

Meson molecules in magnetic fields

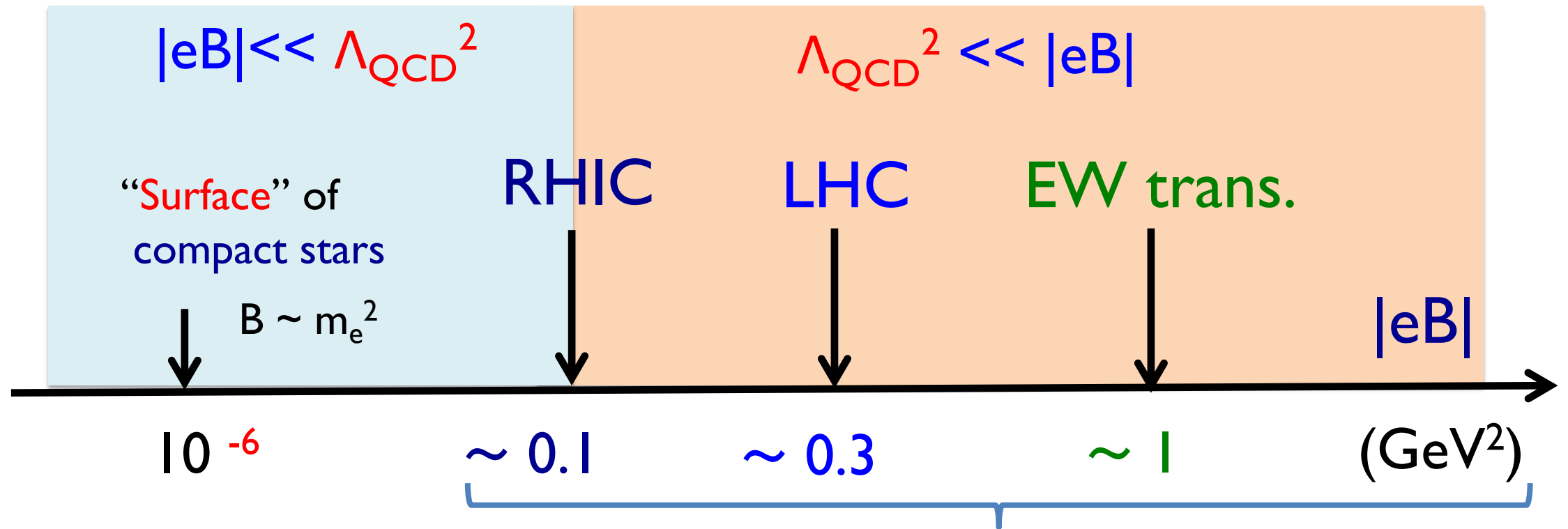
Toru Kojo

(KEK/SOKENDAI)

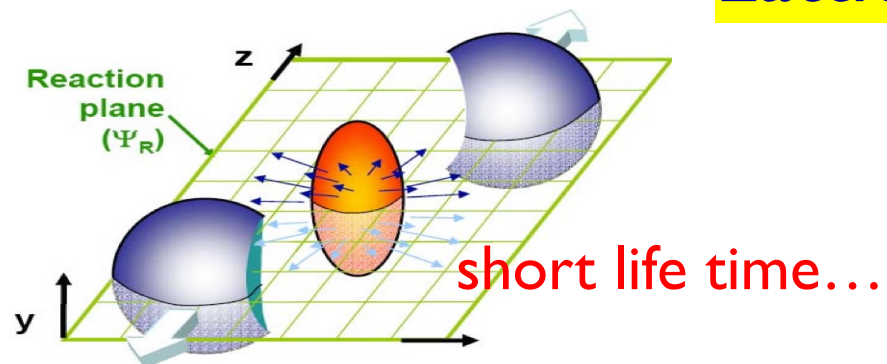
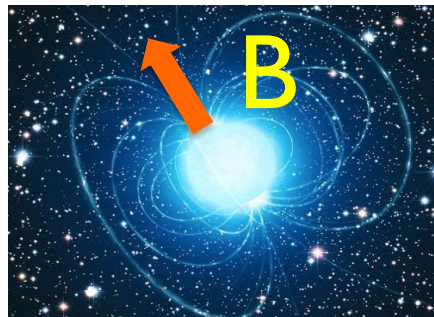
- Refs)
- TK-Su, "The quark mass gap in a magnetic field," PLB (2013)
 - Hattori-TK-Su, "Mesons in strong magnetic fields: (I) general analyses," NPA (2016)
 - TK, "Neutral and charged mesons in magnetic fields," EPA (2021)
 - TK-Itatani, "Delineating neutral and charged mesons in magnetic fields," PRD (2026)
 - TK, "**Meson molecules in magnetic fields**," under review

Strength of B-fields

$$\Lambda_{\text{QCD}} \sim 0.2\text{-}0.3 \text{ GeV}$$



Lattice data are available (“**laboratory**”)

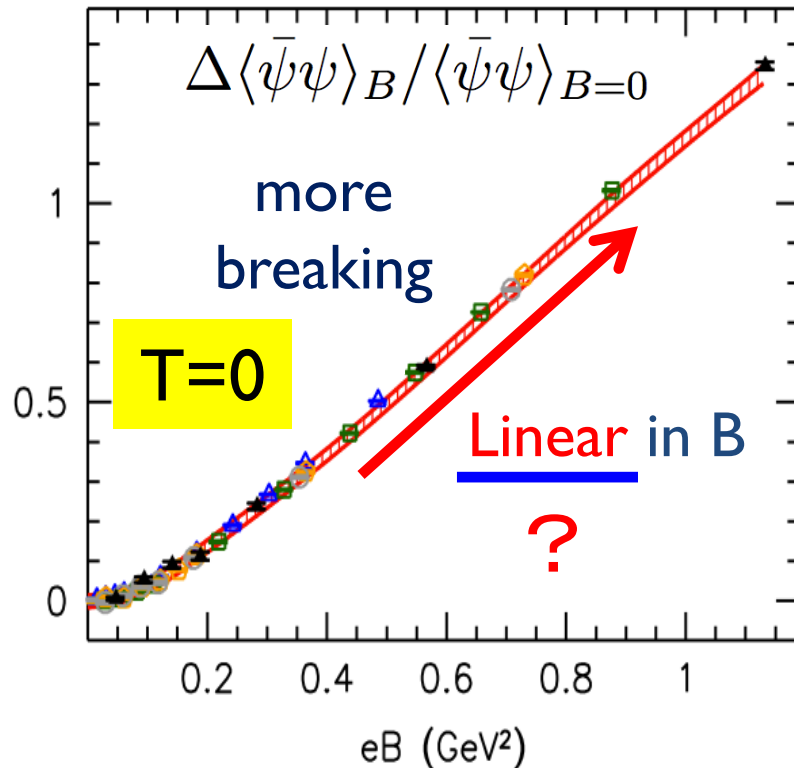


Target of this talk

Unexpected behaviors on the lattice

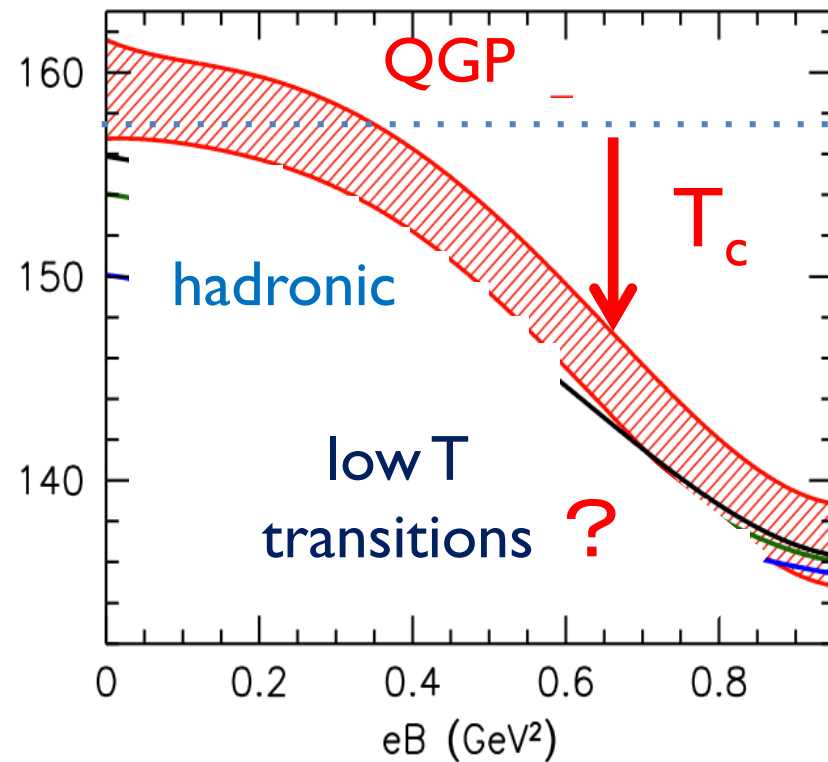
chiral symmetry breaking

(Bali+ '11)



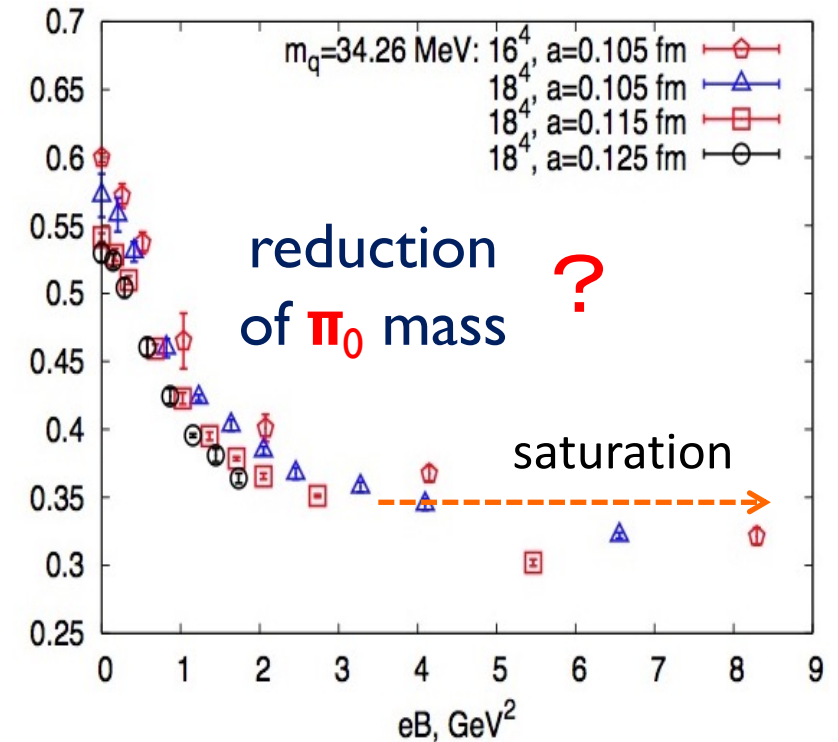
chiral & deconf. T

(Bali+ '11)



neutral hadron mass

(ITEP group '15)

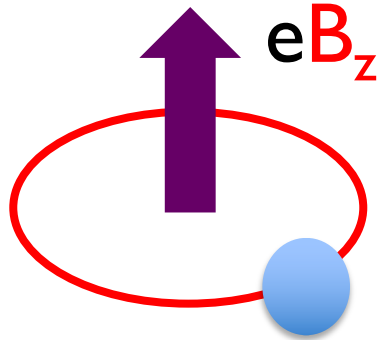


Vacuum & hadronic structures change

Point-like spin 0 particles in magnetic fields

(orbital levels, p_z fixed)

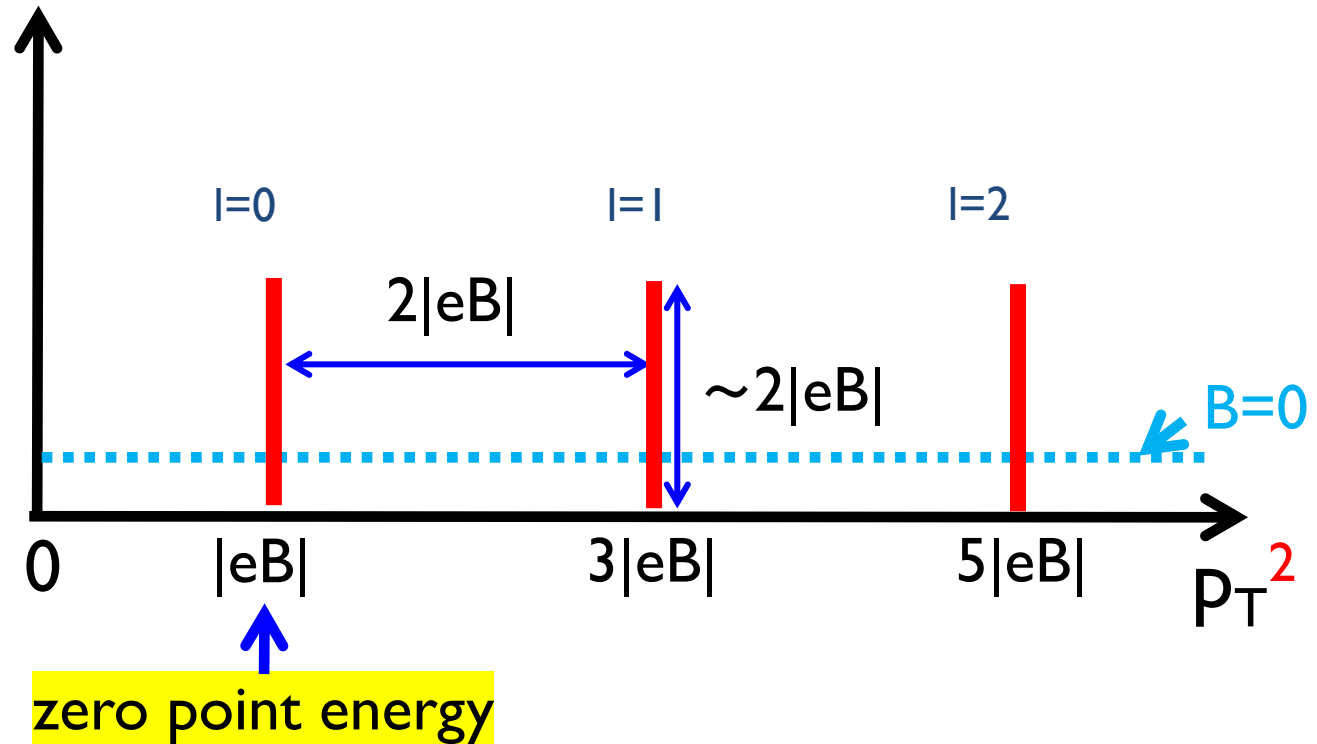
$B \neq 0$



periodic

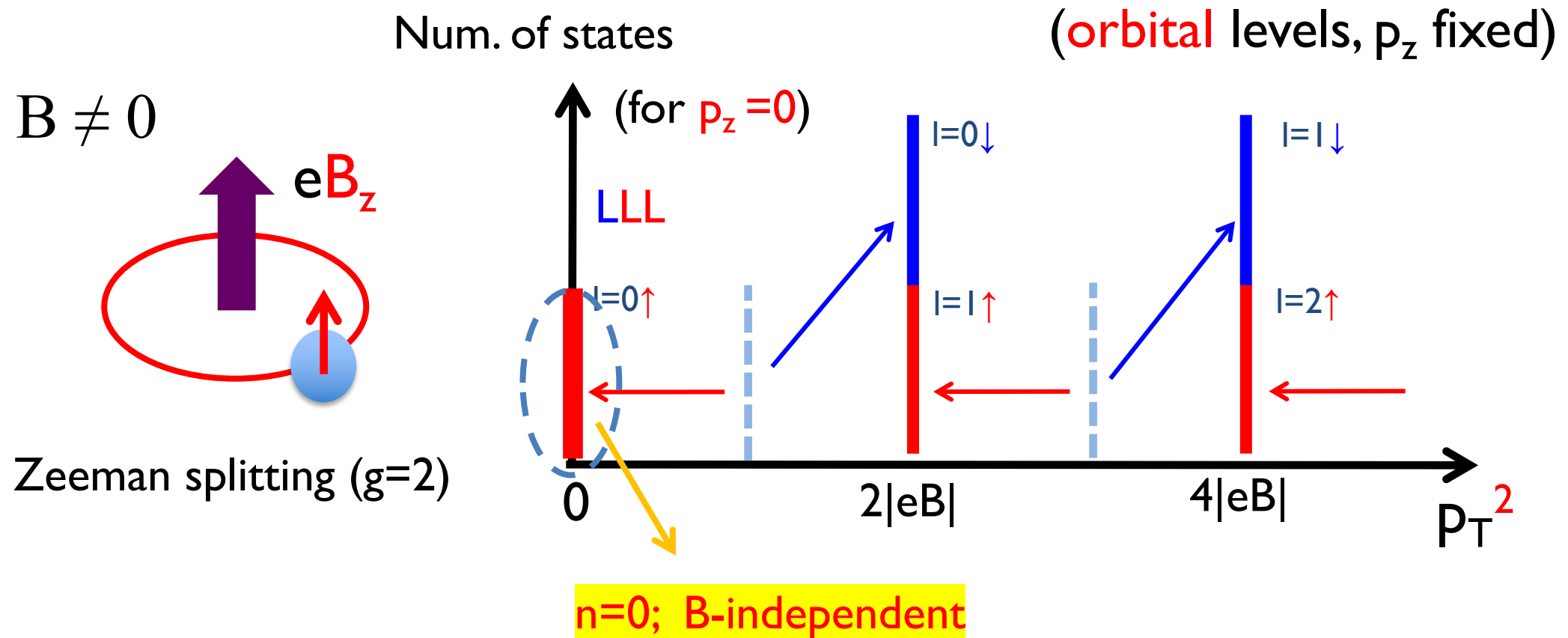
→ quantization

Num. of states



$$E_{n_{LL}}^2 = m^2 + p_z^2 + 2n|eB| + |eB|$$

Point-like spin 1/2 particles in magnetic fields

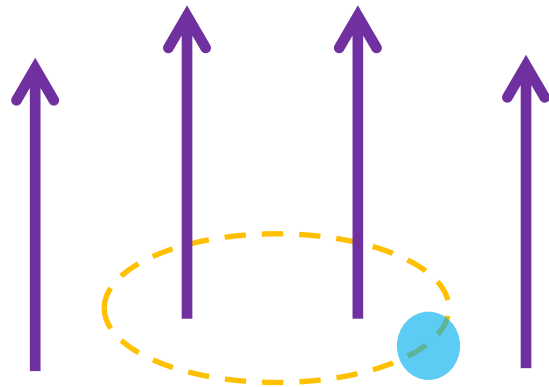


$$E_{nLL}^2 = m^2 + p_z^2 + 2n|eB| + (l - \sigma_z)|eB|$$

Composite particles in magnetic fields

e.g.) spin 0 mesons

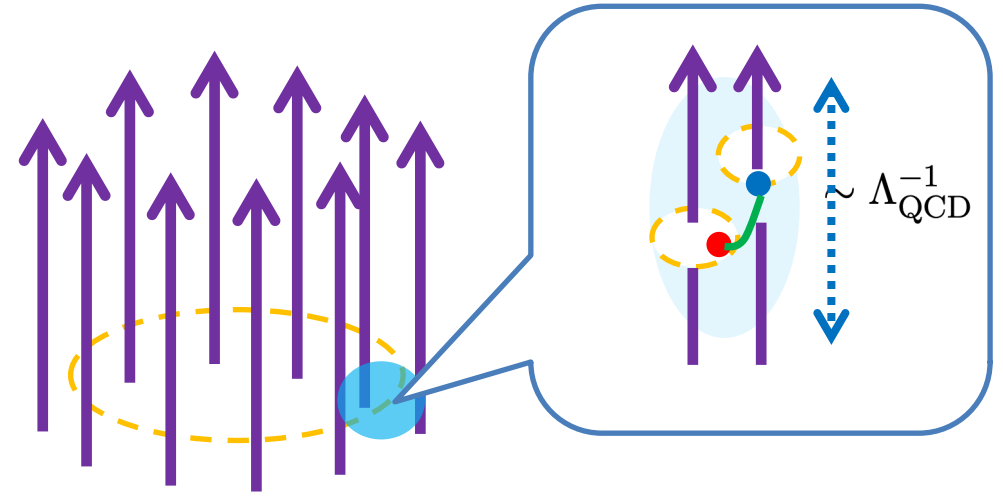
weak B



$$|eB| \gg \Lambda_{\text{QCD}}^2$$



strong B



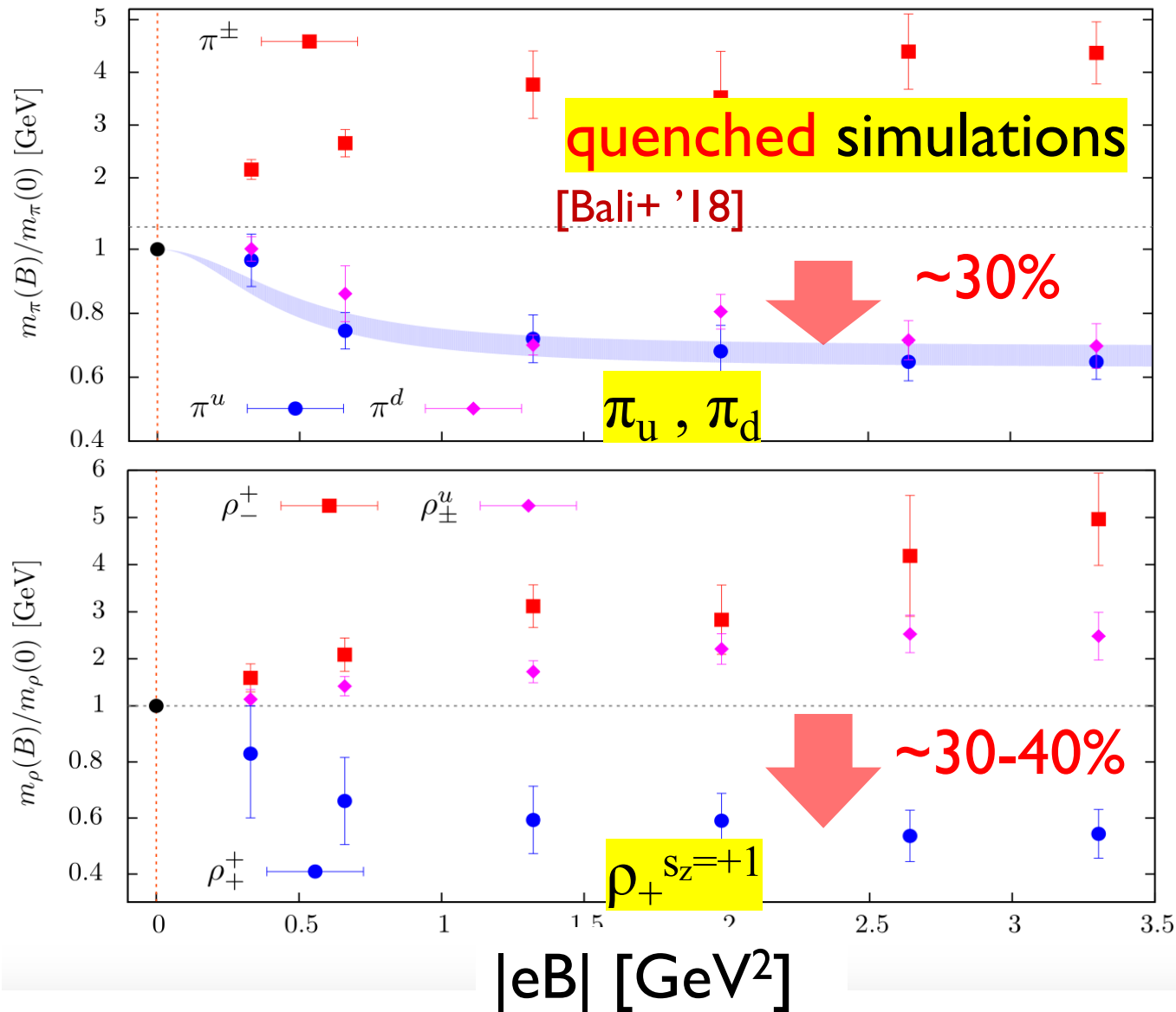
B mainly probes the collective c.m. motion

$$E^2 \approx p_z^2 + m_{B=0}^2 + (2n_{\text{cm}} + 1)|eB|$$

In addition, B directly couples to internal quarks

$$E^2 \approx p_z^2 + m_B^2 + (2n_{\text{cm}} + 1)|eB|$$

Mesons saturated by **LLL** quarks



LLL quarks: $u_\uparrow, d_\downarrow, \bar{u}_\uparrow, \bar{d}_\downarrow$

at large B

light

$$\pi_u \sim u_\uparrow \bar{u}_\uparrow$$

$$\rho_+^{s_z=+1} \sim u_\uparrow \bar{d}_\downarrow$$

heavy

$$\pi_+ \sim u_\uparrow \bar{d}_\uparrow$$

$$\rho_-^{s_z=+1} \sim d_\uparrow \bar{u}_\downarrow$$

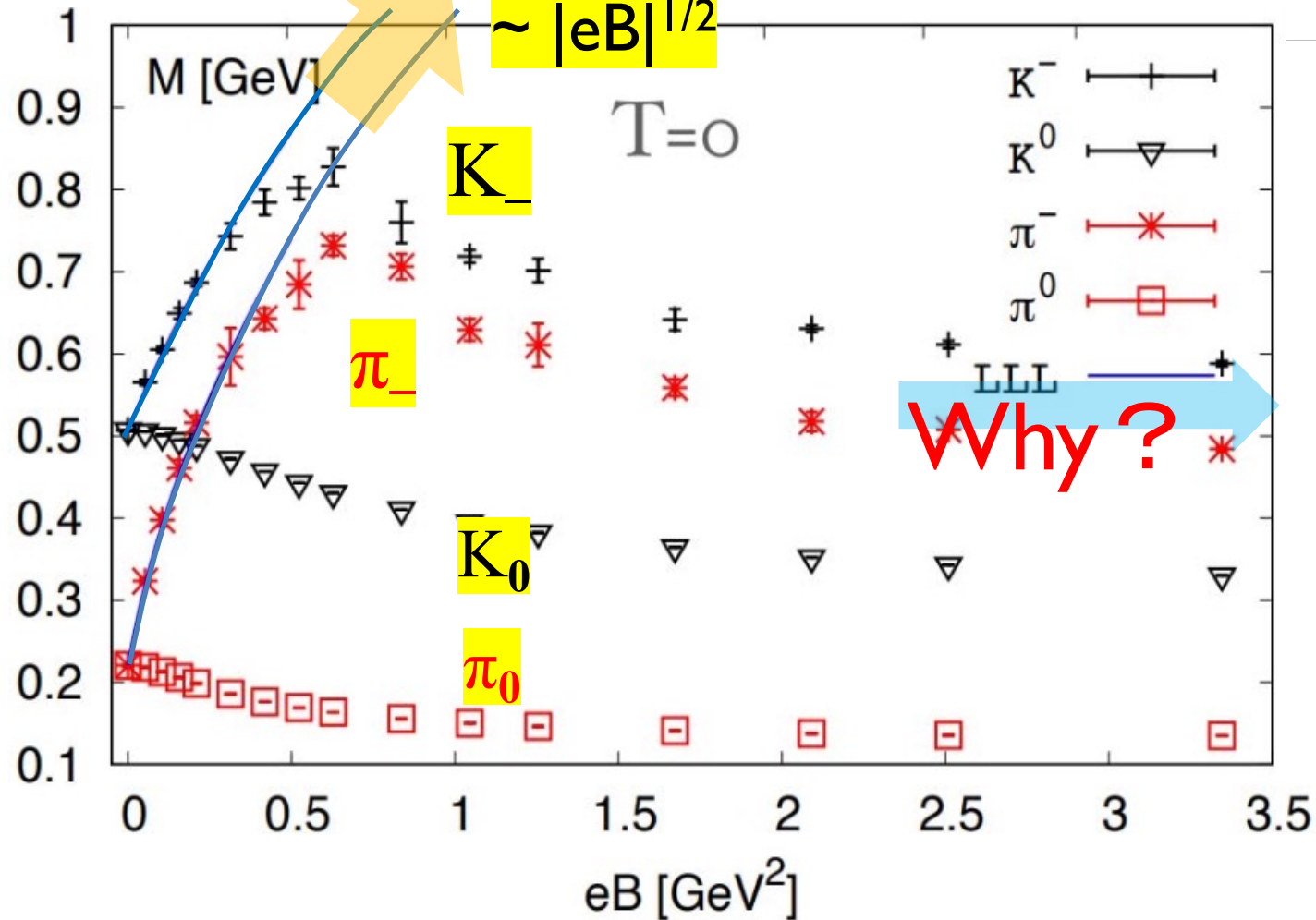
Mesons in **full** simulations

$$m_\pi \approx 220 \text{ MeV}$$

[Ding+ '21]

point-particle or qqbar

$\sim |eB|^{1/2}$



Charged PS mesons

at large B

higher LL

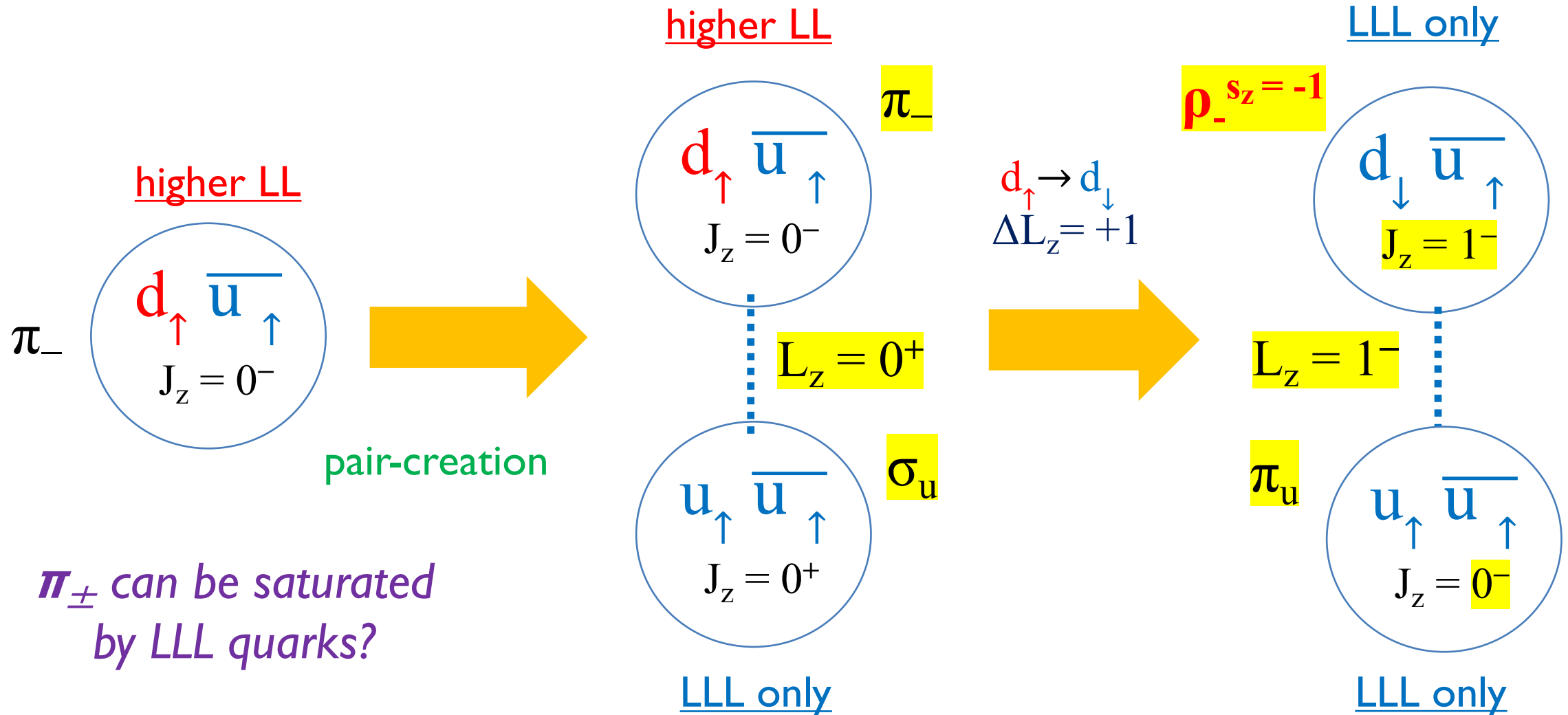
$$\pi_- \sim d_{\uparrow} \bar{u}_{\uparrow} + d_{\downarrow} \bar{u}_{\downarrow}$$

$$K_- \sim s_{\uparrow} \bar{u}_{\uparrow} + s_{\downarrow} \bar{u}_{\downarrow}$$

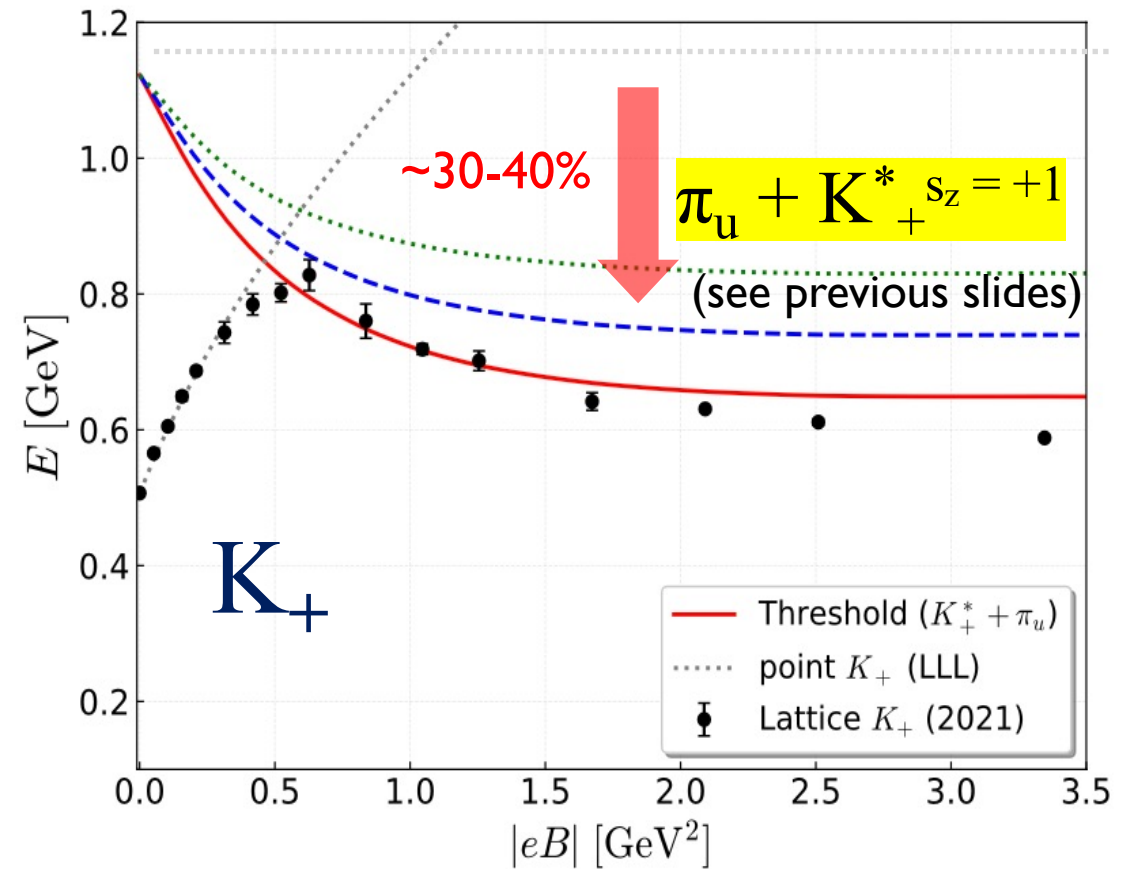
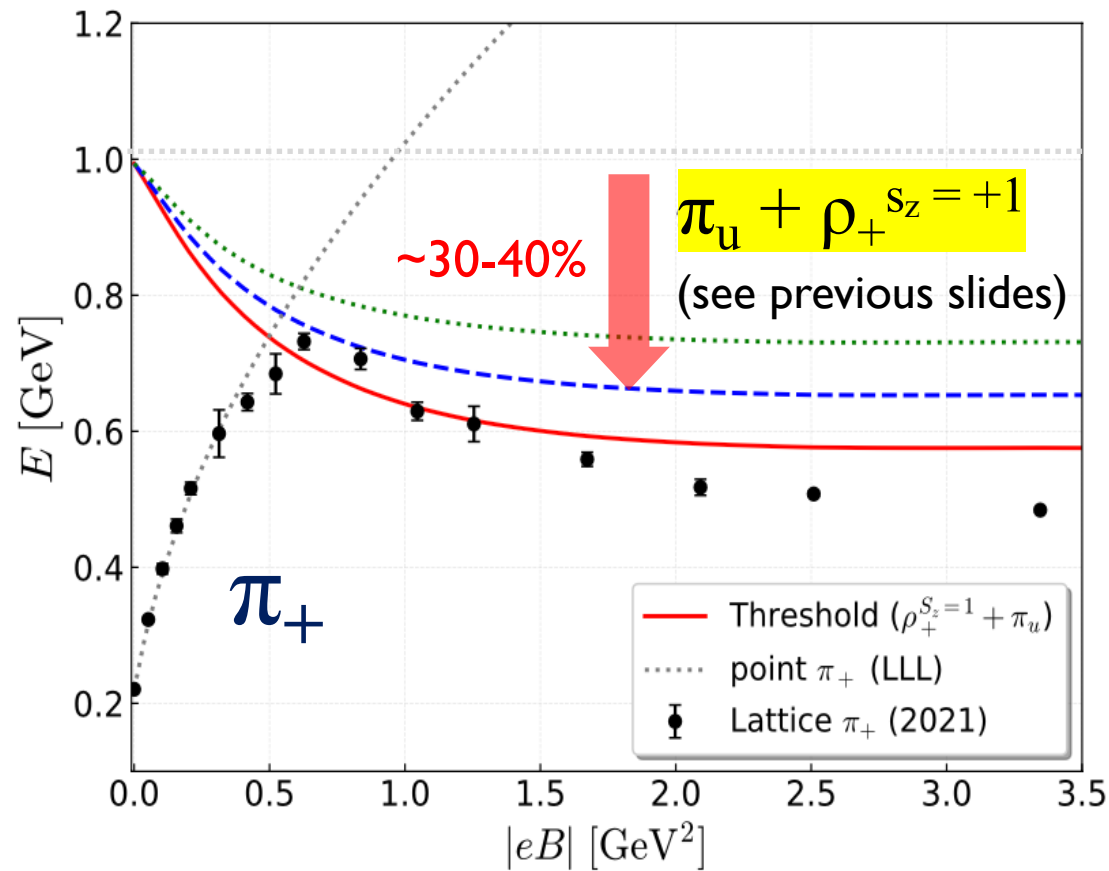
should scale as $E^2 \sim |eB|$ (?)

Why does the energy go down and approach constant?

Quenched $\rightarrow$ full simulations



π_+ , K_+ vs two-meson thresholds



The spectra lie close to the two-meson thresholds (!)

weakly depend on the volume



Meson molecule ?

Near-threshold states

2003-) many **near-threshold** states (bound states and resonances) have been observed in **heavy-quark** systems.

	state	quantum numbers	threshold	$E_{\text{state}} - E_{\text{thre}}$
[Belle (2003)]	$X(3872)$	$I \simeq 0, J^{PC} = 1^{++}$	$D^0 \bar{D}^{*0}$	$-0.05 \pm 0.10 \text{ MeV}$
[LHCb (2021)]	$T_{cc}^+(3875)$	$I = 0, J^P = 1^+$	$D^{*+} D^0$	$-0.36 \pm 0.04 \text{ MeV}$

$$X(3872) \sim c\bar{c}(u\bar{u}, d\bar{d})$$

$$T_{cc}^+(3875) \sim cc\bar{u}\bar{d}$$

Large hadron masses suppress $E_{\text{kin}} \sim p^2/m_h$, favoring shallow binding.

Light-hadron systems? $\rightarrow E_{\text{kin}}$ is larger, disfavoring bound states.

Can magnetic fields enable bound states of light mesons?

Mesons at finite momenta

--- quenching **transverse kin. energy** ---

[Fukushima-Hidaka '12; Hattori+'16; TK-Itatani '26,..]

Warm-up: **I-body** problem in a **NR quark model**

sym. gauge: $A = \frac{\mathbf{B} \times \mathbf{r}}{2}$



$$H_{\text{single}} = m + \frac{\Pi^2}{2m} - \frac{e}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} \quad \Pi = \mathbf{p} - e\mathbf{A}$$

$$H_{\text{single}} = m + \frac{p_z^2}{2m} + \frac{\mathbf{p}_\perp^2}{2m} + \frac{(eB)^2}{8m} r_\perp^2 - \frac{eB}{2m} L_z - \frac{e}{2m} \boldsymbol{\sigma} \cdot \mathbf{B}$$

spectra: $E_{n_r, L_z, \sigma_z}(p_z) = m + \frac{p_z^2}{2m} + \frac{|eB|}{2m} (2n_r + |L_z| + 1) - \frac{eB}{2m} L_z - \frac{eB}{2m} \sigma_z$

$n_r = 0, \sigma_z = +1, L_z \geq 0$ (for $eB > 0$) $E_{\text{g.s.}}(p_z) = m + \frac{p_z^2}{2m}$ **B-indep. (!)**

- remarks:
- 1) the zero-point of $+|eB|/2m$ cancels with the spin Zeeman of $-|eB|/2m$
 - 2) at large B , the NR model fails for excited modes with $\sim B/2m$ ($\gg m$) terms
 - 3) kin. E at $L_z > 0$ cancels with the orbital Zeeman $\rightarrow$ *Landau degeneracy*

Pseudo-momentum/guiding center

B is uniform but A is not $\rightarrow [p, H] \sim B \neq 0$

Conserved quantities?

pseudo-momentum:

$$\mathcal{K} = \Pi + e\mathbf{B} \times \mathbf{r}$$

or

$$\mathbf{X} = -\frac{\hat{z} \times \mathcal{K}}{eB}$$

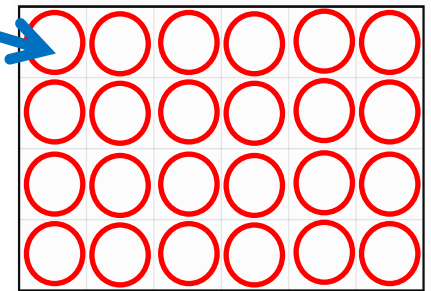
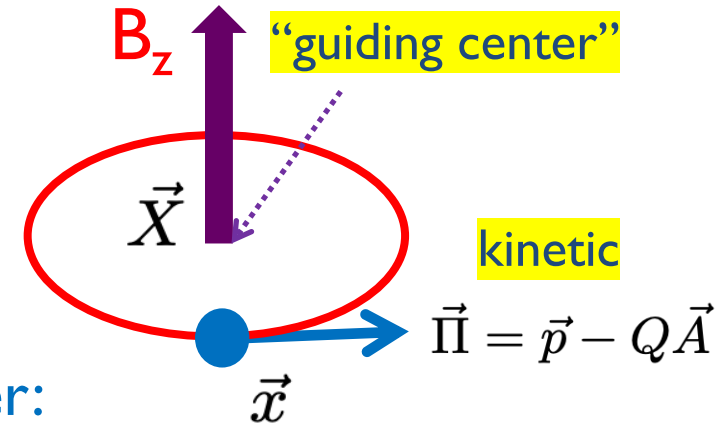
guiding center:

conserved

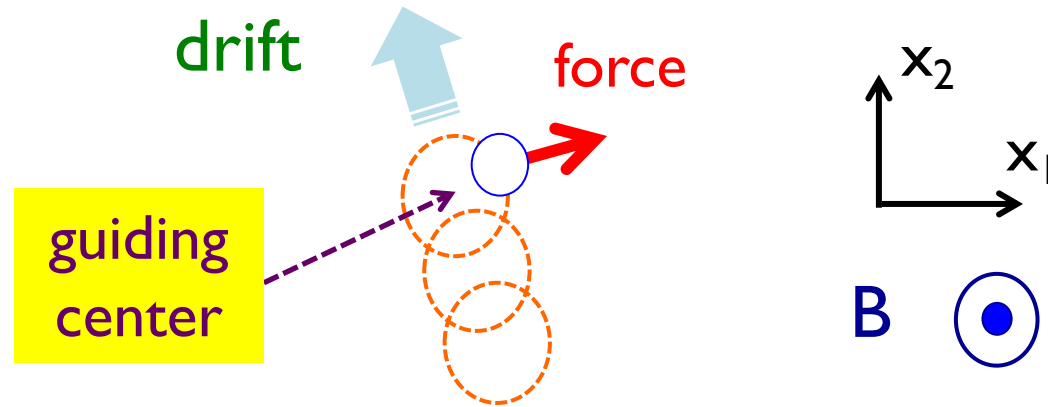
$$[\Pi_\alpha, \mathcal{K}_\beta] = 0 = [\Pi_\alpha, X_\beta] \quad \Rightarrow \quad [\mathcal{K}, H_{\text{single}}] = 0 = [\mathbf{X}, H_{\text{single}}]$$

$$[\mathcal{K}_x, \mathcal{K}_y] = -ieB \quad \text{or} \quad [X, Y] = -\frac{i}{eB}$$

one state per area
 $\Delta X \Delta Y \sim 1/B$
 $\rightarrow$ density of states $\sim B$
 (degeneracy in energy)

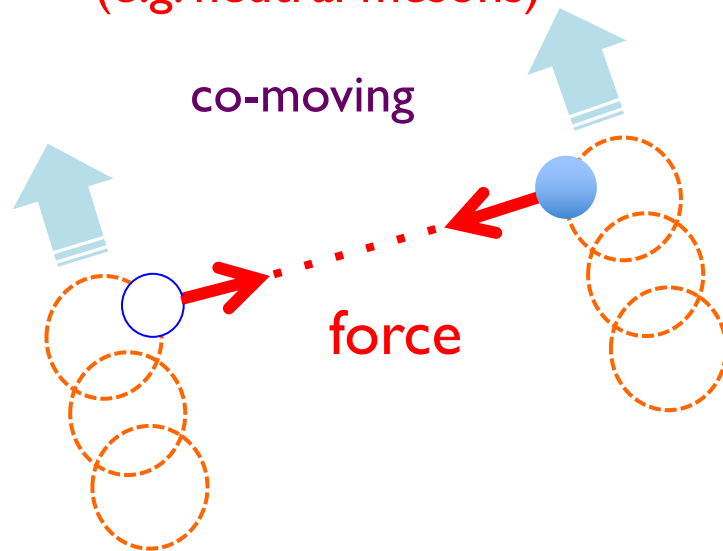


2-body problems: Hall drift (classical)



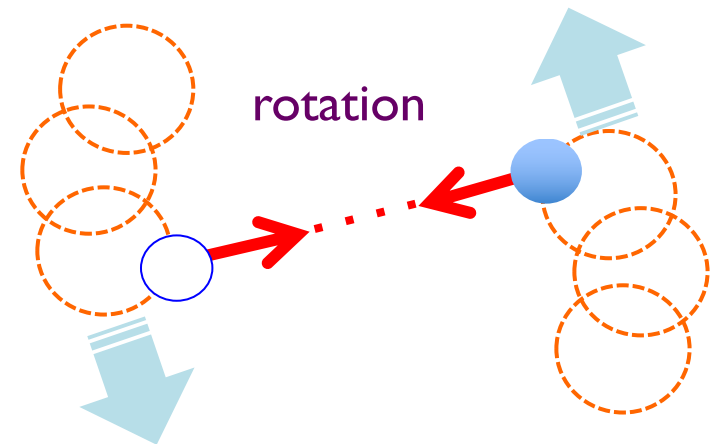
$$Q = -Q'$$

(e.g. neutral mesons)



$$Q = Q'$$

(e.g. di-electrons)



2-body problem for neutral mesons

$$H = \sum_{j=1,2} \left[m_j + \frac{\mathbf{\Pi}_j^2}{2m_j} - \boldsymbol{\mu}_j \cdot \mathbf{B} \right] + V(r_{12}) \quad \text{for } m_1 = m_2$$

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

total pseudo-momentum is conserved: $[\mathcal{K}_R, V(\mathbf{r})] = 0$

special property in neutral case:

$$\mathcal{K}_R = \sum_i \mathcal{K}_i = \mathbf{P} + \frac{e_1 + e_2}{2} \mathbf{B} \times \mathbf{R} + \frac{e_1 - e_2}{4} \mathbf{B} \times \mathbf{r} \quad \text{q} = -e_1 = e_2 \quad \text{no } R$$

$$\longrightarrow \mathbf{P} - \frac{\mathbf{B}_q}{2} \times \mathbf{r}$$

$$[\mathcal{K}_R^x, \mathcal{K}_R^y] = -i(e_1 + e_2)B \longrightarrow 0$$

both $\mathcal{K}_R^x, \mathcal{K}_R^y$ are
good quantum numbers

eigenvalue equation: $\mathcal{K}_R \Psi_K(\mathbf{R}, \mathbf{r}) = \bar{\mathbf{K}} \Psi_K(\mathbf{R}, \mathbf{r})$

$$\Psi_K(\mathbf{R}, \mathbf{r}) = \exp \left[i \left(\mathbf{K} + \frac{\mathbf{B}_q}{2} \times \mathbf{r} \right) \cdot \mathbf{R} \right] \phi_K(\mathbf{r}) \equiv e^{i\Theta(\mathbf{R}, \mathbf{r})} \phi_K(\mathbf{r})$$

2-body problem for neutral mesons

unitary trans.

P & r entangled

p & R entangled

$$e^{-i\Theta} \frac{\mathbf{\Pi}_1^2 + \mathbf{\Pi}_2^2}{2m} e^{i\Theta} = e^{-i\Theta} \left[\frac{1}{4m} \left(\mathbf{P} + \mathbf{B}_q \times \frac{\mathbf{r}}{2} \right)^2 + \frac{1}{m} \left(\mathbf{p} + \frac{\mathbf{B}_q}{2} \times \mathbf{R} \right)^2 \right] e^{i\Theta}$$

$$= \frac{1}{4m} (\underline{\mathbf{K}} + \mathbf{B}_q \times \mathbf{r})^2 + \frac{\mathbf{p}^2}{m} \quad \text{disentangled}$$

Hamiltonian at a given K:

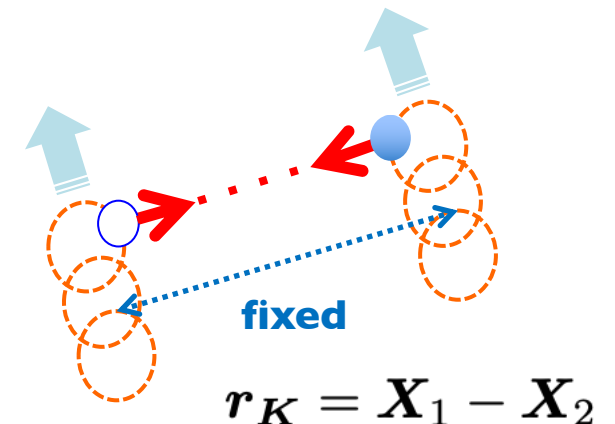
$$r_K \sim K_{\perp}/B$$

$$\tilde{H}_K = \frac{K_z^2}{2M} + \frac{p_z^2}{m} + \frac{\mathbf{p}_{\perp}^2}{m} + \frac{B_q^2}{4m} (\mathbf{r}_{\perp} - \underline{\mathbf{r}_K})^2 + H_{Zee} + V(r)$$

$K_{\perp}$ -dep.

$K_{\perp}$ -dep. arises only through shift of the center

larger $K_{\perp} \Rightarrow$ larger displacement



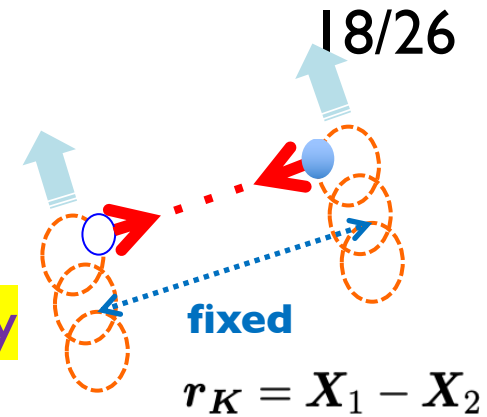
Potentials → the $K_{\perp}$ -dep. of spectra

conf. pot. (unpert.)

pert.

$$V = a_{\text{ho}}(z^2 + \mathbf{r}_{\perp}^2) + V_{\text{short}}$$

→ breaks Landau degeneracy



G.S. spectrum:

$\sim a_{\text{ho}} r_K^2$

zero-point E

Zeeman

color- E&M

$$E_{g.s.}(\mathbf{K}) \simeq \frac{K_z^2}{2M} + \underbrace{a_{\text{ho}} \frac{K_{\perp}^2}{B_q^2 + 2Ma_{\text{ho}}}}_{\beta_q^2 \equiv B_q^2 + 4ma_{\text{ho}}} + \sqrt{\frac{a_{\text{ho}}}{m}} + \frac{\beta_q}{m} - \frac{B_q}{m} \sigma_z + \underbrace{\langle V_{\text{short}} \rangle_{K_{\perp}}}_{\sim \alpha_s \frac{\Lambda_{\text{QCD}}}{|B_q|} K_{\perp}^2}$$

remarks: 1) $V=0$: flat Landau levels (degeneracy) → finite V : $K_{\perp}$ -dependent bands

2) large B → the $K_{\perp}$ -dep. terms are suppressed by some powers of B

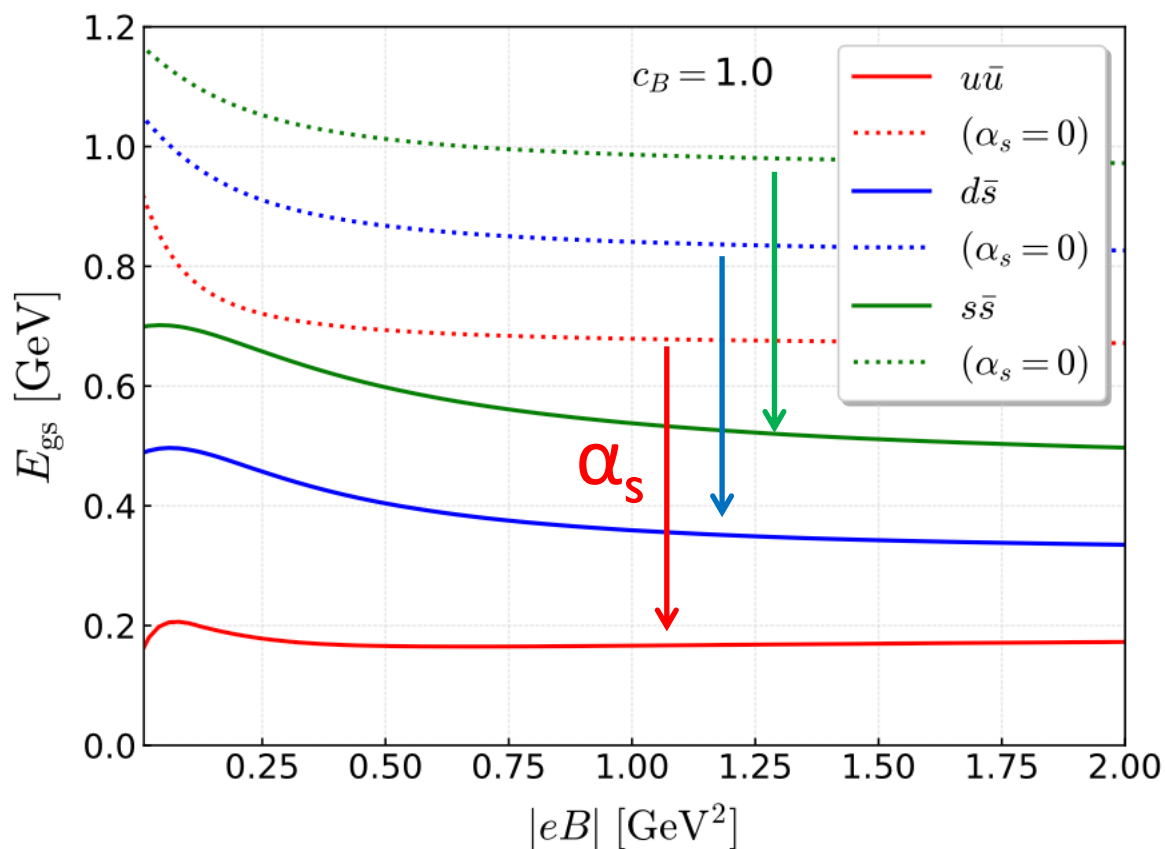
3) the zero-point E of $\sim |B_q|/m$ cancels with the Zeeman of $-|B_q|/m$

Spectra: model results

19/26

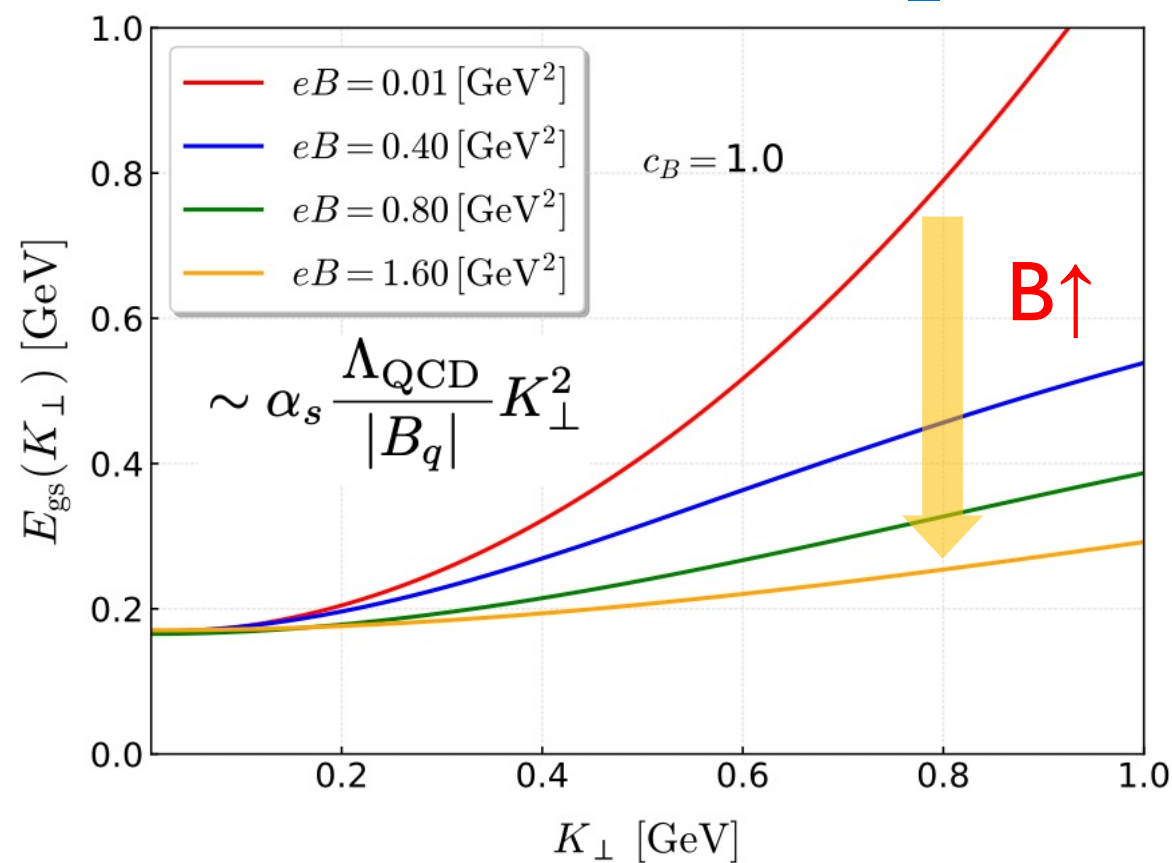
[TK-Itatani '26]

spectra at $K_{\perp} = 0$



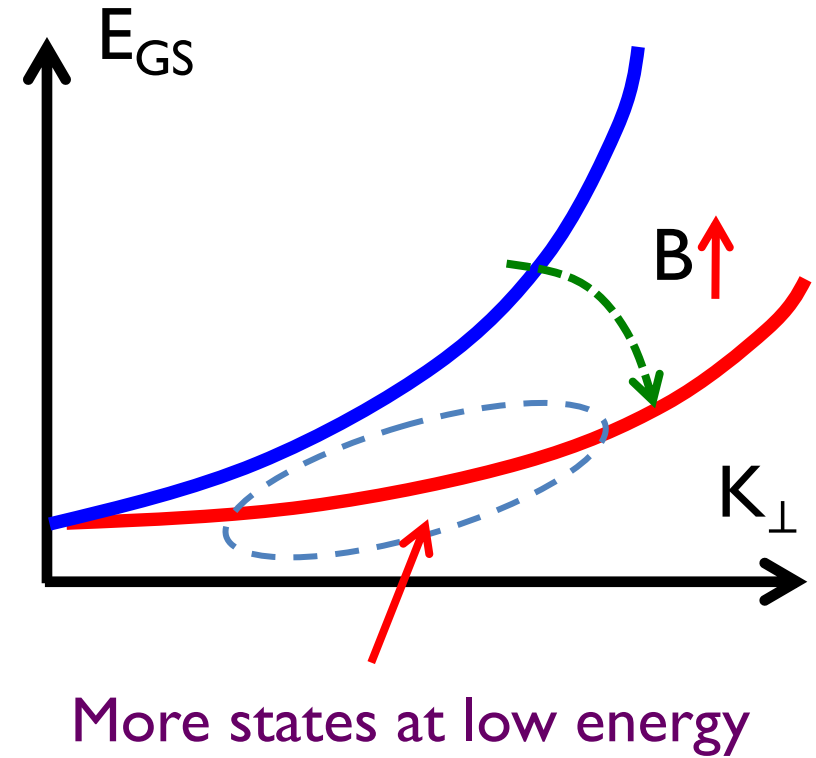
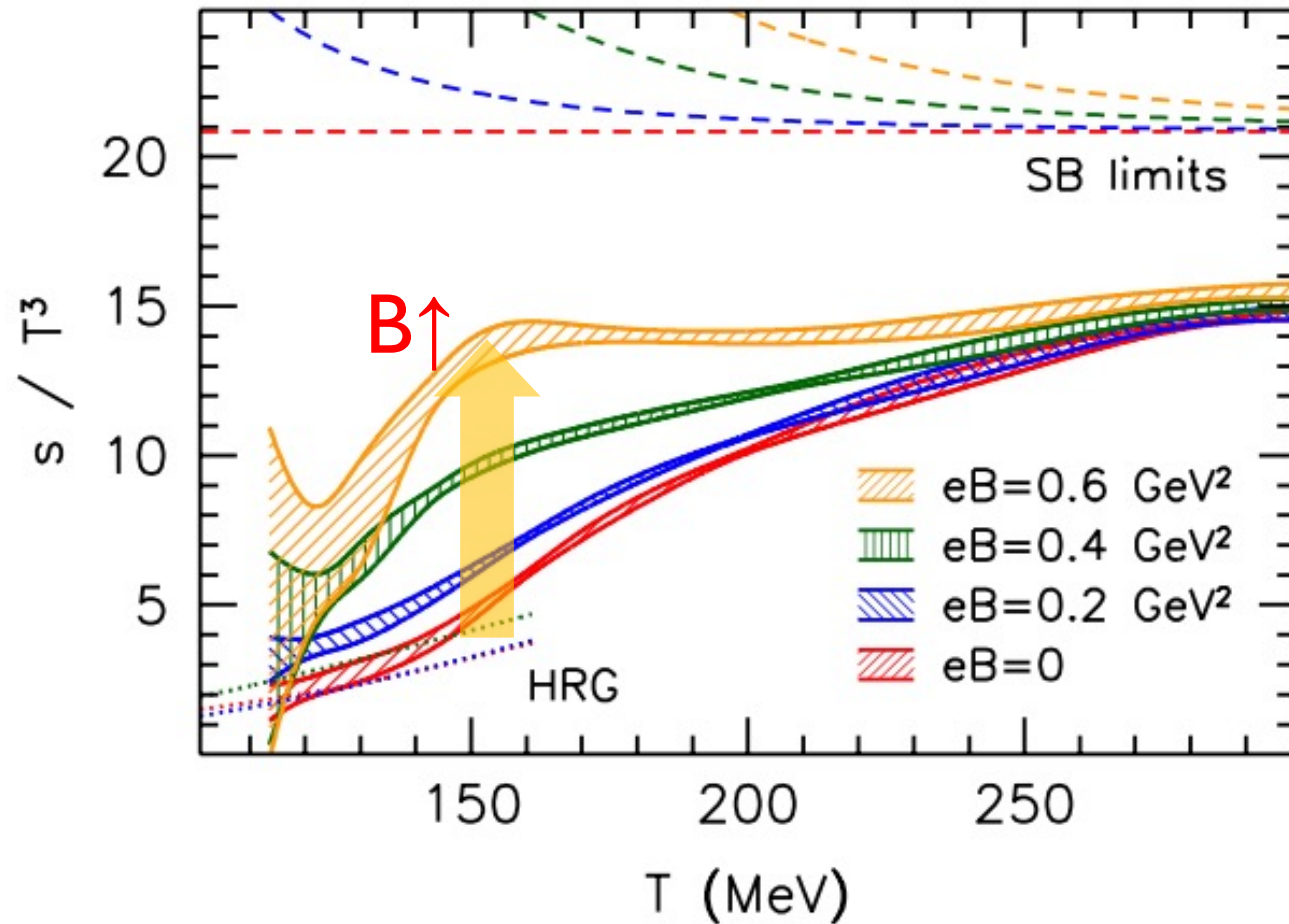
20-30% reduction at $B \sim 1 \text{ GeV}^2$

spectra at finite $K_{\perp}$



quenching of trans. kin. terms

Hints of $K_{\perp}$ -quenching?

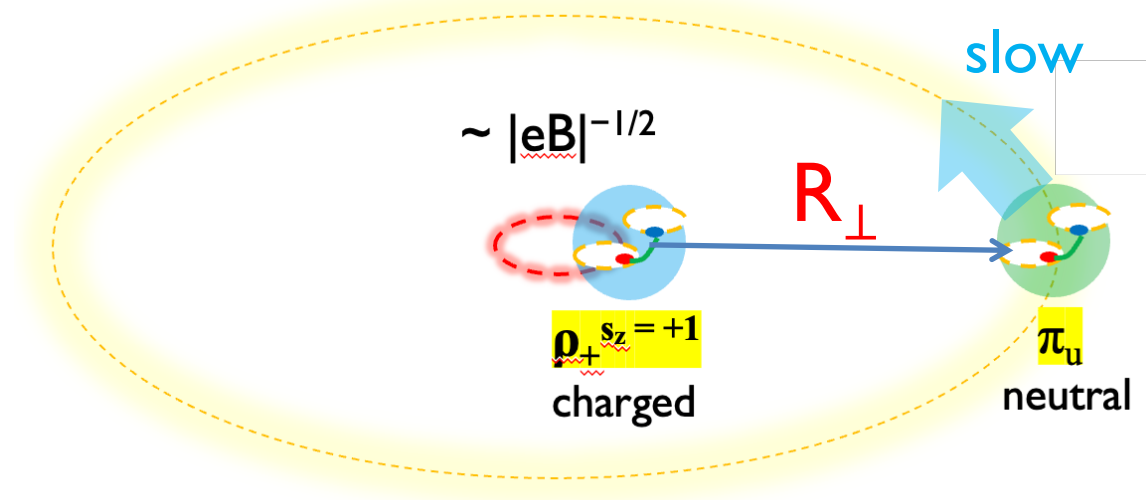


Entropy grows faster than expected from HRG with mass reduction

HRG = hadron resonance gas

Meson molecules

Can **light** mesons form bound states ?



at large B

internal quarks → **fast**

c.m. motion (π_u) → **slow**

π_u or π_d (**neutral**) **too light to bind** ? But...

[Fukushima-Hidaka '12; Hattori+'16; TK-Itatani '26,..]

$$\epsilon_n(\mathbf{K}) = \sqrt{m_\pi^2 + K_z^2 + K_\perp^2} \quad \xrightarrow{\text{large } B} \quad \epsilon_n(\mathbf{K}) = \sqrt{m_\pi^2(B) + K_z^2 + \underbrace{c \frac{\Lambda_{\text{QCD}}^2}{B} K_\perp^2}_{\text{suppressed}} + \dots}$$

can superpose **wide range of $K_\perp$** → w.f. **highly localized in $R_\perp$**

Born-Oppenheimer approach

$$H = \overset{\text{neutral}}{H_n} + \overset{\text{charged}}{H_c} + \overset{\text{M-M potential}}{U_{nc}(\mathbf{R}_n - \mathbf{R}_c)} \quad \mathbf{R}_n, \mathbf{R}_c: \text{ center-of-mass}$$

internal 2-body Hamiltonian: $H_M(e_1, e_2) = \sum_{i=1,2} \frac{\Pi_i^2 - e_i \sigma_z B}{2m} + V(\mathbf{r})$

fast modes:
spectra **at a given $\mathbf{K}$**

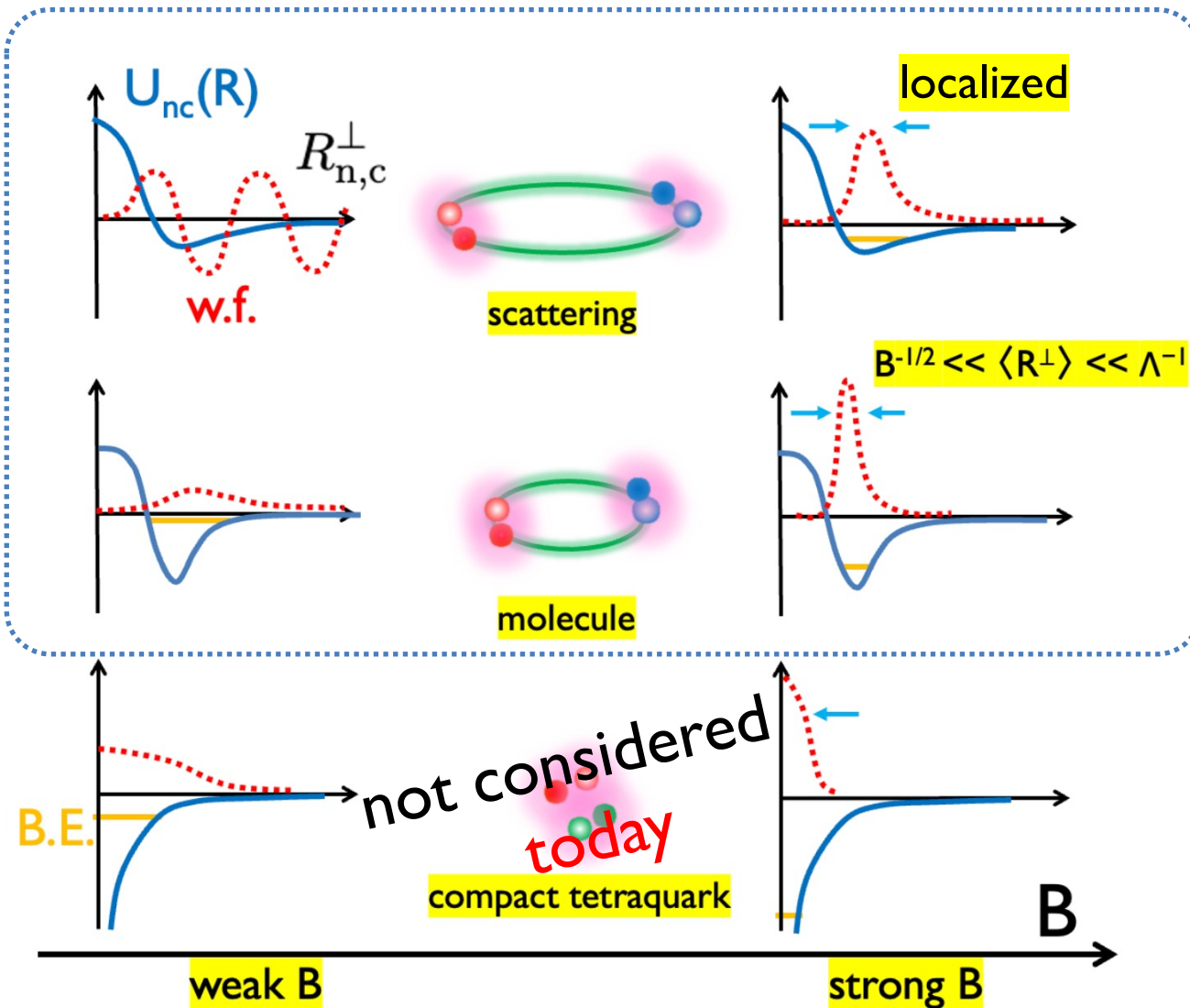
$$\left\{ \begin{array}{ll} \text{neutral:} & H_n \varphi_{\mathbf{K}}(\mathbf{r}) = \epsilon_n(\mathbf{K}) \varphi_{\mathbf{K}}(\mathbf{r}) \\ \text{charged:} & [H_c^{K_x} - \epsilon_c(\tilde{\mathbf{K}})] e^{iK_z R_z} \varphi_{K_x}(\mathbf{r}, R_y) = 0 \end{array} \right.$$

slow modes: effective Hamiltonian

$$\left[E_{nc} - \frac{(K_{\text{tot}}^z)^2}{2M_{nc}} - \frac{(\partial_{nc}^z)^2}{2\mu_{nc}} - \underbrace{c \frac{\Lambda_{\text{QCD}}}{2|eB|}}_{\text{suppressed}} \partial_{n\perp}^2 - U_{nc}(\mathbf{R}_{nc}) \right] \Phi_{nc}(\mathbf{R}_{nc}) = 0$$

$\mathbf{R}_{nc} = \mathbf{R}_n - \mathbf{R}_c$

Impacts of meson-meson potential



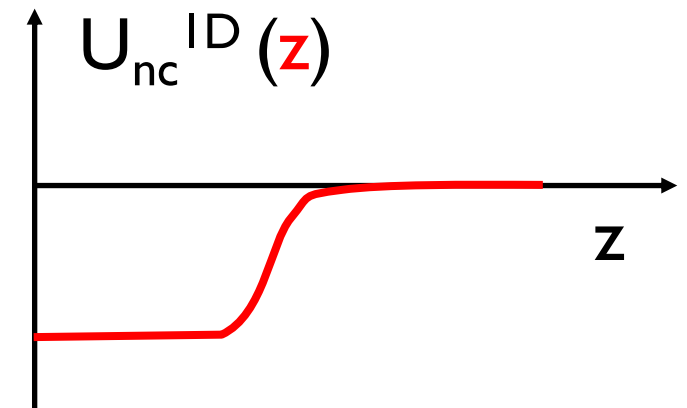
$$U_{nc}(\sqrt{R_\perp^2 + z^2})$$

large B

$$U_{nc}(\sqrt{\langle R_\perp^2 \rangle + z^2}) \equiv U_{nc}^{1D}(z)$$

frozen

3D $\rightarrow$ 1D



Existence of bound states

$$\text{ID: } \left[E_{\text{nc}} - \frac{(K_{\text{tot}}^z)^2}{2M_{\text{nc}}} - \frac{(\partial_{\text{nc}}^z)^2}{2\mu_{\text{nc}}} - U_{\text{nc}}(\langle \mathbf{R}_{\text{nc}}^\perp \rangle, R_{\text{nc}}^z) \right] \Phi_{\text{nc}}(\mathbf{R}_{\text{nc}}) \simeq 0$$

~frozen

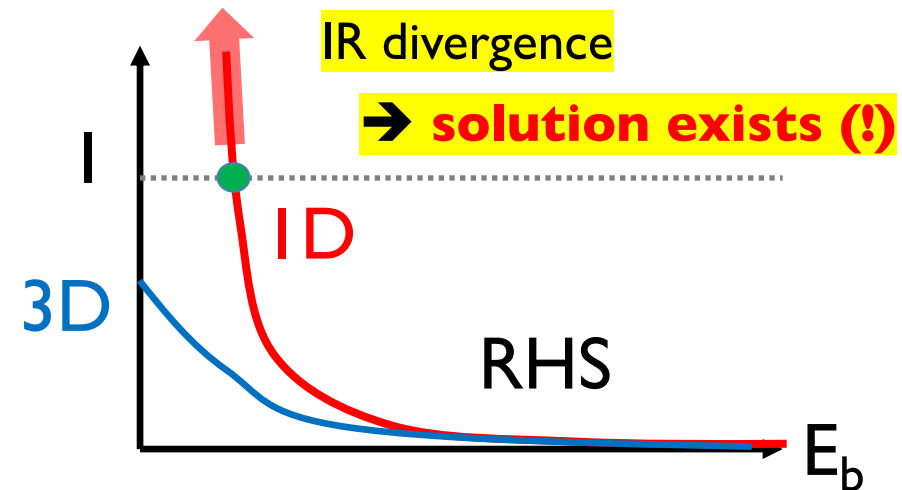
Lippmann-Schwinger: $\Phi(K_z) = \frac{1}{E_b + K_z^2/2\mu_{\text{nc}}} \int_{K'_z} \tilde{U}_{\text{BO}}(K_z - K'_z) \Phi(K'_z)$
(E_b : binding E)

e.g.) for loosely bound state: $\tilde{U}_{\text{BO}}(K_z - K'_z) \simeq -G_0 + O[(K_z - K'_z)^2]$

$$1 \simeq \int_{K_z} \frac{G_0}{E_b + K_z^2/2\mu_{\text{nc}}}$$

➔

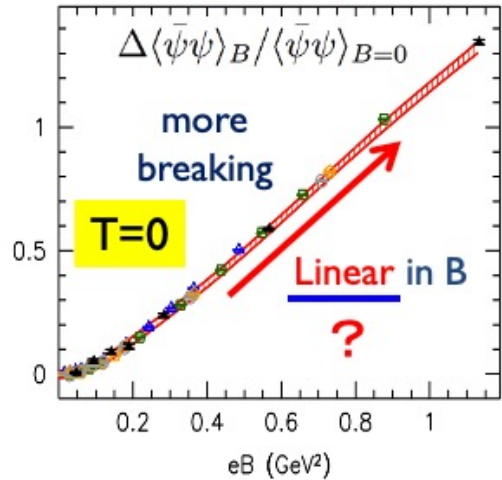
$$E_b \simeq \frac{\mu_{\text{nc}} G_0^2}{2}$$



Summary

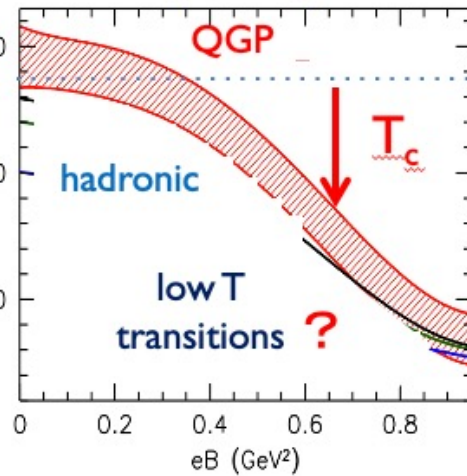
chiral symmetry breaking

(Bali+ '11)



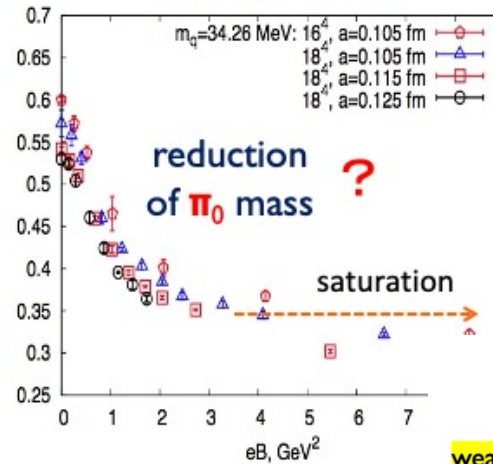
chiral & deconf. T

(Bali+ '11)



neutral hadron mass

(ITEP group '15)



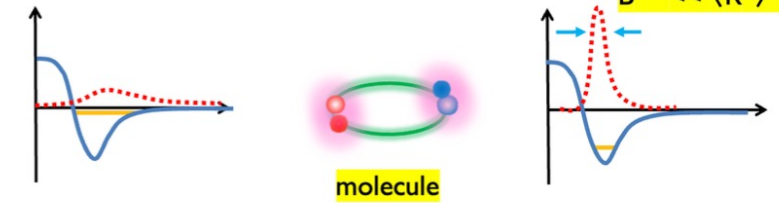
~2011- QCD vacuum
& phase structure
many studies (!)

A new avenue to study exotic hadrons
& inter-hadron potentials

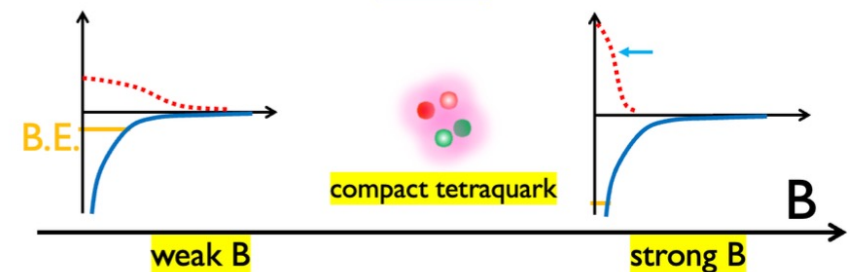
weak
attraction



moderate
attraction



strong
attraction



weak B

strong B

$B^{-1/2} \ll \langle R^\perp \rangle \ll \Lambda^{-1}$

Backup

Matrix elements

$$\langle O \rangle = \frac{\Lambda_z \Lambda_\perp^2}{\pi^{3/2}} \int dz \int_0^\infty dr_\perp r_\perp \int_0^{2\pi} d\theta \, O e^{-\Lambda_z^2 z^2} e^{-\Lambda_\perp^2 (\mathbf{r}_\perp - \tilde{\mathbf{r}}_K)^2}$$

$$\langle O \rangle = \langle O \rangle_0 - \frac{\Lambda_\perp^2}{2|B_q|} \frac{\partial \langle O \rangle_0}{\partial \Lambda_\perp^2} K_\perp^2 + \dots$$

$$\langle O \rangle_0 \equiv \frac{\Lambda_z \Lambda_\perp^2}{\pi^{1/2}} \int dz \int_0^\infty dr_\perp^2 \, O e^{-\Lambda_z^2 z^2} e^{-\Lambda_\perp^2 r_\perp^2}$$

$$\langle V_E \rangle \sim -\Lambda_{\text{QCD}} \times \alpha_s \ln \frac{|B_q|}{\Lambda_{\text{QCD}}^2} + \alpha_s \frac{\Lambda_{\text{QCD}}}{|B_q|} K_\perp^2 + \dots$$