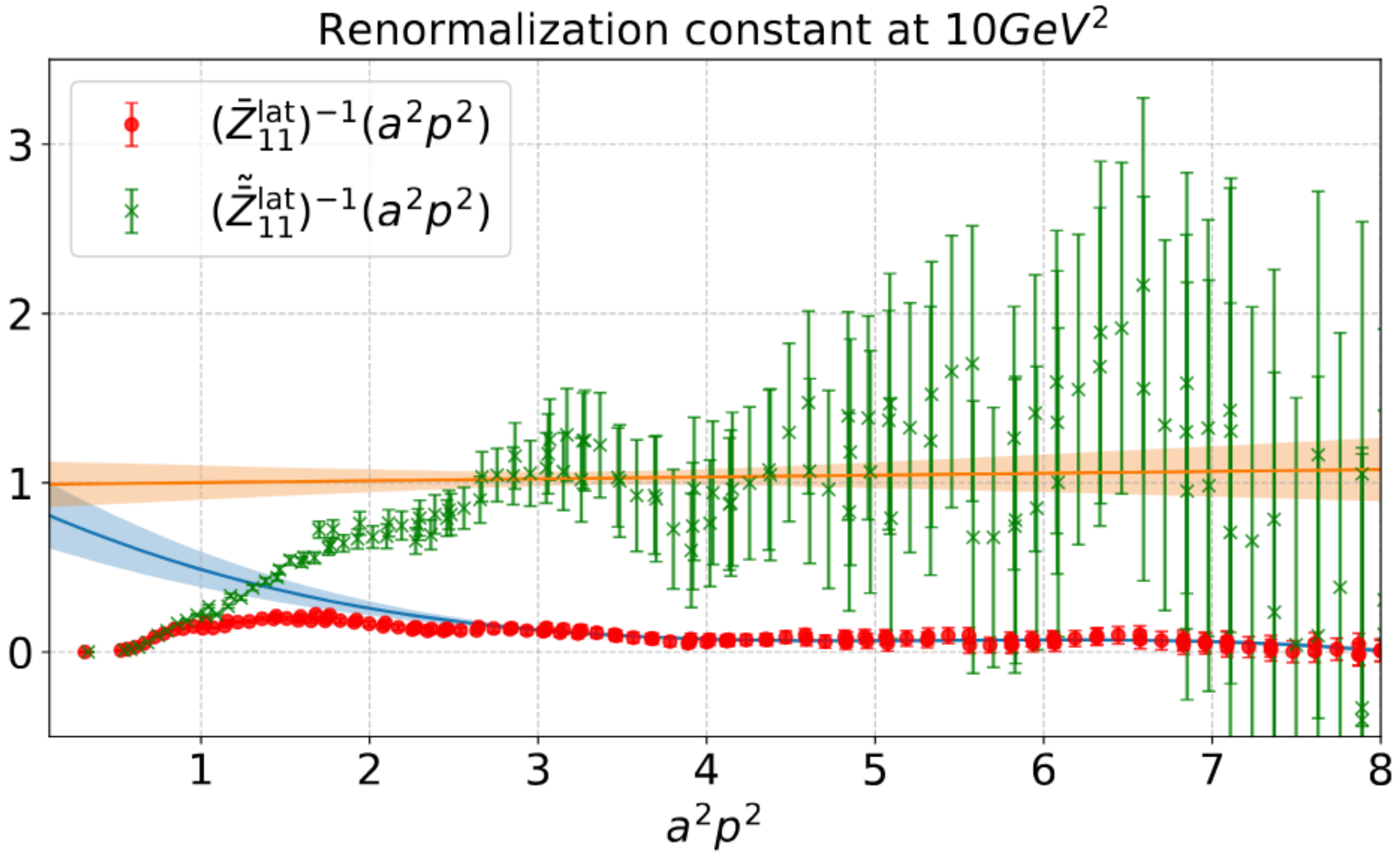
a^2p^2 poly-subtraction of $\bar{Z}_{11}^{\text{lat}}$

$\bar{Z}_{11}^{\text{lat}} \equiv R_{11}^{\text{3loop}} Z_{11}^{\text{RI}}$

$$\tilde{\bar{Z}}_{11}^{\text{lat.}}(\mu^2, a^2p^2) = \bar{Z}_{11}^{\text{lat.}}(\mu^2, a^2p^2) f(a^2p^2 \to 0)/f(a^2p^2)$$

where $f(a^2p^2) = \dfrac{\langle \text{Tr}[A_\mu^{n-\text{HYP}}(p) A_\mu^{n-\text{HYP}}(-p)] \rangle}{\langle \text{Tr}[A_\mu(p) A_\mu(-p)] \rangle}$

Y.-B. Yang, et al., Phys. Rev. Lett. 121, 212001

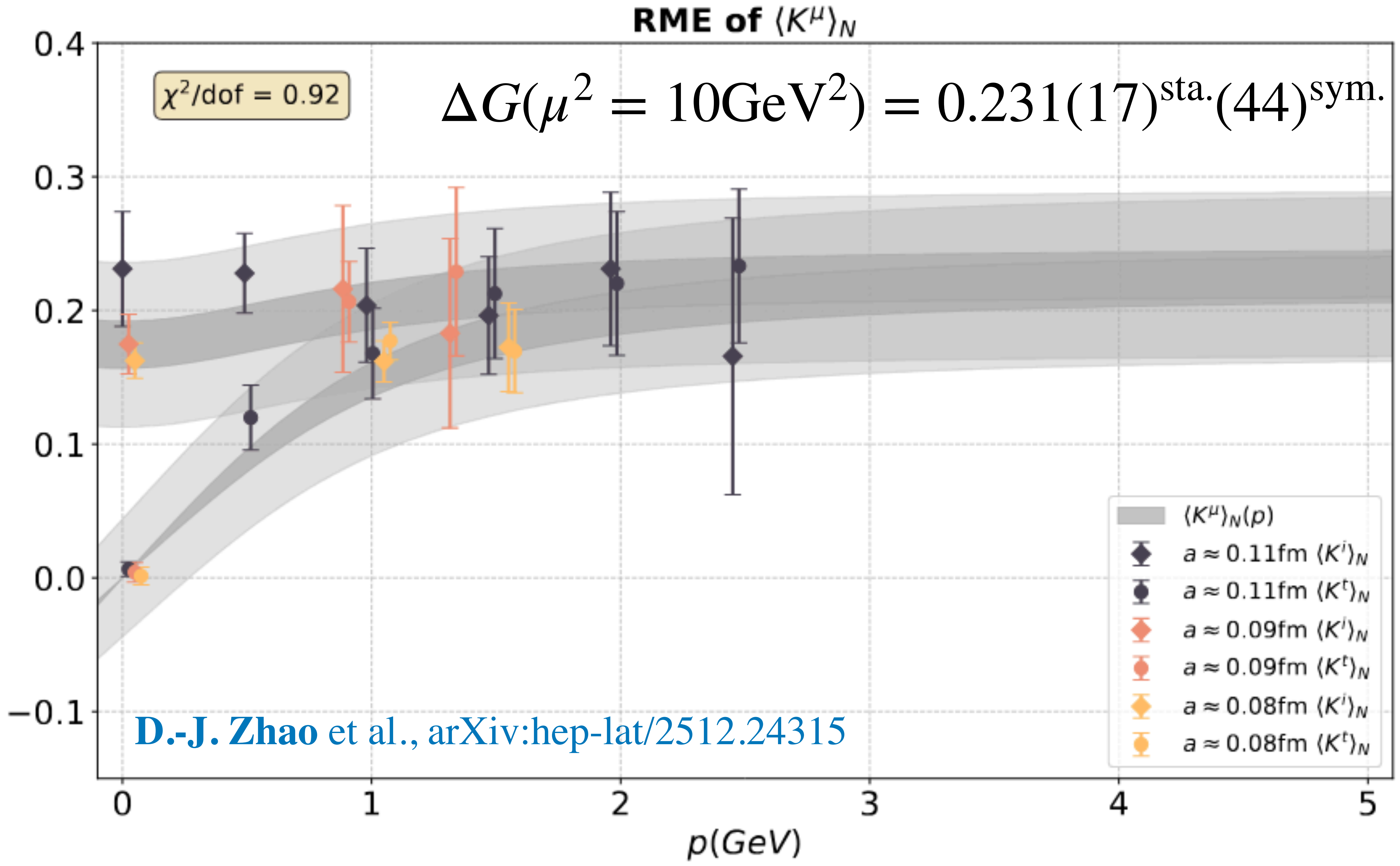


Continuous limit extrapolation

$$\langle K^\mu \rangle_N^{\overline{\mathrm{MS}}} = \bar{Z}_{11}^{\mathrm{lat.}} \langle K^\mu \rangle_N^{\mathrm{B.}} = \frac{S^\mu}{E}\left(\Delta G + \frac{c_{\mathrm{h.t.}}}{E^2}\right) + c_a a^2$$

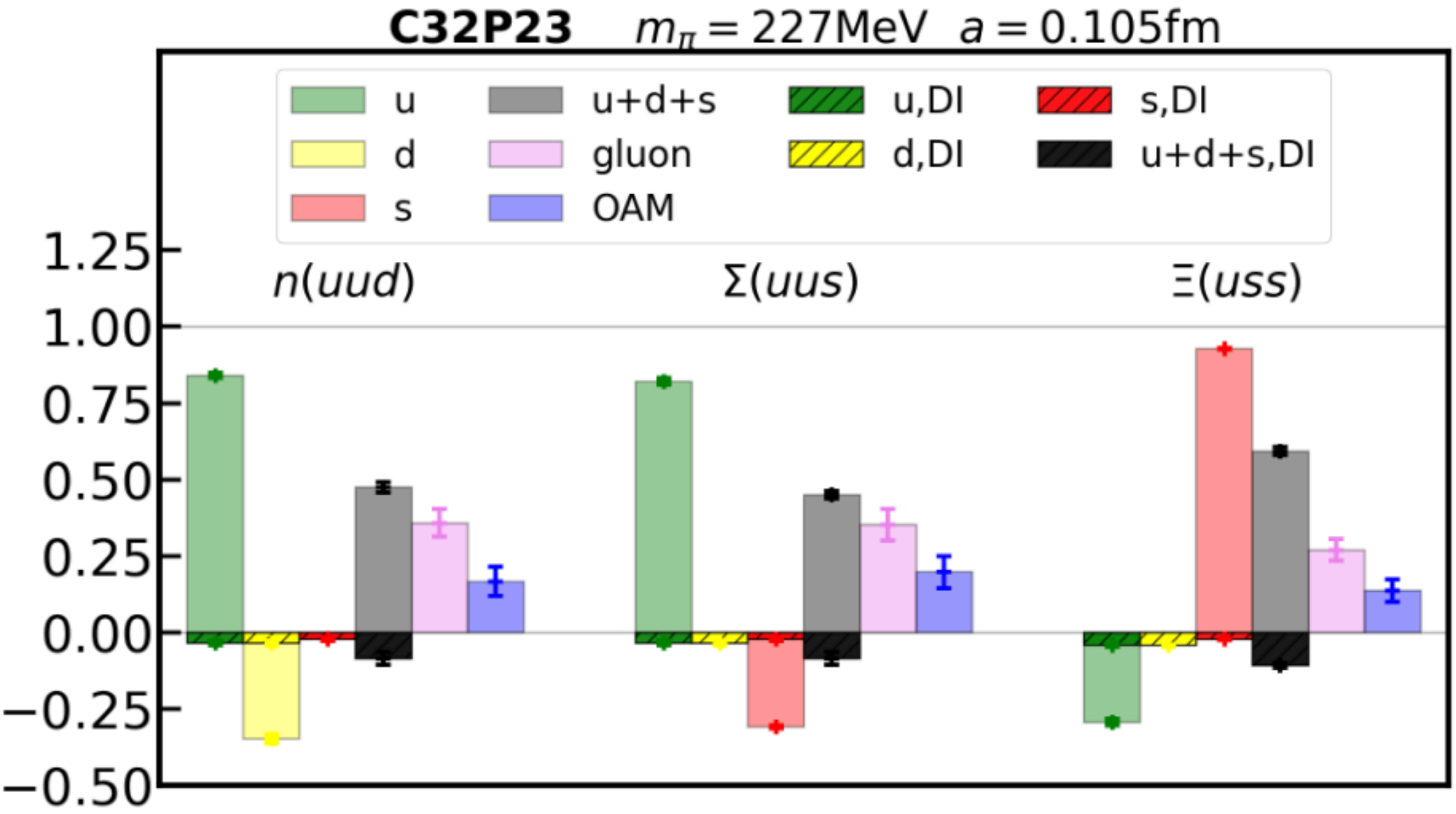
Systematic error sources:

1. Zero-momentum external state data points
2. Finite lattice spacing
3. Choice of the UV window in the RI/MOM scale
4. Perturbative inputs



If we really take '**spin structure**' as the goal instead of '**helicity**' as the endpoint, then the next step is not another helicity observable, but **GPD**.

Preliminary exploration of octet states



If the quark masses which make up hadrons increases, then gluon helicity will be squeezed.

Summary and Outlook

Summary

1. **Distillation + Momentum smear** (for B.M.E.) and **CDER** (for Renorm.) scheme.

2. After non-perturbative renormalization, $\Delta G(\mu^2 = 10\text{GeV}^2) = 0.231(17)^{\text{sta.}}(44)^{\text{sym.}}$, which accounts for $46(9)\ \%$ of proton spin.

Outlook

1. Lattice methodology for the OAM calculations is required to give accurate results for proton spin components.

2. Combine $\frac{1}{2}\Delta\Sigma$ using Blending to provide octet spin decomposition.

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