

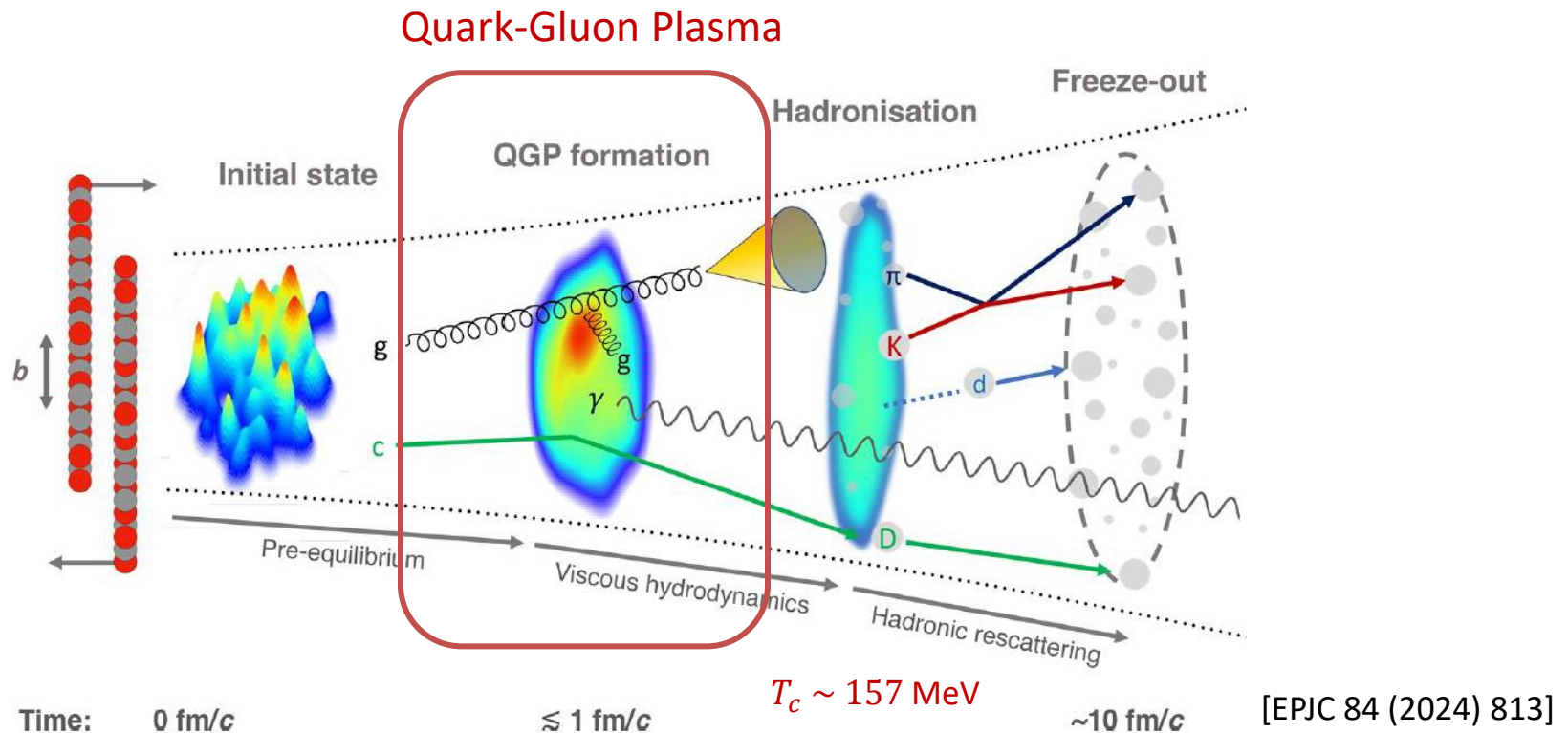
Nonequilibrium approach to heavy-quark transport

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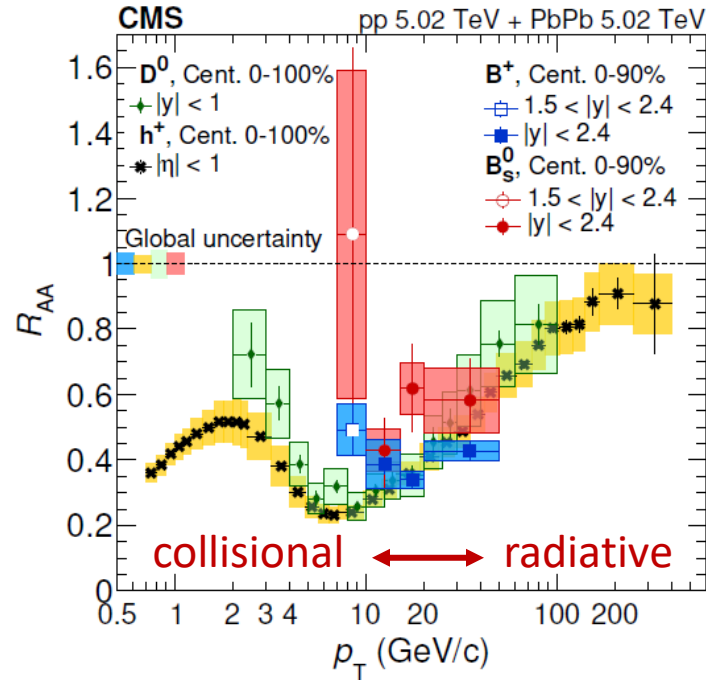
arXiv:2607.15618

Relativistic heavy-ion collisions



- Nonequilibrium QCD matter with high temperature and density
- Heavy quark production at an early stage and conservation

Heavy quark suppression



[JHEP 02 (2025) 195]

- Collisional/radiative energy loss dominant at low/high momentum
- At intermediate momentum: transition from collisional to radiative
- Heavy-quark transport models based on the Boltzmann equation:

$$\frac{P}{E_p} \cdot \frac{\partial}{\partial X} f = C_{col}[f] + C_{rad}[f]$$

Motivation

- Semiclassical Boltzmann equation:
on-shell quasiparticle, Markovian dynamics
- Strongly interacting correlated QCD plasmas:
finite lifetimes and thermal masses, quantum correlations
- Nonequilibrium quantum dynamics:
off-shell, memory effects in QGP
- Quantum extension of heavy-quark kinetic theory:
Kadanoff-Baym equation

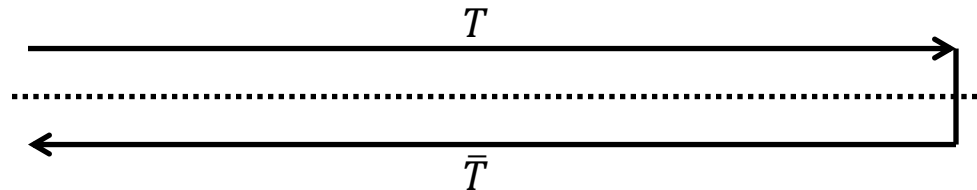
Outline

- Nonequilibrium Green's function approach to heavy-quark transport:
Kadanoff-Baym equation
- Heavy-quark self-energy with hard-thermal-loop resummed propagators:
elastic scattering and medium-induced gluon emission
- Quasiparticle approximation:
Boltzmann equation with collisional and radiative energy loss
- Quantum effects:
the interaction rate in QGP of off-shell partons,
memory effects on relaxation of a single-particle excitation

Kadanoff-Baym equation

Nonequilibrium approach

- Schwinger-Keldysh contour:



- Two-point Green's function: $\begin{pmatrix} S_F & S^< \\ S^> & S_{\bar{F}} \end{pmatrix}$

$$S_F(x_1, x_2) = \langle T \psi(x_1) \bar{\psi}(x_2) \rangle : \text{time-ordered}$$

$$S_{\bar{F}}(x_1, x_2) = \langle \bar{T} \psi(x_1) \bar{\psi}(x_2) \rangle : \text{antitime-ordered}$$

$$S^>(x_1, x_2) = \langle \psi(x_1) \bar{\psi}(x_2) \rangle$$

$$S^<(x_1, x_2) = -\langle \bar{\psi}(x_2) \psi(x_1) \rangle$$

- $S_F + S_{\bar{F}} = S^> + S^<$

Green's function

- In $(\psi_r = \frac{\psi_1 + \psi_2}{2}, \psi_a = \psi_1 - \psi_2)$ basis: $\begin{pmatrix} \frac{S^> + S^<}{2} & S_R \\ S_A & 0 \end{pmatrix}$

- Retarded and advanced Green's function: spectral information

$$\begin{aligned} S_R(x_1, x_2) &= \theta(t_1 - t_2)[S^>(x_1, x_2) - S^<(x_1, x_2)] \\ S_A(x_1, x_2) &= -\theta(t_2 - t_1)[S^>(x_1, x_2) - S^<(x_1, x_2)] \end{aligned}$$


spectral density ρ

- Quasiparticle $S^< \sim \rho f$: statistical information
- In Wigner space, $S^<$ satisfies a kinetic equation that describes the nonequilibrium distribution function

Kadanoff-Baym equation

- Wigner function: Fourier transform w. r. t. $r = x_1 - x_2$

$$S^<(X, P) = - \int d^4r e^{iP \cdot r} \left\langle \bar{\psi} \left(X - \frac{r}{2} \right) \psi \left(X + \frac{r}{2} \right) \right\rangle \quad X = \frac{x_1 + x_2}{2}$$

- Kadanoff-Baym equation:

$$P \cdot \frac{\partial}{\partial X} \text{Tr} [S^<(X, P)] = \frac{1}{2} \text{Tr} [(\not{P} + m) \{ \Sigma^<(X, P) S^>(X, P) - \Sigma^>(X, P) S^<(X, P) \}]$$

- Quasiparticle approximation: on-shell heavy quarks

$$S^<(X, P) = f(X, \mathbf{p}) (\not{P} + m) \rho(P) \quad \rho(P) = 2\pi \delta(P^2 - m^2)$$

- In isotropic and unpolarized plasmas:

$$\frac{P}{E_p} \cdot \frac{\partial}{\partial X} \boxed{f(X, \mathbf{p})} = \frac{1}{2} \int_0^\infty \frac{dp^0}{2\pi} \text{Tr} [\boxed{\Sigma^<(X, P)} S^>(X, P) - \boxed{\Sigma^>(X, P)} S^<(X, P)]$$

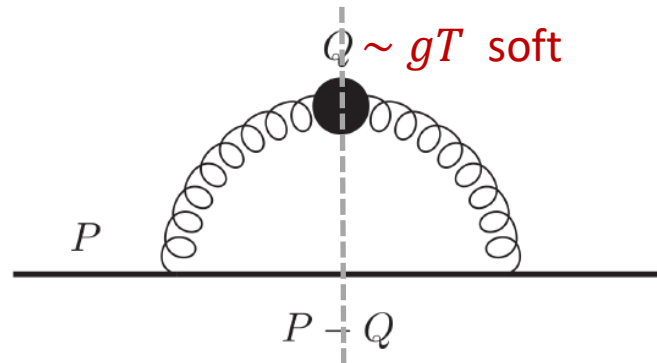
phase-space distribution function

heavy-quark self-energy

Heavy-quark self-energy

Leading order

Hard-Thermal-Loop (HTL) resummation



Braaten, Thoma (1991)

- Weakly coupled plasma at high T : $g \ll 1$
- Heavy quark: $p, m \gg T$
- At leading order: one-loop self-energy with a HTL resummed propagator

$$\Sigma_{\text{LO}}^>(P) = g^2 C_F \int \frac{d^4 Q}{(2\pi)^4} \gamma^\mu S^>(P - Q) \gamma^\nu G_{\mu\nu}^>(Q)$$

LO self-energy

- Self-energy: $\Sigma_{\text{LO}}^>(P) = g^2 C_F \int \frac{d^4 Q}{(2\pi)^4} \gamma^\mu S^>(P - Q) \gamma^\nu G_{\mu\nu}^>(Q)$

- Cut propagators by the Kadanoff-Baym ansatz

- Heavy-quark propagator:

$$S^>(P - Q) = [1 - f(\mathbf{p} - \mathbf{q})](\not{P} - \not{Q} + m)\rho(P - Q) \quad \rho(P - Q) = \frac{1}{2E_p} 2\pi \delta(\omega - \mathbf{q} \cdot \mathbf{v})$$

- Gluon propagator for soft spacelike $Q \sim gT$:

$$G_{\mu\nu}^>(Q) = [1 + f(\omega)][\rho_L(Q)g_{\mu 0}g_{\nu 0} + \rho_T(Q)(\delta_{ij} - \hat{q}_i \hat{q}_j)]$$

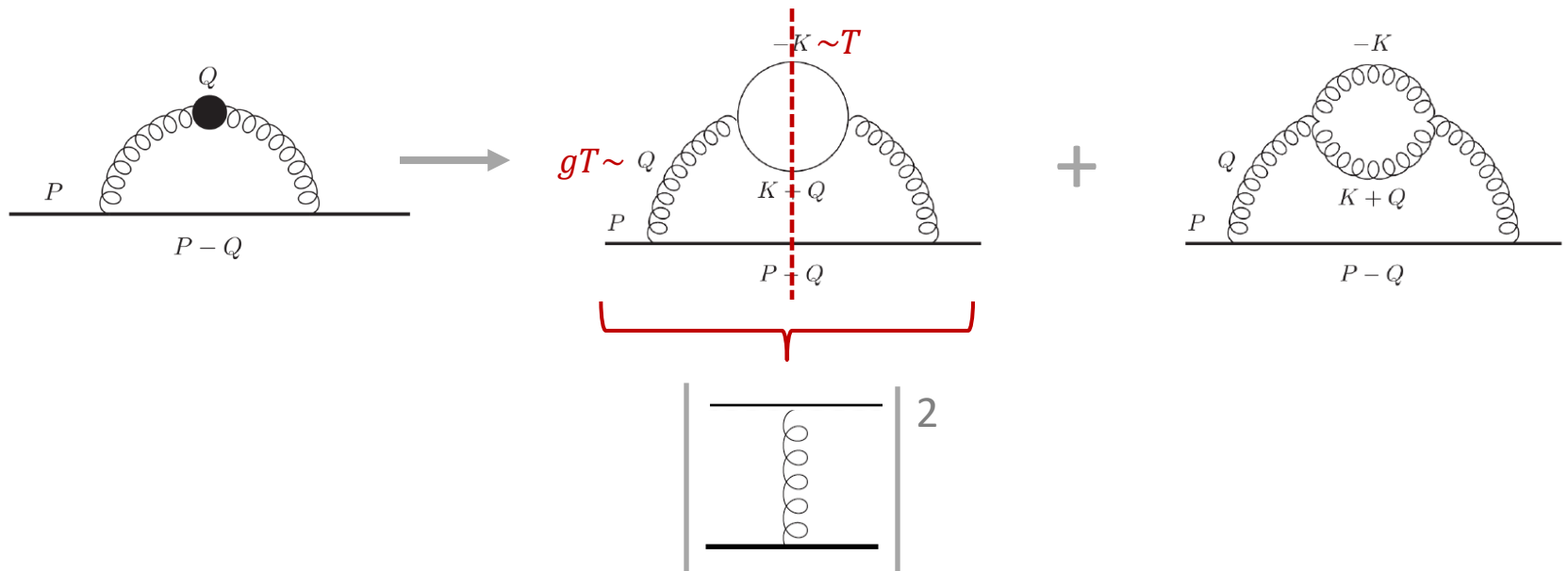
HTL resummed propagator in Coulomb gauge

$$G^{00}(Q) = \frac{i}{q^2 + \Pi_L(Q)} \quad \Pi_L(Q) = m_D^2 \left[1 - \frac{\omega}{2q} \left(\ln \frac{q + \omega}{q - \omega} - i\pi \right) \right]$$

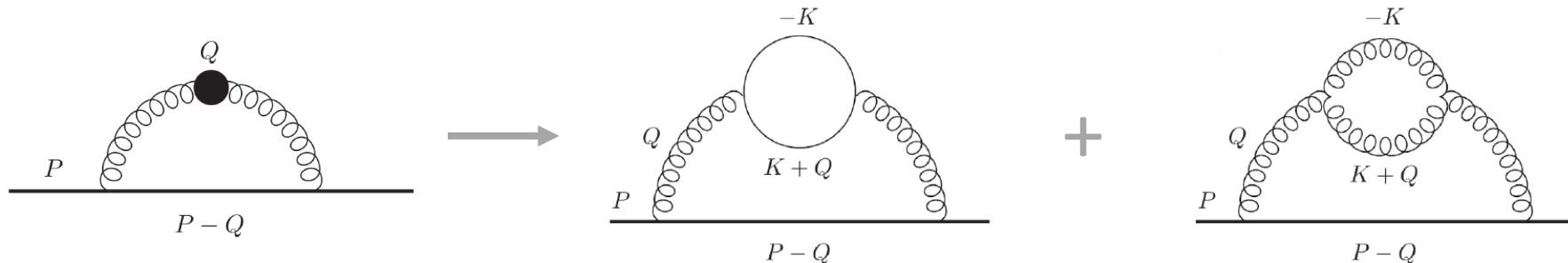
$$G^{ij}(Q) = \frac{i(\delta^{ij} - \hat{q}^i \hat{q}^j)}{\omega^2 - q^2 - \Pi_T(Q)} \quad \Pi_T(Q) = \frac{m_D^2}{2} \left[\frac{\omega^2}{q^2} + \frac{\omega(q^2 - \omega^2)}{2q^3} \left(\ln \frac{q + \omega}{q - \omega} - i\pi \right) \right]$$

Interaction rate

- Interaction rate: $\Gamma_{\text{LO}} = \frac{1}{4E_p} \text{Tr} [(\not{P} + m) \Sigma_{\text{LO}}^>(P)]$
- Kinetic equation: $\frac{P}{E_p} \cdot \frac{\partial}{\partial X} f(X, \mathbf{p}) = \frac{1}{2} \int_0^\infty \frac{dp^0}{2\pi} \text{Tr} [\Sigma^<(X, P) S^>(X, P) - \underbrace{\Sigma^>(X, P) S^<(X, P)}_{\Gamma_{\text{LO}} f(\mathbf{p}) [1 - f(\mathbf{p} - \mathbf{q})]}]$
- Resummed gluon propagator: quark- and gluon-loop contributions



HTL resummed propagator



- Quark- and gluon-loop contribution:

$$G_{>}^{\mu\nu}(Q) = N_f g^2 \int \frac{d^4 K}{(2\pi)^4} \text{Tr}[\gamma_\alpha S^>(-K) \gamma_\beta S^>(K+Q)] G_F^{\mu\alpha}(Q) G_{\bar{F}}^{\nu\beta}(Q) + (g\text{-loop})$$

- Quark propagator:

$$S^>(-K) = f(k)(\not{K} + m_q) \rho(K) \rightarrow f(k) \frac{\not{K}}{2k} 2\pi \delta(k^0 - k)$$

$$S^>(K+Q) = [1 - f(k')](\not{K}' + m_q') \rho(K') \rightarrow [1 - f(k')] \frac{\not{K}}{2kq} 2\pi \delta\left(\cos \theta_k - \frac{\omega}{q}\right)$$

spectral function

quasiparticle

Elastic scattering

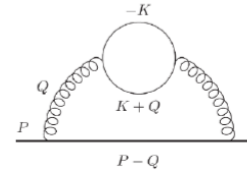
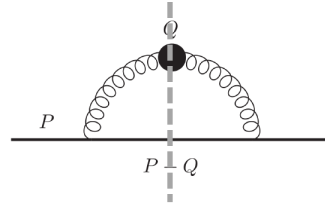
- Phase-space integral:

$$\frac{1}{2} \int_{p^0} \text{Tr}[\Sigma_{LO}^> S^<] = \underbrace{\Gamma_{LO}} f_p (1 - f_{p-q})$$

$$\frac{1}{4E_p} \text{Tr} [(\not{P} + m) \underbrace{\Sigma_{LO}^>(P)}]$$

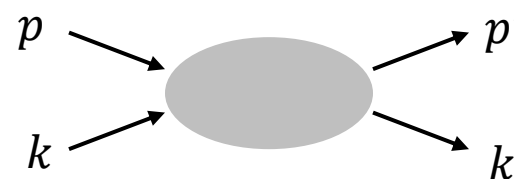
$$g^2 C_F \int_Q \gamma^\mu S^>(P - Q) \gamma^\nu \underbrace{G_{\mu\nu}^>(Q)}$$

$$g^2 \int_K \text{Tr}[\gamma^\alpha S^>(-K) \gamma^\beta S^>(K + Q)] G_{\alpha\mu}^F(Q) G_{\beta\nu}^{\bar{F}}(Q)$$



- Momentum conservation: $\int d^3\mathbf{p}' \delta^3(\mathbf{p} + \mathbf{k} - \mathbf{p}' - \mathbf{k}') = 1$
- Quasiparticle approximation: loss term in the Boltzmann equation

$$\frac{1}{2} \int_{p^0} \text{Tr}[\Sigma_{LO}^> S^<] = \frac{1}{2E_p} \int_{\mathbf{p}', \mathbf{k}, \mathbf{k}'} |M|^2 (2\pi)^4 \delta^4(P_{tot}) f_p f_k (1 - f_{p'}) (1 \pm f_{k'})$$



Boltzmann equation

- Self-energy: $\Sigma_{LO}^<(P) = g^2 C_F \int_Q \gamma^\mu S^<(P - Q) \gamma^\nu G_{\mu\nu}^<(Q)$

$$S^<(P - Q) = f(\mathbf{p} - \mathbf{q})(\not{P} - \not{Q} + m)\rho(P - Q)$$

$$G_{\mu\nu}^<(Q) = f(\omega) [\rho_L(Q) g_{\mu 0} g_{\nu 0} + \rho_T(Q) (\delta_{ij} - \hat{q}_i \hat{q}_j)]$$

- Quasiparticle approximation: gain and loss terms in Boltzmann equation

$$\frac{P}{E_p} \cdot \frac{\partial}{\partial X} f(X, \mathbf{p}) = \frac{1}{2} \int_0^\infty \frac{dp^0}{2\pi} \text{Tr} \left[\underbrace{\Sigma^<(X, P) S^>(X, P)}_{\text{gain term}} - \underbrace{\Sigma^>(X, P) S^<(X, P)}_{\text{loss term}} \right]$$

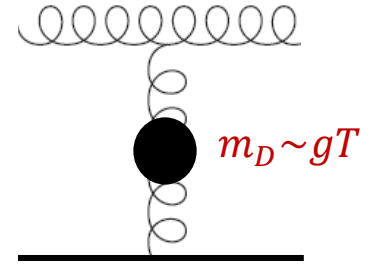
- LO: Boltzmann equation with elastic scattering

$$\frac{P}{E_p} \cdot \frac{\partial}{\partial X} f = C_{col}[f]$$

Collisional energy loss

- For soft gluon exchange: Fokker-Planck equation

$$C_{\text{col}}[f] = \frac{\partial}{\partial p^i} \underbrace{[\eta(\mathbf{p})]}_{\text{drag}} p^i f(\mathbf{p}) + \frac{1}{2} \frac{\partial^2}{\partial p^i \partial p^j} \underbrace{[\kappa^{ij}(\mathbf{p})]}_{\text{diffusion}} f(\mathbf{p}) \quad \text{Svetitsky (1988)}$$



- Momentum diffusion coefficients:

$$\kappa^{ij}(\mathbf{p}) = \kappa_L(p) \hat{p}^i \hat{p}^j + \kappa_T(p) (\delta^{ij} - \hat{p}^i \hat{p}^j)$$

$$\kappa_L(p) = \int d^3 \mathbf{q} \frac{d\Gamma(\mathbf{q})}{d^3 \mathbf{q}} q_z^2$$

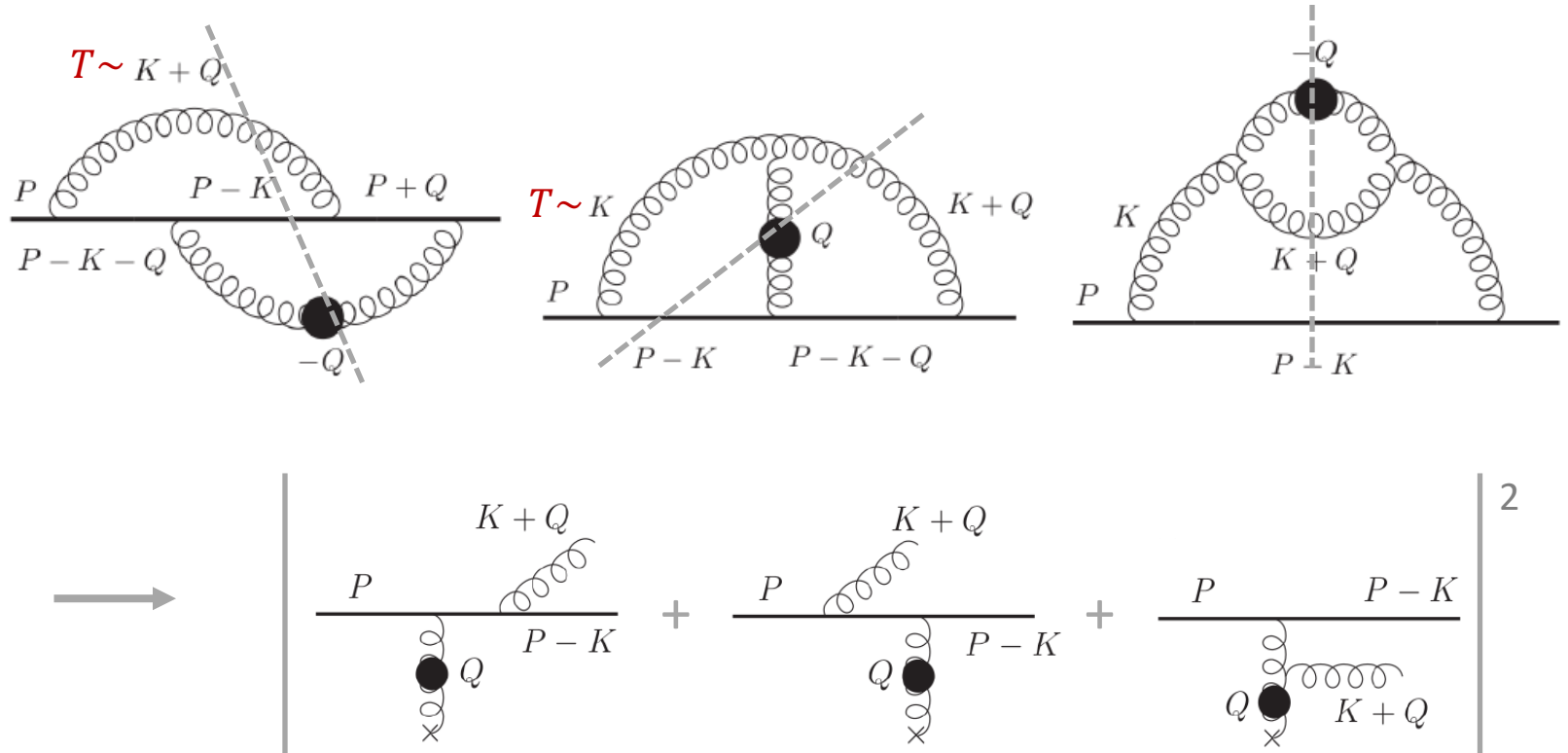
$$\kappa_T(p) = \frac{1}{2} \int d^3 \mathbf{q} \frac{d\Gamma(\mathbf{q})}{d^3 \mathbf{q}} q_T^2$$

- $t \rightarrow \infty$: approach the thermal equilibrium, $f(\mathbf{p}) \propto e^{-E/T}$

$$\eta(p) = \frac{\kappa_L(p)}{2TE} \quad \text{up to } O\left(\frac{T}{E}\right)$$

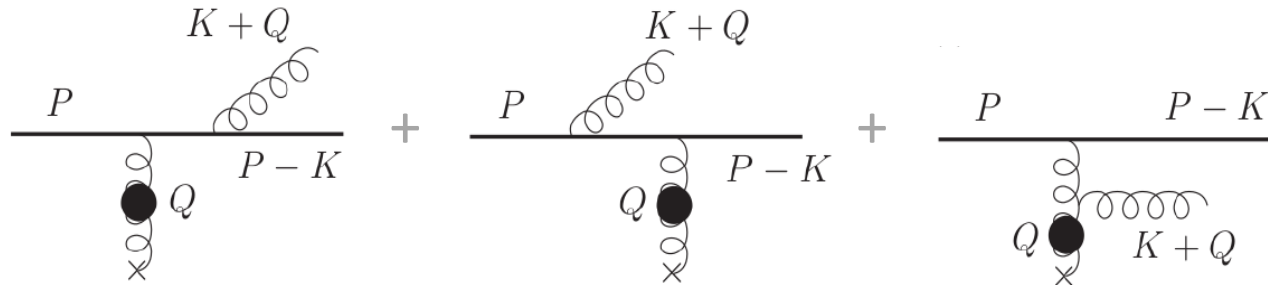
Radiative processes

Next-to-leading order



- For $p^z \gg m$: lightcone coordinates $[p^+, p^-, \mathbf{p}_T] = \left[\frac{p^0 + p^z}{2}, p^0 - p^z, \mathbf{p}_T \right]$
- Semicollinear approximation: $p^+ \gg k^+ \sim T \gg k_T \gg q^+, q_T \sim gT$

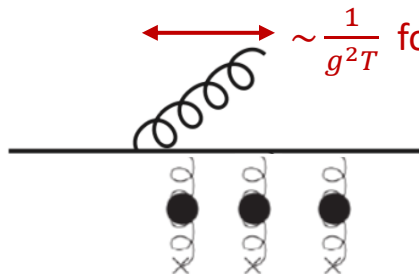
Medium-induced gluon emission



- Semicollinear approximation: $p^+ \gg k^+ \sim T \gg k_T \gg q^+, q_T \sim gT$
- Formation time $\left(\sim \frac{2k^+}{k_T^2 + m^2 x^2} \right) \ll$ mean free path $\left(\sim \frac{1}{g^2 T} \right)$: $\left(x = \frac{k^+}{p^+} \right)$

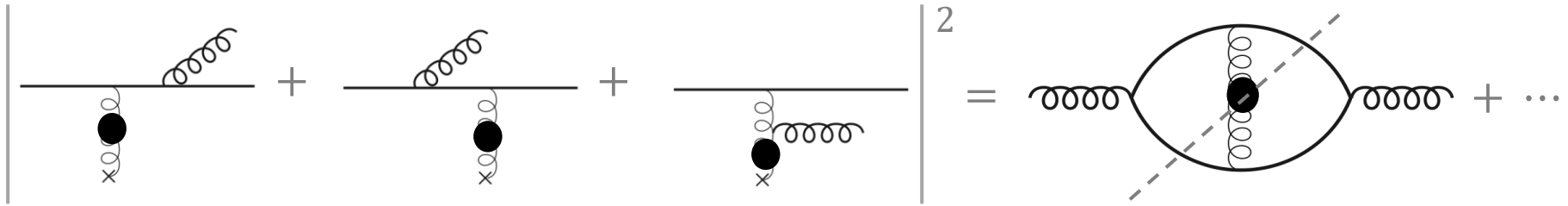
medium-induced gluon emission from a single scattering
transition between collisional and radiative energy loss

no coherence (LPM) effect $\longleftrightarrow \sim \frac{1}{g^2 T}$ for jet: multiple soft scatterings

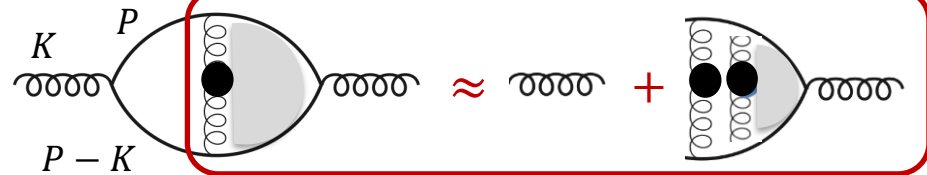


Gluon emission rate

Arnold, Moore, Yaffe (2001,2002)



- Bethe-Salpeter equation:



- Pinching poles:
$$\int \frac{dp^0}{2\pi} \frac{1}{p^0 - k^0 - E_{p-k} + i\Gamma_{LO}/2} \frac{1}{p^0 - E_p - i\Gamma_{LO}/2} \approx \frac{1}{i\delta E + \Gamma_{LO}}$$

$$E_{p-k} + k^0 - E_p$$

- Integral equation:
$$F(\mathbf{p}_T) = \frac{2\mathbf{p}_T}{i\delta E + \Gamma_{LO}} + \frac{1}{i\delta E + \Gamma_{LO}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} C(\mathbf{q}) F(\mathbf{p}_T + \mathbf{q}_T)$$

$$\Gamma_{LO}$$

Radiative energy loss

- Integral equation summing ladder diagrams:

$$2\mathbf{p}_T = i\delta E F(\mathbf{p}_T) + \int \frac{d^3\mathbf{q}}{(2\pi)^3} C(\mathbf{q}) [F(\mathbf{p}_T) - F(\mathbf{p}_T + \mathbf{q}_T)]$$

- Solve the equation perturbatively for $\Gamma_{LO} \ll \delta E$:

$$F^0 = \frac{2\mathbf{p}_T}{i\delta E(p_T)} \rightarrow \text{Re } F^0 = 0$$

$$\text{Re } F(\mathbf{p}_T) = \frac{2}{\delta E(\mathbf{p}_T)} \int \frac{d^3\mathbf{q}}{(2\pi)^3} C(\mathbf{q}) \left[\frac{\mathbf{p}_T}{\delta E(\mathbf{p}_T)} - \frac{\mathbf{p}_T + \mathbf{q}_T}{\delta E(\mathbf{p}_T + \mathbf{q}_T)} \right]$$

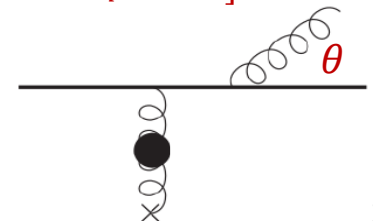
- Gluon emission rate: $\frac{d\Gamma}{dk} = \frac{g^2 C_F}{4\pi k^3} [1 + f(k)] \int \frac{d^2\mathbf{p}_T}{(2\pi)^2} \mathbf{p}_T \cdot \text{Re } F(\mathbf{p}_T)$

$$= \frac{2g^2 C_A}{\pi k} [1 + f(k)] \int \frac{d^2\mathbf{k}_T}{(2\pi)^2} \frac{\kappa_T [q^- = \delta E]}{(k_T^2 + m^2 x^2)^2}$$

momentum diffusion coefficient: $\kappa_T = \frac{g^2 C_F}{2} \int \frac{d^4 Q}{(2\pi)^3} \delta(q^-) \mathbf{q}_T^2 \boxed{G_{>}^{--}(Q)}$

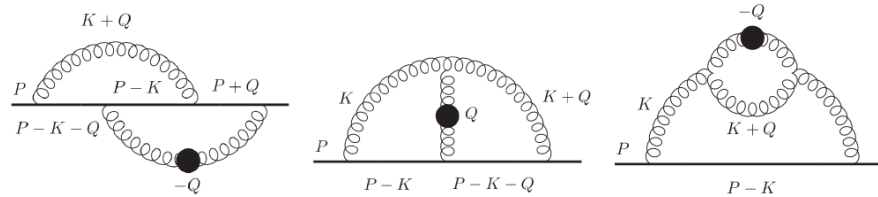
$$[1 + f(w)] \left[\rho_L(Q) + \frac{\mathbf{q}_T^2}{q^2} \rho_T(Q) \right]$$

- Dead cone for $\theta < \frac{m}{E}$: emission is suppressed if $k_T < mx$



NLO self-energy

- Self-energy:



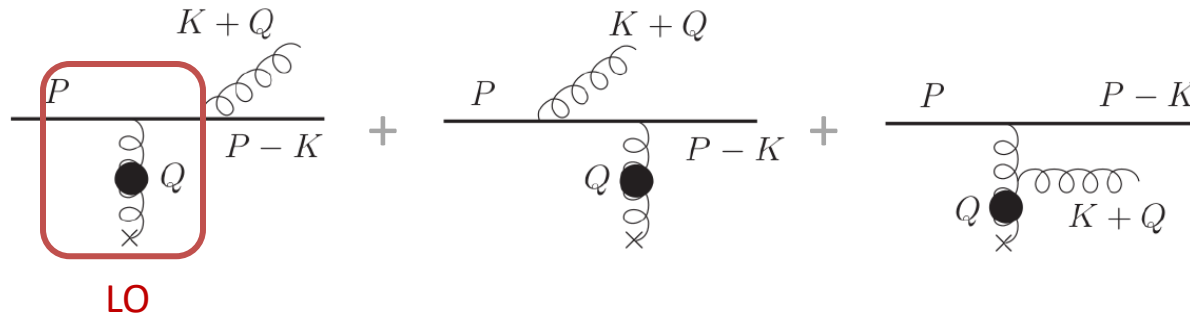
$$\Sigma_{\text{rad}}^>(P) = 2g^4 C_F C_A \int \frac{dk^+}{k^+} \int \frac{d^2 \mathbf{k}_T}{(2\pi)^2} \int \frac{d^4 Q}{(2\pi)^4} \delta\left(q^- - \frac{\mathbf{k}_T^2 + m^2 x^2}{2k^+}\right) [1 + f(k^+)]$$

$$\times G_{>}^{--}(-Q) \frac{\mathbf{q}_T^2}{(\mathbf{k}_T^2 + m^2 x^2)^2} [1 - f(\mathbf{p} - \mathbf{k})]$$

$$\Gamma_{\text{LO}} = g^2 C_F \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \int d\omega \delta(\omega - \mathbf{q} \cdot \mathbf{v}) G_{>}^{--}(Q) \quad \text{for } p^z \gg m$$

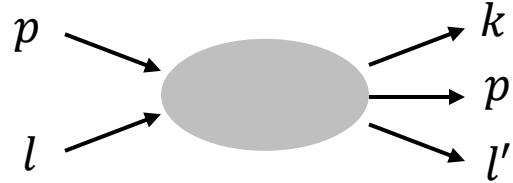
approach the leading-order soft-scattering rate for a jet parton

- Medium-induced gluon emission:



Boltzmann equation

- Phase-space integral: loss term in the Boltzmann equation



$$\frac{1}{2} \int_{p^0} \text{Tr}[\Sigma_{rad}^> S^<] = \frac{1}{2E_p} \int_{\mathbf{k}, \mathbf{p}', l, l'} |M|_{rad}^2 (2\pi)^4 \delta^4(P_{tot}) f_p f_l (1 - f_{p'}) (1 \pm f_{l'}) (1 + f_k)$$

$$|M|_{rad}^2 = \frac{4g^2 C_A q_T^2}{(\mathbf{k}_T^2 + m^2 x^2)^2} |M|^2 : \text{ (g-emission)} \times \text{(LO elastic)}$$

- Integrate over small transverse momenta: splitting 

$$\frac{1}{2} \int_{p^0} \text{Tr}[\Sigma_{rad}^> S^<] = \frac{1}{2p^2} \int dk \int dp' \delta(p - p' - k) f_p (1 - f_{p'}) (1 + f_k)$$

longitudinal momentum

- Quasiparticle approximation: radiation term in the Boltzmann equation

$$\frac{1}{2} \int_{p^0} \text{Tr}[\Sigma_{rad}^> S^<] = \int dk \left[\frac{d\Gamma}{dk} \right] f_p$$

$$\frac{2g^2 C_A}{\pi k} [1 + f(k)] \int \frac{d^2 \mathbf{k}_T}{(2\pi)^2} \frac{\kappa_T [q^- = \delta E]}{(\mathbf{k}_T^2 + m^2 x^2)^2}$$

Quantum transport

- Heavy-quark transport equation:

$$\frac{P}{E_p} \cdot \frac{\partial}{\partial X} f = \frac{1}{2} \int_{p^0} \text{Tr}[\Sigma_{LO+rad}^< S^> - \Sigma_{LO+rad}^> S^<]$$

- Quasiparticle approximation:

Boltzmann equation with collisional and radiative energy loss

$$\frac{P}{E_p} \cdot \frac{\partial}{\partial X} f = C_{col}[f] + C_{rad}[f]$$

- Quantum effects on heavy-quark transport:

building on the self-energy terms (constructed from interacting Green's functions), generalize the kinetic equation to include quantum effects

Off-shell effects

QGP of off-shell partons

- Strong interactions in QCD plasmas
- Partons acquire a finite width and thermal mass
- $m_{g,q}$ and $\gamma_{g,q}$ increase with T :

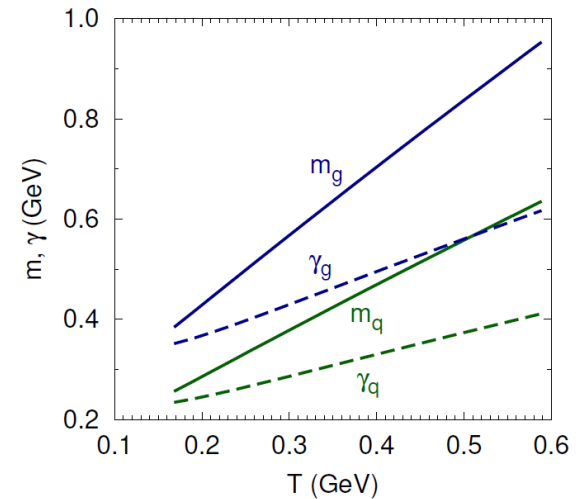
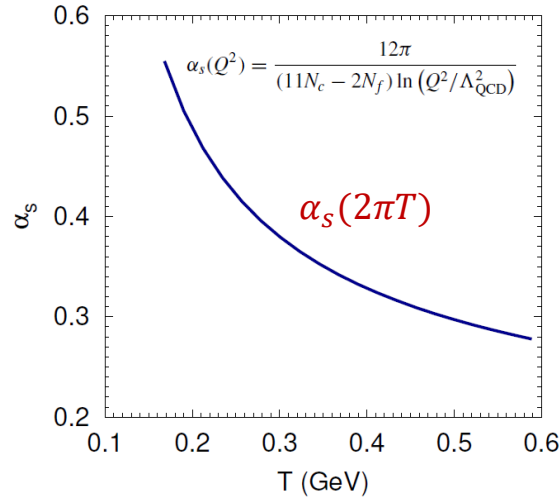
$$m_g^2 = \left(N_c + \frac{N_f}{2}\right) \frac{g^2 T^2}{6}$$

$$m_q^2 = \frac{g^2 T^2}{N_c}$$

$$\gamma_g = 0.3 g^2 T$$

$$\gamma_q = 0.2 g^2 T$$

Braaten, Pisarski (1990,1992)



- Broadened spectral density: Lorentzian

$$\rho_{\text{free}}(K) = 2\pi\delta(k_0^2 - k^2) \quad \rightarrow \quad \rho(K) = \frac{4\gamma_{g,q}k^0}{(k_0^2 - k^2 - m_{g,q}^2)^2 + (2\gamma_{g,q}k_0)^2}$$

Interaction rate

- Resummed gluon propagator:

$$G_{>}^{\mu\nu}(Q) = N_f g^2 \int \frac{d^4 K}{(2\pi)^4} \text{Tr} \left[\underbrace{\gamma_\alpha S^>(-K)}_{f(k)(K+m_q)\rho(K)} \underbrace{\gamma_\beta S^>(K+Q)}_{[1-f(k')](K'+m'_q)\rho(K')} \right] G_F^{\mu\alpha}(Q) G_{\bar{F}}^{\nu\beta}(Q) + (g\text{-loop})$$

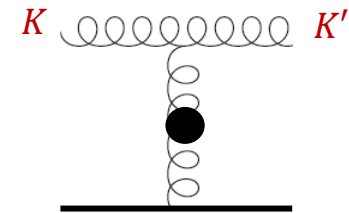
spectral function

- In off-shell QGP: additional energy integrations

$$\Gamma_{\text{LO}} = \frac{1}{2E_p} \int \frac{d^4 K}{(2\pi)^4} \rho(K) \int \frac{d^4 K'}{(2\pi)^4} \rho(K') \int \frac{d^3 \mathbf{p}'}{(2\pi)^3 2E_{p'}} |\mathcal{M}|^2$$

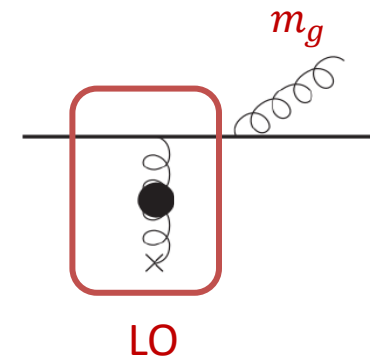
$$\times (2\pi)^4 \delta^4(P + K - P' - K') f(k^0) [1 \pm f(k'^0)]$$

off-shell partons

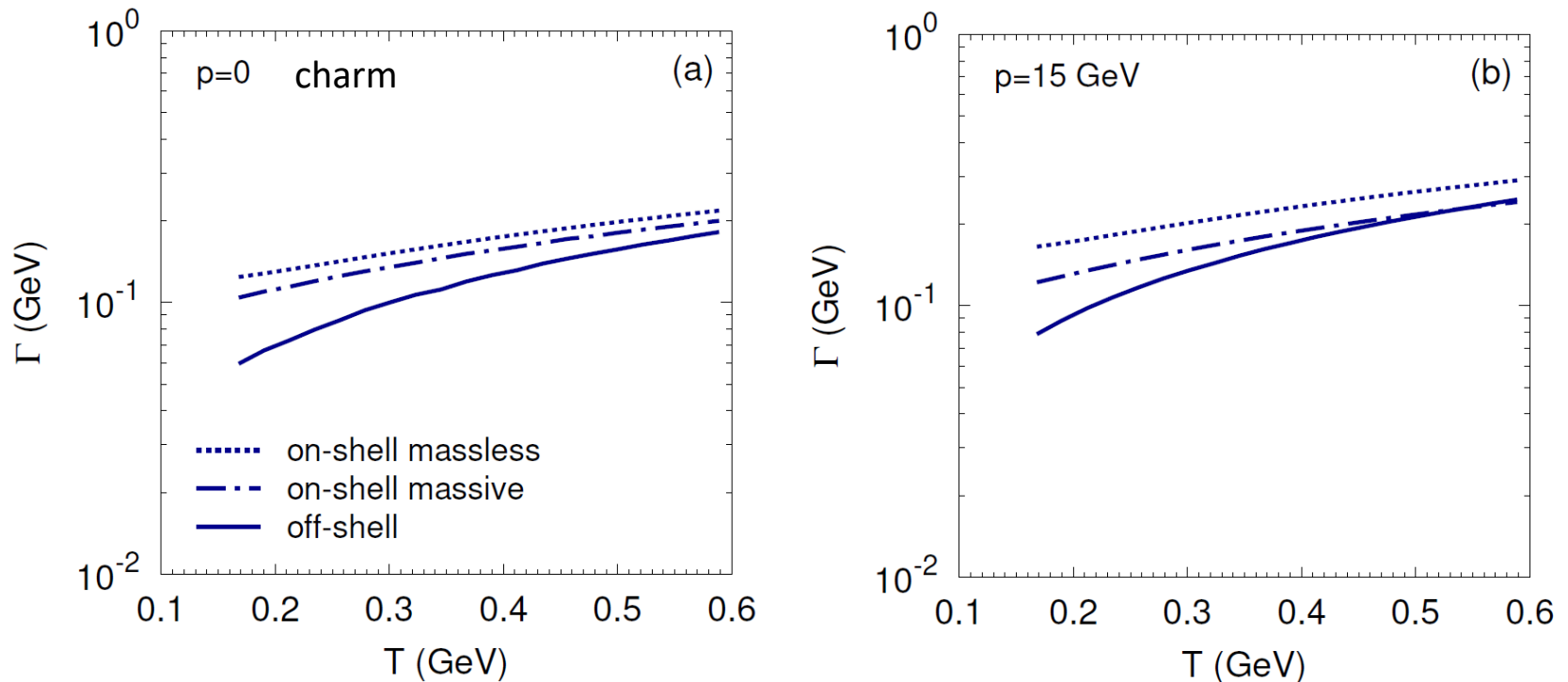


- Gluon emission rate:

$$\frac{d\Gamma}{dk} \sim \underbrace{\Gamma_{\text{LO}} \frac{g^2 C_A}{\pi k} [1 + f(k)] \int \frac{d^2 k_T}{(2\pi)^2} \frac{q^2 (1 - \cos^2 \theta_p)}{(k_T^2 + m^2 x^2 + m_g^2)^2}}_{\text{emission}}$$

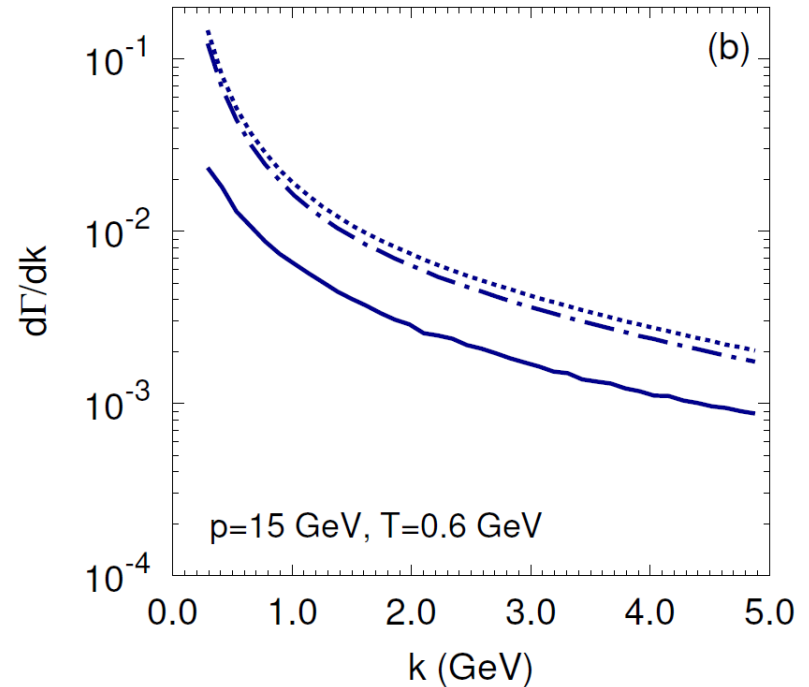
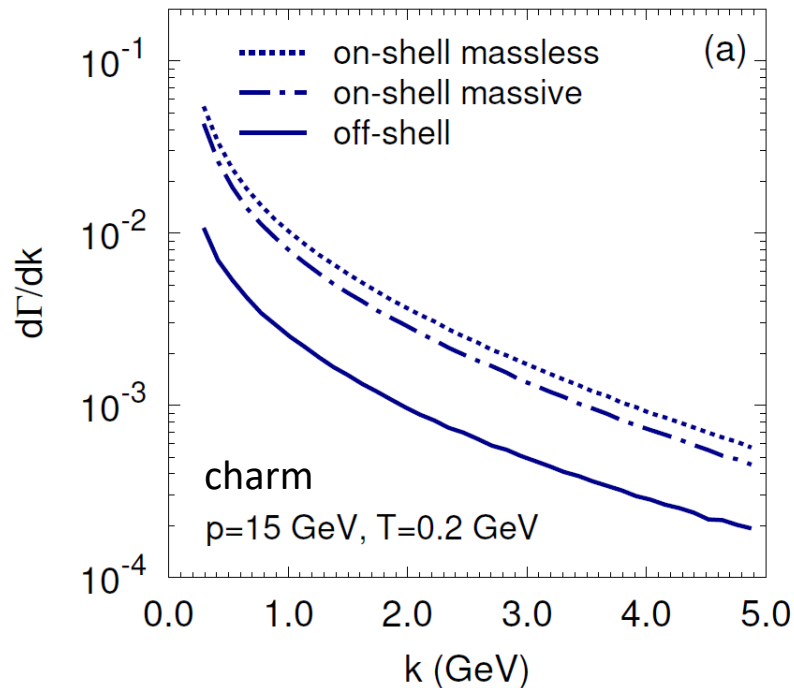


Off-shell effects on LO rate



- Thermal masses reduce the parton densities while the spectral functions modify the phase space
- T -dependent running coupling $\alpha_s(2\pi T)$: greater off-shell effects at lower T with stronger coupling

Off-shell effects on gluon emission



- Thermal masses reduce the parton densities
- Reduced by the underlying LO rate, further suppressed by the thermal mass of the emitted gluon
- Greater off-shell effects at lower T with stronger coupling

Memory effects

Non-Markovian dynamics

- In strongly coupled QCD plasmas:
the collision time not much shorter than the mean time between collisions
- Interactions nonlocal in time: non-Markovian dynamics
- Memory effects: correlations persist over time and affect later dynamics
- Kinetic equation with nonlocal collision terms:
in spatially uniform QGP,

$$\frac{\partial}{\partial t} f(t, \mathbf{p}) = \frac{1}{2} \int_{t_0}^t dt' \int_0^\infty \frac{dp^0}{2\pi} \text{Tr} [\Sigma^<(t, t', P) S^>(t', t, P) - \Sigma^>(t, t', P) S^<(t', t, P)]$$

- Kadanoff-Baym equation for a two-time Green's function

Relaxation of a single-particle excitation

$$\frac{\partial}{\partial t} f(t, \mathbf{p}) = \frac{1}{2} \int_{t_0}^t dt' \int_0^\infty \frac{dp^0}{2\pi} \text{Tr} [\Sigma^<(t, t', P) S^>(t', t, P) - \Sigma^>(t, t', P) S^<(t', t, P)]$$

- Causal generalized Kadanoff-Baym ansatz:

Lipavsky, Spicka, Velicky (1986)

$$S^>(t, t', \mathbf{p}) = [1 - f(t', \mathbf{p})] \frac{\not{P} + m}{2E_{\mathbf{p}}} e^{-iE_{\mathbf{p}}(t-t')}$$

$$S^<(t, t', \mathbf{p}) = \underbrace{f(t', \mathbf{p})}_{\text{earlier time}} \frac{\not{P} + m}{2E_{\mathbf{p}}} e^{-iE_{\mathbf{p}}(t-t')}$$

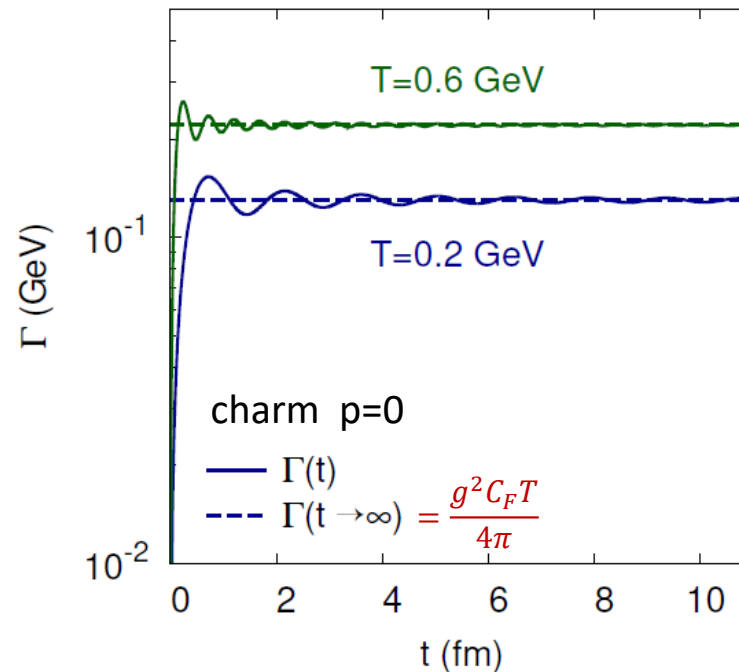
earlier time

- Excitation by adding a particle at $t_0 = 0$ to a system initially in equilibrium
- To leading order in perturbation: $\Sigma^> \sim \Gamma (1 - f_{\text{eq}})$, $\Sigma^< \sim \Gamma f_{\text{eq}}$ Blaizot, Iancu (2002)
- Linearized equation for $\delta f = f - f_{\text{eq}}$:

without memory, $\frac{\partial}{\partial t} \delta f(t) = -\Gamma \delta f(t) \rightarrow \text{exponential decay}$

with memory, $\frac{\partial}{\partial t} \delta f(t) = -\Gamma(t) \delta f(0) - \int_0^t dt' \Gamma(t - t') \frac{\partial}{\partial t'} \delta f(t')$

Time-dependent interaction rate

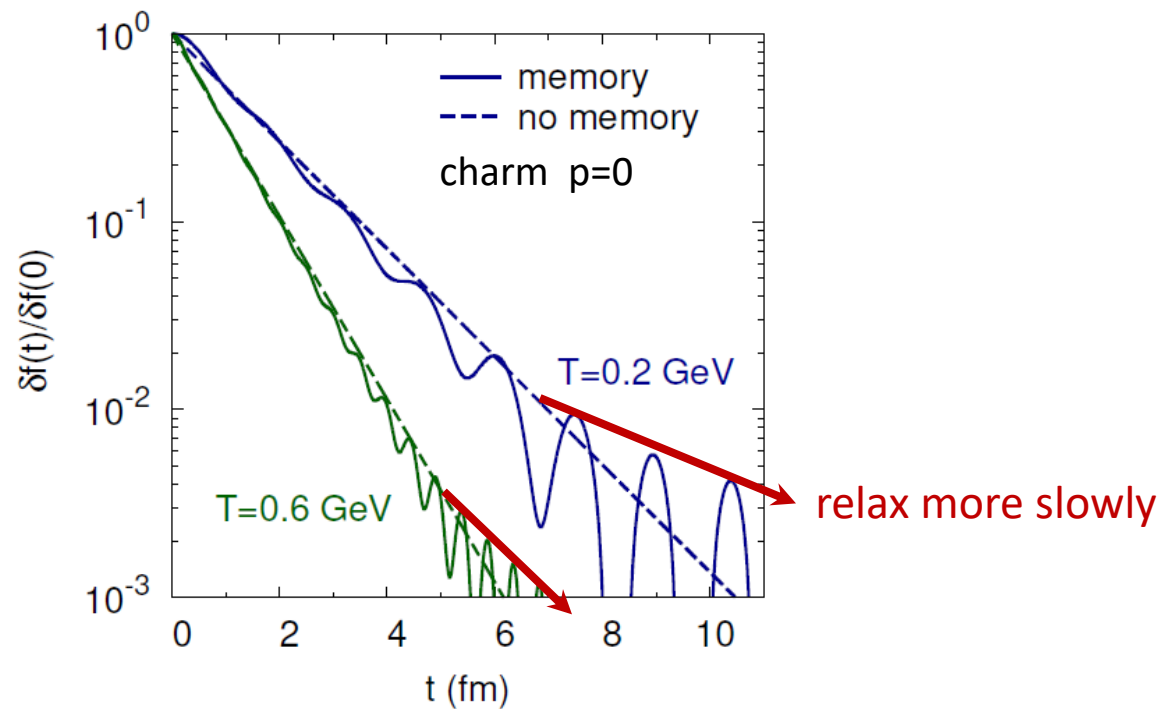


- Interaction rate at $p = 0$:

$$\Gamma(t) = 2\pi g^2 C_F T \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{m_D^2}{q(q^2 + m_D^2)^2} \int \frac{d\omega}{2\pi} \underbrace{\frac{\sin[(\omega + E_{\mathbf{p}-\mathbf{q}} - E_{\mathbf{p}})t]}{\omega + E_{\mathbf{p}-\mathbf{q}} - E_{\mathbf{p}}}}_{\rightarrow \pi \delta(\omega - \mathbf{q} \cdot \mathbf{v})}$$

- In the limit $t \rightarrow \infty$: energy conservation in the Boltzmann equation

Memory effects



- Without memory effects: exponential decay
- With memory effects: larger magnitude, slower relaxation
- Excitation decays faster at higher T
- Greater memory effects at lower T with stronger coupling $\alpha_s(2\pi T)$

Summary

- Nonequilibrium Green's function approach to heavy-quark transport within the Kadanoff-Baym equation
- Heavy-quark self-energy with HTL resummations
- Quasiparticle approximation: Boltzmann equation with elastic scattering and medium-induced gluon emission from a single scattering
- Apply a generalized Kadanoff-Baym ansatz to the heavy-quark self-energy constructed from interacting Green's functions
- Off-shell and memory effects in QGP influence heavy-quark dynamics near T_c with strong coupling
- Quantum transport with realistic parameters and spectral functions

Thank you for your attention