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Spherical Domain Wall formed by Field Dynamics of Hawking Radiation and Spontaneous Charging-up of Black Hole

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Abstract

We discuss the Hawking radiation in the gauge-Higgs-Yukawa theory. The ballistic model is proposed as an effective description of the system. We find that a spherical domain wall around the black hole is formed by the field dynamics rather than the thermal phase transition. The formation of the wall is a general property of the black hole whose Hawking temperature is equal to or greater than the energy scale of the theory. The formation of the electroweak wall and that of the GUT wall are shown. We also find a phenomenon of the spontaneous charging-up of the black hole with the wall. The Hawking radiation drives a mechanism of the charge-transportation into the black hole when C- and CP-violation are assumed. A black hole whose Hawking-temperature is on the electroweak scale can obtain a large amount of the hyper-charge by the mechanism.

Key words: Hawking radiation, Spherical domain wall, Spontaneous charging-up
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1 Introduction

The radiation from a black hole known as the Hawking radiation has a thermal spectrum [1]. Its temperature, namely, the Hawking temperature, is inversely proportional to mass of the Schwarzschild black hole. By the Hawking radiation the Schwarzschild black hole is losing its mass and is increasing its Hawking temperature. The Hawking temperature and the intensity of the radiation increase explosively at the final stage of the evaporation, therefore the black hole with light mass is very interesting to particle physicists [2].

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Several authors discussed that a heating-up by the Hawking radiation can thermalize the neighborhood of the Schwarzschild black hole and a thermal phase transition around the black hole can arise. The QCD phase transition [3] and the electroweak (EW) phase transition [4,5] have been shown. The local phase-transition around the black hole makes a spherical domain wall which separates the symmetric-phase-region from the broken-phase background. Several mechanisms of the baryon-number-production by a black hole with the EW wall have been shown and a cosmological baryogenesis scenario by using one of the mechanisms has been proposed [4,5,6].

The thermal phase transition on some local volume requires a local thermal equilibrium which is confirmed by an enough interaction rate in the considered volume. The mean free paths of the radiated particles are much longer than the Schwarzschild radius. The radius of the wall should be much larger than the Schwarzschild radius. Hence the formation of the wall by the thermal phase transition needs much higher Hawking-temperature than the critical temperature of the transition [4,5]. However, it is natural to ask if the Hawking radiation whose temperature is equal to or slightly greater than the critical temperature influences the Higgs vacuum-expectation-value (vev) locally.

In this paper we discuss the Hawking radiation with such a temperature in the gauge-Higgs-Yukawa theories, e.g., the Standard Model (SM) or the Grand Unified Theory (GUT). We show that a structure of the spherical domain wall around the black hole arises by the field dynamics of the system. The mean free paths of the radiated particles with gauge interaction are much longer than the radius of the black hole [4,5]. The mean wavelength of the radiated particles is approximately given by the radius of the black hole and is shorter than the length-scale of the wall-structure which we will consider outside the black hole. These simplify the analysis. The Hawking-radiated particles can be regarded as ballistic because of their long mean-free-paths and their short wavelengths. Their masses are proportional to the Higgs vev $|\langle\phi\rangle|$. Our basic idea of the wall-formation-mechanism is schematically shown in the Figure 1. When once a radius-depending Higgs-vev-structure $|\langle\phi(r)\rangle|$, namely the wall-structure, has been formed, the Hawking-radiated particles run rush up the slope on the wall-structure and they push the wall-structure outside. Concretely when a particle with momentum $p(r_1) = \sqrt{E^2 - m(r_1)^2}$ ascends a small part of the slope from the position r_1 to r_2, the momentum of the particle becomes $p(r_2) = \sqrt{E^2 - m(r_2)^2}$ with $m(r_2) = m(r_1) + \Delta m$. The parameter E means the conserved energy of the particle. The part of the slope receives momentum $\Delta p = p(r_1) - p(r_2) \simeq (m/p)\Delta m$. Therefore there arises a pressure, named the Hawking pressure, which is acting on the spherical wall-structure to expand its radius. On the other hand the wall-structure also feels its wall-tension and the Higgs-potential-force, which are acting on the wall-structure to shrink its radius. A balance of these pressures seems to be able to keep the wall-structure stationary. The relevant interaction in this mechanism

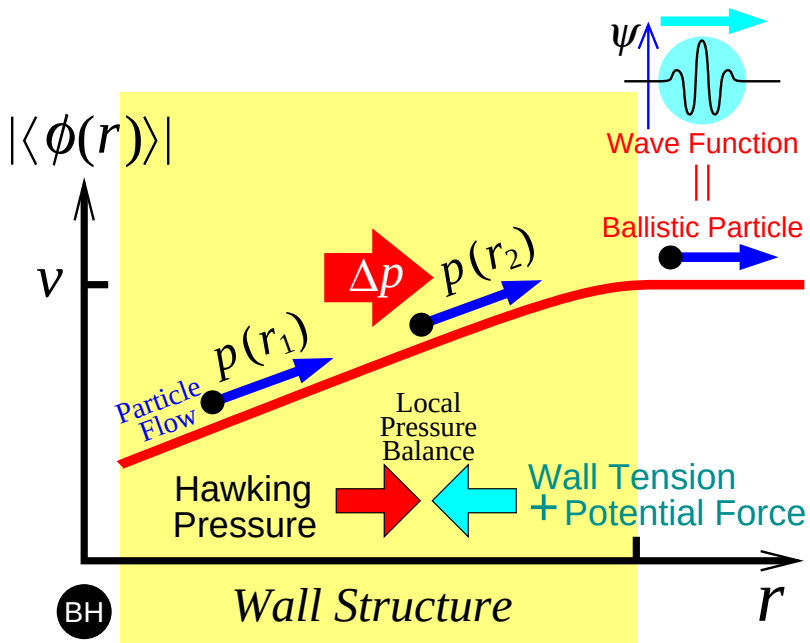

Fig. 1. Our basic idea for the wall-formation-mechanism by a balance of pressures. The parameter r is the distance from the center of the black hole and $|\langle\phi(r)\rangle|$ means the absolute value of the Higgs vev. When once a wall-structure is formed, mass of the particle is depending on the distance r. A part of the momentum Δp of the Hawking-radiated particle is transfered to the wall because the momentum is depending on the distance r in the wall-structure: $p(r) = \sqrt{E^2 - m(r)^2}$. The transfered momentum causes a Hawking-radiation-pressure acting on the wall. A local balance of the Hawking-radiation-pressure and the wall-tension can stabilize the wall-structure.

is only the Yukawa interaction between a ballistic particle and the Higgs vev.

To discuss the idea of the wall-formation systematically we propose a *ballistic model* as an effective description of the system. Our model consists from the action for the Higgs field and that for the relativistic point particles radiated from the black hole. We derive an effective potential for the Higgs vev which determines the structure of the Higgs vev around the black hole. By considering the effective potential, a formation of the spherical domain wall around the black hole is shown when the Hawking temperature is equal to or greater than the energy scale of the Higgs-Yukawa theory. We will call this mechanism *the dynamical formation of the spherical wall* in distinction from the thermal formation of the spherical wall. Here the dynamical formation of the EW wall and that of the GUT wall are shown.

We also discuss a mechanism of *the spontaneous charging-up of the black hole* as an application of the spherical wall. The Hawking radiation is neutral for any charge, however, the following assumptions allow us to consider the charging-up mechanism. We assume (i) a CP-broken phase in the spherical wall and (ii) a chiral charge assignment of the fermions in the field theory, i.e.,

a C-violation of the theory. These assumptions induce a reflection-asymmetry for the charge on the wall. Therefore a charge-transport-mechanism from the wall into the black hole works and it charges-up the black hole. This mechanism is a variant application of the charge-transport-mechanism of the electroweak baryogenesis proposed by Cohen, Kaplan and Nelson [7,8]. The assumptions are equivalent to two of the Sakharov's conditions for the baryogenesis [9]. When the Hawking temperature is on the EW scale, the black hole can obtain a large amount of the hyper-charge. The black hole with the GUT temperature can also obtain several charge.

The paper is organized as follows. In Section 2 the formation mechanism of the spherical wall is discussed. In Section 3 a mechanism for the spontaneous charging-up of the black hole is discussed. In section 4 we provide a conclusion and discussions.

2 Dynamical Formation of Spherical Domain Wall

We consider a Hawking radiation in the gauge-Higgs-Yukawa theory like the Standard Model (SM). Especially we discuss dynamics of the Higgs field around the black hole whose Hawking temperature is similarly equal to or greater than the energy scale of the theory, e.g., the critical temperature. In the situation the mean free paths of the Hawking-radiated particles are much greater than the length scale of the black hole [4]. Because the mean wavelength of the radiated particle is approximately given by the Schwarzschild radius, the mean wavelength is similar to or smaller than the length-scale of the domain wall which we will consider around the black hole. We can regard these radiated particles as ballistic. The particles are almost freely running away from the black hole except for the interaction with the Higgs vacuum-expectation-value (vev) as their background. Only the Yukawa interactions of the heavy fermions with the Higgs vev and the gauge interactions of the heavy gauge bosons with the Higgs vev are relevant. These particles obtain their mass from the Higgs vev through the interactions. Inversely we can expect the deformation of the Higgs vev, i.e., the symmetry restoration or the wall formation, by the particles with both high energy and high density around the black hole. The deformation can be approximately treated by a combination of the relativistic kinematics of each radiated ballistic particle and the dynamics of the Higgs field with the relevant interactions. The approximated system should have Lorentz invariance and each ballistic particle should obtain its mass by the Higgs vev. To discuss the subject we propose a *ballistic model* which satisfies the required conditions. Our model is described by the action for a Higgs field $\phi(x)$ with a Higgs potential $V(\phi)$ and the action for trajectories $\{y_i^\mu(s)\}$ of relativistic point particles $\{i\}$ as the ballistic particles. We use a metric convention $g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. The action is given

by [1]:

$$S[\phi, y] = \int d^4x \ \left[(\partial\phi)^2 - V(\phi)\right] \ - \ \sum_i \int ds_i \ Y_i \, |\phi\,(y_i(s_i))\,| \sqrt{|\dot{y}_i(s_i)|^2}, \quad (1)$$

where $\dot{y}_i^\mu \equiv dy_i^\mu/ds_i$ and Y_i is a "extended Yukawa coupling constant" for the point particle i. The summation in the action (1) is performed over all particles. The Higgs potential in the vacuum is given by the double-well form

$$V(\phi) = -\frac{1}{2}\,\mu^2\phi^2 \ + \ \frac{1}{2}\frac{\mu^2}{v^2}\,\phi^4, \quad (2)$$

which has a minimum at $|\phi| = v/\sqrt{2}$ and the constant $\mu^2 > 0$ is the square of the Higgs mass. Mass of the particle i is proportional to the Higgs vev:

$$m_i(x) = Y_i |\,\langle\phi(x)\rangle\,| \quad (3)$$

and which reproduces the relation in the ordinary gauge-Higgs-Yukawa theory. Mass of the particle in the vacuum is given by $m_i = Y_i v/\sqrt{2}$. Here we note that the summation in the action (1) is performed over not only the fermions but also the massive gauge bosons. Because mass of the massive gauge boson in the gauge-Higgs theory is also proportional to the Higgs vev, we have extended the concept of the Yukawa-coupling-constant Y_i to the massive gauge boson in the action (1) [2]. Such a massive gauge boson as the ballistic particle also obeys the mass relation (3). The general relativistic (GR) effect is omitted. The reason will be discussed in section 4.

To calculate the effective potential for the Higgs field, we fix gauge for the point particles as $s_i = x^0 = y_i^0(x^0)$. Then we have $\sqrt{|\dot{y}_i(x^0)|^2} = \sqrt{1 - |\mathbf{v}_i(x^0)|^2} \equiv 1/\gamma_i(x^0)$, where $\mathbf{v}_i(x^0)$ is 3-velocity for the particle i. The action becomes

$$S[\phi, \mathbf{y}] = \int d^4x \left[(\partial\phi)^2 - V(\phi) \ - \ \sum_i \ \delta^{(3)}(\mathbf{x} - \mathbf{y}_i(x^0)) Y_i \, \frac{|\phi\,(x)\,|}{\gamma_i(x^0)}\right]. \quad (4)$$

By using the definition of the 4-momentum for the particle i:

[1] The action for the ordinary relativistic point particle with mass m is given by $-\int ds \ m\sqrt{|\dot{y}(s)|^2}$. Our action describes point particles which obtain their mass by the Higgs vev.

[2] In the simplest gauge-Higgs model with a gauge coupling constant g, we have W-boson mass $m_W = \frac{1}{\sqrt{2}} g |\,\langle\phi\rangle\,|$. Then the extended Yukawa coupling constant is given by $Y_W = \frac{1}{\sqrt{2}} g$.

$$p_{i\,\mu} = \frac{\partial L}{\partial \dot{y}_i^\mu} \;=\; -Y_i\,|\phi(y_i)|\frac{\dot{y}_{i\,\mu}}{\sqrt{|\dot{y}_i|^2}} \;=\; -Y_i\,|\phi(y_i)|\gamma_i(x^0)\,(1,\,-\mathbf{v}_i)$$
$$\equiv (-E_i,\,\mathbf{p}_i), \tag{5}$$

the equation of the motion for the particle becomes

$$\dot{p}_{i\mu} \;+\; Y_i\frac{1}{\gamma_i(x^0)}\partial_\mu|\phi|(y_i) = 0, \tag{6}$$

where L is the integrand of the action (4), namely, the Lagrangian. The parameter $E_i(x^0)$ of the particle motion means a positive energy of the particle i. The equation of the motion for the Higgs field is derived from the gauge-fixed action (4) and it becomes

$$\partial^2\phi = -\frac{1}{2}\frac{\partial}{\partial\phi}\left[V(\phi) \;+\; \sum_i\;\delta^{(3)}(\mathbf{x}-\mathbf{y}_i(x^0))Y_i\,\frac{|\phi\,(x)\,|}{\gamma_i(x^0)}\right]. \tag{7}$$

Here we find that the right hand side of the field-equation includes an absolute value $|\phi|$ which is not a differentiable function. This fact looks unnatural but this difficulty is resolvable. By using the positive energy parameter (5), the field equation becomes a regular form of the field:

$$\partial^2\phi = -\frac{1}{2}\frac{\partial}{\partial\phi}\left[V(\phi) \;+\; \frac{1}{2}\sum_i\;\delta^{(3)}(\mathbf{x}-\mathbf{y}_i(x^0))Y_i^2\,\frac{\phi^2(x)}{E_i(x^0)}\right]. \tag{8}$$

If we perform the differentiation before substituting the energy parameter, we find a Heaviside theta function $\theta(\phi) = \pm 1$ temporarily but we will obtain the same result: $|\phi|\frac{\partial}{\partial\phi}|\phi| = |\phi|\theta(\phi) = \phi$. We can consider that the non-differentiability of the absolute-value-function $|\phi|$ in the action is relaxed by the effect of the particle motion $\sqrt{|\dot{y}_i|^2}$.

The Higgs field is decomposed into the time-independent vev part and the fluctuating part:

$$\phi(x^0,\mathbf{x}) = \langle\phi(\mathbf{x})\rangle \;+\; \frac{1}{\sqrt{2}}\rho(x^0,\mathbf{x}). \tag{9}$$

The fluctuating part $\rho(x^0,\mathbf{x})$ can be regarded as the Higgs particles which are propagating in the background of the position-depending Higgs vev $\langle\phi(\mathbf{x})\rangle$. According to our concept of the ballistic model we consider that the propagating mode $\rho(x^0,\mathbf{x})$ can be treated as the massive particles and be taken into the ballistic particles $\{y_i\}$. We will precisely discuss a treatment of the ballistic Higgs particle in the Appendix. Mass of the Higgs propagating mode

does not obey the mass relation (3), however, we can show that the action (1) correctly describes the ballistic Higgs particles by choosing the Yukawa-coupling-constant $Y_i = \sqrt{3}\mu/v$ for the Higgs particles. Here we redefine $\phi(\mathbf{x})$ as the position-depending Higgs vev $\langle\phi(\mathbf{x})\rangle$ and $\{y_i\}$ as the set of the ballistic particles including the ballistic Higgs particles. The redefined Higgs field satisfies $\partial_0\phi = 0$. The equation of the field-motion (8) becomes an effective equation of the Higgs vev by the redefinition. The property $\partial_0\phi = 0$ simplifies the equation of the particle-motion (6) as

$$E_i = Y_i \, |\phi(\mathbf{y}_i(x^0))| \, \gamma_i(x^0), \tag{10}$$

where E_i becomes a constant of the particle motion. The constant is the positive-definite energy of the particle i at the rest frame.

When we put the trajectories $\{\mathbf{y}_i(x^0)\}$ for all particles with their energy-constants $\{E_i\}$ into the equation (8), we obtain an effective potential for the Higgs field

$$V_{\text{eff}}(\phi) = V(\phi) \; + \; \frac{1}{2}\sum_i \; \delta^{(3)}(\mathbf{x} - \mathbf{y}_i(x^0)) Y_i^2 \, \frac{\phi^2(\mathbf{x})}{E_i}, \tag{11}$$

which determines the static structure of the Higgs vev by the field-equation $\triangle\phi = \frac{1}{2}\frac{\partial}{\partial\phi}V_{\text{eff}}$. Here we adopt a differential particle number-density $dE \times N_f(E;\mathbf{x})$ for particle species f. The effective potential for the Higgs field can be written down as

$$V_{\text{eff}}(\phi) = V(\phi) \; + \; \frac{1}{2}\phi^2 \sum_f Y_f^2 \int \frac{dE}{E} N_f(E;\mathbf{x}), \tag{12}$$

where the summation in the equation (12) is performed over all particle-species in the theory. Then the effective potential should be

$$V_{\text{eff}}(\phi, \mathbf{x}) = +\frac{1}{2}\,\mu_{\text{eff}}^2(\mathbf{x})\,\phi^2 \; + \; \frac{1}{2}\frac{\mu^2}{v^2}\,\phi^4, \tag{13}$$

where we have defined the effective μ^2-parameter as

$$\mu_{\text{eff}}^2(\mathbf{x}) = -\mu^2 \; + \; \sum_f Y_f^2 \int \frac{dE}{E} N_f(E;\mathbf{x}). \tag{14}$$

Next we consider that the particle distribution $dE \times N_f(E;\mathbf{x})$ is produced by the Hawking radiation from the Schwarzschild black hole with mass m_{BH}.

By the Hawking process all kinds of the particles in the field theory are radiated from the horizon whose radius is given by the Schwarzschild radius $r_{\rm BH} = \frac{1}{4\pi}\frac{1}{T_{\rm BH}}$. The radiated particles have thermal spectrum with the Hawking temperature $T_{\rm BH} = \frac{1}{8\pi}\frac{m_{\rm pl}^2}{m_{\rm BH}}$. The particle distribution around the black hole is approximately given by

$$dE\ N_f = \frac{1}{4}\frac{g_f}{(2\pi)^3}\ f_{T_{\rm BH}}(E)\ 4\pi E^2 dE\ \times\ \left(\frac{r_{\rm BH}}{r}\right)^2, \tag{15}$$

where

$$f_{T_{\rm BH}}(E) := \frac{1}{e^{E/T_{\rm BH}} \pm 1} \tag{16}$$

is the Bose-Einstein or the Fermi-Dirac distribution function with the temperature $T_{\rm BH}$ and g_f means a degree of freedom for the particle-species f. The leading contribution from the Hawking radiation is given by the distribution (15). Later we will discuss the formation of the wall by the Hawking-radiated particles. The wall influences the trajectories of the particles and slightly deforms the distribution of the particles. In the distribution (15) we have ignored this backreaction. The ignored contribution can be evaluated by solving the equation of the particle-motion $|\dot{\mathbf{y}}_i|^2 = 1 - (Y_i\phi(\mathbf{y}_i)/E_i)^2$ derived from the equation (10) and it will be a future subject.

By substituting the distribution (15) into the effective Higgs mass (14) we obtain

$$\mu_{\rm eff}^2(r) = -\mu^2\ +\ \frac{A^2}{r^2}, \tag{17}$$

where we have defined a dimensionless "wall-formation-constant"

$$A^2 \equiv \frac{1}{768\pi^2}\sum_f \tilde{g}_f Y_f^2 \tag{18}$$

which is depending on the field theory and we have defined an effective g_f as

$$\tilde{g}_f = \begin{cases} g_f & (f\ :\ \text{boson}) \\ \frac{1}{2}g_f & (f\ :\ \text{fermion}) \end{cases}. \tag{19}$$

Here we find that the sign of $\mu_{\rm eff}^2(r)$ changes at the "characteristic wall-radius"

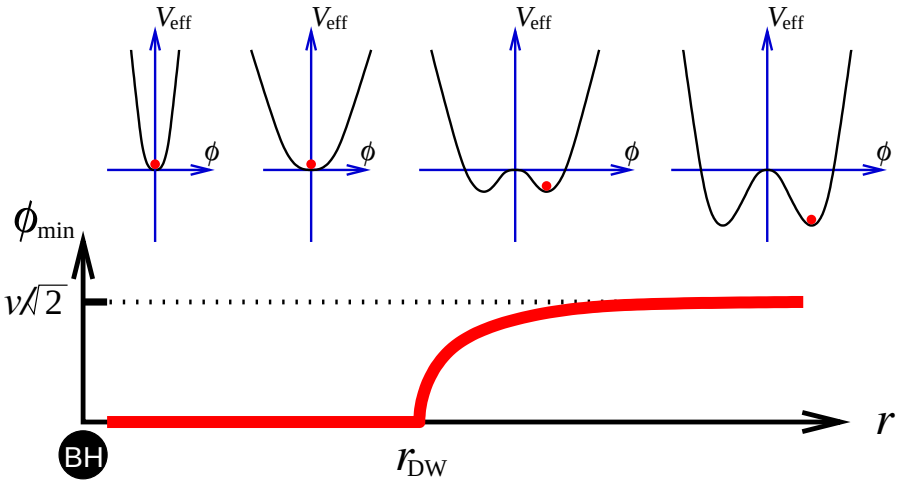

Fig. 2. The distribution of the effective Higgs potential $V_{\rm eff}(\phi, r)$ around a black hole. The parameter r is the distance from the center of the black hole and $r_{\rm DW} = A/\mu$ is the radius where the sign of $\mu^2_{\rm eff}(r)$ changes. The thick curve indicates the value of the Higgs field $\phi_{\rm min}(r)$ which minimizes the effective Higgs potential at each point r. Since the value $v/\sqrt{2}$ is the ordinary Higgs vacuum expectation value, a boundary condition $\phi_{\rm min}(r \to \infty) = v/\sqrt{2}$ is required.

$$r_{\rm DW} \equiv \frac{A}{\mu} \tag{20}$$

as $\mu^2_{\rm eff}(r)$ is negative for $r > r_{\rm DW}$ and is positive for $r < r_{\rm DW}$. The distribution of the Higgs effective-potential $V(\phi, r)$ around the black hole is shown in the Figure 2. The effective potential is minimized at each position r by

$$\phi_{\rm min}(r) = \begin{cases} \dfrac{v}{\sqrt{2}} \left[1 - \left(\dfrac{r_{\rm DW}}{r} \right)^2 \right]^{1/2} & (r > r_{\rm DW}) \\ 0 & (r \leq r_{\rm DW}) \end{cases} . \tag{21}$$

Therefore there arises a local-vacuum depending on r continuously and we can expect the formation of the spherical domain wall with the radius $r_{\rm DW}$ when $r_{\rm DW} > r_{\rm BH}$. The vacuum continuously varying with the dependence on the position is one of the distinctive features in the non-equilibrium system.

The form of the Higgs vev around a black hole, namely, the structure of the wall, is calculated as a spherically-symmetric stationary solution of the motion-equation for the Higgs field with the effective potential (13):

$$\triangle \phi(r) = \frac{1}{2} \frac{\partial}{\partial \phi} V_{\rm eff}(\phi, r), \tag{22}$$

where a boundary condition $\phi(r \to \infty) = v/\sqrt{2}$ is required. The equation (22)

can be solved numerically and the result is shown in the Figure 3. The form of the Higgs vev is depending on the wall-formation-constant A^2 defined in (18). Generally speaking, the curve of the Higgs vev $\phi(r)$ approaches $\phi_{\rm min}(r)$ when A^2 becomes large. Here we find that the Hawking radiation can form the spherical wall or wall-like structure around the black hole for the non-zero A^2 and the radius of the spherical wall is characterized by the parameter $r_{\rm DW}$.

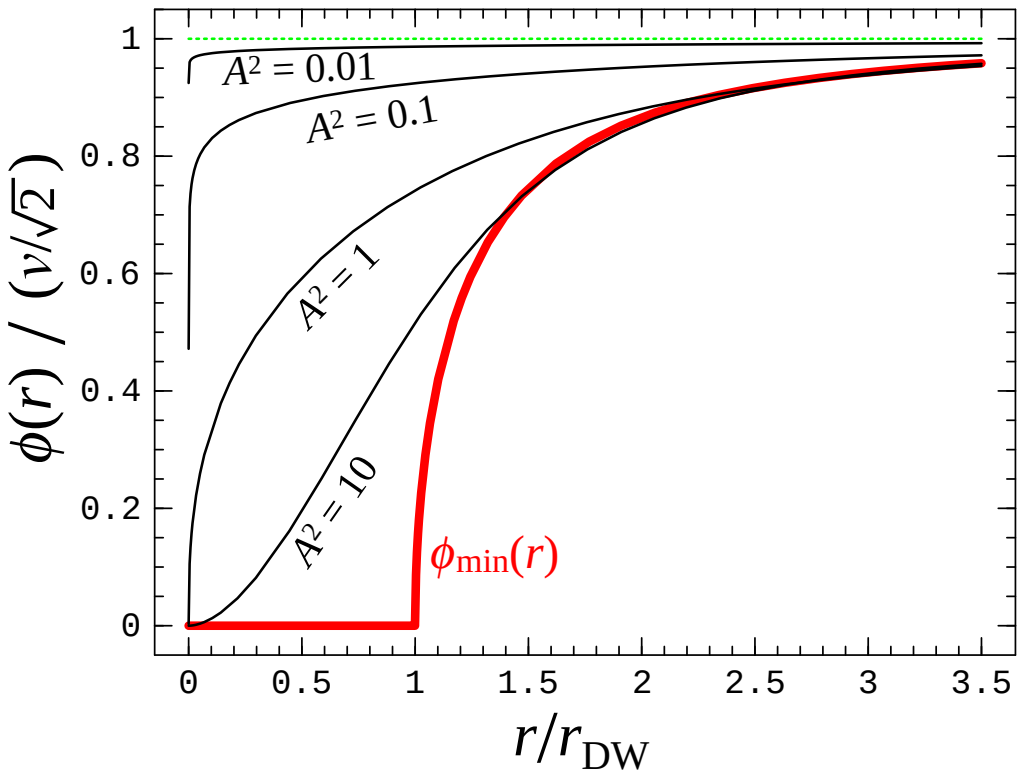

Fig. 3. The profiles of the spherical-wall around a black hole. The parameter r is distance from the center of the black hole and $r_{\rm DW}$ is a characteristic-wall-radius. The thin curves $\phi(r)$ are resultant Higgs vevs depending on the wall-formation-constant A^2. They are spherically-symmetric stationary solutions of the effective field-equation (22) which associates with the effective potential $V_{\rm eff}(\phi, r)$ in (13). The thick curve $\phi_{\rm min}(r)$ indicates the value of the Higgs field which minimizes the effective Higgs potential at each point. The dotted line means the ordinary Higgs vev $\langle\phi\rangle = v/\sqrt{2}$.

The condition for the wall-formation becomes

$$r_{\rm BH} < r_{\rm DW}, \tag{23}$$

because the Schwarzschild radius $r_{\rm BH}$ should be smaller than the characteristic-wall-radius $r_{\rm DW}$. This condition can be rewrite as

$$T_{\rm BH} > T^*_{\rm BH}, \tag{24}$$

where we have defined the critical Hawking-temperature

$$T^*_{\rm BH} \equiv \frac{\mu}{4\pi A} \tag{25}$$

for the dynamical wall-formation. It should be noticed that the characteristic-wall-radius $r_{\rm DW}$ in (20) is not depending on the Hawking temperature. Once the wall is formed, the shape of the wall hardly changes in spite of the change of the Hawking temperature during the evaporation of the black hole. The integrated particle-number-flux from the black hole which is derived from (15) is proportional to the Hawking temperature $T_{\rm BH}$. On the other hand, the effect of the ballistic particles upon the Higgs vev is inversely proportional to the Hawking temperature because of the form of the effective potential (11) with the relation $E \sim T_{\rm BH}$. Therefore the influence of the Hawking radiation on the wall is not depending on the Hawking temperature.

Now we can discuss the formation of the wall in the realistic field theories, e.g., the electroweak (EW) theory or the grand unified theory (GUT). In the EW theory the heavy particles which contribute to create the wall are the top quarks, the weak bosons Z^0, $W^\pm$ and the Higgs bosons. Approximately we have the wall-formation-constant for the EW wall:

$$\begin{aligned} A^2_{\rm EW} &\equiv \frac{1}{768\pi^2}\sum_f \tilde{g}_f Y_f^2 \\ &= 10^{-3} \times \left[0.979 + 0.065\left(\frac{\mu_{\rm EW}}{100\ {\rm GeV}}\right)^2\right] \simeq 1/960 \sim 1/800 \end{aligned} \tag{26}$$

which is depending on the Higgs mass $\mu_{\rm EW} = 100 \sim 200$ GeV. The critical Hawking-temperature for the dynamical formation of the EW wall becomes

$$T^{*\rm EW}_{\rm BH} \equiv \frac{\mu_{\rm EW}}{4\pi A_{\rm EW}} \sim 2\,\mu_{\rm EW}. \tag{27}$$

Consequently the Hawking radiation produces the spherical EW domain wall whose structure is given in the Figure 3 with $A^2 = A^2_{\rm EW} \simeq 0.001$ when the Hawking temperature of the black hole is similar to or greater than the EW scale. The characteristic radius of the wall is given by

$$r^{\rm EW}_{\rm DW} = A_{\rm EW}/\mu_{\rm EW} \simeq 0.03/\mu_{\rm EW} \tag{28}$$

In the $SU(5)$-GUT there are, at least, lepto-quarks X, Y and **24**-Higgs Φ as heavy particles with the GUT scale mass $\mu_{\rm GUT} \simeq 10^{16}$ GeV. The same analysis results in

$$A^2_{\rm GUT} \simeq 1/200, \tag{29}$$
$$T^{*\rm GUT}_{\rm BH} \simeq \mu_{\rm GUT}, \tag{30}$$
$$r^{\rm GUT}_{\rm DW} \simeq 0.07/\mu_{\rm GUT}. \tag{31}$$

Then the spherical GUT wall is also formed when the Hawking temperature is greater than the GUT scale. Finally we can conclude that the spherical wall around a black hole is dynamically formed in most of the field theories which have a Higgs mechanism when the Hawking temperature is greater than the energy scale of the field theory.

3 Spontaneous Charging-up of the Black Hole

We discuss that the Hawking radiation can charge up the black hole by the effect of the spherical wall discussed in the previous section. We need two assumptions such that (i) the domain wall has a CP-broken phase and (ii) the field theory has fermions with chiral charge-assignment. The Standard Model (SM) satisfies the second assumption because the left-handed quark and the right-handed quark have different hyper-charges. The SM with the two Higgs extension (2HSM) or the Minimal Supersymmetric SM (MSSM) can satisfy the first assumption. When the first assumption is satisfied, the reflection-rate on the wall of the left-handed fermions is different from that of the right-handed fermions. The domain wall has a charge-reflection-asymmetry when both assumptions are satisfied. The Hawking radiation is charge-neutral, however, the black hole obtains net charge because the reflected particle-flux is not charge-neutral due to the asymmetry and a part of the reflected particles return into the black hole. Consequently the net charge is transported into the black hole. This process is similar to the "charge transport mechanism on the thin wall regime" in the electroweak baryogenesis proposed by Cohen, Kaplan and Nelson [7,8]. In the charge-transport-mechanism of the electroweak baryogenesis, the hyper-charge is transported from the thin EW wall to the symmetric-phase-region and it boosts up the baryon-number-creation by the sphaleron process. On the other hand our mechanism of the charge-transportation charges up the black hole.

The rate for the charge-transportation into the black hole is given by

$$\frac{dQ}{dt} = \sigma_{\rm BH} \times C_{\rm fcs} F_Q, \tag{32}$$

where $\sigma_{\rm BH}$ is the cross section for the absorption into the black hole, F_Q is the reflected charge flux on the wall and the dimensionless parameter $C_{\rm fcs}$ is a focusing factor. We adopt the horizon area as the absorption-cross-section:

$$\sigma_{\rm BH} = 4\pi r_{\rm BH}^2. \tag{33}$$

Strictly speaking, the absorption-cross-section $\sigma_{\rm BH}$ is slightly different from the horizon area (33) with a factor of $O(1)$ and is depending on the energy of the particle E due to the general relativistic effect. The absorption-cross-section for the high-energy limit (the eikonal limit) is given by $\sigma_{\rm BH}(E \to \infty) = \frac{27\pi}{4} r_{\rm BH}^2$. The spectrum of the Hawking radiation is slightly different from the thermal distribution because the flux of the Hawking radiation is proportional to the $\sigma_{\rm BH}(E)$, which is known as the gray-body effect. The absorption-cross-section (33) is consistent with the flux of the Hawking radiation given in the distribution (15).

In the spherical reflector any particles radiated from a neighborhood of the center of the reflector return into the same neighborhood when they are reflected. Then the incoming flux to the horizon from the spherical wall is greater than the reflected flux F_Q on the wall. This is a focusing effect by the spherical reflector. The flux on the horizon can be written down as $C_{\rm fcs} F_Q$ by the focusing factor $C_{\rm fcs} \geq 1$. We have $(r_{\rm BH}/r_{\rm DW})^2 \times C_{\rm fcs} = 1$ when the effect maximally works and we have $C_{\rm fcs} = 1$ in the absence of the effect.

The charge flux on the wall is given by

$$F_Q = \sum_{f \in \chi} \int_{m_f}^{\infty} dE \; N_f(E; r_{\rm DW}) \, \Delta Q_f \, \Delta R_f(E). \tag{34}$$

The summation is performed over all species χ of the chiral-charged fermions, where the particle-species $f \in \chi$ does not distinguish both the chirality of the particle and the particle/anti-particle. The number $\Delta Q_f \equiv Q_{f_L} - Q_{f_R}$ means the difference between the charge of the left-handed fermion f_L and that of the right-handed fermion f_R. This number is related to the C-violation for the theory. The function

$$\Delta R_f(E) \equiv R_{f_R \to f_L}(E) - R_{\bar{f}_R \to \bar{f}_L}(E) \tag{35}$$

describes the difference between the reflection-probabilities, where the reflection-probability $R_{f_R \to f_L}(E)$ is a probability which the right-handed fermion f_R is reflected to the left-handed fermion f_L and the reflection-probability $R_{\bar{f}_R \to \bar{f}_L}(E)$ is that for the anti-fermion: $\bar{f}_R \to \bar{f}_L$. These probabilities are depending on the particle-energy E. The non-zero $\Delta R_f(E)$ is related with the CP-violation of the wall. When we put the structure of the CP-broken phase in the wall, we can numerically evaluate $\Delta R_f(E)$ according to the method proposed by Cohen, Kaplan and Nelson [7].

The wall-solution (see the Figure 3) shows the absolute value of the Higgs

vev $|\langle\phi(r)\rangle|$, namely, the wall profile without a complex phase. To evaluate the reflection-asymmetry $\Delta R_f(E)$ we assume the structure of the CP-broken phase in the wall:

$$\langle\phi(r)\rangle = f(r) \times \exp\left[i\,\Delta\theta_{\rm CP}\left(1-\frac{f(r)}{v/\sqrt{2}}\right)\right], \tag{36}$$

where $\Delta\theta_{\rm CP}$ is the amount of the CP-broken phase in the wall and the function $f(r) \equiv |\langle\phi(r)\rangle|$ is a profile of the Higgs vev in the wall. The wall-structure (36) is defined for $r > r_{\rm BH}$. The profile-function $f(r)$ should be a solution of the effective-field-equation (22). The complex-phased wall-structure (36) is the same as that in the EW baryogenesis [7,8] except for the profile function [3]. The reflection-asymmetry $\Delta R_f(E)$ of the wall is depending on the CP-broken phase $\Delta\theta_{\rm CP}$, the wall-formation-constant A^2, the energy scale of the field-theory μ, the energy of the fermion E and mass of the fermion m_f.

By substituting the particle-distribution (15), the charge flux (34) becomes

$$F_Q = \left(\frac{r_{\rm BH}}{r_{\rm DW}}\right)^2 C_{\rm fcs} \times \sum_{f\in\chi} g_f\,\Delta Q_f\,m_f^3\,\Delta R(m_f, T_{\rm BH}), \tag{37}$$

where we have defined a reflection-asymmetry integrated over the thermal distribution

$$\Delta R(m_f, T_{\rm BH}) = \frac{1}{8\pi^2}\int_1^\infty d\xi\,\xi^2\,f_{T_{\rm BH}}(m_f\xi)\,\Delta R_f(m_f\xi). \tag{38}$$

We have numerically calculated $\Delta R_f(E)$ and we show the resultant reflection-asymmetry $\Delta R_f(m_f, T_{\rm BH})$ in the Figure 4 and Figure 5. In these calculation we have assumed $\mu = m_f$ for simplicity.

The black hole with the Hawking temperature $T_{\rm BH}$ has a finite lifetime $\tau_{\rm BH} = \frac{20}{\pi^2 g_*}\frac{m_{\rm pl}^2}{T_{\rm BH}^3}$, where g_* is the total degree of freedom with a fermion correction [4]. The total charge transported into the black hole in its lifetime is given by

[3] In the EW baryogenesis $f(z) = [1+\tanh(z/z_{\rm DW})]/2$ is assumed as the profile function of the wall formed by the first order phase transition, where $z_{\rm DW}$ is a thickness of the wall.

[4] The definition is $g_* \equiv g_{\rm b} + (7/8)g_{\rm f}$, where $g_{\rm b}$ is the total degree of the freedom of bosons and $g_{\rm b}$ is that of the fermions. In the SM with the gravity, we have $g_* = 108.75$.

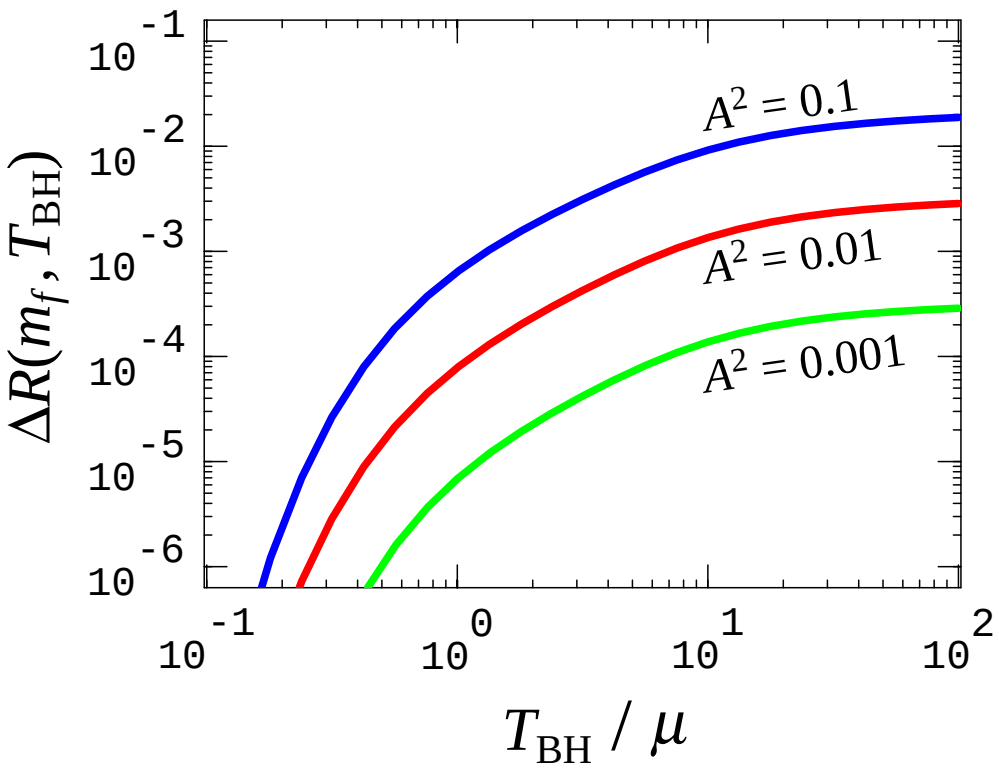

Fig. 4. Numerical results of the dimensionless integrated reflection-asymmetry $\Delta R(m_f, T_{\rm BH})$ for the wall given in the equation (36) with wall-formation-constants $A^2 = 0.1$, $A^2 = 0.01$ and $A^2 = 0.001$ and with a maximum CP-violated phase $\Delta\theta_{\rm CP} = \pi$. We have assumed $\mu = m_f$ for simplicity.

$$
\begin{aligned}
Q &= \int^{\tau_{\rm BH}} dt \frac{dQ}{dt} \\
&= \frac{15}{\pi^3 g_*} m_{\rm pl}^2 \sum_{f \in \chi} g_f \Delta Q_f m_f^3 \int_{T_{\rm BH}}^{\infty} \frac{dT}{T^6} \Delta R(m_f, T),
\end{aligned}
\tag{39}
$$

where we have assumed the maximum focusing effect. The integration in the equation (39) can be performed by using the numerical form of $\Delta R(m_f, T)$. According to the numerical results in the Figure 4 and the Figure 5, the function $\Delta R(m_f, T)$ has meaningful value for $T_{\rm BH} > \mu$ and is exponentially dumping for $T_{\rm BH} < \mu$. Therefore we approximately obtain the total charge transported into the black hole whose initial Hawking temperature $T_{\rm BH}$ is similar to or smaller than μ:

$$
Q \simeq \frac{3}{\pi^3} \Delta R(\mu, \mu) \left(\frac{m_{\rm pl}}{\mu} \right)^2 \sum_{f \in {\rm H}\chi} \frac{g_f}{g_*} \Delta Q_f,
\tag{40}
$$

where we have put a simplification $m_f = \mu$ for all related fermions f, i.e., the summation is performed over only species of the heavy fermions Hχ. Our

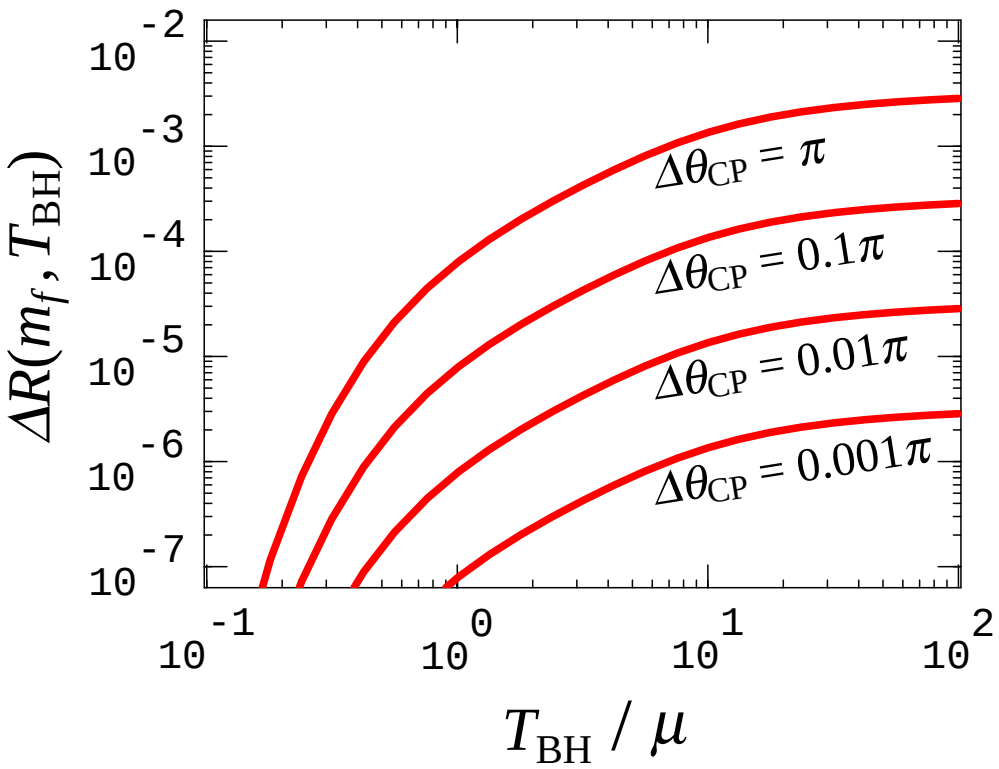

Fig. 5. Numerical results of $\Delta R(m_f, T_{\rm BH})$ with a typical wall-formation-constant $A^2 = 0.01$ and with several CP-violated phases $\Delta\theta_{\rm CP}/\pi = 1,\ 0.1,\ 0.01,$ and 0.001.

numerical analysis results in $\Delta R(\mu, \mu) \simeq 10^{-4} \times \Delta\theta_{\rm CP}/\pi$ for $A^2 = 1/100$ as the EW wall or as the GUT wall, then we obtain

$$Q \simeq 10^{-5} \times \frac{\Delta\theta_{\rm CP}}{\pi} \left(\frac{m_{\rm pl}}{\mu}\right)^2 \sum_{f \in {\rm H}_\chi} \frac{g_f}{g_*} \Delta Q_f. \tag{41}$$

Finally we conclude that the spherical wall can charge up the black hole non-trivially when the energy scale of the wall satisfies $\mu \lesssim 10^{-3} \times m_{\rm pl} \simeq 10^{16}$ GeV.

When we consider a black hole whose Hawking temperature is on the EW scale $\mu \simeq 100$ GeV, the spherical EW wall arises and the hyper-charge Q_Y is transported into the black hole. Mainly the top quark, $g_f = 3$ and $\Delta Q_f = 1/2$, carries the hyper-charge into the black hole. The total transported hyper-charge becomes $Q_Y \simeq 10^{25} \times \Delta\theta_{\rm CP}$. The macroscopic hyper-charge can be transported into the black hole, then we can conclude that the spontaneous charging-up mechanism of the black hole works strongly. When a black hole with the GUT temperature is considered, several charge can be transported into the black hole. The amount of the charge is depending on the chiral charge assignment ΔQ_f of the GUT.

4 CONCLUSION AND DISCUSSIONS

In this paper the Hawking radiation in a gauge-Higgs-Yukawa theory is discussed by an effective model based on the ballistic property of the Hawking-radiated particles. A dynamical formation of the spherical domain wall around the black hole is shown. Our model, named ballistic model, is described by the ordinary Higgs action with a double well potential and the action for the many relativistic point particles coupled to the Higgs field. The ballistic model quite simplifies the analysis of the Higgs vev structure around the black hole as compared with directly analyzing the field-equations for the gauge-Higgs-Yukawa theory.

The ballistic model is valid when (i) the mean free path of the Hawking-radiated particle is greater than the wall radius $r_{\rm DW}$ and (ii) the mean wavelength λ of the particle is smaller than $r_{\rm DW}$. The mean wavelength λ is approximately given by the Schwarzschild radius $r_{\rm BH}$. The radius of the wall we have concluded in (23) satisfies $r_{\rm DW} > \lambda \sim r_{\rm BH}$, then our analysis is consistent with the second condition. If the background of the black hole is not thermalized, the first condition is always satisfied. When the Hawking temperature $T_{\rm BH}$ is so much high that the Hawking radiation thermalizes its neighborhood and the plasma-temperature in the neighborhood locally exceeds the critical temperature of the phase transition, the thermal formation of the spherical wall arises [4,5] instead of the dynamical formation. The transition from the dynamical wall to the thermal wall may be interesting subject.

General relativistic (GR) effects are omitted in our analysis. The black hole has been assumed as an particle-source with a thermal spectrum of the Hawking temperature. No reliable treatment for the GR corrections to the Hawking-radiated particles has been known. For example, a blue-shift effect near the horizon arises when we consider that the trajectory of the radiated particles obeys the Schwarzschild metric. The effect implies that the energy of the radiated particles near horizon is much higher than the Hawking temperature. No matter how low the Hawking-temperature is, the effect allows us to consider the formation of the spherical wall near the horizon dynamically or thermally. This effect is interesting, however, has difficulties to treat properly. The problem of the GR-effect will be the future subject.

We have evaluated the influence from the radiated particles into the Higgs vev and have found the formation of the wall. On the other hand, there should arise the backreaction from the wall into the motion of the particles. The largest backreaction is the reflection of the low-energy particles by the wall. When $T_{\rm BH} \gg m_f$, the backreaction can be ignored because most of the radiated particle pass through the wall. In the case the form of the wall is insignificantly deformed by the backreaction and the Higgs vev in the wall becomes slightly

smaller. When $T_{\rm BH} \sim m_f$, the backreaction cannot be ignored because many of the radiated particles are reflected on the wall. We will subsequently develop the argument of the backreaction.

One may consider that our solution for the Higgs field shown in the Figure 3 is not a domain wall according to the usual terminology — a domain wall is a region of a false vacuum which separates into different real vacua. However our terminology "domain wall" is suitable because of the following reasons. The local vacuum $\phi_{\rm min}(r)$ in (21) is continuously varying (see the Figure 2) because the effective potential $V_{\rm eff}(\phi, \mathbf{x})$ in (13) is apparently depending on the position $\mathbf{x}$. The continuous vacuum is roughly divided into two vacua, namely, the symmetry-restored local-vacuum $\phi_{\rm min} = 0$ for the neighborhood of the black hole as $r < r_{\rm DW}$ and the symmetry-broken vacuum $\phi_{\rm min} = v/\sqrt{2}$ for $r \gg r_{\rm DW}$. Our solution separates these vacua. Our solution $\phi(r)$ does not correspond to the continuous vacuum $\phi_{\rm min}(r)$ for $r_{\rm BH} < r \lesssim r_{\rm DW}$ (see the Figure 3), i.e., the region can be considered as a false vacuum. Consequently the properties of our solution agree with the ordinary definition of the domain wall.

We have also discussed the mechanism of the spontaneous charging up of the black hole by the effect of the spherical domain wall. The mechanism can work when C-violation of the field theory and CP-broken phase in the wall are assumed. Our analysis results that the mechanism can work when the initial Hawking temperature of the black hole is smaller than about 10^{16} GeV. The EW wall by the black hole with the EW temperature can transport a large amount of the hyper-charge into the black hole. The black hole with the GUT temperature can obtain several charge.

On the other hand, a mechanism for the spontaneous charge-loss of the black hole has been discussed. Gibbons first proposed this subject by the semi-classical method [10,11] and recently Gabriel discussed that this result can be confirmed by the functional method [12]. Their works were briefly as follows. The strong electric field around the black hole induces the pair creation of the charged particles. One of the pair-created particles falls into the black hole and the other runs away from the black hole. The outgoing particles carry away the charges from the black hole. Consequently they concluded that a kind of Schwinger process could work for discharging the black hole. These calculation are reliable when the radius of the black hole is greater than the Compton wave length of the electron, i.e., $r_{\rm BH} > 1/m_e$. Because the radius of the black hole discussed in this paper is smaller than the EW length-scale, we can not directly apply their process of the charge-loss to our system. Several authors discussed the subject for the charge loss of the smaller black hole $r_{\rm BH} < 1/m_e$ [13,14], however, we do not know a relevant treatment in this case.

The subjects of the charge-up and the charge-loss are related to the problem of the remnant; "after the completion of the Hawking radiation, whether something is left or not?" and "if something is left, what is it?". Zel'dovich discussed that a black hole leaves a remnant with a Planck mass scale [15]. If the process of the charge-loss does not work, the extremal charged black hole is left as the remnant because the black hole obtains charges by our process, loses its mass by the Hawking radiation and becomes the extremal charged black hole which is quantum-mechanically stable. If the charge-loss process works for the large charged black hole and later the process stops as the charge becomes small, a Planck-massive remnant with several (hyper-)charges is left after the end of the Hawking radiation. Such a remnant can be applicable to explain the dark matter of the universe [5]. The scenario is as follows. Primordial black holes are created by the density-fluctuation in the very early universe and are later made to dominate over the universe by the universe expansion. The Hawking radiation of the black hole reheats up the universe and the spherical EW wall thermally formed around the black hole provides the baryon number for the universe. The black hole obtains some charge by our charge-up process and becomes the Planck-massive remnant as the main constituent of the dark matter. Finally we expect that the formation of the spherical wall around the black hole plays crucial roles in the cosmology.

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APPENDIX: Ballistic Model for Higgs Particle

In this appendix we discuss a treatment of the Higgs particle as the ballistic particle. The action for the Higgs field ϕ is given by

$$S[\phi] = \int d^4x \left[(\partial\phi)^2 - V_{\text{eff}}(\phi) \right] \tag{42}$$

with the effective potential

$$V_{\text{eff}}(\phi) = \frac{1}{2}\mu_{\text{eff}}^2(\mathbf{x})\phi^2 + \frac{1}{2}\frac{\mu^2}{v^2}\phi^4. \tag{43}$$

The Higgs field is decomposed into

$$\phi(x^0, \mathbf{x}) = \phi_s(\mathbf{x}) + \frac{1}{\sqrt{2}}\rho(x^0, \mathbf{x}), \tag{44}$$

where $\phi_s(\mathbf{x}) \equiv \langle\phi(\mathbf{x})\rangle$ is a time-independent Higgs vev and $\rho(x^0, \mathbf{x})$ is a massive Higgs propagating mode in the background of the Higgs vev. When we consider the ordinary vacuum given by the potential with $-\mu_{\text{eff}}^2(x) = \mu^2 > 0$, we obtain a constant Higgs vev $\phi_s(\mathbf{x}) = v/\sqrt{2}$ and a Higgs mass $m_\rho = \mu$.

The Higgs vev $\phi_s(\mathbf{x})$ is determined by the solution of the field equation with $\partial_0\phi_s = 0$:

$$0 = \triangle\phi_s - \frac{1}{2}\left.\frac{\partial V_{\text{eff}}}{\partial\phi}\right|_{\phi=\phi_s}. \tag{45}$$

The equation of the motion for the Higgs propagating mode $\rho(x^0, \mathbf{x})$ becomes

$$\begin{aligned}\frac{1}{\sqrt{2}}\left[\partial_0^2\rho - \triangle\rho\right] &= \triangle\phi_s - \frac{1}{2}\left.\frac{\partial V_{\text{eff}}}{\partial\phi}\right|_{\phi=\phi_s+\frac{1}{\sqrt{2}}\rho} \\ &= \triangle\phi_s - \frac{1}{2}\left.\frac{\partial V_{\text{eff}}}{\partial\phi}\right|_{\phi=\phi_s} - \frac{1}{2}\left.\frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}\right|_{\phi=\phi_s}\frac{\rho}{\sqrt{2}} + \cdots.\end{aligned} \tag{46}$$

In the last deformation in (46) we have expanded the right hand side by $\rho/\sqrt{2}$ around ϕ_s. By substituting (45) into (46) we obtain

$$\partial_0^2\rho - \triangle\rho = -\frac{1}{2}\left.\frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}\right|_{\phi=\phi_s}\rho + \cdots, \tag{47}$$

therefore, we find that mass of the Higgs propagating mode is given by

$$m_\rho^2(\mu^2, \phi_s^2) \equiv \frac{1}{2}\left.\frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}\right|_{\phi=\phi_s} = \frac{1}{2}\mu_{\text{eff}}^2 + 3\frac{\mu^2}{v^2}\phi_s^2 \tag{48}$$

and is not proportional to the Higgs vev.

The mass relation for the Higgs particle becomes (48) rather than the linear relation (3) derived from the action (1). We should consider a new ballistic model which includes the Higgs particles as the ballistic particles and derives

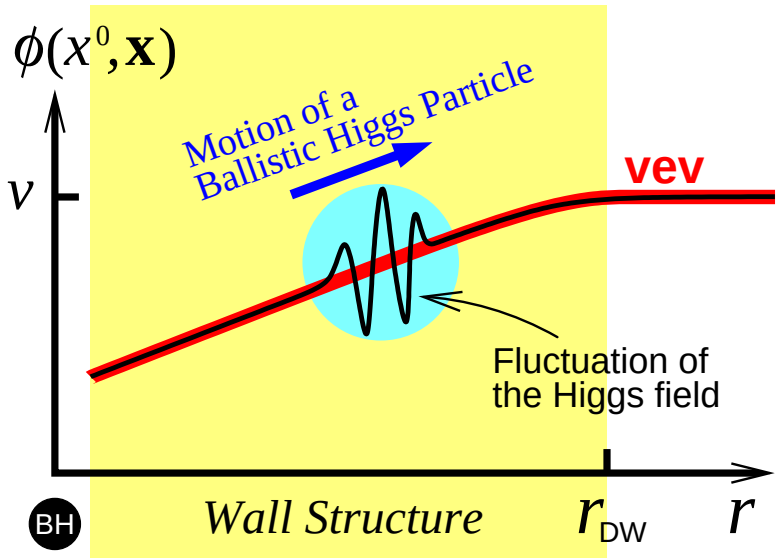

Fig. 6. A schematic picture of the ballistic Higgs particle. The thick curve is the Higgs vev $\phi_s(r) \equiv \langle \phi(r) \rangle$ which is the time-independent spherically-symmetric solution of (45) and thin curve is the Higgs field $\phi(x^0, \mathbf{x})$. The fluctuation $\rho(x^0, \mathbf{x})$ of the Higgs field in (44) is regarded as a ballistic Higgs particle propagating in the background of the Higgs vev.

the mass relation (48). Our concept of the ballistic Higgs particle is schematically shown in the Figure 6. The model is described by the action for the Higgs vev $\phi_s(\mathbf{x})$ and the action for the trajectories $\{y_i\}$ of the ballistic Higgs particles $\{i\}$:

$$S[\phi_s, y] = \int d^4x \ \left[(\partial \phi_s)^2 - V_{\rm eff}(\phi_s) \right] \\ - \sum_i \int ds_i \ m_\rho(\mu^2(\mathbf{y}_i), \phi_s^2(\mathbf{y}_i)) \sqrt{|\dot{y}_i(s_i)|^2}, \tag{49}$$

where the summation is performed over all Higgs particles around the black hole. The action (49) is obtained by changing the factor $Y_i|\phi(y_i)|$ in the action (1) into $m_\rho(\mu^2(\mathbf{y}_i), \phi_s^2(\mathbf{y}_i))$. The field $\phi_s(\mathbf{x})$ does not include the propagating mode, then the condition $\partial_0 \phi_s = 0$ on the field equation is required. The parameter $\mu^2_{\rm eff}(\mathbf{x})$ in the effective potential $V_{\rm eff}(\phi_s)$ has been determined by evaluating the effect of all the ballistic particles around the black hole except for the Higgs particles. We will consider the additional effect on $V_{\rm eff}(\phi_s)$ by the ballistic Higgs particles. The equation of the motion for the ballistic Higgs particle becomes

$$E_i = m_\rho(\mu^2(\mathbf{y}_i), \phi_s^2(\mathbf{y}_i)) \, \gamma_i(x^0). \tag{50}$$

The equation for Higgs vev ϕ_s becomes

$$0 = \triangle\phi_s - \frac{1}{2}\frac{\partial V_{\text{eff}}}{\partial\phi_s} - \frac{1}{2}\sum_i \delta^{(3)}(\mathbf{x}-\mathbf{y}_i)\frac{1}{\gamma_i}\frac{\partial m_\rho}{\partial\phi_s}. \tag{51}$$

The factor in (51) is computed by using (50):

$$\frac{1}{\gamma_i}\frac{\partial m_\rho}{\partial\phi_s} = \frac{m_\rho}{E}\frac{\partial m_\rho}{\partial\phi_s} = \frac{1}{2}\frac{1}{E}\frac{\partial}{\partial\phi_s}m_\rho^2 = \frac{\partial}{\partial\phi_s}\left(\frac{3}{2}\frac{1}{E}\frac{\mu^2}{v^2}\phi_s^2\right). \tag{52}$$

The (51) becomes

$$0 = \triangle\phi_s - \frac{1}{2}\frac{\partial}{\partial\phi_s}\tilde{V}_{\text{eff}}(\phi_s), \tag{53}$$

where we have defined a new effective potential

$$\tilde{V}_{\text{eff}}(\phi_s) = V_{\text{eff}}(\phi_s) + \frac{1}{2}\sum_i \delta^{(3)}(\mathbf{x}-\mathbf{y}_i(x^0))\, 3\frac{\mu^2}{v^2}\,\frac{\phi_s^2(\mathbf{x})}{E_i} \tag{54}$$

including the effect of the ballistic Higgs particles. By comparing the effective potential (11) and (54), the difference between the ordinary ballistic particle and the ballistic Higgs particle is only constant factor. These effective potentials are identical by defining the Yukawa-coupling-constant for the ballistic Higgs particle as

$$Y_\rho = \sqrt{3}\frac{\mu}{v}. \tag{55}$$

Both the action (1) with the Yukawa-coupling-constant (55) for the Higgs particles and the action (49) for the ballistic Higgs particles result the same effective potential for the wall-formation. Finally we conclude that the action (1) also describes the ballistic Higgs particles by choosing the Yukawa-coupling-constant (55) for the Higgs particles.

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