Holography in the Large-J Limit of AdS/CFT Correspondence and Its Applications

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We give a brief expository discussion on the holographic correspondence of correlation functions in the large-J limit of AdS/CFT conjecture. We first review our proposals on the interpretation of the so-called GKPW relation in the large-J limit or BMN limit, which are based upon a tunneling picture in relating the AdS bulk to its boundary. Some concrete results, explicitly confirming our picture, are summarized. We then proceed to comment on various issues related to this subject, such as extension of the present picture to nonconformal Dp-brane backgrounds, the correlators of deformed Wilson loops, spinning-string/spin-chain correspondence, and the inclusion of higher string-loop effects. In particular, as for the deformation of Wilson loops, we present a typical tunneling world-sheet solution which can be used for direct derivation of the expectation values of deformed Wilson loops following our picture.

§1. Introduction

The AdS/CFT (or more generally *string/gauge*) correspondence has been one of central themes in recent developments of string theory, since its first proposal by Maldacena[1)] in 1997. It is a remarkable conjecture that makes possible to connect the gravitational physics of strings in bulk spacetimes to that of non-gravitational field theories living on their boundaries. Establishing such a 'holographic' correspondence would mean that we may achieve an entirely new synthesis of various different field theories using physics of strings and branes. On one hand, various old attempts, starting from the 70's, towards the derivation of string picture from gauge theories, are precursors to recent developments. A new perspective gained is the surprising possibility that an important part of gauge-theory dynamics may be encoded into spacetime physics in higher dimensions. On the other hand, from the viewpoint of string theory, gravitational physics in the bulk is equivalently described by lower dimensional systems associated with various branes. It is important to understand this correspondence, including its origin, applications, and extensions, as deeply as possible from complementary standpoints of field theory and of string theory.

However, it has been very difficult to check this correspondence when it involves really nontrivial dynamical aspects, since we do not yet have any systematic way of relating both sides from first principles and therefore we would be required to solve the dynamics to the extent that is necessary to check the correspondence for final physical outcomes. For instance, we do not have appropriate (or *intrinsic*) understanding on how the gauge principles of both sides, namely, general coordinate

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invariance in the bulk and the gauge invariance in internal space on the boundary, are related to each other. Because of this difficulty, we could gather some evidence for the correspondence only in special circumstances where (conformal) supersymmetry protects simple lowest order results against various complicated corrections.

In this situation, it is useful to study appropriate limiting cases where we can justifiably use some approximation schemes in deriving physical quantities on both sides and compare them directly. The large-J limit[2)] has been one of such limits by which some of most interesting results have been obtained in recent few years. It means that we consider physical states with very large angular momentum (J) along some direction in the bulk, and, correspondingly, gauge-invariant operators with large R-symmetry charges in the boundary theory. In the bulk, we can then have certain semi-classical treatments for string states, while on the boundary, as it turns out, we can apply perturbation theory with respect to a rescaled coupling constant $\tilde{\lambda} \sim g_{\rm YM}^2 N/J^2$ for gauge theories.

Even in such limiting cases, however, the large majority of investigations[3)] in this area have been concentrated to comparison of only the spectrum of conformal dimensions for some special sets of BPS or non-BPS operators. In spite of its importance, discussions on correlation functions have been very scarce. In this exposition, we first review main lines of our works[4)5)6)7),8)] in which we have been trying to put the correspondence in the large-J limit to the spirit of the original proposal in[9)10)] of direct holographic relation for correlators. Clarification of relevant issues from this viewpoint is indeed useful for better understanding of the meaning and the role of the large-J limit. For example, we have been able to extend the correspondence beyond the spectrum of conformal dimensions by proposing a concrete relation[7)] that expresses the OPE coefficients of non-BPS operators in terms of 3-point interaction vertices of string-field theory in the bulk. In the present article, we also give a new solution of infinitely extended string-world sheet which should be useful for studying the deformation of Wilson-loop operators from the viewpoint of bulk string theory. The author hopes that the present compact account of our woks and various comments on related issues serve the reader as a guide to our original works and as a stimulus for further investigations.

§2. How to reconcile PP-wave limit with holography?

2.1. *Standard argument of the PP-wave limit*

Let us start from recalling the standard argument[2)] of the PP-wave limit and the BMN limit associated with it. In the $\mathrm{AdS}_5\times\mathrm{S}^5$ metric using the global coordinate

$$ds^2 = R^2\Big[-\cosh^2\rho(dt_g)^2 + (d\rho)^2 + \sinh^2\rho d\Omega_3^2 + \cos^2\theta(d\psi)^2 + (d\theta)^2 + \sin^2\theta d\tilde{\Omega}_3^2\Big], \tag{2.1}$$

we first choose a null geodesic

$$t_g = \psi, \quad \rho = \theta = 0 \tag{2.2}$$

which traverses a large circle of S^5 at the center of the AdS_5 spacetime. If we are interested only in a vicinity of the geodesic, we can enlarge the region around the trajectory by redefining the coordinates as $r \equiv R\rho, y \equiv R\theta$, $x^+ = (t_g + \psi)/2, x^- = R^2(t_g - \psi)/2$ and taking the limit of large R. The metric then reduces to

$$ds^2 = -4dx^+ dx^- - x_i^2 (dx^+)^2 + (dx_i)^2 \tag{2.3}$$

where the index i runs from 1 to 8, of which the first 4 ($i = 1, \ldots, 4$) correspond to the part $(dy)^2 + y^2 d\tilde{\Omega}_3^2$ of the reduced metric and the last 4 ($i = 5, \ldots, 8$) to $(dr)^2 + r^2 d\Omega_3^2$. If we choose the light-cone coordinate x^+ as the world-sheet time τ, the metric is quadratic with respect to these transverse coordinates. We can then treat the quantum theory of strings with this background in terms of a free field theory on the world sheet as in flat spacetime.

A natural question is when this approximation is justified. Let us first study the center-of-mass degrees of freedom of strings moving along the null geodesic. The angular momentum along ψ is $J = \alpha R^2 \frac{d\psi}{d\tau}$ with α being the string-length ($0 \le \sigma \le 2\pi\alpha$) parameter in the sense of world-sheet in the light-cone gauge. The null geodesic can be regarded as a point-like classical solution of string equation of motion: $t_g = \psi = \frac{J}{\alpha R^2}\tau \equiv \frac{\mu}{\alpha'}\tau$ or equivalently, $x^+ = \frac{\mu}{\alpha'}\tau, x^- = x_i = 0$ with $\mu\alpha = \alpha' J/R^2$. We can consider the limit of large $R^2 = \sqrt{g_{\rm YM}^2 N}\alpha' (\gg \alpha')$ with $\mu\alpha$ being kept fixed. The limit $R^2/\alpha' \gg 1$ allows us to regard the quadratic form of the metric to be a semi-classical approximation to full string theory on the AdS background. Note that this limit can be taken with a finitely fixed α', since we have an independent parameter N which controls the higher-genus effect for a large but fixed AdS radius. We can consider stringy excitations, just as we do in flat background.

The bosonic part of the string effective action for the transverse part is then

$$S = \frac{1}{4\pi\alpha'} \int d\tau \int_0^{2\pi\alpha} d\sigma \Big[\Big(\frac{\partial x_i}{\partial \tau}\Big)^2 - \Big(\frac{\partial x_i}{\partial \sigma}\Big)^2 - \mu^2 x_i^2 \Big]. \tag{2.4}$$

As discussed in[11),12)] the full light-cone gauge action in the Green-Schwarz formalism is supersymmetric with 32 generators, as it should be since the above limit does not violate supersymmetry of the original AdS$\times S^5$ background, owing to the presence of the 5-form field strength. The latter is responsible for a mass term, the partner of the bosonic mass term in (2.4), for the world-sheet fermionic fields and reduces in the same limit as above to

$$F_{+1234} = F_{+5678} = 2\mu. \tag{2.5}$$

Among the 32 susy generators, a half of them correspond, in the language of boundary theory, to the ordinary maximal global supersymmetry of $\mathcal{N} = 4$ super Yang-Mills theory, and the other half correspond to the global supertranslation of the massless fermion fields.

The energy of a single free string in this background takes the form

$$P^- = \frac{1}{\alpha} \sum_{n=-\infty}^{\infty} \omega_n (a_{ni}^\dagger a_{ni} + b_{ni}^\dagger b_{ni}), \quad \omega_n = \sqrt{n^2 + (\mu\alpha)^2} \tag{2.6}$$

with n being the mode number with respect to the spatial world-sheet momentum and $a(b)$-operators come from bosonic (fermionic) world-sheet fields. As in the flat spacetime, we have to impose the level-matching condition,

$$\sum_{n=-\infty}^{\infty} n(a_{ni}^{\dagger}a_{ni} + b_{ni}^{\dagger}b_{ni})|\Psi\rangle = 0. \tag{2.7}$$

In particular, states with only zero mode excitations constructed from $a_{0i}^{\dagger}, b_{0i}^{\dagger}$ constitute the supergravity states. Namely, the tower of states generated by acting fermion operators to each given bosonic state that is constructed only by bosonic oscillators gives a $\mathbf{2^8} = \mathbf{128} + \mathbf{128}$ multiplet of Kaluza-Klein supergravity fields, which are 1/2-BPS states. The spectrum of these states

$$\Delta = \frac{P^-}{\mu} + J = N + J, \quad N = a_{0i}^{\dagger}a_{0i} + b_{0i}^{\dagger}b_{0i}$$

precisely coincides with that of conformal dimensions of the corresponding half-BPS operators of Yang-Mills theory. The ground state $N = 0$ corresponds to the operator $Tr(Z(x)^J)$ with $Z \equiv (\phi_5 + i\phi_6)/\sqrt{2}$ representing the basic unit of the U(1) R-charge corresponding to the large circle of the geodesic in S^5. A 'scalar' excitation of $a_{0i}^{\dagger}$, $i = 1, \ldots, 4$ corresponds for the 1/2-BPS operators to the insertion of an 'impurity' field $\phi_i(x)$ in a completely symmetric way. Similarly, a 'vector' excitation of $a_{0i}^{\dagger}$, $i = 5, \ldots, 8$ corresponds to the insertion of a derivation with respect to one of 4 base spacetime coordinates x_{i-4}.

In the case of nonzero-mode operators, the world-sheet momentum n is interpreted, in the language of boundary theory, as the discrete momentum associated with the insertions of scalar fields or of base spacetime derivatives along the 'string' of local operator products of approximate length J in the large J limit.

A crucial observation in[2)] is that the spectrum of P^-/μ for higher excited states with nonzero mode n gives the anomalous dimensions of non-BPS operators which are obtained by general *non*-symmetric and a finite number of insertions of impurity scalar fields and of base-spacetime derivatives. This has been explicitly confirmed to all orders within the planar approximation, at least for the case of scalar excitations, in perturbative expansion of the energy

$$\omega_n = \sqrt{n^2 + (\mu\alpha)^2} = \mu\alpha + \frac{n^2}{2\mu\alpha} + \cdots$$

with respect to $1/(\mu\alpha)^2 = R^4/J^2 = g_{\rm YM}^2 N/J^2$. Thus we can explicitly see the effect of higher stringy modes in the holographic relation between bulk string theory and gauge theory on its boundary by considering the large J limit appropriately.

The correspondence has been extended further to more general operators[13)] in which the number of Z and impurities are of the same order of magnitude. The corresponding strings in this generalization are interpreted[14)] as 'spinning strings' that are characterized by two or more independent angular momenta in the bulk, while on the gauge-theory side the spectrum of anomalous dimensions is obtained by mapping the mixing matrix of anomalous dimensions to a Hamiltonian of integrable spin-chains in statistical mechanics: different spin states in the latter are interpreted as different components within the strings of local products of scalar fields.

2.2. *Puzzles and resolution*

Let us now discuss this remarkable correspondence from the viewpoint of holographic principle of AdS/CFT correspondence. In the standard interpretation, the boundary which constitutes a holographic screen is located at the conformal boundary $\rho \sim \infty$ in terms of the AdS metric (2.1). Then the (3+1)-dimensional coordinates (t_g, Ω_3) on the boundary are identified with the $\mathrm{R}\times S^3$ foliation (or 'radial quantization') of the base spacetime of super Yang-Mills theory. In this sense, energies in the bulk are related to conformal dimensions in the boundary theory, since evolution operator in the latter can be identified with the dilatation operator in the Euclidean formulation.

Comparing this interpretation with the above ansatz for the correspondence between PP-wave string theory and super Yang-Mills theory, we notice apparent contradictions.[4] The BMN ansatz assumes that the four $(i = 5, \ldots, 8)$ of transverse directions which must be by definition orthogonal to the time direction are identified with the directions of base spacetime. According to the above idea of holography, however, the time direction of bulk and boundary is one and the same global time t_g. Also, while the BMN ansatz requires an Euclidean metric of base spacetime for the boundary, the PP-wave limit as formulated above clearly assumes a Minkowski metric. As a matter of fact, one might forget about the problematical four transverse directions by focusing attention only to the scalar impurities. But then we would lose our way to a systematic understanding of correlation functions from the viewpoint of holography, as we shall discuss below.

A possible attitude facing this perplexing situation could be to try to find a different interpretation of holography and study the boundary[15] for the PP-wave background *per se*, independently of its ancestor, AdS/CFT. We do not take this viewpoint, since it seems more natural to seek for a resolution by trying a reconciliation between the coincidence of anomalous conformal dimensions and the original interpretation of holography connecting AdS bulk and its boundary, since after all the BMN operators are nothing other than the operators of original Yang-Mills theory. In particular, it seems very desirable to keep the direct relation, embodied as the famous GKPW formula, between the correlation functions of boundary theory and the partition function of bulk theory.

From our standpoint, a crucial problem with respect to the holographic principle is that the null geodesic (2.2), sitting at the center of the AdS, does never reach the AdS boundary $\rho = \infty$. Thus one naively inclines to conclude that it would not be feasible to establish any direct link between the PP-wave geometry and the original AdS/CFT correspondence. That is not correct.

To get a useful perspective, it is instructive to consider this trajectory using the Poincaré metric $ds_{ads}^2 = R^2((dz)^2 + (d\vec{x})^2)/z^2$ for AdS_5 with $(d\vec{x})^2$ being the metric of $R^{3,1}$ as the base spacetime of the boundary, in terms of which the geodesic (2.2) is given as (t being the time coordinate of $R^{3,1}$)

$$z = 1/\cos t_g, \quad t = \tan t_g, \quad \psi = t_g. \tag{2.8}$$

Here for brevity, we dropped the physical parameters J and E from the equations. This is always possible by a suitable rescaling of the coordinates. The conformal boundary corresponds to $z = 0$, and $z = \infty$ is the horizon of the Poincaré patch. The periodicity in t_g reflects the fact that the AdS_5 background can be regarded as a universal covering of a hyperboloid embedded in $R^{5,1}$. In this picture, the global time coordinate t_g plays the role of the affine parameter along the trajectory. The trajectory does not reach the boundary because of the restriction $z \geq 1$ arising from the null condition (Virasoro condition) $(\dot{z}^2 - \dot{t}^2)/z^2 + \dot{\psi}^2 = 0$ with $\dot{t}/z^2 = 1$ and $\dot{\psi} = 1$. Thus we have effectively the one-dimensional constraint $\dot{z}^2 = z^2(z^2 - 1)$, which exhibits the existence of a centrifugal wall at $z = 1$ separating the central region $z \to \infty$ and the conformal boundary $z = 0$.

Now, if we make a Wick rotation $t_g \to -i\tau$, $t \to -ix_4$, for the global time and the boundary time simultaneously, the solution turns into

$$z = 1/\cosh\tau, \quad x_4 = \tanh\tau \tag{2.9}$$

since the null condition becomes $\dot{z}^2 = z^2(1 - z^2)$. Obviously, this corresponds to considering a 'tunneling' trajectory on the opposite side $z \leq 1$ of the wall, and correspondingly the trajectory now starts at $\tau = -\infty$ from the boundary and returns at $\tau = +\infty$ to the boundary, traversing a finite distance $\Delta x_4 = 2$ in the Euclideanized time direction. Thus we are led to treat the Euclideanized AdS (EAdS) background. The Wick rotation requires us a further rotation $\psi \to -i\psi$ along a large circle of S^5, which is understandable in the large J limit.

The fact that the bulk-boundary correspondence is indeed a tunneling phenomenon is easily seen from the behavior of bulk fields near the boundary: A massless scalar field in the bulk, for instance, behaves as $\phi(z, x) \to z^{-J}\phi(x)$ or $z^{J-4}\phi(x)$ near the boundary. According to,[10] the former *non-normalizable* solution gives correlation functions on the boundary with conformal dimension $\Delta = J+4$. The exponential behavior $\exp[\pm J\tau]$ for large $|\tau|$ is indicative of tunneling. When we do not consider the PP-wave limit, whether we use Minkowskian or Euclidean signature is more or less a matter of convention and is not a big problem for usual purposes. However, after the limit being taken, the topological character of the limiting spacetime in relation to the original AdS background critically depends on these choices. If one only considers the regions near the turning points $\tau \sim 0$ or $t_g \sim 0$, both of these trajectories pass through essentially the same region of the AdS spacetime. In this latter sense, both pictures may be connected. In particular, the free spectrum of strings in the PP-wave limit should be derivable equally from either viewpoint. But the tunneling picture is crucial for discussing correlation functions from the viewpoint of GKPW relation.

Near the boundary, a translation $\tau \to \tau + c$ with respect to the affine time is equivalent to the scaling $z \to \mathrm{e}^{\pm c}z$ of radial coordinate of the EAdS background. Because of the conformal isometry of the EAdS, this scaling is equivalent to a dilatation in R^4 on the boundary. This naturally explains the fact that the spectrum of the Hamiltonian for evolution with respect to the affine time coincides with that of the dilatation operator on the Yang-Mills side. This also gives a resolution for the question why we can identify the half ($i = 5, \ldots, 8$) of transverse directions with the base spacetime directions at the boundary, since now the direction of the tunneling trajectory is manifestly orthogonal to the boundary. Thus we have a very simple and natural resolution of the puzzles of the BMN correspondence and its holographic interpretation. It is straightforward to check that we obtain essentially the same metric and effective action for string propagation by blowing up the spacetime along the tunneling geodesic as (2.3), with an appropriate rearrangement of variables. Note that the light-cone time in this formulation flows along the tunneling trajectory. We refer the reader to[4)7)] for details of such calculations.

§3. Large J limit of the GKPW relation

The direct connection of the tunneling null geodesic to the EAdS boundary enables us to study the large J limit using the GKPW relation

$$Z[\phi]_{\rm bulk} \sim \left\langle \exp\Big[\int d^4x \sum_i \phi_i(\vec{x}) O_i(\vec{x})\Big] \right\rangle_{\rm boundary} \tag{3.1}$$

and thereby to examine the behavior of the holographic relation of correlators explicitly. Remember that this type of correspondence has further been generalized to (generalized) Wilson loop operators in[16)17)] and has led to many intriguing results. It is a matter of course that when we consider the limit of small Wilson loops the prescription of computing correlators of Wilson loops from the bulk viewpoint must reduce to (3.1).

This conjectural relation has been essentially confirmed at least in the case of two-and three-point functions of chiral primary operators by explicit computations on both sides. On the bulk side, the correlators are obtained diagrammatically by the so-called Witten diagram. For instance, in the leading planar approximation, a 3-point function of scalar operator takes the form

$$\int \frac{d^4\vec{x}dz}{z^5} K_{\Delta_1}(z,\vec{x};\vec{x}_1) K_{\Delta_2}(z,\vec{x};\vec{x}_2) K_{\Delta_3}(z,\vec{x};\vec{x}_3)$$

where

$$K_\Delta(z,\vec{x};\vec{y}) = \frac{\Gamma(\Delta)}{\pi^2 \Gamma(\Delta-2)} \Big(\frac{z}{z^2+(\vec{x}-\vec{y})^2}\Big)^\Delta \tag{3.2}$$

is the bulk-boundary propagator of scalar field, connecting a bulk position $(z, \vec{x})$ to a boundary point $(0, \vec{y})$ and satisfying

$$\lim_{z\to 0} z^{\Delta-4} K_\Delta(z,\vec{x};\vec{y}) = \delta(\vec{x}-\vec{y}). \tag{3.3}$$

Since, in the large J limit, the conformal dimension is also of order J, we can apply saddle-point method[7)] in evaluating the integral in the above expression. Let us first consider the simpler case of a 2-point function.

$$G_2(\vec{x}_1,\vec{x}_2)\equiv\int\frac{d^4\vec{x}dz}{z^5}\,K_\Delta(z,\vec{x};\vec{x}_1)K_\Delta(z,\vec{x};\vec{x}_2)z^\epsilon, \tag{3.4}$$

where the factor z^ϵ is introduced as a regularization. The saddle-point equation is then

$$\frac{\partial}{\partial z}\Big(\ln[\frac{z}{z^2+(\vec{x}-\vec{x}_1)^2}]+\ln[\frac{z}{z^2+(\vec{x}-\vec{x}_2)^2}]\Big)=0, \tag{3.5}$$

$$\frac{\partial}{\partial \vec{x}}\Big(\ln[\frac{z}{z^2+(\vec{x}-\vec{x}_1)^2}]+\ln[\frac{z}{z^2+(\vec{x}-\vec{x}_2)^2}]\Big)=0. \tag{3.6}$$

It is easy to see that the general solution takes the form

$$z_{saddle}=\frac{|\vec{x}_1-\vec{x}_2|}{2\cosh\tau},\quad \vec{x}_{saddle}=\frac{1}{2}(\vec{x}_1+\vec{x}_2)-\frac{1}{2}(\vec{x}_1-\vec{x}_2)\tanh\tau \tag{3.7}$$

with τ being an undermined integration constant. Evidently, this is precisely the tunneling trajectory (2.9), after making suitable scaling, shift and rotation of coordinates, provided that we identify the 'collective' coordinate τ with the affine time. This implies that in the large $J(\sim\Delta)$ limit the bulk spacetime can be effectively replaced by the vicinity around a tunneling trajectory which connects two points $\vec{x}_1,\vec{x}_2$ of AdS boundary where the local operators are inserted, as it should be from the discussion of the previous section.

The integration over the collective coordinate and over Gaussian fluctuations around the trajectory can be performed in a standard way. The measure factor is transformed as

$$\frac{dzd^4\vec{x}}{z^5}\Rightarrow d\tau d\tilde{z}d^3\vec{\tilde{x}}_\perp J(\tau) \tag{3.8}$$

with $\tilde{z}$ and $\vec{\tilde{x}}$ being the linearized fluctuation in the directions orthogonal ($\frac{dz_{saddle}}{d\tau}\delta\tilde{z}+\frac{d\vec{x}_{saddle}}{d\tau}\cdot\delta\vec{\tilde{x}}=0$) to trajectory and

$$J(\tau)=\sqrt{\Big(\frac{4\cosh^2\tau}{|\vec{x}_1-\vec{x}_2|^2}\Big)^4\cosh^2\tau}. \tag{3.9}$$

It turns out that the Jacobian factor $J(\tau)$ is precisely cancelled by the determinant coming from the integral over fluctuations, and the final result takes the form

$$G_2(\vec{x}_1,\vec{x}_2)\Rightarrow\frac{\Delta^2}{\pi^2}|\vec{x}_1-\vec{x}_2|^{-2(\Delta-\epsilon)}\int_{-\infty}^{+\infty}d\tau\,(2\cosh\tau)^{-\epsilon}. \tag{3.10}$$

By making a suitable renormalization in the limit $\epsilon\to 0+$, this gives the correct two-point function.

It seems clear that in order to utilize the same *single* tunneling trajectory for two-point functions for higher-point functions too, we have to arrange the points of operator insertions at the boundary $z\sim 0$ into two small bunches. Each insertion must be located close to either $\vec{x}_1$ or $\vec{x}_2$. In the case of a 3-point function, we can choose $\vec{x}_2$, renamed as $\vec{x}_c$, to be the center-of-mass point of two operator insertions (with R-charges J_2 and J_3 satisfying $J_1 = J_2 + J_3$) whose distance is 2δ. For sufficiently small δ, the same single trajectory as above can be a good approximation to the saddle point solution, until we approach too close the point $\vec{x}_c$. It turns out that, with respect to the affine time parameter τ, the range of validity of this approximation is controlled by the condition

$$\mathrm{e}^{-2\tau} > \frac{J_2 J_3}{J_1} \frac{(2\delta)^2}{|\vec{x}_1 - \vec{x}_c|^2},$$

and the final integral form of 3-point function after integration over the fluctuations is

$$\begin{aligned} &\frac{\pi^2}{J_1^2} |\vec{x}_1 - \vec{x}_c|^{-(\Delta_1+\Delta_2+\Delta_3)} \int_{-\infty}^{+\infty} d\tau \, \mathrm{e}^{-(\Delta_1-\Delta_2-\Delta_3)\tau} \exp[-\frac{J_2 J_3}{J_1} \frac{(2\delta)^2}{|\vec{x}_1 - \vec{x}_c|^2} \mathrm{e}^{2\tau}] \\ &= \frac{\pi^2}{J_1^2} |\vec{x}_1 - \vec{x}_c|^{-2\Delta_1} (2\delta)^{-(\Delta_2+\Delta_3-\Delta_1)} \Big(\frac{J_2 J_3}{J_1}\Big)^{-(\Delta_2+\Delta_3-\Delta_1)/2} \frac{\Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2}+1)}{-\Delta_1+\Delta_2+\Delta_3}. \end{aligned} \tag{3.11}$$

Note that in our small δ limit this is consistent with the general form of 3-point functions of operators with definite conformal dimensions with OPE coefficient C_{123},

$$\frac{C_{123}}{|\vec{x}_1 - \vec{x}_2|^{2\alpha_3} |\vec{x}_2 - \vec{x}_3|^{2\alpha_1} |\vec{x}_3 - \vec{x}_1|^{2\alpha_2}} \sim \frac{C_{123}}{|\vec{x}_1 - \vec{x}_c|^{2\Delta_1} |2\vec{\delta}|^{2\alpha_1}} \tag{3.12}$$

where $\alpha_1 = (\Delta_2 + \Delta_3 - \Delta_1)/2$ etc.

It is instructive to rewrite (3.11) in the form

$$\frac{\epsilon^{\Delta_1-\Delta_2-\Delta_3}}{-\Delta_1+\Delta_2+\Delta_3} \times \Big(\frac{J_2 J_3}{J_1}\Big)^{\frac{\Delta_1-\Delta_2-\Delta_3}{2}} \Gamma(\frac{-\Delta_1+\Delta_2+\Delta_3}{2}+1), \quad \epsilon = 2\delta.$$

The first factor has been predicted on the basis of the tunneling picture in our first paper[4)] in which, owing to an ambiguity of regularization, we could not have derived the second factor originating from the exponential cutoff factor in the integral form (3.11). The cutoff factor leads to the fact that the energies (conformal dimensions) are not conserved even though we are considering 'S-matrix' along the tunneling trajectory. By contrast, if we have naively considered S-matrix in the PP-wave background in the standard interpretation, energies must be conserved, and hence we would not be able to relate scattering matrix elements in the Minkowski signature to the correlation functions of gauge theory in any meaningful manner.

Thus we have now established a concrete way of dealing with correlators in the PP-wave background by reconciling the large J limit with the original formulation of AdS/CFT holography. We would like to invite the interested readers to our original papers[7)8)] which contain all the details and necessary extensions of the above arguments.

For scalar chiral primary operators, the precise relation between the OPE coefficient and the three-point interaction vertex of effective field theory in the bulk has been known from the work.[18] The bulk effective action is

$$S_{bulk} = \frac{4N^2}{(2\pi)^5}\int d^5x\sqrt{g}\Big[\sum_I \frac{1}{2}(\nabla\phi_I)^2+\frac{1}{2}k(k-4)\phi_I^2-\frac{1}{3}\sum_{I_1,I_2,I_3}\frac{\mathcal{G}_{I_1I_2I_3}}{\sqrt{A_{I_1}A_{I_2}A_{I_3}}}\phi_{I_1}\phi_{I_2}\phi_{I_3}\Big] \tag{3.13}$$

where we used Euclidean metric and assumed a particular normalization for the scalar fields:

$$A_I = 2^{6-k}\pi^3\frac{k(k-1)}{(k+1)^2}, \tag{3.14}$$

where $k = \Delta = J + \tilde{k}$ is the conformal dimension with $\tilde{k}$ being the number of impurities exciting in the 4 directions $i = 1, \ldots, 4$. In our large J limit, the known relation between the 3-point vertex to the OPE coefficient reduces to ($\tilde{\Sigma} = \tilde{k}_1 + \tilde{k}_2 + \tilde{k}_3$)

$$C^{I_1I_2I_3} = \frac{1}{N}\frac{2^{J_1+\frac{\tilde{\Sigma}}{2}-9}}{\pi^3}\frac{\sqrt{J_1J_2J_3}}{J_1^2}\Big(\frac{J_1}{J_2J_3}\Big)^{\alpha_1}\frac{\alpha_1!}{\alpha_1}\mathcal{G}_{I_1I_2I_3}. \tag{3.15}$$

It is easy to check that, apart from the normalization factor which is independent of $\alpha_1 = (\Delta_2 + \Delta_3 - \Delta_1)/2 = (\tilde{k}_2 + \tilde{k}_3 - \tilde{k}_1)/2 \equiv \tilde{\alpha}_1$, the J dependence of this relation coincides with the result of our saddle-point computation (3.11). In particular, the prefactor $\frac{\alpha_1!}{\alpha_1} \propto \Gamma(1 + (\Delta_2 + \Delta_3 - \Delta_1)/2)/(\Delta_2 + \Delta_3 - \Delta_1)$ is responsible for the fact that the CFT coefficient $C^{I_1I_2I_3}$ can be *finite* even when the 3-point vertex $\mathcal{G}_{I_1I_2I_3}$ *vanishes* in the extremal case $\alpha_1 = 0$. By this check, we can now fix the absolute normalization of 3-point correlators in relating 3-point correlators on the boundary to 3-point vertex of the effective field theory in the bulk.

§4. Holographic relation and string field theory in the BMN limit

The foregoing discussion is a basis for our proposal[7] of general holographic relation between bulk string field theory and the OPE coefficients of BMN operators. First we derive the effective theory around the tunneling geodesic in supergravity approximation, by making a redefinition of scalar field in S_{bulk} as

$$\phi(\tau,\vec{y}) = \frac{(2\pi)^{5/2}}{2N}\frac{1}{\sqrt{2J}}\mathrm{e}^{-J\tau}\sum_n \phi_n^{(J)}(\vec{y})\exp[-\frac{1}{2}J\vec{y}^{\,2}]\psi_n(\tau), \tag{4.1}$$

$$\overline{\phi}(\tau,\vec{y}) = \frac{(2\pi)^{5/2}}{2N}\frac{1}{\sqrt{2J}}\mathrm{e}^{+J\tau}\sum_n \phi_n^{(J)}(\vec{y})\exp[-\frac{1}{2}J\vec{y}^{\,2}]\overline{\psi}_n(\tau), \tag{4.2}$$

where $\phi_n^{(J)}(\vec{y})$ together with the exponential factor $\mathrm{e}^{-J\tau-\frac{1}{2}J\vec{y}^2}$ are the wave functions for the fluctuations along the trajectory whose Hamiltonian is the four-dimensional harmonic oscillator,

$$h = -\frac{1}{2}\partial_y^2 + \frac{1}{2}J^2\vec{y}^{\,2}. \tag{4.3}$$

Note that the S^5 part of wave function is already factorized in these expansions. Taking the large J limit, we derive the effective action

$$\int d\tau \sum_I \Big[\frac{1}{2}(\overline{\psi}_I \partial_\tau \psi_I - \partial_\tau \overline{\psi}_I \psi_I) + \overline{\psi}_I{:}h{:}\psi_I\Big] + \frac{1}{2}\int d\tau \sum_{I_1,I_2,I_3} \lambda_{\overline{I}_1,I_2,I_3}(\overline{\psi}_{I_1}\psi_{I_2}\psi_{I_3} + h.c.), \tag{4.4}$$

$$\lambda_{\overline{I}_1,I_2,I_3} = \frac{1}{\pi^3 N} 2^{-8+J_1} \frac{2^{\tilde{\Sigma}/2}}{J_1^2}\sqrt{J_1 J_2 J_3}\, \mathcal{G}_{\overline{I}_1,I_2,I_3}, \tag{4.5}$$

with $J_1 = J_2 + J_3$. The zero-point energy is precisely cancelled by the contribution coming from τ-differentiation of the energy factor $e^{\pm J\tau}$. It is not difficult to extend this action by including the remaining 4 directions corresponding to the derivatives of BMN operators with respect to the 4 dimensional base spacetime. In the oscillator basis, the final form of the effective action turns out to be

$$S_{eff} = S_2 + S_3, \tag{4.6}$$

$$S_2 = \int d\tau \Big[\frac{1}{2}\langle\overline{\psi}|\partial_\tau|\psi\rangle - \frac{1}{2}(\partial_\tau\langle\overline{\psi}|)|\psi\rangle + \langle\overline{\psi}|h_{sv}|\psi\rangle\Big] \tag{4.7}$$

with

$$h_{sv} = h_s + h_v, \quad h_s = \frac{1}{R}\sum_{i=1}^{4} a_i^\dagger a_i, \quad h_v = \frac{1}{R}\sum_{j=5}^{8} a_j^\dagger a_j, \tag{4.8}$$

$$S_3 = \frac{1}{2N}\int d\tau\, {}_{(1)}\langle\overline{\psi}|_{(2)}\langle\psi|_{(3)}\langle\psi|\sqrt{J_1 J_2 J_3}(h_s^{(2)} + h_s^{(3)} - h_s^{(1)})|v_0\rangle + h.c.\,, \tag{4.9}$$

$$|v_0\rangle = \exp\Big[-\frac{1}{2}\sum_{r,s=1}^{3}\Big(\sum_{i=1}^{4} a_{i(r)}^\dagger n_{00}^{rs} a_{i(s)}^\dagger + \sum_{j=5}^{8} a_{j(r)}^\dagger n_{00}^{rs} a_{j(s)}^\dagger\Big)\Big]|0\rangle \tag{4.10}$$

where n_{00}^{rs} are the supergravity approximation of the well-known Neumann functions of string-field theory restricted to the zero-mode part in the zero-slope limit $\alpha' \to 0$:

$$n_{00}^{rs} = \delta^{rs} - \sqrt{\frac{J_r J_s}{J_1^2}}, \quad n_{00}^{r1} = n_{00}^{1r} = -\sqrt{\frac{J_r}{J_1}} \qquad \text{for} \quad r,s = 2,3 \quad \text{and} \quad n_{00}^{11} = 0. \tag{4.11}$$

The most characteristic feature of this result is that the pre-factor in the 3-point vertex does not contain the Hamiltonian h_v of the vector excitations. This conclusion is obtained[7)] by a detailed examination of descendents of chiral primary operators with respect to base spacetime derivatives.

The emergence of string field theory in the zero-slope limit is expected, since we know that supergravity approximation is sufficient to reproduce the correlator in the case of chiral primary operators. To treat non-BPS operators, however, we have to go beyond the zero-slope limit. Actually, even for lowest supergravity modes, the Neumann functions[19)] are subject to α'-corrections as

$$n_{00}^{rs} \to N_{00}^{rs} = f n_{00}^{rs} \quad \text{for } r = 2,3, \quad f = 1 - 4\mu(\alpha')^3 \frac{J_1 J_2 J_3}{R^3} K$$

with K being a complicated function expressible in terms of the Neumann coefficient. In the limit of large μ with finite α', f behaves as $\frac{R^2 J_1}{4\pi\alpha' J_2 J_3}$. Thus, the holographic relation should be generalized even for chiral primaries to finite α' such that the OPE coefficients for the chiral primary operators are not modified but the 3-point vertex are replaced by string field theory of finite α' corresponding to the above replacement.

In this way, we are led to propose the following ansatz for the holographic relation between OPE coefficient and 3-point vertex of string field theory in the PP-wave background in our Eculidean or tunneling picture:

$$C_{123} = \frac{\tilde{\lambda}_{123}}{\Delta_2 + \Delta_3 - \Delta_1}, \tag{4.12}$$

$$\tilde{\lambda}_{123} = \Big(f\frac{J_2 J_3}{J_1}\Big)^{-(\Delta_2+\Delta_3-\Delta_1)/2} \Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2}+1)\lambda_{123} \tag{4.13}$$

where λ_{123} is the 3-point interaction vertex of string field theory whose kinetic part is normalized in the same way as the supergravity effective action given above. For chiral primary operators in the left hand side and hence for states with only zero-mode oscillators in the right hand side, the additional factor of f in this ansatz is precisely cancelled by the modification of Neumann functions as above. However, for non-BPS states and hence for higher excited oscillators, the factor f plays nontrivial roles[8] in relating bulk and boundary.

It is important here to emphasize that our discussion in fact puts also a nontrivial constraint in exploring the correct 3-point interaction vertex of string field theory. Namely, for states with only zero-mode excitations, the 3-point vertex must be equal to

$$\lambda_{123} = {}_{(1)}\langle 1|_{(2)}\langle 2|_{(3)}\langle 3|\frac{\sqrt{J_1 J_2 J_3}}{N} R(h_s^{(2)} + h_s^{(3)} - h_s^{(1)})|V_0\rangle, \tag{4.14}$$

and the corresponding interaction part of the action is

$$S_3 = \frac{1}{2}\int d\tau\, {}_{(1)}\langle\overline{\psi}|_{(2)}\langle\psi|_{(3)}\langle\psi|\frac{\sqrt{J_1 J_2 J_3}}{N}(h_s^{(2)} + h_s^{(3)} - h_s^{(1)})|V_0\rangle + h.c. \tag{4.15}$$

with $|V_0\rangle$ being the one obtained from $|v_0\rangle$ by replacing the supergravity approximation of the Neumann functions by the exact ones N_{00}^{rs}.

The interaction vertex must also respect supersymmetry. As has already been obvious in our first work, supersymmetry is not sufficient to fix the form of 3-point vertex in string field theory in the PP-wave limit. Indeed, there have been different proposals. Unfortunately, neither of previous proposals alone were consistent with the above condition. It turned out that the particular combination of two different types of vertex satisfies the above condition. With the standard normalization, the correct choice of the vertex state for general string states can be expressed as

$$|H_3\rangle_h \equiv \frac{1}{2}(|H_3\rangle_{SV} + |H_3\rangle_D) \tag{4.16}$$

where first term is the most familiar vertex discussed in[20] and the second term is the one advocated in.[21] If we restrict our attention to states with only zero-mode oscillators, this form reduces to (4.15),

$$|H_3\rangle_h \to (h_s^{(2)} + h_s^{(3)} - h_s^{(1)})|V_0\rangle.$$

Thus the 3-point vertex of full 'holographic' string field theory should be

$$\lambda_{123} = {}_{(1)}\langle 1|_{(2)}\langle 2|_{(3)}\langle 3|\frac{\sqrt{J_1 J_2 J_3}}{N}|H_3\rangle_h. \tag{4.17}$$

The difference between two terms in (4.16) lies in the choice of the so-called prefactor that multiplies the familiar overlap function $|V_0\rangle$ representing the continuity of string field configurations. In the former choice $|H\rangle_{SV}$, the prefactor is determined by making a straightforward generalization of flat space result. In the latter $|H\rangle_D$, the prefactor is essentially the energy factor $H^{(2)} + H^{(3)} - H^{(1)}$ using the free worldsheet Hamiltonian $H^{(r)}$ corresponding to three external lines $r = 1, 2, 3$. In fact, this particular form has already been suggested in our first paper.[4] Each of these two proposals has been associated with a different way of relating quantities between bulk and boundary. Our proposal is therefore useful to disentangle different ways of relating bulk and boundary, and explain the origins of such seemingly *ad hoc* prescriptions on the basis of the original AdS/CFT correspondence as discussed detail in.[7]

References[7][8] presented nontrivial explicit checks of our holographic relation formulated as the formulae (4.12) $\sim$ (4.17) in the leading large μ expansions. In particular, it was confirmed that the above holographic relation is valid for the case of impurity *non*-preserving 3-point amplitudes at least to our leading expansion, as well as for the cases of preserved impurities. The structure of the above particular combination of possible prefactors of string field theory and also the additional factor f in the holographic relation play key roles in obtaining these results. We emphasize that, with respect to this universality of the relation of OPE coefficients of BMN operators and string field theory, there has been no other competing proposal. Previously, there has been no discussion on impurity non-preserving processes.

It should also be mentioned that our particular choice of string-field theory vertex may not be completely unique, since we have motivated it by studying the zero-mode part. Although we have many nontrivial checks for the cases with small number of insertions of impurities with nonzero modes, they are not yet sufficient for excluding the possibility of modifying string-field theory vertex further. In fact, the work[22] have suggested a possible modification which does not affect all of explicit checks mentioned above but may affect more complicated insertions of fermionic impurities. In principle, of course, it would be possible to derive the string field theory in this particular PP-wave background, once we have a firm candidate of string field theory on the original $\mathrm{AdS}_5 \times S^5$ background. However, in the situation that the latter program is impracticable at the present time, our result in a limiting case may hopefully provide a useful data.

§5. Further applications and extensions

In this final section, we summarize some applications or extensions of our ideas, and then discuss remaining issues.

5.1. *Case of Dp-brane background*

It is generally believed that the AdS/CFT correspondence should be understood as a special typical case of more universal string/gauge correspondence. From the viewpoint of string theory, the origin of the correspondence is the world-sheet string duality between closed strings and open strings. The origin of the world-sheet string duality is directly neither conformal symmetry, nor (spacetime) supersymmetry, that are effective in the AdS/CFT case. Moreover, in the case of bosonic string, it is known that Witten's version of open string field theory in perturbative genus expansion covers the whole moduli space of Riemann surfaces including the effects of exchange of closed strings. Therefore, it is not at all unreasonable to imagine the situation where we can generalize the correspondence to more general *non*symmetric backgrounds and their boundaries. Recently, in fact, there has been many 'phenomenological' attempts towards construction of such holographic duals to realistic QCD and its variants.

Before going down to such a level, it would also be interesting to study the case of backgrounds produced by general Dp-branes that have in general smaller supersymmetries and have no conformal symmetry in the usual sense. For instance, in the case of $p = 0$, the gauge theory is nothing but the Yang-Mills super quantum mechanics which can be interpreted as a discretized light-cone formulation of M-theory or just as the effective low-energy theory of D-particles. In the work,[6] an extension of tunneling picture to such cases has been investigated in order to predict the behaviors of two-point functions for corresponding boundary gauge theories.

Consider the general Dp-brane background

$$ds^2 = q_p^{1/2}\Big[H^{-1/2}(-dt^2 + d\tilde{x}_a^2) + H^{1/2}(dr^2 + r^2 d\psi^2 + r^2\cos^2\psi d\Omega_{7-p}^2)\Big], \qquad (5.1)$$

with $H = 1/r^{7-p}$ and $q_p \propto g_{\rm YM}^2 N$, where a runs from 1 to p. For general $p(\neq 3)$, the horizon $r = 0$ is a real singular point. Thus it is not meaningful to consider the correspondent of the ordinary null geodesic located at the 'center' of the background. But for $p < 5$, we can consider tunneling trajectories after making the Wick rotation as in the case of $p = 3$, which satisfy

$$\dot{r} = \pm H^{-1/2}\sqrt{\ell^2 r^{-2} - H} = \pm r_0^{-(5-p)/2}\sqrt{r^{5-p} - r_0^{5-p}}. \qquad (5.2)$$

For $p < 3$, there is a difference from the case $p \geq 3$ in that in terms of the natural affine parameter used in the above equation, the time interval $2T_b$ connecting boundary ($r = \infty$) and boundary is finite,

$$T_b = \left(\frac{2}{5-p}\right)^{\frac{7-p}{5-p}} \int_0^{z_0} dz \frac{z^{-\frac{2}{5-p}}}{\sqrt{z_0^2 - z^2}}$$

where the integration variable z is related with the original radial coordinate r by $z = 2r^{-(5-p)/2}/(5-p)$. But conceptually this is not an obstacle for applying our basic ideas.

The background metric in the PP-wave limit can be obtained by the similar methods as in the case of D3-branes, and the effective string action for fluctuating string coordinates turns out to be

$$S^{(2)} = \frac{1}{4\pi}\int d\tau \int_0^{2\pi\alpha} d\sigma \Big\{ \dot{\tilde{x}}_a{}^2 + \tilde{x}_a'{}^2 + m_{\tilde{x}}^2(\tau)\tilde{x}_a^2 + \dot{x}^2 + x'^2 + m_x^2(\tau)x^2 + \dot{y}_l{}^2 + y_l'^2 + m_y^2(\tau)y_l^2 \Big\}, \tag{5.3}$$

where[23)]

$$m_{\tilde{x}}^2 = m_x^2 = -\frac{(7-p)}{16r^2}\left\{(3-p) + (3p-13)\ell^2 r^{5-p}\right\}, \tag{5.4}$$

$$m_y^2 = -\frac{(7-p)}{16r^2}\left\{(3-p) - (p+1)\ell^2 r^{5-p}\right\}. \tag{5.5}$$

Here the p-dimensional vector $\tilde{x}_a$ can roughly be regarded as the fluctuations in the spatial directions of Dp-brane, x in the direction of Euclideanized time, and the $(7-p)$-dimensional vector y_ℓ in the directions transverse to Dp-branes. Comparing to the case of D3-branes, a complication is that the world-sheet mass parameters now depend on the affine time through the function $r = r(\tau)$ of the above classical solution. For $p < 3$, the masses asymptotically increase indefinitely as we approach the boundary, while for $5 > p > 3$ they vanish asymptotically. This leads[6)] to physical consequences that are drastically different from the conformal case, but are consistent with expected IR properties of non-conformal Yang-Mills theory.

We can apply the same picture as the case $p = 3$ to compute 2-point functions by semiclassical quantization of string theory around the tunneling trajectory. In,[6)] we have developed a systematic method for dealing with the time dependence of mass for the purpose of deriving S-matrix along the tunneling trajectory. The general form of two-point functions for operators involving only zero-modes obtained there is

$$\langle O(x_1)O(x_2)\rangle \sim (|x_1 - x_2|\Lambda)^{-\frac{4}{5-p}J - 2N_0^x - \frac{4}{5-p}N_0^y - 2C_0(p)} \tag{5.6}$$

(Λ being a renormalizable regularization parameter which is always necessary in deriving two-point functions from the bulk viewpoint) in the case of bosonic excitations. For $p \neq 3$, there is no conformal symmetry, but the above general form is consistent with a generalized conformal symmetry proposed in.[24)] Here $N_0^{x,y}$ denote the number of excitations along the directions longitudinal (x) or transverse (y), respectively, to the directions of Dp-branes. The constant C_0 which is related to the zero-point energy on the world sheet is calculated by taking into account the fermionic excitations to be $C_0(p) = -(3-p)^2/2(5-p)$, which does not vanish except for $p = 3$ indicating that the world-sheet supersymmetry is in general violated. This result is consistent with some previous results obtained on the basis of standard linearized analyses of supergravity, in particular with.[25)]

Extension to the case involving higher string excitations has also been discussed in the first and third references in[6)] together with some interesting implications for the IR behaviors of the boundary gauge theories.

A suggestive observation related to these results is that we can extract effective dimensions $d_{eff} = (14-2p)/(5-p)$ from the exponent of (5.6) for transverse excitations by interpreting the two-point functions as those of free scalar fields living on d_{eff}-dimensional effective base spacetime, $d_{eff} - 2 = 4/(5-p)$. Of course, for $p = 3$, we have $d_{eff} = 4$ as it should be. Magically enough, we have $d_{eff} = 3$ or 6 for $p = 1$ and $p = 4$, respectively. It is tempting to relate this phenomena as being a manifestation of M2 and M5 branes, respectively, emerging from the large N limit of effective D1 and D4 branes. In fact, in the case $p = 1$, we can adopt the IIA matrix-string interpretation for D1 branes of type IIB theory. Then the Yang-Mills coupling constant is reversed, and hence the present weak coupling limit must be reinterpreted as the strong coupling M-theory limit, that corresponds to a decompactification of the radius $R_{11} \propto 1/g_{\rm YM}^2 \to \infty$ along the 11-th direction. In fact, we have discussed in[26] how 11-dimensional supermembrane is related to the matrix-string theory. Interpretation of $p = 4$ case seems more mysterious.

5.2. *Case of spinning-string/spin-chain correspondence*

Let us go back again to the conformal case $p = 3$. In principle, it is straightforward to extend the analysis explained in sections 3 and 4 to the case of spinning strings.[13)14)] Only difference from the BMN limit is the existence of two or more independent angular momenta that are sent to infinity. The dominant tunneling trajectory in the EAdS background describes the center-of-mass motion and therefore can be taken to be the same (2.9) or (3.7) as in the case of a single angular momentum, since its emergence depends only on the limit of large conformal dimensions. Therefore, only the treatment of the degrees of freedom along the S^5 directions is affected. The ground state in the case of a single angular momentum corresponds to a point-particle solution moving along a great circle of the S^5. In the case of two or more angular momenta, by contrast, we have to take into account stringy extension (hence, the naming of 'spinning strings') even in obtaining classical solutions as shown in.[27)]

Since almost all of previous discussions of spinning strings/spin chain correspondence are focused on the comparison of the spectrum between the dilatation operator and the string energy, it is a useful check to derive two-point correlators directly using the same method as the argument in section 4 or as in the previous subsection. This has recently been done in[28)] to which we refer the reader. It would also be an interesting exercise to further study 3-point functions of spin-chain operators from this viewpoint and to investigate how the string field theory of spinning strings should look like.

5.3. *Deformed Wilson loop operators*

Any local operators corresponding to those spinning states can be regarded as arising from Wilson loops extending along the R-charge directions. Thus, computations being discussed here can also be regarded as computing the correlators of two small Wilson loops in the boundary gauge theory. From this viewpoint, it would also be interesting to investigate large Wilson loops and its infinitesimal deformations, corresponding to insertions of local gauge-covariant operators midst the loops, such as

$$W_{12,J} = \mathrm{Tr}\Big[\mathcal{P}\exp\Big(\oint d\sigma A_\mu(\vec{x}(\sigma))\dot{x}^\mu + \phi^i(\vec{x})\sqrt{(\vec{x}')^2}\theta^i\Big)Z(\vec{x}(\sigma_1))^J\overline{Z}(\vec{x}(\sigma_2))^J\Big]. \quad (5.7)$$

Some examples of this type have been considered recently in[29] for the case of circular or straight-line Wilson loops. Reconsidering this case from the viewpoint of our tunneling picture seems useful for clarifying the holographic interpretation in a way consistent with the picture of references.[16)17)*)]

The string world sheet corresponding to a (doubled) straight-line Wilson loop is an infinite plane (actually of double sheet) extending from the EAdS boundary to its center (and back to the boundary again). Insertion of a local gauge-covariant operator amounts to a deformation of the plane at the vicinity around the point of insertion on the boundary. Such local deformations in general propagate into the bulk of the world-sheet plane and are expected to produce a similar (or perhaps the same) trajectory as our tunneling geodesics in the limit of large J.

It is indeed not difficult to construct the typical form of desired solution, which represents the particular deformation corresponding to (5.7). In the conformal gauge, we find

$$(z, x_4)_\pm = \Big(\frac{\sinh\sigma}{\cosh\sigma\cosh\tau \pm 1}, \frac{\cosh\sigma\sinh\tau}{\cosh\sigma\cosh\tau \pm 1}\Big), \quad (5.8)$$

$$(\cos\theta, \psi) = (\tanh\sigma, \tau) \quad (5.9)$$

where the angle coordinates (θ, ψ) are those used in (2.1) with the Wick rotation $\psi \to -i\psi$. Here, the (single) infinite string world sheet is represented by two coordinate patches corresponding to the sign $\pm$ in (5.8), each of which is parametrized in terms of the world-sheet coordinates τ and σ, running from $-\infty$ and 0, respectively, to ∞ and is connected smoothly to each other along a one-dimensional line defined by $\sigma = \infty$. The two patches are related by an inversion $(z, x_4) \to (\frac{z}{z^2+x_4^2}, \frac{x}{z^2+x_4^2})$ which is an isometry of the EAdS. The trajectory along which this connection is made coincides with our tunneling trajectory which is nothing but a unit circle in the (z, x_4) plane; namely, we have

$$\lim_{\sigma\to\infty}(z, x_4)_\pm = \Big(\frac{1}{\cosh\tau}, \tanh\tau\Big).$$

The equation (5.9) together with this last property means that, at $x_4 = \pm 1, z = 0$ corresponding to the ends (at $\tau = \pm\infty$) of this trajectory reaching the conformal boundary in EAdS, the string rotates around a large circle $\theta = 0$ in S^5 in the same manner as in the case of subsection 2.2, while, away from these two points, θ

*) In ref.,[29] the authors use the 'standard' picture in the interpretation of the PP-wave limit. We cannot therefore use their solution in order to study the expectation value of the deformed Wilson loop operator. Their solution is subject to no deformation near the conformal boundary, since the points of deformations are sent to $\pm\infty$ on the boundary. The effect of insertions of local gauge-covariant operators is interpreted to correspond to the property that solution exhibits rotation around an S^5 circle when we approach the AdS center. In fact, one of two patches of the solution discussed below is related to their solution by our Wick rotation, which drastically changes the topology of the solution as emphasized before.

approaches the pole ($\theta = \pi/2$) of S^5, corresponding to no rotation. This is precisely the required property corresponding to the above deformation occurring at $x_4 = \pm 1$ of the boundary of the string world sheet which is simultaneously at the EAdS boundary. The solution representing the plane without deformation is simply given by a single patch $(z, x_4) = (\frac{\sigma}{\tau^2+\sigma^2}, \frac{\tau}{\tau^2+\sigma^2}), (\theta, \psi) = (\frac{\pi}{2}, 0)$ with the same range of world-sheet coordinates. It is straightforward to apply similar arguments as for the correlators of local operators to obtain the expectation value of the deformed Wilson-loop operator (5.7). More detailed study will be reported elsewhere.

5.4. *Bubbling 'E'AdS and correlators*

Another interesting path of investigation seems to consider the Euclideanized version of the so-called bubbling AdS configurations. As is now well known, such configurations with 1/2 of the full supersymmetry and with SO(4)×SO(4)×R isometry can be characterized, in the Minkowski signature, by droplet configurations[30] defined on a two-dimensional plane embedded in the bulk. The case of a single circular droplet, the ground state, corresponds to the $AdS_5 \times S^5$ background itself. The perimeter of this circular droplet coincides with the trajectory of the same null geodesic as the one used in obtaining the PP-wave limit. This implies that if we make the Wick rotation in the way discussed in this exposition, the perimeter of circular droplet becomes a hyperbola which extends to the boundary.*) Thus the droplet shows a characteristic feature of the old $c = 1$ matrix model. In particular, the motion of the droplets of general excited states propagates to the EAdS boundary. Together with our tunneling picture, the amplitudes of this propagation from boundary to boundary are expected to give two-point functions of chiral primary operators which correspond to the excited states. Namely, the S-matrix of $c = 1$ matrix model may be related directly to the correlators of chiral primary operators. It would be nice to try to formulate this concretely and to see to what extent the exact solvability of $c = 1$ matrix model can guide us the properties of 1/2 BPS operators in $\mathcal{N} = 4$ super Yang-Mills theory.

5.5. *Higher-genus effects*

All our discussions so far have been restricted to the leading large N limit, namely the planar limit in the language of gauge theory. There have been attempts to derive corrections to conformal dimensions beyond the planar approximation. On the gauge-theory side, the corrections to the formula (2.1) is computed to be[32][33]

$$\Delta - J = 2(1 + \tilde{\lambda} n^2 + \cdots) + \frac{J^4}{4\pi^2 N^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)(\tilde{\lambda} + \cdots), \quad \tilde{\lambda} = \frac{g_{\rm YM}^2 N}{J^2}. \tag{5.10}$$

The second term is the first genus correction. Unfortunately, even this leading correction has never been derived correctly from the viewpoint of the bulk string theory. In fact, a claim has been made that this correction could be obtained by assuming the particular string vertex that retains only the first term $|H_{SV}\rangle$ of (4.16). However, a general consensus at the present time is that this was not correct. Aside from some errors, this type of calculations is based on an unjustified basic assumption that the

*) We have already suggested this possibility in.[31]

sum over the intermediates states in the second order calculation is truncated up to excited string states with only 2 impurities.

In a more recent computation[34] to which we refer the reader for more detailed references on previous works, it has been clarified that these assumptions do not only lead to wrong coefficients, but also to nonperturbative terms of the form $\tilde{\lambda}^{1/2}, \tilde{\lambda}^{3/2}, \ldots$, which cannot appear in usual perturbative expansions on the gauge-theory side.

Most recently, a calculation using our proposal (4.16) instead of the term $|H_{SV}\rangle$ alone, under the same truncation, has been reported in.[35] According to this computation, the situation is improved with respect to the coefficients at the first order in $\tilde{\lambda}$. In addition to this, unwanted nonperturbative terms of low orders which are found in[34] are also eliminated by 'miraculous' cancellations, occurring by this particular form of the interaction vertex. But the complete agreement with the gauge-theory result is still *not* attainable if one considers $\tilde{\lambda}^2$ terms and highers. Therefore, we should definitely take into account other possible contributions involving more impurities.

Another possible source of disagreement is that in principle we do not yet fixed the next order string-interaction term of the string action. The discussion of the string vertex reviewed in section 4 is effective only at the first order with respect to the string coupling. As emphasized in our works[4],[7] supersymmetry is not sufficient to fix the higher order corrections to the interaction Hamiltonian. Higher genus corrections would crucially depend on the structure of the higher order interaction terms. For example, if the interaction Hamiltonian were obtained solely by a unitary transformation from the free Hamiltonian, the conformal dimensions would not be corrected by higher genus effect. The second part $|H_D\rangle$ of the interaction term in (4.16) indeed is of this form. Similarly, some part of the higher interaction terms may also be of this form, as suggested in our first paper[4] in a different context. This might also be somehow related to the truncation of higher impurity states, if that is after all destined to be justified.

We can also raise a subtle question as to the dependence on the order of various limits involved in this problem. For instance, from the viewpoint of string theory in the AdS background, it is not at all obvious whether we can take the large J limit before completing summation over infinite number of intermediate states in higher-order calculations, since two-body states with a large total angular momentum J can consist of one-body states with large angular momentum of the same order as J and those with smaller angular momentum of order $O(1)$. For the latter part, we cannot have any *a priori* justification for using the large J limit. In any case, the fact that the situation is much improved with our proposal is a good news, but further painstaking but rewarding efforts, hopefully, are required to clarify this issue.

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References

1) J. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. 2 (1998) 231.
2) D. Berenstein, J. M. Maldacena, H. Nastase, *Strings in flat space and pp waves from N=4 super Yang-Mills*, JHEP0204(2002)013, hep-th/0202021.
3) For a review, see, *e.g.* N. Beisert, *The Dilatation operator of $\mathcal{N}$ = 4 Super Yang-Mills theory and integrability*, hep-th/0407277 and references therein.
4) S. Dobashi, H. Shimada, and T. Yoneya, *Holographic reformulation of string theory on $AdS_5 \times S^5$ background in the PP-wave limit*, Nucl.Phys.B665(2003)94, hep-th/0209251.
5) T. Yoneya, *What is holography in the plane wave limit of AdS(5)/SYM(4) correspondence?*, hep-th/0304183 (expanded from the paper published in the Proceedings, Prog. Theor. Phys. 152 (2003) 108).
6) M. Asano, Y. Sekino, and T. Yoneya, *PP wave holography for Dp-brane backgrounds*, Nucl.Phys.B678(2004)197, hep-th/0308024; M. Asano and Y. Sekino, *Large N limit of SYM theories with 16 supercharges from superstrings on Dp-brane backgrounds*, Nucl. Phys. B705(2005)33, hep-th/0405203; M. Asano, *Stringy effect of the holographic correspondence for Dp-brane backgrounds*,JHEP 0412(2004)029, hep-th/0408030.
7) S. Dobashi and T. Yoneya, *Resolving the holography in the plane-wave limit of AdS/CFT correspondence*, Nucl. Phys. B711(2005) 3, hep-th/0406225.
8) S. Dobashi and T. Yoneya, *Impurity non-preserving 3-point correlators of BMN operators from PP-wave holography I: bosonic excitations.* Nucl. Phys. B711(2005)54, hep-th/040908; S. Dobashi, *Impurity non-preserving 3-point correlators of BMN operators from PP-wave holography II: fermionic excitations*, hep-th/0604082.
9) S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, Phys.Lett.B428(1998)105, hep-th/9802109.
10) E. Witten, *Anti-de Sitter space and holography*, Adv.Theor.Math.Phys.2(1998)253, hep-th/9802150.
11) R.R. Metsaev,*Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, Nucl. Phys. B625 (2002) 70, hep-th/0112044.
12) R. R. Metsaev and A. A. Tseytlin, *Exactly solvable model of superstring in AdS(5) x S(5) background*, Phys. Rev. D65 (2002) 126004, hep-th/0202109.
13) J. A. Minahan and K. Zarembo, *The Bethe-ansatz for $\mathcal{N}$ = 4 super Yang-Mills*, JHEP 0303 (2003)013, hep-th/0212208.
14) M . Kruczenski, *Spin chains and string theory*, Phys. Rev. Lett. 93 (2004)161602: M. Kruczenski, A. V. Ryshov and A. A. Tseytlin, *Large spin limit of AdS(5)$\times S^5$ string theory and low energy expansion of ferromagnetic spin chains*, Nucl. Phys. B682 (2004) 3, hep-th/0403120.
15) For an example of such approach, see *e. g.* D. Berenstein and H. Nastase, hep-th/0205048.
16) S. J. Rey and J. T. Yee, *Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity*, Eur. Phys. J. C22 (2001) 379, hep-th/9803001.
17) J. M. Maldacena, *Wilson loops in large N field theories*, Phys. Rev. Lett. 80(1998)4859, hep-th/9803002.
18) S.-M. Lee, S. Minwalla, M. Rangamani, and N. Seiberg, *Three point functions of chiral operators in D=4, N=4 SYM at large N*, Adv.Theor.Math.Phys.2(1998)697, hep-th/9806074.
19) For a review on string field theory for the PP-wave background, see, *e. g.* A. Pankiewicz, *Strings in plane wave backgrounds*, hep-th/0307027 and references therein.
20) M. Spradlin and A. Volovich, *Superstring interactions in a pp wave background*, Phys.Rev.D66(2002)086004,, hep-th/0204146; M. Spradlin and A. Volovich, *Superstring interactions in a pp wave background 2*, JHEP0301(2003)036, hep-th/0206073; hep-th/0208209; A. Pankiewicz and B. Stefanski,Jr., *PP wave light cone superstring field theory*, Nucl.Phys.B657(2003)79, hep-th/0210246; A. Pankiewicz and B. Stefanski,Jr., *On the uniqueness of plane wave string field theory*, hep-th/0308062. See also.[19)]
21) P. Di Vecchia, J.L. Petersen, M. Petrini, R. Russo, and A. Tanzini, *The three string vertex and the AdS/CFT duality in the pp wave limit*, hep-th/0304025.
22) S. Lee and R. Russo, *Holographic cubic vertex in the pp-wave*, Nucl. Phys. B705(2005) 296,

hep-th/0409216.

23) E. G. Gimon, L. A. Pando Zayas, J. Sonnenschein, hep-th/0206033; H. Fuji, K. Ito and Y. Sekino, JHEP 0211 (2002) 005, hep-th/0209004.
24) A. Jevicki and T. Yoneya, *Spacetime uncertainty principle and conformal symmetry in D-particle dynamics*, Nucl. Phys. B535(1998) 335, hep-th/8905069; A. Jevicki, Y. Kazama and T. Yoneya, *Generalized conformal symmetry in D-brane matrix models*, Phys. Rev. D59(1999)060010-1-12, hep-th/9810146.
25) Y. Sekino and T. Yoneya, *Generalized AdS/CFT correspondence for matrix theory in the large N limit*, Nucl. Phys. B570 (2000) 174, hep-th/9907029.
26) Y. Sekino and Y. Yoneya, *From supermembrane to matrix string*, Nucl. Phys. B619 (2001) 22, hep-th/0108176.
27) S. Frolov and A. A. Tseytlin, *Multi-spin string solutions in AdS(5)× S(5)*, Nucl. Phys. B668 (2003) 77, hep-th/0304255.
28) A. Tsuji, *Holography of Wilson loop correlator and spinning strings*, hep-th/0606030.
29) N. Drukker and S. Kawamoto, *Small deformations of supersymmetric Wilson loops and open spin-chains*, hep-th/0604124.
30) H. Lin, O. Lunin and J. M. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, JHEP 10 (2004)025, hep-th/0409174.
31) T. Yoneya, *Extended fermion representation of muti-charge 1/2 BPS operators in AdS/CFT–Towards field theory of D-branes*, JHEP 12(2005)028, hep-th/0510114.
32) C. Kristjansen, J. Plefka, G. W. Semenoff and M. Staudacher, *A new double-scaling limit of N=4 super Yang-Mills theory and PP-wave strings*, Nucl. Phys. B643(2002)3, hep-th/0205033; N. R. Constable, D. Z. Freedman, M. Headrick, S. Minwalla, L. Motl, A. Postnikov and W. Skiba, *PP-wave string interactions from perturbative Yang-MIlls theory*, JHEP0207(2003)017.
33) N. Beisert, C. Kristjansen, J. Plefka, G. W. Semenoff and M . Staudacher, *BMN correlators and operator mixing in N=4 super Yang-Mills theory*, Nucl. Phys. B650(2003)125, hep-th/0208178; N. R. Constable, D. Z. Freedman, M. Headrick and S. Minwalla, *Operator mixing and the BMN correspondence*, JHEP 0210(2002)068, hep-th/0209002.
34) P. Gutjahr and A. Pankiewicz, *New aspects of the BMN correspondence beyond the planar limit*, Nucl. Phys. B704 (2005)583; See also, G. Grignani, M. Orselli, B. Ramadanovic, G. W. Semenoff and D. Young, *Divergence cancellation and loop corrections in string field theory on a plane wave background*, JHEP0512(2005)017, hep-th/0508126.
35) G. Grignani, M. Orselli, B. Ramadanovic, G. W. Semenoff and D. Young, *AdS/CFT v. s. string loops*, hep-th/0605080.