Searching for Extremely High-Energy Behaviours from Accelerator-Energy Regions

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We use rich information on πp total cross sections below N(~ 10 GeV) in addition to high-energy data in order to discriminate whether these cross sections increase like $\log\nu$ or $\log^2\nu$ at high energies. A finite-energy sum rule (FESR) which is derived in the spirit of the P$'$ sum rule as well as the n=1 moment FESR have been required to constrain the high-energy parameters. We then searched for the best fit of $\sigma_{tot}^{(+)}$ above 70 GeV in terms of high-energy parameters constrained by these two FESR. We have shown from this analysis that the $\log^2\nu$ behaviours are preferred to the $\log\nu$ behaviours. We also propose to search for $\bar{\text{p}}$p, pp total cross sections at extremely high-energy from accelerator-energy regions using the similar technique.

§1. Introduction

As you all know, the sum of π^-p and π^+p total cross sections has a tendency to increase above 70 GeV experimentally.[1)] It is well-known as the Froissart-Martin unitary bound[2)] that the increase of of total cross sections is at most $\log^2\nu$.

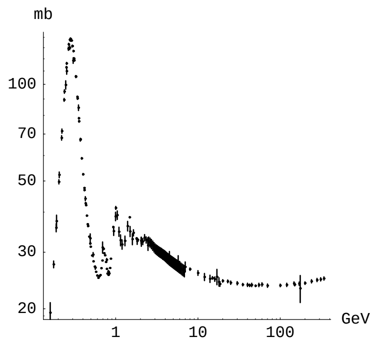

Fig. 1. πN Total cross section: Whole-energy region

It had not been possible,[3)] however, to discriminate between asymptotic $\log\nu$ and $\log^2\nu$ fits if one uses high-energy data alone above 70 GeV.

So, Muneyuki Ishida and myself proposed[4)] to use rich information of πp total cross sections at low and intermediate-energy regions to investigate high-energy behaviours of πp total cross sections above 70 GeV using a kind of FESR as constraints.

Such a kind of attempt to investigate high-energy behaviours from those at low and intermediate-energy regions had been initiated[5)] by the author. In the early days of the Regge pole theory, there were hot controversies if there are other singularities with the vacuum quantum numbers except for the Pomeron(P). Under the assumption that no J singularities extend above α=0 except for the Pomeron, we were led to the sum rule for the s-wave πN scattering length $a^{(+)}$ of the crossing-even amplitude as

$$\left(1+\frac{\mu}{M}\right) \mathrm{a}^{(+)} = -\frac{g_r}{4\pi}^2 \left(\frac{\mu}{2M}\right)^2 \frac{1}{M} \frac{1}{1-\left(\frac{\mu}{2M}\right)^2} + \frac{1}{2\pi^2}\int_0^\infty dk[\sigma_{tot}^{(+)}(k)-\sigma_{tot}^{(+)}(\infty)] \quad (1.1)$$

Here, M is the nucleon mass and μ is the pion mass.

The evidence that the sum rule(1.1) was not satisfied implied that the original assumption was not correct, i.e., there has to be some singularities with vacuum quantum numbers for 1> $\alpha \geq 0$.

§2. The P$'$sum rule

Let us assume that there are no singularities between 0 and 1 except for the Pomeron and the P$'$ pole with 1> $\alpha_{P'}$ $\geq$0.

Suppose we define

$$\tilde{f}^{(+)}(\nu) = f^{(+)}(\nu) - f_P(\nu) - f_{P'}(\nu) \quad (2.1)$$

which vanishes for large values of ν like ν^α (α<0). Here $f^{(+)}(\nu)$ is the crossing-even πp forward scattering amplitude, $f_P(\nu)$ is the Pomeron term, and $f_{P'}(\nu)$ is the P$'$ term. Then we can write the unsubtracted dispersion relation for $\tilde{f}^{(+)}(\mu)$ as

$$\begin{aligned}\mathrm{Re}\ \tilde{f}^{(+)}(\mu) &= \frac{P}{\pi}\int_{-\infty}^{\infty} d\nu \frac{\mathrm{Im}\tilde{f}^{(+)}(\nu)}{\nu-\mu} \\ &= \frac{2P}{\pi}\int_0^\infty \frac{\nu \mathrm{Im}\tilde{f}^{(+)}(\nu)}{k^2} d\nu \quad (2.2)\end{aligned}$$

Taking the imaginary part of Eq.(2.1), we have

$$\begin{aligned}\mathrm{Im}\tilde{f}^{(+)}(\nu) &= \mathrm{Im}f^{(+)}(\nu) - \mathrm{Im}f_P(\nu) - \mathrm{Im}f_{P'}(\nu) \\ &= \frac{k}{4\pi}\sigma_{tot}^{(+)}(\nu) - \mathrm{Im}f_P(\nu) - \beta_{P'}\left(\frac{\nu}{\mu}\right)^{\alpha_{P'}} \quad (2.3)\end{aligned}$$

Substituting Eq.(2.3) into Eq.(2.2), we obtained

$$\mathrm{Re}\ \tilde{f}^{(+)}(\mu) = -\frac{g_r^2}{4\pi}\left(\frac{\mu}{2M}\right)^2 \frac{1}{M}\frac{1}{1-(\frac{\mu}{2M})^2} + \frac{2P}{\pi}\int_0^N \frac{\nu}{k^2}\left\{\frac{k}{4\pi}\sigma_{tot}^{(+)}(\nu) - \frac{1}{4\pi}\sigma_{tot}^{(+)}(\infty)\nu - \frac{\beta_{P'}}{\mu}\left(\frac{\nu}{\mu}\right)^{\alpha_{P'}}\right\}d\nu \quad (2.4)$$

Therefore, we were led to a kind of finite-energy sum rules (FESR)[6),7)] for the s-wave πN scattering lengths $a^{(+)}$ such as

$$\left(1+\frac{\mu}{M}\right) a^{(+)} = \mathrm{Re}\ f^{(+)}(\mu) = -\frac{\beta_{P'}}{\mu} - \frac{g_r^2}{4\pi}\left(\frac{\mu}{2M}\right)^2 \frac{1}{M}\frac{1}{1-(\frac{\mu}{2M})^2}$$
$$+\frac{2P}{\pi}\int_0^N \frac{\nu}{k^2}\left\{\frac{k}{4\pi}\sigma_{tot}^{(+)}(\nu) - \frac{1}{4\pi}\sigma_{tot}^{(+)}(\infty)\nu - \frac{\beta_{P'}}{\mu}\left(\frac{\nu}{\mu}\right)^{\alpha_{P'}}\right\}d\nu \qquad (2.5)$$

This Eq. (2.5) is the so-called P$'$ sum rule.
Here the relation

$$\mathrm{Re}\tilde{f}^{(+)}(\mu) = \mathrm{Re}\ f^{(+)}(\mu) - \mathrm{Re}f_P(\mu) - \mathrm{Re}f_{P'}(\mu)$$
$$= \mathrm{Re}\ f^{(+)}(\mu) + \frac{\beta_{P'}}{\mu} \qquad (2.6)$$

derived from Eq.(2.1) was substituted into Eq.(2.4).

This is the sum rule which correlates high-energy parameters with low-energy quantities, a kind of FESR. *
The phenomenological analysis based on ref.5 led us to the prediction of the second Pomeranchunk pole, $P'$with $\alpha_{P'}(0)\simeq 0.5$. Later, the f meson(1275MeV) was discovered as a resonance with I=0, J=2 just on this P' trajectory. We refer the reader to the review article[8)] on survey of hadron phenomenology and phenomenological duality.

§3. Search for the behaviours of πN total cross sections at high energies using new FESR

As I talked to you in §1, the sum of $\pi^- p$ and $\pi^+ p$ total cross sections has a tendency to increase above 70 GeV. In order to discriminate logν or log$^2\nu$ at high energies, let us derive FESR in the spirit of the P' article.[5)]

3.1. *Derivation of the FESR (1)*

Let us consider the crossing-even forward πN scattering amplitude $f^{(+)}(\nu)$ similar to §2. Suppose we assume

$$\mathrm{Im}\ f^{(+)}(\nu) \simeq \mathrm{Im}\ R(\nu) + \mathrm{Im}\ f_{P'}(\nu)$$
$$= \frac{\nu}{\mu^2}\left(c_0 + c_1 \log\frac{\nu}{\mu} + c_2\log^2\frac{\nu}{\mu}\right) + \frac{\beta_{P'}}{\mu}\left(\frac{\nu}{\mu}\right)^{\alpha P'} \qquad (3.1)$$

at high energies ($\nu \geq$ N). Since this amplitude is crossing-even, we have

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* Dolen, Horn and Schmid[7)] pointed out that the sum rule is the first example of the $n = -1$ moment FESR.

$$R(\nu) = \frac{i\nu}{2\pi^2}\Big\{2c_0 + c_2\pi^2 + c_1\left(\log\frac{e^{-i\pi}\nu}{\mu} + \log\frac{\nu}{\mu}\right) + c_2\left(\log^2\frac{e^{-i\pi}\nu}{\mu} + \log^2\frac{\nu}{\mu}\right)\Big\}, \qquad (3.2)$$

$$f_{P'}(\nu) = -\frac{\beta_{P'}}{\mu}\left(\frac{(e^{-i\pi}\nu/\mu)^{\alpha_{P'}} + (\nu/\mu)^{\alpha_{P'}}}{\sin\pi\alpha_{P'}}\right), \qquad (3.3)$$

and subsequently we obtain

$$\mathrm{Re}R(\nu) = \frac{\pi\nu}{2\mu^2}\left(c_1 + 2c_2\log\frac{\nu}{\mu}\right), \qquad (3.4)$$

$$\mathrm{Re}f_{P'}(\nu) = -\frac{\beta_P{}'}{\mu}\left(\frac{\nu}{\mu}\right)^{0.5}, \qquad (3.5)$$

substituting $\alpha_{P'} = \frac{1}{2}$ in Eq.(3.3). Let us define

$$\tilde{f}^{(+)}(\nu) = f^{(+)}(\nu) - R(\nu) - f_{P'}(\nu) \sim \nu^{\alpha(0)}\ (\alpha(0) < 0), \qquad (3.6)$$

and write dispersion relation for $\frac{\tilde{f}^{(+)}(\nu)}{\nu-\mu}$. Since this amplitude is superconvergent, we obtain

$$\mathrm{Re}\ \tilde{f}^{(+)}(\mu) = \frac{P}{\pi}\int_{-\infty}^{\infty} d\nu'\frac{\mathrm{Im}\ \tilde{f}^{(+)}(\nu')}{\nu'-\mu} = \frac{2P}{\pi}\int_0^{\infty}\frac{\nu'\mathrm{Im}\ \tilde{f}^{(+)}(\nu')}{k'^2}d\nu' \qquad (3.7)$$

Using Eqs.(3.6) and (3.7), we have

$$\mathrm{Re}\ f^{(+)}(\mu) = \mathrm{Re}\ R(\mu) + \mathrm{Re}\ f_{P'}(\mu) - \frac{g_r^2}{4\pi}\left(\frac{\mu}{2M}\right)^2\frac{1}{M}\frac{1}{1-(\frac{\mu}{2M})^2} + \frac{1}{2\pi^2}\int_0^{\bar{N}}\sigma_{tot}^{(+)}(k)dk - \frac{2P}{\pi}\int_0^N\frac{\nu}{k^2}\Big\{\mathrm{Im}\ R(\nu) + \frac{\beta_{P'}}{\mu}\left(\frac{\nu}{\mu}\right)^{0.5}\Big\}d\nu, \qquad (3.8):FESR(1)$$

where $\bar{N} \equiv \sqrt{N^2-\mu^2} \simeq N$. Let us call this Eq.(3.8) as the FESR(1) which we use as the first constraint. It is very important to notice that Eq.(3.8) reduces to the P' sum rule,[5] Eq. (2.5) if c_1, $c_2 \longrightarrow 0$.

The FESR[5),6),7)]

$$\int_o^N d\nu\ \nu^n Imf(\nu) = \sum_i \beta_i\frac{N^{\alpha_i} + n + 1}{\alpha_i + n + 1} \qquad (3.9)$$

holds for even positive integer n when $f(\nu)$ is crossing odd, and holds for odd positive integer n when $f(\nu)$ is crossing even. We can also derive negative -integer moment FESR. The only significant FESR is a one for $f^{(+)}(\nu)/\nu$ corresponding to n=−1. The FESR (1) belongs to this case.

3.2. *Derivation of the FESR(2)*

The second FESR corresponding to n=1 is:

$$\pi\mu\left(\frac{g_r^2}{4\pi}\right)\left(\frac{\mu}{2M}\right)^3+\frac{1}{4\pi}\int_0^{\bar{N}} k^2\sigma_{tot}^{(+)}(k)dk$$
$$=\int_0^N \nu\mathrm{Im}\ R(\nu)d\nu+\int_0^N \nu\mathrm{Im}\ f_{P'}(\nu)d\nu. \quad (3.10): FESR(2)$$

We call Eq.(3.10) as the FESR(2). It is to be noticed that the contribution from higher energy regions is enhanced.

§4. Analysis:log ν or log^2 ν?

Let us search for the best fit of $\sigma_{tot}^{(+)}$ above 70 GeV in terms of high-energy parameters constrained by the FESR(1) and FESR(2).

4.1. *Data*

We first evaluate the Born terms

$$-\frac{g_r^2}{4\pi}\left(\frac{\mu}{2M}\right)^2\frac{1}{M}\frac{1}{1-(\frac{\mu}{2M})^2}=-0.0854\ \mathrm{GeV}^{-1}, \tag{4.1}$$

$$\pi\mu\frac{g_r^2}{4\pi}\left(\frac{\mu}{2M}\right)^3=0.0026\ \mathrm{GeV}, \tag{4.2}$$

using $\frac{g_r^2}{4\pi}=14.4$. We also obtain the scattering length as

$$\mathrm{Re}\ f^{(+)}(\mu)=\left(1+\frac{\mu}{M}\right)a^{(+)}=\left(1+\frac{\mu}{M}\right)\frac{1}{3}\left(a_{\frac{1}{2}}+2a_{\frac{3}{2}}\right)$$
$$=-(0.014\pm0.026)\mathrm{GeV}^{-1}. \tag{4.3}$$

We have also used rich data[9] of $\sigma^{\pi p}$ to evaluate the relevant integrals of cross sections appearing in FESR(1) and FESR(2). We have obtained

$$\frac{1}{2\pi^2}\int_0^{\bar{N}} dk\ \sigma_{tot}^{(+)}(k)=38.75\pm0.25\ \mathrm{GeV}^{-1}, \tag{4.4}$$

$$\frac{1}{4\pi}\int_0^{\bar{N}} dk\ k^2\sigma_{tot}^{(+)}(k)=1817\pm31\ \mathrm{GeV} \tag{4.5}$$

for $\bar{N}$=10 GeV. The errors of relevant integrals, which are form the errors of each data point, are very small ($\sim$1 percent), and thus, we regard the central values are exact ones in the following analysis.

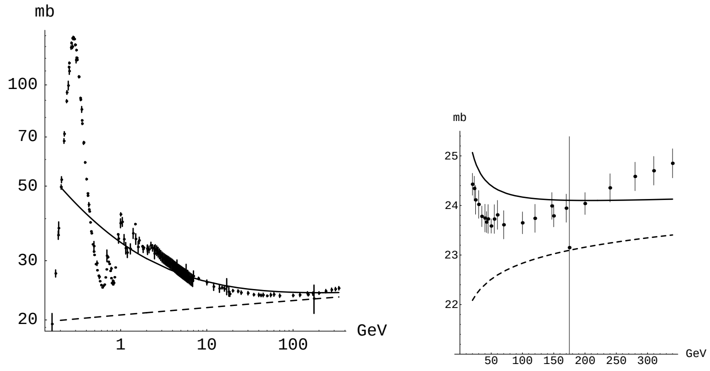
4.2. *Analysis*

Our starting points are the FESR(1) (Eq.(3.8)) and FESR(2)(Eq.(3.10)). Armed with these two, let us express high-energy parameters. c_0,c_1,c_2,$\beta_{P'}$ in terms of the Born term and the πN scattering length $a^{(+)}$ as well as the total cross sections up to N. We then attempt to fit the $\sigma_{tot}^{(+)}$ above 70 GeV. We set N=10 GeV corresponding to ($\sqrt{s}_{p\pi}$=4.43 GeV) since there are no resonances above this energy. We generally assume that the Im$f^{(+)}(\nu)$ behaves as Eq.(3.1) at high energies ($\nu \geq N$).

(1)The logν fit:
Let us discuss the logν fit(with three parameters c_0,$c_1$and $\beta_{P'}$) with two constraints FESR(1),(2). We set N=10 GeV and express c_0, $\beta_{P'}$ as a function of c_1 using the FESR(1)and (2). We obtained

$$\begin{aligned} c_0(c_1) &= 0.0879 - 4.94c_1, \\ \beta_{P'}(c_1) &= 0.1290 - 7.06c_1. \end{aligned} \tag{4.6}$$

Let's try to fit 12 data points of $\sigma_{tot}^{(+)}(k)$ between 70 GeV and 340 GeV. The best fit we obtained is c_1=0.00185. This gives c_0 = 0.0787 and $\beta_{P'}$=0.142 with the bad "reduced χ^2," i.e. $\chi^2/(N_{data}$ -$N_{param})$ =29.03/(12-1)$\simeq$ 2.6. So, it turned out that this fit has difficulties to reproduce the experimental increase of πp total cross sections above 70 GeV (see Fig.2(a) and (b)). Figure2: Fit to the $\sigma_{tot}^{(+)}$ data above



(a) whole-energy regions (b) high-energy regions

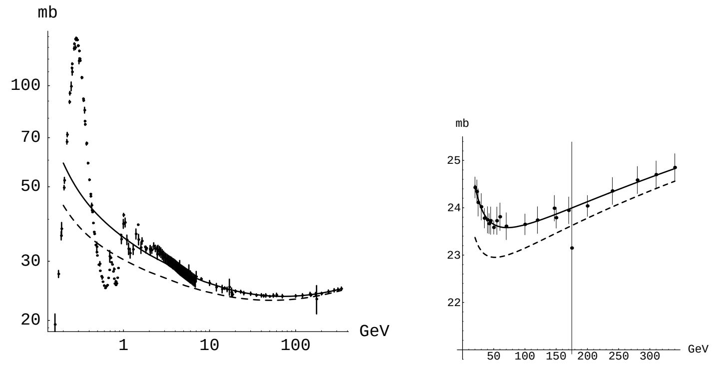
70 GeV by the *log* ν model. The dashed line represents the contributions from Im $R(\nu)$ with c_2=0.

(2) The $\log^2 \nu$ fit:

This fit has four parameters c_0,c_1,c_2 and $\beta_{P'}$ with two constraints FESR(1),(2). (So,the number of independent parameters is two.) We again set $N = 10$ GeV and required both FESR(1)and (2) as constraints. Then, c_0,$\beta_{P'}$ are expressed as functions of c_1 and c_2 as

$$\begin{aligned} c_0(c_1, c_2) &= 0.0879 - 4.94c_1 - 21.50c_2, \\ \beta_{P'}(c_1, c_2) &= 0.1290 - 7.06c_1 - 41.46c_2. \end{aligned} \tag{4.7}$$

We then searched for the fit to 12 data points of $\sigma_{tot}^{(+)}(k)$ above 70 GeV. The best fit in terms of two parameters $c_1$and c_2 led us to greatly improved value of "reduced χ^2", $\chi^2/(\mathrm{N}_{data} - \mathrm{N}_{param}) = 0.746/(12-2) \simeq 0.075$ for $c_1 = -0.0215<0$ and $c_2 = 0.00182 > 0$ which give $c_0 = 0.155$ and $\beta_{P'}$=0.0574. This is an excellent fit to the data. Figure3: Fit to the $\sigma_{tot}^{(+)}$ data above 70 GeV by the $\log^2 \nu$ model.The



(a) whole-energy regions (b) high-energy regions

dashed line represents the contributions from Im $R(\nu)$ with $c_2 > 0$.

4.3. *Remarks*

It is remarkable to notice that the wide range of data(k≥5 GeV) have been reproduced within the error even in the region where the fit has not been made (see fig.3(a) and (b)). The results do not change so much for the value of N. The increase of $\sigma_{tot}^{(+)}$ above 50 GeV is explained via $\log^2 \nu/\mu$ ($c_2 > 0$) and the decrease between 5∼50 GeV is explained by $\log \nu/\mu$ ($c_1 < 0$). Therefore, we emphasize that the comparison of Fig.2($\log \nu$ fit) and Fig.3($\log^2\nu$ fit) clearly idicates the latter fit to be preferred. **

Therefore, we can conclude that our analysis in terms of high-energy parameters constrained by the FESR(1),(2) prefers the $\log^2 \nu/\mu$ behaviours satisfying the

** SELEX collaboration[10)] for $\pi^- N$ total cross section at $(k = 610 \text{ GeV})^2$ was reported. Our $\log^2\nu$ fit ($\log\nu$ fit)predicts 25.9mb(24.2mb) for $\sigma_{tot}^{(+)}$ at 610 GeV which is consistent(inconsistent) with their value on $\pi^- N$,(26.6±0.9)mb.

Froissart-Martin unitary bound.[2)]

§5. Investigations of the p̄p, pp total cross sections at extremely high energies from accelerator-energy regions

As for the p̄p, pp total cross sections, there are a lot of data including cosmic ray data up to $\sqrt{s}\sim$several times of 10^4 GeV compared with data up to $\sqrt{s}\sim30$ GeV for πN scattering.

Therefore, it is worthwhile if you could search for these extremely high-energy regions from accelerator-energy regions.

So, Let's study the p̄p, pp total cross sections, along the similar line as the πNcattering. We define

$$F^{(+)}(\nu) \equiv \frac{f^{\bar{p}p}(\nu) + f^{pp}(\nu)}{2}$$

with

$$\mathrm{Im}\ F^{(+)}(\nu) \equiv \frac{k\sigma^{(+)}_{tot}(\nu)}{4\pi} \tag{5.1}$$

We also assume

$$\begin{aligned}\mathrm{Im}\ F^{(+)}(\nu) &= \mathrm{Im}\ R(\nu) + \mathrm{Im}\ F_{P'}(\nu)\\ &= \frac{\nu}{M^2}\left(c_0 + c_1\log\frac{\nu}{M} + c_2\log^2\frac{\nu}{M}\right) + \frac{\beta_{P'}}{M}\left(\frac{\nu}{M}\right)^{a_{P'}}\end{aligned} \tag{5.2}$$

at high energies($\nu \geq$N). We defined the functions R(ν), F$_{P'}(\nu)$ by replacing μ by M in Eq.(3.1).

As before, let's define

$$\tilde{F}^{(+)}(\nu)=F^{(+)}(\nu) - R(\nu) - F_{P'}(\nu) \approx \nu^{a(0)}\ (\alpha(0) < 0) \tag{5.3}$$

Using the similar technique as Eq.(3.7), we obtain

$$\begin{aligned}\mathrm{Re}\ \tilde{F}^{(+)}(M) &= \frac{2P}{\pi}\int_0^\infty \frac{\nu\mathrm{Im}\ \tilde{F}^{(+)}(\nu)}{k^2}d\nu\\ &= \frac{2P}{\pi}\int_0^M \frac{\nu}{k^2}\mathrm{Im}\ F^{(+)}(\nu)d\nu + \frac{P}{2\pi^2}\int_0^{\bar{N}} \sigma^{(+)}_{tot}(k)dk\\ &- \frac{2P}{\pi}\int_0^N \frac{\nu}{k^2}\left\{\mathrm{Im}\ \mathrm{R}(\nu) + \frac{\beta_{P'}}{M}\left(\frac{\nu}{\mathrm{M}}\right)^{0.5}\right\}d\nu.\end{aligned} \tag{5.4}$$

Two problems occur here.

• Our problem is that the p̄p reaction is exothermic reaction. So, the second term of the right-hand side diverges logarithmically at threshold. To overcome this difficulty, let's take $\nu{=}\nu_1({\gg}\mathrm{M})$. Then we can obtain the n=−1 moment FESR which we call FESR(1):

$$\mathrm{Re}\ \tilde{F}^{(+)}(\nu_1) = \frac{P}{\pi}\int_{-N}^{N}\frac{\mathrm{Im}\ \tilde{F}^{(+)}\Big(\nu\Big)}{\nu-\nu_1}d\nu$$

$$= \frac{P}{\pi}\int_{0}^{N}\Big(\frac{1}{\nu-\nu_1}+\frac{1}{\nu+\nu_1}\Big)\mathrm{Im}\ \tilde{F}^{(+)}\big(\nu\big)d\nu \qquad (5.5):\mathrm{FESR}(1)$$

•We have another problem below the $\bar{\mathrm{p}}$p threshold,i.e.,there are the so-called unphysical regions coming from boson poles below $\bar{\mathrm{p}}$p, which we discuss later.

The second FESR corresponding to $n{=}1$ is

$$\int_{0}^{M}\nu\mathrm{Im}\ \tilde{f}^{(+)}\Big(\nu\Big)d\nu + \frac{1}{4\pi}\int_{0}^{\tilde{N}}k^2\sigma_{tot}^{(+)}dk$$

$$= \int_{0}^{N}\nu\mathrm{Im}\ R\Big(\nu\Big)d\nu + \int_{0}^{N}\nu\mathrm{Im}\ f_{P'}\Big(\nu\Big)d\nu. \qquad (5.6):\mathrm{FESR}(2)$$

Proposal

1. You can use the FESR(1),(2)as constraints as before to search for extremely high-energy regions. Contributions from unphysical regions, however, are not neglible in the FESR(1) in the $\bar{\mathrm{p}}$p scattering although these are negligible[11)] in the FESR(2).

2. Therefore, one can use the FESR(2) as a constraint, assuming the $c_1\log\frac{\nu}{M}+c_2\log^2\frac{\nu}{M}$ behaviours to be satisfied at thigh energy. The ρ ratio of the real to imaginary parts of the forward scattering are also helpful to use as another constraint as was proposed by Block and Halzen.[12)]

3. So, we are planning to search[13)] for extremely high-energy cosmic ray regions[1)] using the FESR(2) as a constraint together with the ρ ratio as another constraint.

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References

1) Practice Data Group, Phys. Letters B592(2004).
2) M. Froissart, Phys.Rev.123,1053(1961). A. Martin, Nuovo Cimento 42, 930(1966).
3) J.R. Cudell et al., Phys. Rev.D61,034019(2000).
4) K.Igi and M.Ishida, Phys.Rev.D66,034023(2002)
5) J.Igi, Phys.Rev. Letters 9,76(1962)
6) K.Igi and S.Matsuda, Phys, Rev. Letters 18,625(1967)
7) R. Dolen, D.Horn and C.Schmid, Phys. Rev.166.1768(1968). This paper includes earlier papers on FESR.

8) K.Igi and M.Fukugita, Physics Reports 31C, 237-338(1976)
9) Particle Data Group, D.E.Groom et al., Eur.Phys. J,C15,235(2000)
10) U.Dersch et al.(SELEX collaboration), Nucl.Phys. B579,277(2000)
11) V. Lengyel, A. Lengyel,Yad.Fiz.(1970)
12) M.M.Block and F.Halzen, hep-ph/0405174(2004)
13) K.Igi and M.Ishida, hep-ph/0505058v2(to be published in Phys. Letters B)