Atomic Structure in Black Hole

Yukinori NAGATANI*)

Okayama Institute for Quantum Physics,
Kyoyama 1-9-1, Okayama 700-0015, Japan

We propose that any black hole has atomic structure in its inside and has no horizon as a model of black holes. Our proposal is founded on a mean field approximation of gravity. The structure of our model consists of a (charged) singularity at the center and quantum fluctuations of fields around the singularity, namely, it is quite similar to that of atoms. Any properties of black holes, e.g. entropy, can be explained by the model. The model naturally quantizes black holes. In particular, we find the minimum black hole, whose structure is similar to that of the hydrogen atom and whose Schwarzschild radius is approximately 1.1287 times the Planck length. Our approach is conceptually similar to the Bohr's model of the atomic structure, and the concept of the minimum Schwarzschild radius is similar to that of the Bohr radius. The model predicts that black holes carry baryon number, and the baryon number is rapidly violated. This baryon number violation can be used as verification of the model.

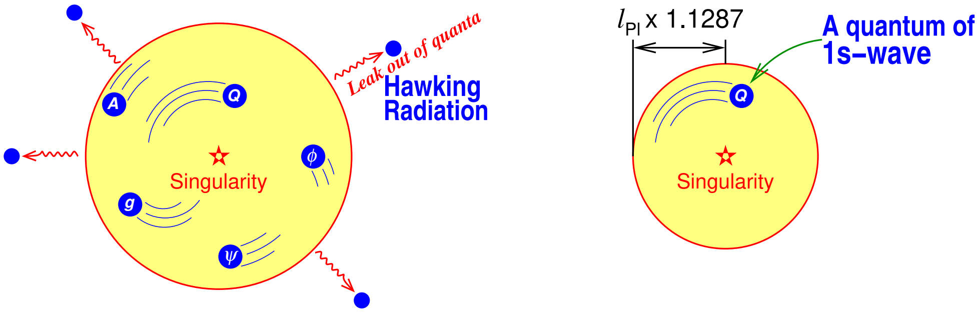
§1. Introduction

The most natural way to solve the information paradox for black holes is to construct a *structure model* of black holes. If we construct a model of a spherical object whose properties, e.g., mass, charges, radius, entropy,[1] emitted radiation,[2] and so on, correspond to those of a black hole, the object can be identified as the black hole for distant observers. If the object has no horizon and has an interior structure, there arises no information paradox.[3)–8)] Such an object is the structure model of a black hole. K. Hotta proposed the Planck solid ball model,[3] which is identical to the structure model of black holes. According to the Planck solid ball model, any black hole consists of a ball consisting of the Planck solid and a layer of thermal radiation around the ball. The Planck solid is hypothetical matter which arises through the stringy thermal phase transition due to the high temperature of the radiation. The temperature of the radiation around the ball becomes very high, because the deep gravitational potential [small $g_{00}(r)$] creates a blue-shift effect $[T(r) \propto 1/\sqrt{g_{00}(r)}]$.[3),4),9)] The entropy of the black holes is carried by the radiation. The Hawking radiation is explained as a leak of the thermal radiation.

Based on the Planck solid ball model without the Planck solid transition hypothesis, we have proposed the radiation ball model[5)–7)] and the atomic structure model.[8] According to these structure model, any black hole consists of a central singularity and quantum fluctuations around the singularity. The generic structure of the model is shown in Fig. 1-a. In this talk, we present our concept of the structure model. The results and prediction of the models are the following: Black holes are naturally quantized, and especially the minimum black hole is found out. The structure of the minimum black hole is similar to that of a hydrogen atom (see Fig. 1-b),

*) E-mail: yukinori_nagatani@pref.okayama.jp

and its radius, namely the minimum Schwarzschild radius, is approximately 1.1287 times the Planck length. The entropy of the structure is carried by the quantum fluctuations. When the radius of the structure is much larger than Planck length, the entropy of the structure is proportional to the surface area of the structure and is almost the same as the Bekenstein entropy. Black holes can carry baryon number, and the Hawking radiation contains the baryon number. The baryon number is rapidly decaying due to the baryon violating interactions in the structure. Our model can be verified by detecting the baryon radiation and its decay curve.



a) The atomic structure of a generic black hole.

b) The hydrogen atomic structure of the minimum black hole.

Fig. 1. The structure model of black holes.

§2. Concept of The Structure Model of Black Holes

Our concept of the structure model of black holes is the following:[8)] We assume Einstein gravity, an electromagnetic field and several other fields. The black hole consists of both a singularity at the center and fluctuating fields around the singularity. The singularity possesses electric (magnetic) charge and creates a spherically symmetric time independent electric (magnetic) field around the singularity. The charge of the singularity causes the singularity to be gravitationally-repulsive, as described by the Einstein equation. The fluctuating fields contain all species of fields in the theory. The repulsive singularity, the static electromagnetic field and the fluctuating fields create a gravitational field as a mean field which obeys the Einstein equation. The gravitational field binds the field fluctuation into a ball structure. The radius of the structure is identical to the Schwarzschild radius of the corresponding charged black hole. The exterior of the structure corresponds to that of the ordinary black hole. The form of the field fluctuations in the structure is determined by quantizing the fluctuating fields in the mean field of the gravity background. The right-hand side of the Einstein equation is the expectation value of the energy-momentum tensor for the quantum-fluctuating fields.

§3. Target Black Hole of the Model Construction

We construct the structure model of the Reissner-Nordström (RN) black hole whose Schwarzschild radius is $r_{\rm BH}$, and whose charge is Q. We introduce a charge-parameter $q := Q/(m_{\rm pl} r_{\rm BH})$ which has a value from 0 to 1, and $q \to 1$ indicates the extremal limit. The metric of the RN black hole is given by

$$ds^2 = F_{\rm BH}(r)dt^2 - G_{\rm BH}(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\varphi^2, \tag{3·1}$$

where

$$F_{\rm BH}(r) = \left(1 - \frac{r_{\rm BH}}{r}\right)\left(1 - q^2 \frac{r_{\rm BH}}{r}\right), \qquad G_{\rm BH}(r) = \frac{1}{F_{\rm BH}(r)}, \tag{3·2}$$

are metric elements. The metric of the exterior region ($r \gtrsim r_{\rm BH}$) of the structure model should agree with that of the black hole metric (3·1). The charge of the structure model also should correspond with that of the black hole.

The Hawking temperature of the charged black hole becomes

$$T_{\rm BH} = \frac{1}{4\pi}\frac{1}{r_{\rm BH}}(1 - q^2). \tag{3·3}$$

When $q = 1$, the black hole becomes the extremal and its Hawking temperature becomes zero. We put the black hole into the background of the radiation with the temperature $T_{\rm BH}$ to consider the stationary situation and the equilibrium of the system.[10] The energy density of the background radiation ($r \to \infty$) is given by the thermodynamical relation:

$$\rho_{\rm BG} = \frac{\pi^2}{30} g_* T_{\rm BH}^4, \tag{3·4}$$

where g_* is the degree of the freedom of the radiation. We assume that g_* is a constant for simplicity.

Here we should introduce the positive cosmological constant $\Lambda = (8\pi/m_{\rm pl}^2)\rho_{\rm vac}$ to stabilize the background universe from the effect of constant energy density $\rho_{\rm BG}$. The background space-time becomes the Einstein static universe by choosing $\rho_{\rm vac} = \rho_{\rm BG}$.

§4. Mean Field Approximation of Gravity

One of the most important concept in the structure model is *the mean field approximation of gravity*. The approximation is defined by decomposition of fields in the base theory which we consider. When there are a gravity field (metric) $g_{\mu\nu}$, an electromagnetic field A_μ, a scalar field ϕ, matter fields ψ_i, and so on in the theory, any field in the theory is decomposed into a spherically-symmetric time-independent part and a fluctuating part:

$$\begin{array}{rcccl} g_{\mu\nu}(t,r,\theta,\varphi) & = & g_{\mu\nu}(r) & + & \delta g_{\mu\nu}(t,r,\theta,\varphi) \\ A_\mu(t,r,\theta,\varphi) & = & A_\mu(r) & + & \delta A_\mu(t,r,\theta,\varphi) \\ \phi(t,r,\theta,\varphi) & = & 0 & + & \delta\phi(t,r,\theta,\varphi) \\ \psi_i(t,r,\theta,\varphi) & = & 0 & + & \delta\psi_i(t,r,\theta,\varphi) \\ \vdots & & \vdots & & \vdots \\ & & \text{mean field} & & \text{fluctuations} \end{array} , \qquad (4{\cdot}1)$$

where r, θ and φ are polar coordinates. The time independent parts are simply mean fields. The metric $g_{\mu\nu}(r)$ describes a background geometry as a mean field. All fields exist in the background space-time described by the mean field metric $g_{\mu\nu}(r)$. The electromagnetic field $A_\mu(r)$ also describes a background field. The metric fluctuation $\delta g_{\mu\nu}$ describes gravitons in the background. The other fluctuations δA_μ, $\delta\phi$, $\delta\psi$, $\cdots$ correspond with photons, scalar bosons, matter particles and so on. All of these fluctuations $\delta g_{\mu\nu}$, δA_μ, $\delta\phi$, $\delta\psi$, $\cdots$ are represented by Φ_i.

In our mean field approximation of gravity, the system is described by combination of the mean field ($g_{\mu\nu}(r)$, $A_\mu(r)$) and the fluctuation fields Φ_i. The fluctuation Φ_i is quantized in the background geometry $g_{\mu\nu}(r)$, and the mean field gravity $g_{\mu\nu}(r)$ is determined by the energy-momentum tensor of the fluctuation Φ_i. The essence of our approach is that the balance amang the mean field of gravity and the fluctuations determines the structure of the model of black holes. When we construct a model of the black hole, we require that the exterior region ($r \gtrsim r_{\rm BH}$) of the structure corresponds with that of the ordinary black hole as a boundary condition. Thus, for the exterior region ($r \gtrsim r_{\rm BH}$), the metric $g_{\mu\nu}(r)$ should correspond with the metric of the corresponding black hole.

The general form of the metric $g_{\mu\nu}(r)$ of the mean field becomes

$$ds^2 = F(r)dt^2 \ - \ G(r)dr^2 \ - \ r^2 d\theta^2 \ - \ r^2 \sin^2\theta d\varphi^2, \qquad (4{\cdot}2)$$

due to the spherical symmetry and time independence. The space time structure of the model is determined by the metric elements $F(r)$ and $G(r)$, and these metric elements are determined by the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda\delta_{\mu\nu} = \kappa\left\{T_{{\rm EM}\mu\nu} + \sum_i \langle\Phi|T_{\mu\nu}[\Phi_i]|\Phi\rangle\right\}. \qquad (4{\cdot}3)$$

The left-hand side of (4·3) is the Einstein tensor of the metric (4·2). The first term of the right-hand side of (4·3) is the energy-momentum tensor of the background electromagnetic fields $A_\mu(r)$. The second term is sum of the energy-momentum tensors of the all fluctuations. Because we construct a model of non-rotating stationary black hole, we only consider the quantum state of fluctuations $|\Phi\rangle$ such that the sum of the energy-momentum tensors $\sum_i \left\langle\Phi|T_\mu{}^\nu[\Phi_i]|\Phi\right\rangle$ becomes diagonal and time-independent.

We assume that the charge density of the total fluctuations is neutral for simplicity. Under this assumption of the neutrality of the fluctuations, the Maxwell equation for the electromagnetic field $A_\mu(r)$ is solved, and the solution of the field strength $F_{\mu\nu}$ is given by

$$F_{\mu\nu}dx^\mu \wedge dx^\nu = E(r)\sqrt{FG}dt \wedge dr \ + \ B(r)r^2 \sin\theta d\theta \wedge d\varphi,$$

where

$$E(r) \ := \ \frac{Q_{\mathrm{E}}}{r^2}, \qquad B(r) \ := \ \frac{Q_{\mathrm{M}}}{r^2} \tag{4·4}$$

are the radial elements of the electric and magnetic fields, respectively. The constants Q_{E} and Q_{M} are the electric and magnetic charges respectively at the central singularity ($r = 0$). The non-trivial elements of the energy-momentum tensor for the electromagnetic field (4·4) are

$$T_{\mathrm{EM}0}{}^{0} \ = \ T_{\mathrm{EM}r}{}^{r} \ = \ -T_{\mathrm{EM}\theta}{}^{\theta} \ = \ -T_{\mathrm{EM}\varphi}{}^{\varphi} \ = \ \frac{Q^2}{8\pi}\frac{1}{r^4}, \tag{4·5}$$

where we have defined $Q^2 := Q_{\mathrm{E}}^2 + Q_{\mathrm{M}}^2$.

§5. Quantization of Fluctuations

We regard the fluctuations Φ_i as free fields in the gravity background, because the gravitational interaction to the fields is essential and the other interactions among the fields are not important in our approach. Therefore, the fluctuation fields Φ_i simply obey the free wave equations in the background gravity $g_{\mu\nu}(r)$, as an essential approximation of the model construction.

The bosonic fields of the fluctuations obey

$$\Box_{(g_{\mu\nu})}\Phi_i = 0, \tag{5·1}$$

where we have defined the d'Alembertian $\Box_{(g_{\mu\nu})}f := \frac{1}{\sqrt{g}}\partial_\mu\left[\sqrt{g}g^{\mu\nu}\partial_\nu f\right]$ in the background metric $g_{\mu\nu}$. The energy-momentum tensor of the field Φ_i becomes

$$T_{\mu\nu}[\Phi_i] = \frac{1}{2}\left(\partial_\mu\Phi_i^\dagger\partial_\nu\Phi_i + \partial_\nu\Phi_i^\dagger\partial_\mu\Phi_i\right) - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\Phi_i^\dagger\partial_\beta\Phi_i. \tag{5·2}$$

Quantization of the fluctuation fields are done by the canonical quantization formalism. The Hamiltonian operator is given by

$$\mathcal{H} := \sum_i \int drd\theta d\varphi\sqrt{FG}r^2\sin\theta \ T_0{}^0[\Phi_i], \tag{5·3}$$

and the conjugate momentum of the field $\Phi_i(t, r, \theta, \varphi)$ is given by

$$\Pi_i := \sqrt{\frac{G}{F}}r^2\sin\theta \ \partial_0\Phi_i. \tag{5·4}$$

By employing the equal-time commutation relations:

$$\begin{aligned}
&[\Phi_i(t, r_1, \theta_1, \varphi_1), \Pi_j(t, r_2, \theta_2, \varphi_2)] = i\delta_{ij}\delta(r_1 - r_2)\delta(\theta_1 - \theta_2)\delta(\varphi_1 - \varphi_2),\\
&[\Phi_i(t, r_1, \theta_1, \varphi_1), \Phi_j(t, r_2, \theta_2, \varphi_2)] = 0,\\
&[\Pi_i(t, r_1, \theta_1, \varphi_1), \Pi_j(t, r_2, \theta_2, \varphi_2)] = 0,
\end{aligned} \tag{5·5}$$

the fluctuation fields are quantized.

By using the real spherically harmonic function $Y_l{}^m(\theta,\varphi)$ and the radial mode function $f_{nl}(r)$, the mode expansion for the field operators is naturally given by

$$\Phi_i = \sum_{n=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} \frac{1}{\sqrt{2\omega_{nl}}} \left\{ a_{inlm}\, e^{-i\omega_{nl}t} \;+\; a^\dagger_{inlm}\, e^{+i\omega_{nl}t} \right\} f_{nl}(r)\, Y_l{}^m(\theta,\varphi), \quad (5\cdot6)$$

where the summation is performed over all of the physical modes $\{nlm\}$ in the background of the gravity (4·2). The index n is numbering of the energy levels for given l. In order for the expansion (5·6) to be solution of the wave equation (5·1), the radial function $f_{nl}(r)$ should satisfy

$$\sqrt{\frac{F}{G}}\frac{1}{r^2}\partial_r\left[\sqrt{\frac{F}{G}}r^2\partial_r f_{nl}\right] \;+\; \omega_{nl}^2 f_{nl} \;-\; l(l+1)\frac{F}{r^2}f_{nl} = 0. \quad (5\cdot7)$$

We require the normalization condition for $f_{nl}(r)$ as

$$\int_0^\infty dr\sqrt{\tfrac{G}{F}}r^2 f_{nl}^2 = 1. \quad (5\cdot8)$$

for the suitable normalization of the creation and annihilation operators. The boundary condition for $r \to 0$ is $f_{nl} \to 0$ or $\partial_r f_{nl}(r) \to 0$, and that for $r \to \infty$ is $f_{nl}(r) \to 0$.

Here, we find the following well-known commutation relations:

$$\begin{aligned} \left[a_{in_1l_1m_1}, a^\dagger_{jn_2l_2m_2}\right] &= \delta_{ij}\delta_{n_1n_2}\delta_{l_1l_2}\delta_{m_1m_2}, \\ \left[a_{in_1l_1m_1}, a_{jn_2l_2m_2}\right] &= 0, \qquad \left[a^\dagger_{in_1l_1m_1}, a^\dagger_{jn_2l_2m_2}\right] \;=\; 0. \end{aligned} \quad (5\cdot9)$$

The vacuum of the system is defined as $a_{nlm}\,|0\rangle = 0$ for all physical modes $\{nlm\}$. The Hamiltonian (5·3) becomes

$$\mathcal{H} = \sum_i\sum_{nlm}\omega_{nl}\left(a^\dagger_{inlm}a_{inlm} \;+\; \frac{1}{2}\right). \quad (5\cdot10)$$

We ignore the vacuum energy in the energy-momentum tensor (5·2) and in the Hamiltonian (5·10) for simplicity. This might be justified by assumption of supersymmetry of the theory.

The general state is given by

$$|\Phi\rangle = \prod_{\{nlm\}} (a^\dagger_{nlm})^{N_{nlm}}\,|0\rangle \quad (5\cdot11)$$

which makes $\langle\Phi|T_\mu{}^\nu|\Phi\rangle$ to be diagonal. The solution of the model is labeled by possible $\{N_{nlm}\}$, and the concrete solution is given by a set of $F(r)$, $G(r)$ and $\{f_{nlm}(r)\}$ which satisfy the Einstein equation (4·3) and the wave equations (5·7).

To obtain a general solution, we should solve a lot of simultaneous differential equations (5·7) with the Einstein equation (4·3) with the boundary conditions, and it seems to be technically difficult. However we can obtain the solutions which consist of a few quanta, because we only need to solve a few simultaneous differential equations.

§6. Minimal Solution

The minimal solution[8)] consists of a single quantum in the lowest-energy in s-wave mode, i.e., $N_{100} = 1$ and the other $N_{nlm} = 0$. We denote the state as $|1s\rangle$. The structure of the minimal solution is just the same as that of the Hydrogen atom.

For simplicity, we consider a minimal model of the near-extremal ($q \to 1$) Reissner-Nordström black hole, and its Hawking temperature is almost zero. Thus we do not need to consider the cosmological constant Λ, because of the absence of the background radiation.

The expectation value of the energy-momentum tensor for the state $|1\mathrm{s}\rangle$ is

$$\begin{aligned}
\langle 1\mathrm{s}|\, T_0{}^0 \,|1\mathrm{s}\rangle = -\langle 1\mathrm{s}|\, T_r{}^r \,|1\mathrm{s}\rangle &= \frac{1}{8\pi}\frac{1}{\omega_{10}C_{10}}\left[\frac{\omega_{10}^2}{F}f_{10}^2 \;+\; \frac{1}{G}(\partial_r f_{10})^2\right], \\
-\langle 1\mathrm{s}|\, T_\theta{}^\theta \,|1\mathrm{s}\rangle = -\langle 1\mathrm{s}|\, T_\varphi{}^\varphi \,|1\mathrm{s}\rangle &= \frac{1}{8\pi}\frac{1}{\omega_{10}C_{10}}\left[\frac{\omega_{10}^2}{F}f_{10}^2 \;-\; \frac{1}{G}(\partial_r f_{10})^2\right], \\
\langle 1\mathrm{s}|\, (\text{non-diagonal-parts}) \,|1\mathrm{s}\rangle &= 0.
\end{aligned} \tag{6·1}$$

The Einstein equation (4·3) becomes

$$\frac{-G + G^2 + rG'}{r^2G^2} = \frac{8\pi}{m_{\rm pl}^2}\left\{+\frac{Q^2}{8\pi}\frac{1}{r^4} \;+\; \langle 1\mathrm{s}|\, T_0{}^0 \,|1\mathrm{s}\rangle\right\}, \tag{6·2}$$

$$\frac{F - FG + rF'}{r^2FG} = \frac{8\pi}{m_{\rm pl}^2}\left\{-\frac{Q^2}{8\pi}\frac{1}{r^4} \;-\; \langle 1\mathrm{s}|\, T_r{}^r \,|1\mathrm{s}\rangle\right\}, \tag{6·3}$$

where the prime represents differentiation with respect r. By subtracting (6·3) from (6·2) with the property $\langle 1\mathrm{s}|\, T_0{}^0 \,|1\mathrm{s}\rangle = -\langle 1\mathrm{s}|\, T_r{}^r \,|1\mathrm{s}\rangle$, we find the relation

$$G = \left(\frac{r}{2}\frac{H'}{H} + 1\right)\left(1 - \frac{Q^2}{m_{\rm pl}^2 r^2}\right)^{-1}, \tag{6·4}$$

where we have defined the new parameter

$$H := \frac{F}{G}. \tag{6·5}$$

By substituting (6·4) into (6·2), the Einstein equation (6·2) becomes

$$\frac{2HH'' - H'^2 + \frac{4}{r}HH'}{2H + rH'} \;-\; 4q^2\frac{r_{\rm BH}^2}{r^4}\frac{H}{1 - q^2\frac{r_{\rm BH}^2}{r^2}} \;-\; \frac{2}{m_{\rm pl}^2}\left(\omega_{10}^2 f^2 + Hf'^2\right) = 0. \tag{6·6}$$

The wave equation (5·7) for the radial mode function f for s-wave becomes

$$Hf'' \;+\; \left(\frac{2}{r}H \;+\; \frac{1}{2}H'\right)f' \;+\; \omega_{10}^2 f = 0. \tag{6·7}$$

The simultaneous equations (6·6) and (6·7) are numerically solved with the initial conditions

$$F(r) \;\to\; F_{\rm sing}(r), \quad G(r) \;\to\; G_{\rm sing}(r), \quad f(r) \;\to\; f_{\rm sing}, \quad f'(r) \;\to\; 0 \tag{6·8}$$

for $r \to 0$, where we have defined the Reissner-Nordström (RN) type singularity: $F_{\rm sing}(r) := \alpha^2 \times \frac{(1-q^2)^2}{12960\sqrt{2\pi}m_{\rm pl}^4 r_{\rm BH}^4} q^2 \left(r_{\rm BH}/r\right)^2$ and $G_{\rm sing}(r) := \frac{1}{q^2}\left(r/r_{\rm BH}\right)^2$. The constant α is red-shift factor of the singularity. We should choose the radius of the solution, $r_{\rm BH}$, the initial value of the mode function, $f_{\rm sing}$, and the frequency of the mode, ω_{10}, so as to satisfy the exterior boundary conditions

$$F(r) \;\to\; F_{\rm BH}(r), \quad G(r) \;\to\; G_{\rm BH}(r), \quad f(r) \;\to\; 0 \tag{6·9}$$

for $r > r_{\rm BH}$ and the normalization condition for f in (5·8). These boundary conditions yield a quantization of $r_{\rm BH}$ as f should not cross zero for $0 < r < r_{\rm BH}$, because the function f describes the lowest s-wave mode (the 1s-mode).

We obtained the solution for $q = 0.99999$ and $\alpha = 1$. The resultant radius of the solution is

$$r_{\rm BH} = 1.12871 \times l_{\rm pl}, \tag{6·10}$$

where $l_{\rm pl} := 1/m_{\rm pl}$ is the Planck length. The frequency of the mode is $\omega_{10} = 0.0253909 \times \alpha\,(1-q^2)\,m_{\rm pl}$. The redshift factor α of the singularity and the factor $(1-q^2)$, which is the shift of the Hawking temperature (3·3), are directly reflected in the frequency ω_{10}. The form of the radial mode function for the scalar field is displayed in Fig. 2, where the initial value of the radial mode function is

$$f_{\rm sing} = f(0) \;=\; 1.268776 \times m_{\rm pl}. \tag{6·11}$$

The distributions of the resultant metric elements $F(r)$ and $G(r)$ are displayed in Fig. 3-a. To contrast our result with that for an ordinary charged black hole, we also plot the exterior part ($r > r_{\rm BH}$) of the Reissner-Nordström metric (3·1) with the same radius $r_{\rm BH}$ and the same charge q (the thick dotted curves in Fig. 3-a).

§7. Many Particle Limit

We consider a many quanta (particles) limit in the structure.[5),6)] In the many particle limit, it is natural to consider behavior of the thermal fluid rather than that of the each motion of the fluctuation fields Φ. Thus we treat the quanta as a thermal fluid of the particles, and this treatment simplifies the analysis.

The fluid is thermalized, because the fluctuation should be time-independent and stationary. We define local temperature of the thermal fluid as $T(r)$ due to spherical symmetry. The thermal fluid is perfect one in the gravity background, because we do not need to consider the interactions among the particles except for the interaction with the mean field of the gravity. Therefore the energy momentum tensor of the fluid is given by $T_\mu{}^\nu(r) = {\rm diag}\left[\rho(r), -P(r), -P(r), -P(r)\right]$, where $\rho(r)$ and $P(r)$ are energy density and pressure respectively. The fluctuation fields are massless, then we have

$$\rho(r) = \frac{\pi^2}{30} g_* T(r)^4, \qquad P(r) = \frac{1}{3}\rho(r). \tag{7·1}$$

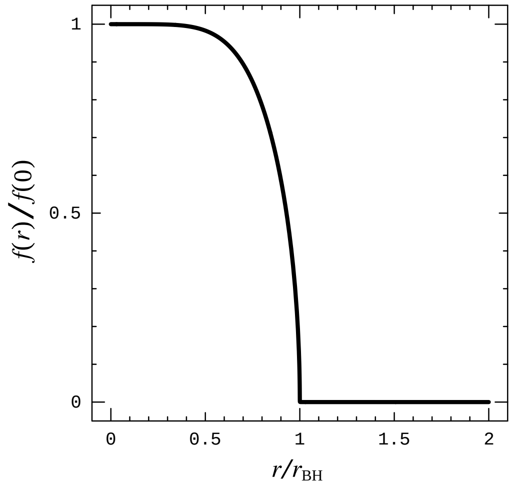
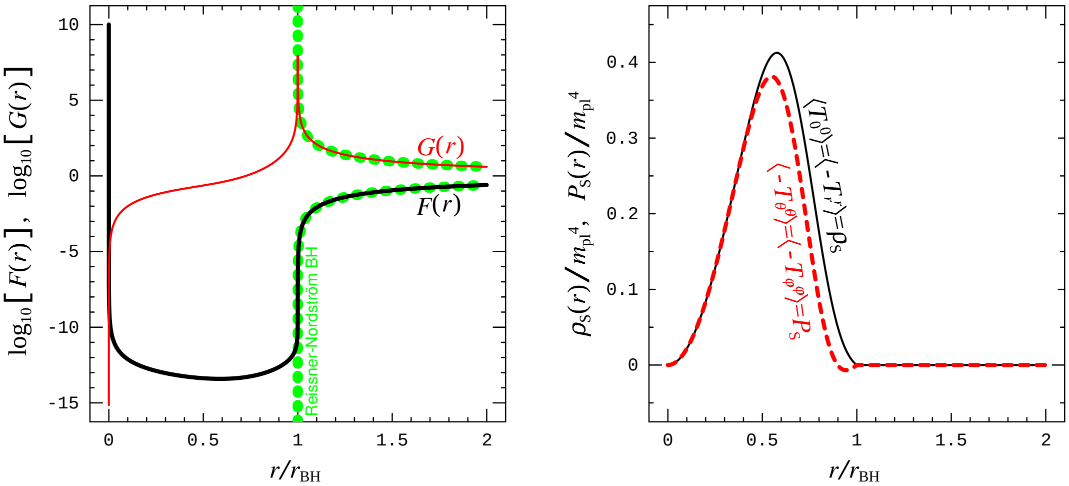


Fig. 2. Distribution of the radial mode function $f(r)$ for the state $|1\mathrm{s}\rangle$. This plot represents the quantum fluctuation of the field in the solution of the minimum structure model for a near-extremal black hole. While we chose the parameters as $q = 0.99999$ and $\alpha = 1$, the curve depends only weakly on the parameters. The vertical axis is normalized by $f(0) = 1.268776 \times m_{\mathrm{pl}}$. The horizontal axis represents the coordinate r normalized by r_{BH}. The radius of the solution, namely the minimal Schwarzschild radius, becomes $r_{\mathrm{BH}} = 1.12871 \times l_{\mathrm{pl}}$, where $l_{\mathrm{pl}} := 1/m_{\mathrm{pl}}$ is the Planck length. We find that the quantum fluctuation is completely bound into the structure.



a) Elements of the metric $F(r)$ and $G(r)$

b) Expectation values of $\left\langle 1\mathrm{s}\left|T_{\mathrm{S}\mu}{}^{\nu}\right|1\mathrm{s}\right\rangle$

Fig. 3. a) The metric elements and b) the energy momentum tensor for the quantized field in the solution of the minimum model. a) The thick solid curve is $F(r)$, and the thin solid curve is $G(r)$. The thick dotted curves indicate the exterior part $(r > r_{\mathrm{BH}})$ of the Reissner-Nordström metric $[F_{\mathrm{BH}}(r)$ and $G_{\mathrm{BH}}(r)$ in (3·2)] with the same radius r_{BH} and with the same charge q. The depth of the potential well $[F(r)$ for $0 < r < r_{\mathrm{BH}}]$ depends on q and α. b) The thin solid curve represents $\rho_{\mathrm{S}} = \left\langle T_0{}^0\right\rangle = -\left\langle T_r{}^r\right\rangle$, and the thick dotted curve represents $P_{\mathrm{S}} = -\left\langle T_\theta{}^\theta\right\rangle = -\left\langle T_\varphi{}^\varphi\right\rangle$. ρ_{S} is positive definite, but P_{S} becomes negative for $r \simeq r_{\mathrm{BH}}$. These curves depend only weakly on the parameters q and α.

Instead of solving the field equations for Φ in the space-time background $g_{\mu\nu}$, we take *the proper temperature ansatz*, i.e., we assume the temperature distribution of the fluid to be[3),4),9)]

$$T(r) = \frac{T_{\rm BG}}{\sqrt{F(r)}}, \tag{7·2}$$

where $F(r)$ is metric element g_{00}, and $T_{\rm BG}$ is the background temperature. This temperature distribution is natural in the thermal equilibrium in the gravity background $g_{\mu\nu}$. It is believed that the temperature ansatz (7·2) can be approximately derived by using a combination of the mean field approximation and the fluid approximation of the particles.

According to our concept of model construction, the background temperature $T_{\rm BG}$ has been chosen to be the Hawking temperature $T_{\rm BH}$. The background universe becomes Einstein static universe.

The Einstein equation (4·3) becomes following equations:

$$\frac{-G + G^2 + rG'}{r^2 G^2} = \frac{8\pi}{m_{\rm pl}^2}\left\{\rho(r) + \rho_{\rm BG} + \frac{Q^2}{8\pi}\frac{1}{r^4}\right\}, \tag{7·3}$$

$$\frac{F - FG + rF'}{r^2 FG} = \frac{8\pi}{m_{\rm pl}^2}\left\{P_r(r) - \rho_{\rm BG} - \frac{Q^2}{8\pi}\frac{1}{r^4}\right\}, \tag{7·4}$$

We obtain the relation

$$G(r) = \left\{1 - \frac{r}{2}\frac{\rho'(r)}{\rho(r)}\right\}\left[1 + \frac{8\pi}{m_{\rm pl}^2} r^2 \left\{\frac{1}{3}\rho(r) - \rho_{\rm BG} - \frac{Q^2}{8\pi}\frac{1}{r^2}\right\}\right]^{-1} \tag{7·5}$$

from the equation (7·4) with (7·1) and (7·2). By substituting (7·5) into the equation (7·3) with (7·1), we obtain the differential equation for the energy density $\rho(r)$ as

$$\begin{aligned} &- 24r\rho^2\left\{\rho - \rho_{\rm BG} + \tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\} \;+\; 12r^2\rho\rho'\left\{\rho + 2\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\} \\ &+ r^3\rho'^2\left\{\rho - 9\rho_{\rm BG} - 9\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\} \;-\; 2r^3\rho\rho''\left\{\rho - 3\rho_{\rm BG} - 3\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\} \\ &+ \frac{3m_{\rm pl}^2}{8\pi}\left\{-4\rho\rho' + 3r\rho'^2 - 2r\rho\rho''\right\} \;=\; 0. \end{aligned} \tag{7·6}$$

The differential equation (7·6) is numerically solved and we find out the solution whose exterior part corresponds with the charged black hole in the Einstein static universe.

The solution is parameterized by $r_{\rm BH}$ and q which are the outer horizon radius and the charge-parameter of the correspondent RN black hole respectively. The element of the metric $F(r)$ is derived by the proper temperature ansatz (7·2) and $G(r)$ is derived by (7·5). The outline of the solution is shown in Fig. 4, and typical numerical solutions are displayed in Fig. 5. The solution indicates that most of the quantum fluctuations are trapped in the structure ($r < r_{\rm BH}$) by the gravitational potential $F(r)$, and that the radius of the structure corresponds with that of the correspondent black hole, namely, $r_{\rm BH}$. The inner horizon of the RN solution becomes completely meaningless.

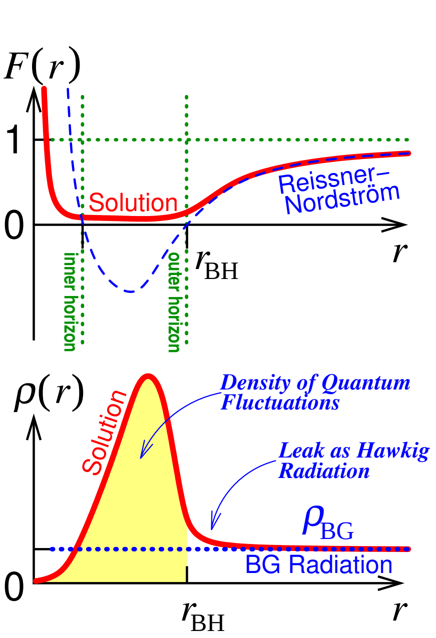
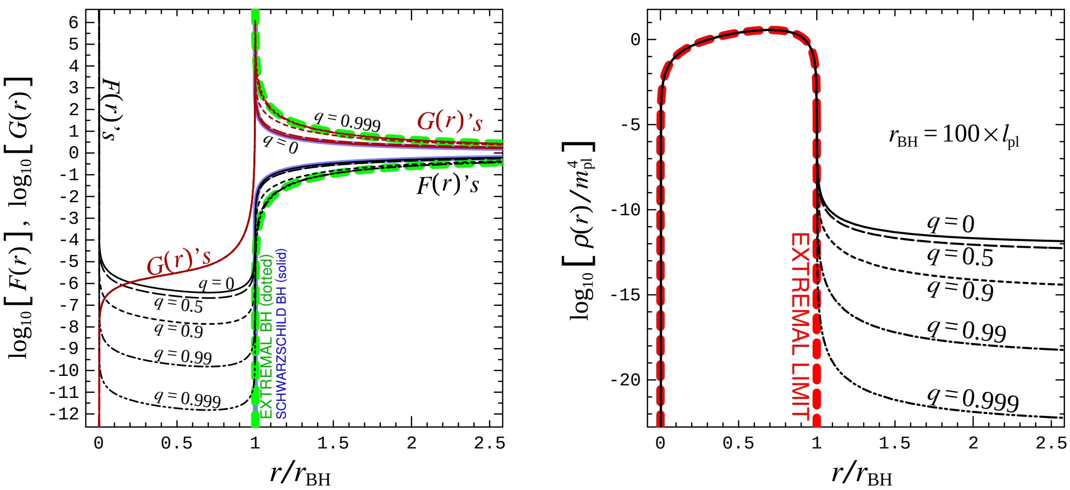


Fig. 4. Outline of the generic solution of the structure model with the many particle limit. The metric elements $F(r)$, $G(r)$ and the density distribution $\rho(r)$ are figured. The blue dotted curves indicate the Reissner-Nordström metric of the black hole corresponding to the model.



a) Elements of the metric $F(r)$ and $G(r)$

b) Distributions of the energy density $\rho(r)$

Fig. 5. a) Elements of the metric $F(r)$ and $G(r)$ and b) Distributions of the energy density $\rho(r)$ of the quantum fluctuations, in the solution of the structure model with the many particle limit. The numerical solutions for the radius $r_{\rm BH} = 100 \times l_{\rm pl}$, $g_* = 4$ and for various charges $q = 0, 0.5, 0.9, 0.99$ and 0.999 are presented. The horizontal axis is the coordinate r normalized by $r_{\rm BH}$. The thick dotted (green) curves and the thick solid (blue) curves indicate the exterior parts of the extremal RN metric ($q \to 1$) and the Schwarzschild metric ($q = 0$) respectively. The distribution in the structure ($r < r_{\rm BH}$) has a universal form. The density $\rho(r)$ approaches to the background density $\rho_{\rm BG}$ defined in (3·4) when r becomes large. We find that the function form of $G(r)$ in the structure is almost independent of q.

§8. Information of Black Hole and Decay of Baryon Number

In our structure model of black holes, any information of the structure is carried by the quantum fluctuation around the singularity. Therefore the Bekenstein entropy is explained by counting of the states of the quantum fluctuation.[5),6)]

We have obtained the temperature distribution $T(r)$ of the thermal fluid in the many body limit, thus the entropy density distribution can be calculated as $s = \frac{2\pi^2}{45} g_* T^3$. We calculate the total entropy of the structure as

$$S \simeq \int_0^{r_{\rm BH}} 4\pi r^2 \sqrt{G(r)}\, s\left(T(r)\right) \;=\; \frac{(8\pi)^{3/4}}{\sqrt{5}} m_{\rm pl}^2 r_{\rm BH}^2 \;\simeq\; 5.0199 \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2}. \qquad (8\cdot1)$$

This result reproduces the area law of the Bekenstein entropy, and is a little greater than the the Bekenstein entropy: $S/S_{\rm Bekenstein} \simeq 1.5978$. This slight difference seems to be caused by the approximation of the many body limit.

The baryon number is also carried by the internal quantum fluctuation in the same way of the entropy.[7)] Therefore we can define the baryon number density $b(r)$ in the structure, and we calculate the total baryon number of the structure as

$$B := \int_0^{r_{\rm BH}} 4\pi r^2 dr \sqrt{G(r)}\, b(r) \qquad (8\cdot2)$$

which is regarded as *the baryon number of the black hole.*

The Higgs scalar vev $\langle\phi\rangle$ in the structure vanishes and the symmetry of the electroweak (EW) theory restores because the temperature in the structure ($\sim$ Planck scale) is much greater than the EW scale ($\sim$ 100 GeV) and the thermal phase transition of the EW theory arises.[11)–13)] Therefore the sphaleron processes are working in the structure, and are changing the baryon number. The sphaleron transition rate for the proper-time is given by $\Gamma_{\rm sph} = \kappa\alpha_{\rm W}^4 T^4$, where $\kappa \sim O(1)$ is a numerical constant and $\alpha_{\rm W}$ is the weak gauge coupling constant at the Planck energy.[14)] When we assume GUT, the GUT interaction also violates the baryon number as the rate $\Gamma_{\rm GUT} = \kappa\alpha_{\rm GUT}^2 T^4$. The time-evolution of the baryon number density on the coordinate-time t is given by the Boltzmann-like equation:[15)] $\frac{d}{dt}b = -\frac{39}{2}\sqrt{F}\frac{\Gamma}{T^3}b$. By using the proper temperature ansatz (7·2), the Boltzmann-like equation becomes

$$\frac{d}{dt}b = -\mathcal{R}b \qquad (8\cdot3)$$

where we have defined the baryon number decay rate:

$$\mathcal{R} = \left\{ \begin{array}{lll} \frac{39}{2}\kappa\alpha_{\rm W}^4 T_{\rm BH} & \simeq 10^{-6}/r_{\rm BH} & \text{(Sphaleron)} \\ \frac{39}{2}\kappa\alpha_{\rm GUT}^2 T_{\rm BH} & \simeq 10^{-3}/r_{\rm BH} & \text{(GUT)} \end{array} \right. . \qquad (8\cdot4)$$

For an astrophysical black hole ($r_{\rm BH} \sim 1$km), the baryon number decay time becomes

$$1/\mathcal{R} = \left\{ \begin{array}{ll} \text{a few seconds} & \text{(Sphaleron)} \\ \text{a few mili-seconds} & \text{(GUT)} \end{array} \right. , \qquad (8\cdot5)$$

therefore, we find a rapid decay of the baryon number in the black hole.[7)]

§9. Conclusions

Our picture of a black hole is quite similar to that of an atom consisting of a charged nucleus and a quantized electron field of several quanta. The radius of the ball structure, namely the Schwarzschild radius, is quantized due to the quantization of the fluctuating field in the structure. Particularly we construct the minimum structure of the black hole. The field fluctuation in the minimul model contains only one quantum in the 1s-wave mode. The minimum model is quite similar to that of a hydrogen atom, whose electron is represented by the 1s-wave function. The radius of the minimum model, namely the minimum Schwarzschild radius, is approximately 1.1287 times the Planck length. There is an obvious analogy between the minimum Schwarzschild radius in our model and the Bohr radius of the hydrogen atom.

At the present time, a quantization of black holes seems to be difficult, because we have no correct theory of quantum gravity. However, the Bohr's model for atomic structure in history of old quantum theory is instructive: Bohr succeeded in quantizing the hydrogen atom by his structure model of the atom before proposition of the Schrödinger equation. Without a theory of quantum gravity, we propose the quantization of black holes by the structure model of black holes based on the Einstein gravity. Our aim is just to construct *the Bohr's quantization* of black holes.

Our model can be verified by the experiment which is schematically shown in Fig. 6. The Hawking radiation contains the internal baryon number, because the Hawking radiation is regard as a leak of the internal quantum fluctuations. If we observe the baryon decay curve (8·4) by the following experiment: 1) prepare a small black hole ($T_{\rm BH} > 1$ GeV) which can radiate protons, 2) input a baryon beam pulse into the black hole, and 3) observe the baryon number in the Hawking radiation, then we confirm our structure model of the black hole.

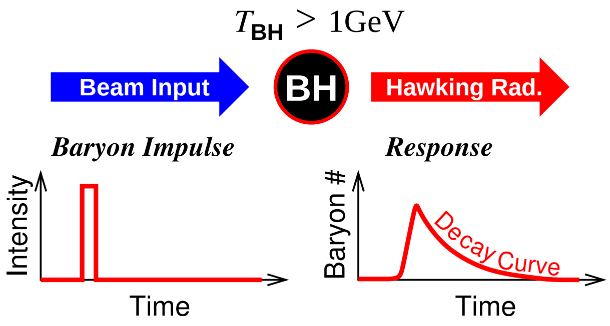


Fig. 6. The experimental verification of the structure model.

Acknowledgements

I would like to thank Ofer Aharony, Micha Berkooz, Hiroshi Ezawa, Satoshi Iso, Hiraku Kawai, Barak Kol, Masao Ninomiya, Kei Shigetomi and Kunio Yasue for useful discussions.

References

1) J. D. Bekenstein, Phys. Rev. D **7** (1973), 2333.
2) S. W. Hawking, Commun. Math. Phys. **43** (1975), 199; Nature **248** (1974), 30.
3) K. Hotta, Prog. Theor. Phys. **99** (1998), 427 [arXiv:hep-th/9705100].
4) N. Iizuka, D. Kabat, G. Lifschytz and D. A. Lowe, arXiv:hep-th/0306209.
5) Y. Nagatani, arXiv:hep-th/0310185.
6) Y. Nagatani, arXiv:hep-th/0311133.
7) Y. Nagatani, arXiv:hep-th/0312060.
8) Y. Nagatani, Prog. Theor. Phys. **113** (2005), 1123 [arXiv:hep-th/0501021].
9) Y. Nagatani, arXiv:hep-th/0307295.
10) G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
11) Y. Nagatani, Phys. Rev. D **59** (1999), 041301 [arXiv:hep-ph/9811485].
12) Y. Nagatani, Nucl. Phys. B **679** (2004), 363 [arXiv:hep-th/0307294].
13) Y. Nagatani, arXiv:hep-ph/0104160.
14) P. Arnold, D. Son and L. G. Yaffe, Phys. Rev. D **55**, 6264 (1997) [arXiv:hep-ph/9609481]; P. Huet and D. T. Son, Phys. Lett. B **393**, 94 (1997) [arXiv:hep-ph/9610259]; G. D. Moore, C. r. Hu and B. Muller, Phys. Rev. D **58**, 045001 (1998) [arXiv:hep-ph/9710436].
15) J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko and M. E. Shaposhnikov, Phys. Rev. D **60**, 123504 (1999) [arXiv:hep-ph/9902449].