Superstring Theory as seen through an Old Man's Eyes

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Is the superstring theory an ultimate theory to replace the quantized field theory? Is it physics or mathematics? An old man's point of view of the superstring theory is given.

§1. Introduction

Generally old people cannot follow developments of new theories and often speak ill of them. However, most of their comments are irrelevant. Being an old man, I wish to follow these people and try to say something against the superstring theory. I must be careful not to be irrelevant, but if I am wrong, please advise me.

§2. S-matrix in the Field theory

Before going into the string theory, let me state a result derived in the field theory.

The current high energy theory is based on two principles, the quantum mechanics and the special relativity. Quantum mechanics describes phenomena in terms of probability amplitudes, and the probability is conserved. That is, the S-matrix must be unitary. The special relativity requires, in addition to the Lorentz invariance, that signals should not travel faster than light. This fact is often called causality.

The unitarity of the S-matrix and the causality are necessary conditions. However, these conditions are severe enough and are almost sufficient to determine the S-matrix completely. The situation is simple in the case of a single variable. Let

$$S = 1 - iT.$$

Then the unitarity means

$$\mathrm{Im}T = \frac{1}{2}T^{\dagger}T. \tag{1}$$

Causality requires that the S-matrix element should satisfy a certain analytic behavior in the complex energy plane so that a dispersion relation of the form

$$\mathrm{Re}T(E) = \frac{1}{\pi}\int \frac{\mathrm{Im}T(E')}{E'-E}\mathrm{d}E'. \tag{2}$$

holds. Given a first order T, from eq. (1) we calculate the second order $\mathrm{Im}T$, and from eq. (2) the $\mathrm{Re}T$. In this way we can calculate higher order T successively.

As an example let us calculate the S in the neutral scalar theory. All particles are free; they satisfy the relation $p_i^2 = m^2$.

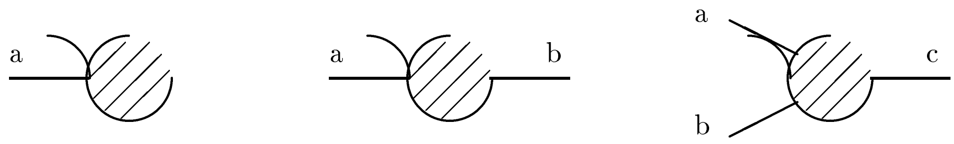

Fig.1

One-legged T is zero since energy-momentum is conserved:

$$T_{\mathrm{a}} = 0.$$

Two-legged T is also zero; the one-particle state is stable and does not make any transition:

$$T_{\mathrm{ab}} = 0.$$

Three-legged T is a function of $p_{\mathrm{a}},\ p_{\mathrm{b}}$. By Lorentz invariance it is a function of

$$s = (p_{\mathrm{a}} + p_{\mathrm{b}})^2.$$

Since $s = p_{\mathrm{c}}^2 = m^2$, the quantity T is a constant. Denote it by g,

$$T_{\mathrm{abc}} = g.$$

This g is called coupling constant.

For four-legged T (scattering amplitude) we first calculate its imaginary part by eq. (1). We insert one-particle, two-particle, $\cdots$ into the intermediate state of the right-hand side.

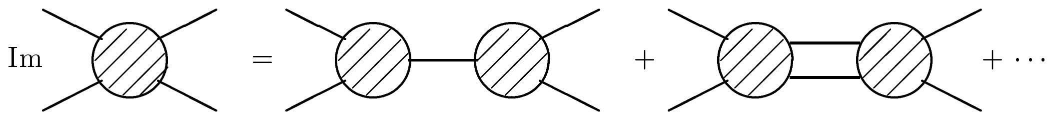

Fig 2

T for the one-particle intermediate state is non-vanishing only when $s = m^2$.

$$\mathrm{Im} T_2(s) = g^2 \delta(s - m^2).$$

Here we have suppressed meaningless constants such as 1/2 . By eq. (2), we obtain

$$T_2 = \frac{g^2}{m^2 - s}. \tag{3}$$

To derive this propagation function we do not need the Schrödinger equation or Klein-Gordon quation. Only the unitarity of S and the causality suffice.

For the two-particle intermediate state we insert two T_2's of eq.(3) to obtain ImT_4 and then evaluate ReT_4. The result is identical to the Feynman-graph calculation (after mass and coupling constant renormalization) of Fig. 3

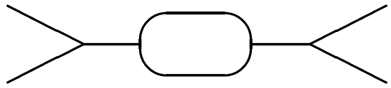

Fig 3

In general, four-legged T, i.e., scattering amplitudes depend, not only on s but also on t, the momentum transfer,

$$t = (p_{\rm a} + p_{\rm c})^2.$$

The function theory of many variables is very complicated. While for fixed $t < 0$ the relations of the form (1) and (2) hold, for t dependence we must use the unitarity and causality in t- (and also in the crossed u-) channels. Scattering can be managed, but in more than five-legged cases it is too complicated. The unitarity relation in the case of complex variables cannot be formulated in a straightforward way. However, it is not impossible to expect that, in principle, S can be constructed in this way. That is, in general,

Unitarity and causality determine the S-matrix.

Here the masses of the particles and coupling constants are assumed to be given.

The quantized local field theory is causal and the S-matrix (*à la* Feynman-Dyson) is unitary. Therefore the S obtained above must coincide with this S. Namely,

THEOREM The S-matrix that satisfies unitarity and causality is limited to that of the local field theory.

The construction of S-matrix given in this section is not proved (although proved for lowest order terms) and there is a space for doubt as to whether on-shell quantities only determine the whole S-matrix. However, the THEOREM is widely accepted as true.

Some caution must be paid when there are bound states among the free particles. Mass and coupling constant of the bound state cannot be given freely but must be the result of a bound state problem. Bound states are extended in space, but when we hit one part of it a signal (energy) is carried by particles so the situation is causal.

§3. String theory

With the THEOREM of the previous section in mind, let us consider the string theory. The theorem states that if a theory is consistent, that is, if it is causal and gives unitary S-matrix, it must be a local field theory. There is no room for a new theory to come in. In fact, if one writes a Lagrangian of a string he cannot, in general, develop a consistent theory. Only in 10- or 26-dimensional space one can formulate a trouble-free theory. Such a theory in 10-dimensional space fails to satisfy causality or unitarity or both if one tries to compactify it into 4-dimensional Minkowski space.

However, it was found that in some special cases one can construct consistent string theories in four dimensional space. Since the theory satisfies causality and unitary, the S-matrix must be identical to the one obtained from a local field theory.

In fact, the first order S-matrix, i.e., scattering amplitude, obtained in this string theory, is the Veneziano amplitude of the form,

$$V = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}.$$

This amplitude was invented in field theory thirty years ago. It has poles of the form eq.(3) at $\alpha(s) =$ integer and is the sum of propagaters of point particles.

The next order approximation in the string theory is the one-hole calculation which is parallel to the dispersion integral of the previous section. The result will be the sum of fourth order graphs in the field theory. A string theory does not produce new type of amplitude. It gives identical results as a field theory does. So far as the S-matrix is concerned,

string theory = field theory.

Here the right-hand side means the local field theory of the particles which are the eigenstates of the string (left-hand side) with local interactions.

An immediate consequence is that in a string theory renormalization constants diverge. At the beginning stage of strings some people expected that, since interactions are extended, they would get a finite theory. That was not true. It was recognized half a century ago that a relativistic cutoff always fails.

§4. Superstring theory

A string produces massless spin two field which can be identified with the gravitational field. Its dynamics, being the same as the local field theory, is unrenormalizable. In order to make the theory renormalizable, people introduce supersymmetry. The supersymmetry requires that every particle has partners of different spins. In general, the sign of divergence depends on the value of the spin, and so it is possible to cancel out the divergences in the presence of particles of different spin values. Before the invention of the renomalization theory, particle mixing was investigated in an attempt to eliminate divergences. The conclusion then was that not all divergences could be cancelled by mixing of particles.

It is possible, however, to cancel out most severe divergences. Remaining logarithmic divergences can be renormalized. By particle mixing a renormalizable gravitational theory can be constructed. The superstring theory is of this type.

To make the gravitational theory renormalizable, the supersymmetry will not be a necessity. To cancel out severe divergences particles have to be mixed, but, divergences being ultraviolet, there is no need that particle masses should be equal. In other words, what is needed is a particle mixing but not the supersymmetry. Anyway, it is clear that the renormalizable gravitation theory cannot be regarded as an achievement of the string theory.

§5. Mass spectrum

So far as the S-matrix is concerned, string theory is the same as field theory. This does not imply that string theory is meaningless. In field theory masses of particles and coupling constants must be given, while in string theory these constants are calculated. If these values fit experimental values it is a big success of the string theory.

A string produces, corresponding to its infinite degrees of freedom, infinite energy levels, i.e., infinite mass spectrum. This spectrum is, however, completely in disagreement with that of our known particles. The only thing one can do is to assign all our particles to the zero mass level of the string which is highly degenerate due to the supersymmetry. It is a pity that infinite levels that are the characteristic feature of the string are not utilized at all. The top quark mass is 150GeV which is regarded as zero in this scheme. The mass scale of the string theory must be fantastically large compared with 150GeV. People assume it to be of the order of the Planck mass $M_{\rm P}$

Is it possible to calculate the masses $M_{\rm SM}$ of our standard model particles in the string theory? Of course, the supersymmetry must be slightly broken (spontaneously or artificially). There is a difference of the order 10^{-15} in the mass scales. Is it possible to calculate such a small quantity?

Hikaru Kawai says it is not impossible and writes down an equation,

$$M_{\rm SM} = \exp(-1/g^2)M_{\rm P}. \tag{4}$$

The equation certainly holds if g^2 is small. It is, however, hard to believe that such an equality holds. Even if this is true, a theory that deals with quantities of the order $\sim 10^{-15}$ cannot be regarded as well-defined.

The physics of the standard model is not governed by a string physics but by a new physics with the mass scale $\sim$ GeV. This physics dresses particles of the string origin particles, renormalizes coupling constants, and, more important, produces bound states. In this way elementary particles of the string region are completely different from those of the standard model particles. For example, in the atomic physics electron and atomic nuclei are elementary particles. In the celestial dynamics that is twenty digits larger in magnitude, planets and the sun are elementary. It is hard to speculate that string region particles have any similarity with the standard model particles. It is not possible to compare mass spectrum of string with experiment.

Do the cosmological phenomena supply any information of excited states of strings (assuming they exist)? Unfortunately the cosmology is not sound enough to speak anything on this. The cosmological physics is now struggling with unsolved problems.

§6. True nature of string

What is the true nature of a string? Is it an ultimate object of matter?

If a string is point-like in the Minkowski space but extended into extra dimensions, things are simple (but not interesting). It is said, however, that a string is really extended in the Minkowski space, and that a signal travels in the string with the velocity of light. This is a bound state. The string must be a bound state composed of point particles. The Lagrangian of a string is a reduced one written in terms of collective coordinates. Or, the Lagrangian is said to be a phenomenological one.

string = composite state of particles.

We are considering translations and rotations in the Minkowski space. Plane waves with spin constitute the fundamental representation of this group (which is often called the Poincaré group). In terms of physical language, in the Minkowski space ultimate objects are (point) particles and anything that moves in this space is a composite state of ultimate particles. A string Lagrangian cannot be regarded as a first principle.

If string exists it must be a bound state of particles. We must investigate this bound structure and ascertain its constituents. String-like objects exist in nature, e.g., polymers and macroscopic strings. Any of these can be used as a model in investigating the bound structure of a superstring. Take macroscopic string as a model which is made of atoms. Then the superstring in the Planck mass region is made of particles about 10^{10} times smaller. This is too much for us. I do not believe that such a structure exists in Nature.

Extended objects like strings have been considered for a long time. The motivation was two-fold. One, to construct a divergence-free theory by extended interactions. Two, to identify known particles with eigenstates of the extended object. We know that under the principles of unitarity and causality, the first motivation is never fulfilled. For the second motivation, there is no experimental indication that the current string theory succeeds. Nevertheless people stick to strings. Is that due to a mathematical motivation to try one-dimensional string next to the zero-dimentional point particle?

§7. Is nature supersymmetric?

The continuous Lorentz invariance holds exactly. On the other hand, discrete transformations like space inversion or time reversal break the invariance. Supersymmetry belongs to this category. There is no indication of the supersymmetry in nature. One might expect that even though the supersymmetry is broken at the present stage, it will hold exactly at the ultimate stage. However, I do not believe so.

A superalgebra is an extension of a Lie algebra. Supersymmetry is a superextension of the Lorentz symmetry and is by no means a fundamental symmetry. In physics we can argue in the following way.

Under the supersymmetry the fundamental representation consists of particles of integral spins and half-odd integral spins. That is to say, the fundamental particles are a mixture of bosons and fermions. This mixture cannot be an ultimate member of fundamental particles. A first principle must start from a minimum set of objects, where bosons must be composite states of fermions. The supersymmetry cannot be an ultimate one.

§8. Conclusions

Some time ago Hideki Yukawa proposed nonlocal field as opposed to the local field $\psi(x)$. The simplest was the bi-local filed $\psi(x,y)$. In order to go further he needed some principle. If he had introduced the causality as the principle, his theory would have become identical to the local field theory, and the bi-local field would have become wave function of a two-body bound state. Yukawa, unsatisfied with the local field theory, refused to take the causality and invented the "reciprocity" principle. This turned out unsuccessful.

The current string theory easily adopted the causality and the theory has been reduced to a phenomenological one. Actually there is no physical phenomenon which corresponds to this string and the theory is to be called a mathematical one.

In order to pursue ultimate objects other than particles or fields one has to get out of the framework of the unitarity and causality. For instance, one should consider extra dimensions, do non-relativistically in three-dimension, allow superphotonic velocity, etc.. In any case one must be consistent with the fact that the special relativity exactly holds so far as we know. How about the general relativity? Is the quantum mechanics alright?

In these attempts one might find a breakthrough and a new physics will be developed. That will be, however, completely different from the superstring theory.