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Radiation Ball as a Black Hole

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Abstract

A structure of the radiation-ball which is identified as a Schwarzschild black hole is found out by investigating the backreaction of Hawking radiation into space-time. The structure consists of the radiation which is gravitationally bounded in the ball and of a singularity. The entropy of the radiation in the ball is proportional to the surface-area of the ball and nearly equals to the Bekenstein entropy. The Hawking radiation is regarded as a leak-out from the ball. There arises no information paradox because there exists no horizon in the structure.

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1 Introduction

One of the most natural approaches to understand the quantum-mechanical properties of a black hole, e.g. the Hawking radiation [1, 2] and the origin of the Bekenstein entropy [3], is considering the structure of the black hole. The membrane-descriptions [4], the stretched horizon models [5, 6] and the quasi-particles model [7] were proposed as this kind of approach. The D-brane descriptions of the (near) extremal-charged black hole succeeded in deriving the entropy [8] and the Hawking radiation [9]. These approaches are just considering the dual structure of the black hole.

Hotta proposed the Planck solid ball model [10] based on the stretched horizon model. The structure of the Schwarzschild black hole consists of a Planck solid ball and a layer of radiation around the ball. There is no horizon and the radius of the ball is slightly greater than the Schwarzschild radius. The Planck solid is a hypothetical matter which arises by the stringy thermal phase transition due to the high temperature of the radiation. The temperature of the radiation around the ball (near the horizon-scale) becomes very high $T(r) \propto 1/\sqrt{g_{tt}(r)}$ because of the blue-shift effect by the deep gravitational potential [6, 7, 10, 11]. Both the Bekenstein entropy and the mass of the black hole are carried by the radiation.

The Planck solid ball is proposed as a backreaction of the high temperature radiation into the space-time structure. If there arises no Planck solid transition, it is natural to ask what occurs as the backreaction. In this paper we consider the backreaction of the thermal radiation by solving the Einstein equation. We obtain a structure of *the gravitationally-bounded radiation-ball with a singularity* as a solution. The structure is spherically symmetric and is characterized by a parameter $r_{\rm BH}$ which means the radius of the radiation-ball. The structure has no horizon and the gravitational potential in the ball becomes quite deep $g_{tt} \sim 10^{-3}/(m_{\rm pl}^2 r_{\rm BH}^2)$. The exterior ($r > r_{\rm BH}$) of the ball is identified as the Schwarzschild black hole with radius $r_{\rm BH}$. In the interior ($r < r_{\rm BH}$) there is a high density radiation $\rho \sim m_{\rm pl}^4$ with the Planck temperature $T \sim m_{\rm pl}$. The radiation is trapped in the ball by the deep gravitational potential. The total entropy of the radiation is proportional to the surface-area of the ball and is near the Bekenstein entropy. The Hawking radiation is regarded as a leak-out of the radiation from the surface of the ball. The singularity in the structure is naked and has quite large negative mass, however, the total mass of the structure (the radiation and the singularity) becomes the ordinary mass of the black hole with radius $r_{\rm BH}$. Therefore the Schwarzschild black hole with the Hawking radiation is identified as our structure and the Bekenstein entropy is carried by the radiation in the ball.

The plan of the rest of the paper is the following. In the next section we present a setup and why the backreaction should be considered. In Section 3, we derive the solution of the radiation-ball. In Section 4, we calculate the entropy of the radiation. In Section 5, we

consider the properties of the singularity. In Section 6, we consider the motion of the test particle in the structure. In the final section we give discussions.

2 Setup and Backreaction

The generic metric of the spherically symmetric static space-time is

$$ds^2 = F(r)dt^2 - G(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2, \tag{1}$$

which is parameterized by the time coordinate t and the polar coordinates r, θ and φ. The elements $F(r)$ and $G(r)$ are functions depending only on r.

The Hawking temperature of a Schwarzschild black hole with radius $r_{\rm BH}$ is given by

$$T_{\rm BH} = \frac{1}{4\pi}\frac{1}{r_{\rm BH}}. \tag{2}$$

We put the black hole into the background of the radiation with the temperature $T_{\rm BH}$ to consider the stationary situation (or equilibrium) of the system [12]. The energy density of the background radiation ($r \to \infty$) is given by the thermodynamical relation:

$$\rho_{\rm BG} = \frac{\pi^2}{30} g_* T_{\rm BH}^4, \tag{3}$$

where g_* is the degree of the freedom of the radiation. We assume that g_* is a constant in order to simplify the analysis. In the situation we expect the local temperature distribution of the radiation as

$$T(r) := \frac{T_{\rm BH}}{\sqrt{F(r)}} \tag{4}$$

due to the effect of the gravitational potential $F(r) = g_{tt}(r)$ [10, 11]. Therefore the energy-density and the pressures of the radiation become

$$\rho(r) = \frac{\rho_{\rm BG}}{F^2(r)} \tag{5}$$

$$P_r(r) = \frac{1}{3}\frac{\rho_{\rm BG}}{F^2(r)}, \tag{6}$$

$$P_{\rm tan}(r) = \frac{1}{3}\frac{\rho_{\rm BG}}{F^2(r)}, \tag{7}$$

respectively. $P_r(r)$ is the pressure in the r-direction and $P_{\rm tan}(r)$ is the pressure in the tangential direction (θ- and φ- direction).

If we fix the space-time structure in the Schwarzschild metric:

$$F_{\rm BH}(r) = 1 - \frac{r_{\rm BH}}{r}, \tag{8}$$

$$G_{\rm BH}(r) = \left[1 - \frac{r_{\rm BH}}{r}\right]^{-1}, \tag{9}$$

then both the density (5) and the pressures ((6) and (7)) diverge on the horizon $r = r_{\rm BH}$. The total radiation-energy around the black hole $\int_{r_1}^{r_2} 4\pi r^2 dr \rho(r) = 1/(r_1 - r_{\rm BH}) + \cdots$ also diverges as $r_1 \to r_{\rm BH}$. This problem requires that we should consider the backreaction of the radiation into the space-time structure.

3 Radiation-Ball Solution

The space-time structure of the black hole including the backreaction from the radiation is computed by solving the Einstein equation $R_\mu{}^\nu - \frac{1}{2}R\delta_\mu{}^\nu - \Lambda\delta_\mu{}^\nu = (8\pi/m_{\rm pl}^2)T_\mu{}^\nu$ with the metric (1) and with the energy-momentum-tensor $T_\mu{}^\nu(r) = {\rm diag}\,[\,\rho(r), -P_r(r), -P_{\rm tan}(r), -P_{\rm tan}(r)\,]$ whose elements are given by (5), (6) and (7). We have introduced the positive cosmological constant $\Lambda = (8\pi/m_{\rm pl}^2)\rho_{\rm vac}$ to stabilize the universe from the radiation background. The background space-time becomes the Einstein static universe by choosing $\rho_{\rm vac} = \rho_{\rm BG}$. The Einstein equation becomes three equations:

$$\frac{-G + G^2 + rG'}{r^2G^2} = \frac{8\pi}{m_{\rm pl}^2}\left\{\rho(r) + \rho_{\rm BG}\right\}, \tag{10}$$

$$\frac{F - FG + rF'}{r^2FG} = \frac{8\pi}{m_{\rm pl}^2}\left\{P_r(r) - \rho_{\rm BG}\right\}, \tag{11}$$

$$\frac{-r(F')^2G - 2F^2G' - rFF'G' + 2FG(F' + rF'')}{4rF^2G^2} = \frac{8\pi}{m_{\rm pl}^2}\left\{P_{\rm tan}(r) - \rho_{\rm BG}\right\}, \tag{12}$$

where $m_{\rm pl}$ is the Planck mass. We obtain the relation

$$G(r) = \frac{1 - \frac{r}{2}\frac{\rho'(r)}{\rho(r)}}{1 + \frac{8\pi}{m_{\rm pl}^2}r^2\left\{\frac{1}{3}\rho(r) - \rho_{\rm BG}\right\}} \tag{13}$$

from the equation (11) with (6). By substituting (13) into the equation (10) with (5), we obtain the differential equation for the energy density $\rho(r)$ as

$$r\rho\left[-24\rho^2\left\{1 - \frac{\rho_{\rm BG}}{\rho}\right\} + 12r\rho\rho' + r^2\rho'^2\left\{1 - 9\frac{\rho_{\rm BG}}{\rho}\right\} - 2r^2\rho\rho''\left\{1 - 3\frac{\rho_{\rm BG}}{\rho}\right\}\right] + \frac{3m_{\rm pl}^2}{8\pi}\left\{-4\rho\rho' + 3r\rho'^2 - 2r\rho\rho''\right\} = 0. \tag{14}$$

We also obtain the same differential equation (14) by substituting (13) into the equation (12) with (7), therefore, the Einstein equation in (10), (11) and (12) and the assumption of the energy-momentum tensor in (5), (6) and (7) are consistent. Although the number of the differential equations exceeds the number of the unknown functions, there exists a solution. This is a non-trivial feature of the system.

The differential equation (14) is numerically solved and we find out the solution whose exterior part corresponds to the exterior of the Schwarzschild black hole with the Hawking

radiation in the Einstein static universe. The Mathematica code for the numerical calculation can be downloaded on [13]. The solution is parameterized by a radius $r_{\rm BH}$ which is the Schwarzschild radius of the correspondent black hole. The numerical solutions for various $r_{\rm BH}$ are displayed in Figure 1. The element of the metric $F(r)$ is derived by (5) and $G(r)$ is derived by (13). A typical form of the metric elements is displayed in Figure 2. The solution indicates that most of the radiation is trapped in the sphere by the gravitational potential $F(r)$ and the radius of the sphere is given by $r_{\rm BH}$. We call the structure the radiation-ball.

For the external region ($r > r_{\rm BH}$) the differential equation (14) is approximated to

$$-4\rho\rho' + 3r\rho'^2 - 2r\rho\rho'' = 0 \tag{15}$$

when $r_{\rm BH}$ is much greater than the Planck length $l_{\rm pl} := m_{\rm pl}^{-1}$. The approximated equation has a solution

$$\rho_{\rm ext}(r) = \rho_{\rm BG} \times \left(1 - \frac{r_{\rm BH}}{r}\right)^{-2}, \tag{16}$$

and the elements of the metric become

$$F_{\rm ext}(r) = 1 - \frac{r_{\rm BH}}{r}, \tag{17}$$

$$G_{\rm ext}(r) = \left[F_{\rm ext}(r) + \frac{8\pi}{3m_{\rm pl}^2}\rho_{\rm BG} r^2 \left\{F_{\rm ext}^{-1}(r) - 3F_{\rm ext}(r)\right\}\right]^{-1}. \tag{18}$$

The approximated solution $\rho_{\rm ext}(r)$ is corresponding to (5) with the Schwarzschild metric (8). The resultant metric ($F_{\rm ext}(r)$ and $G_{\rm ext}(r)$) is also consistent with the external Schwarzschild metric ((8) and (9)) with a background correction of the Einstein static universe. The function-form of the approximated solution is displayed in Figure 3.

For the internal region ($r < r_{\rm BH}$) the differential equation (14) is approximated to

$$-24\rho^2 + 12r\rho\rho' + r^2\rho'^2 - 2r^2\rho\rho'' = 0 \tag{19}$$

for $r_{\rm BH} \gg l_{\rm pl}$. We obtain the solution of the equation (19) as

$$\rho_{\rm int}(r) = \frac{135}{\pi}\frac{1}{g_*} m_{\rm pl}^4 \left(\frac{r}{r_{\rm BH}}\right)^2 \left[1 - \left(\frac{r}{r_{\rm BH}}\right)^5\right]^2, \tag{20}$$

and also obtain the elements of the metric

$$F_{\rm int}(r) = \frac{g_*}{720\sqrt{2\pi}}\frac{1}{m_{\rm pl}^2 r_{\rm BH}^2}\left(\frac{r_{\rm BH}}{r}\right)\left[1 - \left(\frac{r}{r_{\rm BH}}\right)^5\right]^{-1}, \tag{21}$$

$$G_{\rm int}(r) \simeq \frac{g_*}{72}\frac{1}{m_{\rm pl}^2 r_{\rm BH}^2}\left(\frac{r}{r_{\rm BH}}\right)\left[1 - \left(\frac{r}{r_{\rm BH}}\right)^5\right]^{-3}, \tag{22}$$

where the coefficients of the solution are determined by matching to the numerical solution. The density $\rho_{\rm int}(r)$ has the maximum value $\rho_{\rm max} = (125 \cdot 3^{3/5})/(4\pi \cdot 2^{2/5})\, m_{\rm pl}^4/g_* \simeq 14.57 \times$

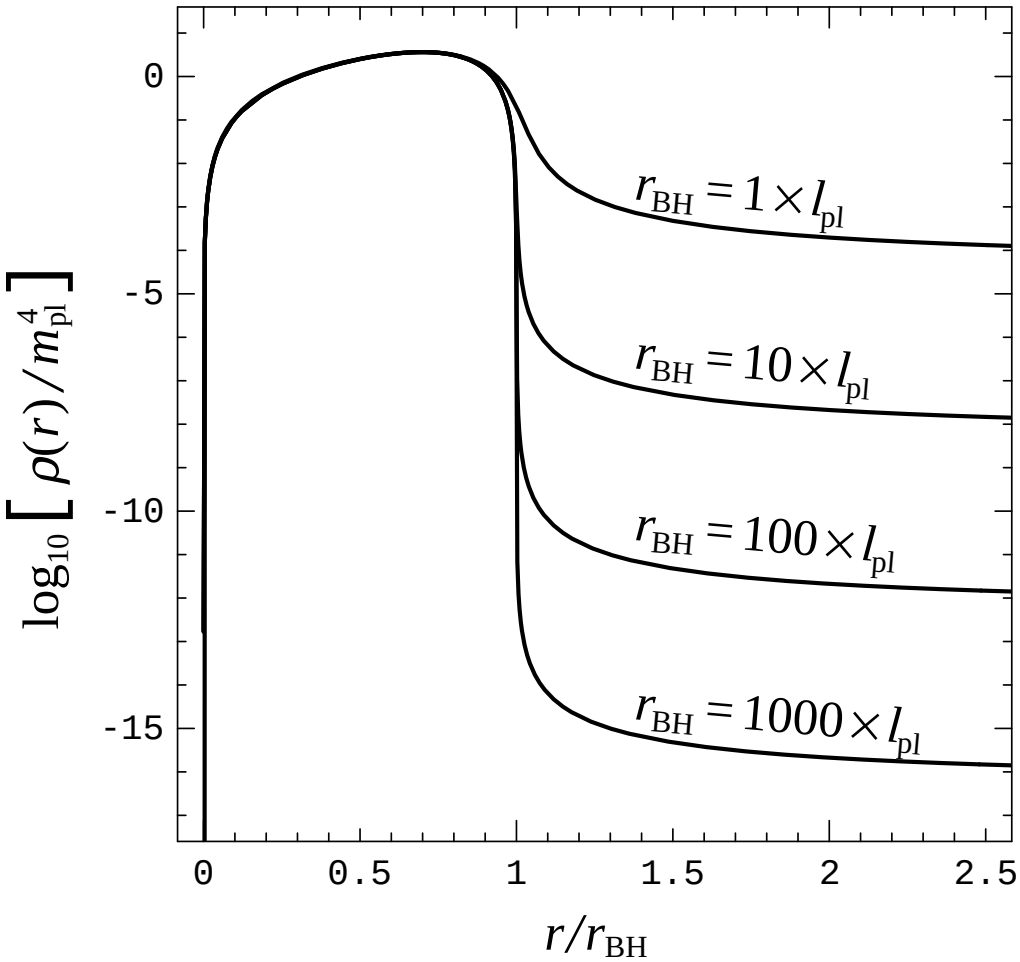

Figure 1: Distributions of the radiation-energy-density $\rho(r)$ in the solution of the radiation-ball for various $r_{\rm BH}$. The horizontal axis is the coordinate r normalized by $r_{\rm BH}$. The solutions for $r_{\rm BH}/l_{\rm pl} = 1, 10, 100$ and 1000 are displayed, where $l_{\rm pl} := m_{\rm pl}^{-1}$ is the Planck length. In the figure the degree of the freedom $g_* = 4$ is assumed. We find that the distribution in the ball ($r < r_{\rm BH}$) has a universal form. The density $\rho(r)$ approaches to the background density $\rho_{\rm BG}$ defined in (3) when r becomes large.

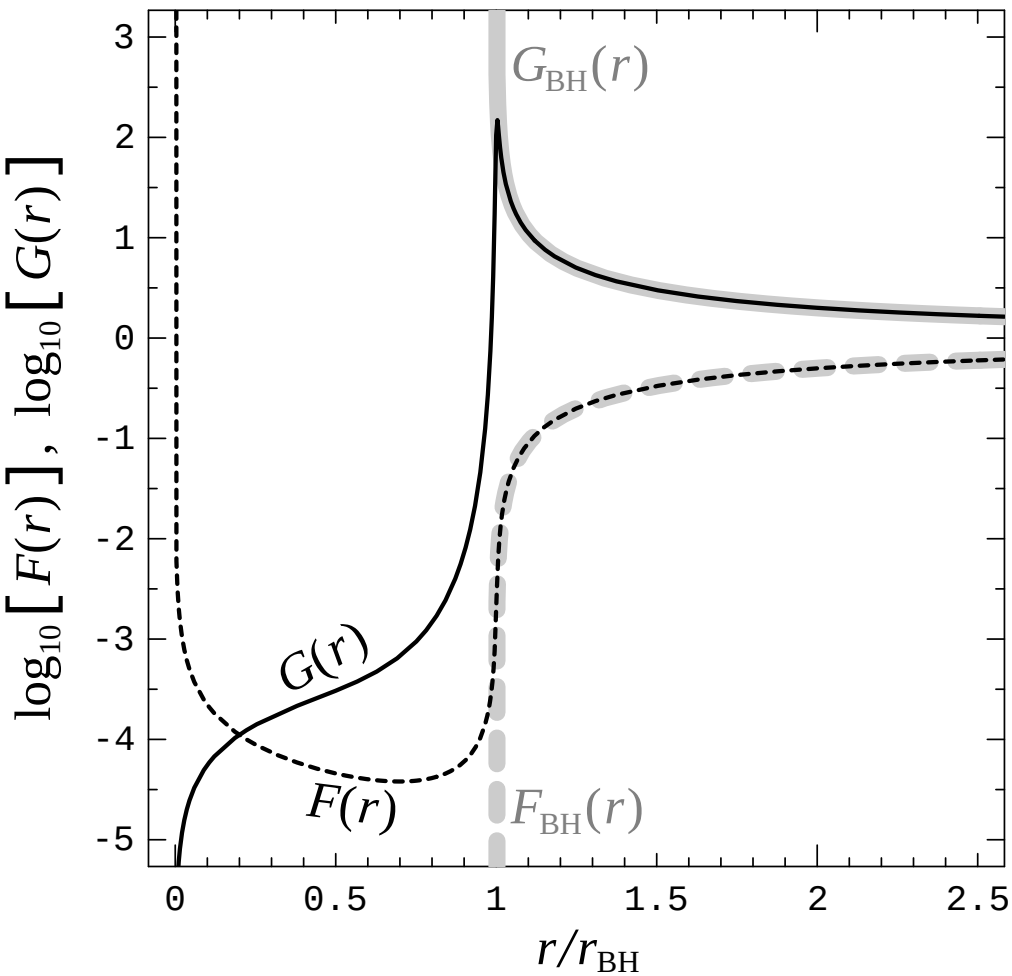

Figure 2: Elements of the metric $F(r)$ and $G(r)$ in the solution of the radiation-ball with $r_{\rm BH} = 10 \times l_{\rm pl}$ and $g_* = 4$. The thin dotted curve is $F(r)$ and the thin solid curve is $G(r)$ in the solution. The thick dotted gray curve $F_{\rm BH}(r)$ and the thick solid gray curve $G_{\rm BH}(r)$ show the external part of the Schwarzschild metric with the radius $r_{\rm BH}$. Curves for more various parameters are shown in [13].

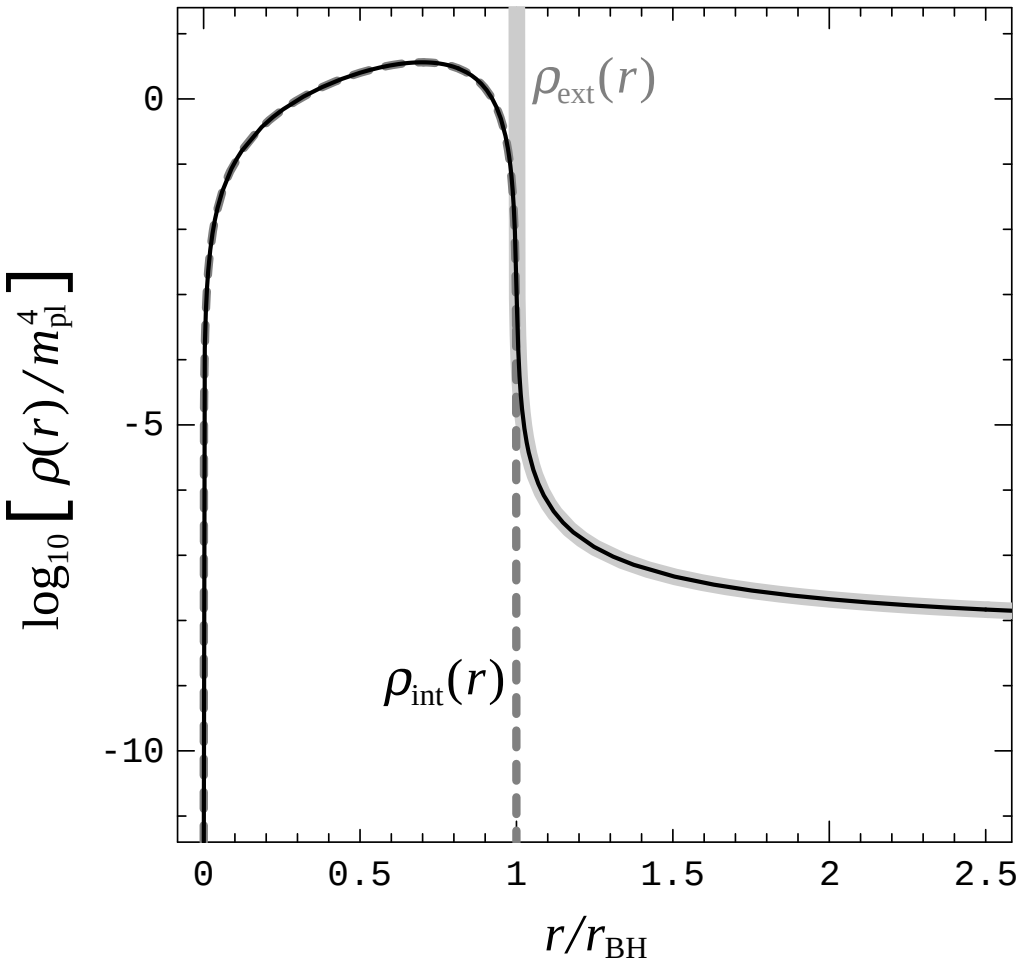

Figure 3: A numerical solution $\rho(r)$, an approximated solution $\rho_{\rm ext}(r)$ for the exterior ($r > r_{\rm BH}$) and an approximated solution $\rho_{\rm int}(r)$ for the interior ($r < r_{\rm BH}$). The thin solid curve is $\rho(r)$, the thick gray curve is $\rho_{\rm ext}(r)$ and the dotted gray curve is $\rho_{\rm int}(r)$. In the figure we have assumed $r_{\rm BH} = 10 \times l_{\rm pl}$ and $g_* = 4$.

$m_{\rm pl}^4/g_*$ on the radius $r_{\rho\text{-peak}} = 6^{-1/5} r_{\rm BH} \simeq 0.6988 \times r_{\rm BH}$. On the same radius, $F(r)$ has the minimum value $F_{\rm min} = g_*/(200 \cdot 2^{3/10} \cdot 9^{4/5} \sqrt{\pi} m_{\rm pl}^2 r_{\rm BH}^2) \simeq 9.515 \times 10^{-4} g_*/(m_{\rm pl}^2 r_{\rm BH}^2)$.

On the transitional region ($r \sim r_{\rm BH}$) the approximated form is a little complicated:

$$\rho_{\rm trans}(r) = \frac{3}{8\pi}\frac{m_{\rm pl}^2}{r_{\rm BH}^2} + \frac{(825)^2}{128\pi^2}\frac{m_{\rm pl}^4}{g_*}\frac{r - r_{\rm BH}}{r_{\rm BH}^2}\left[(r - r_{\rm BH}) - \sqrt{(r - r_{\rm BH})^2 + \frac{96\pi}{(825)^2}\frac{g_*}{m_{\rm pl}^2}}\right]. \quad (23)$$

The element of the metric $G(r)$ has the maximum value $G_{\rm peak} \simeq 30.86 \times m_{\rm pl} r_{\rm BH}/\sqrt{g_*}$ at the radius $r_{G\text{-peak}} \simeq r_{\rm BH} + 0.01215 \times \sqrt{g_*} l_{\rm pl}$ which is quite slightly greater than $r_{\rm BH}$ (see Figure 2).

4 Entropy of the Radiation in the Ball

The entropy density of the radiation with the temperature T is given by the thermodynamical relation:

$$s = \frac{2\pi^2}{45} g_* T^3. \quad (24)$$

By combining the form of the energy density $\rho = \frac{\pi^2}{30} g_* T^4$, the entropy density is described as a function of the energy density:

$$s(\rho) = \frac{2 \cdot (30)^{3/4}\sqrt{\pi}}{45} g_*^{1/4} \rho^{3/4}. \quad (25)$$

The total entropy of the radiation inside ($r < r_{\rm BH}$) of the ball becomes

$$\begin{aligned} S_{\rm int} &\simeq \int_0^{r_{\rm BH}} 4\pi r^2 dr \sqrt{G_{\rm int}(r)}\, s\left(\rho_{\rm int}(r)\right) \\ &= \frac{(8\pi)^{3/4}}{\sqrt{5}} m_{\rm pl}^2 r_{\rm BH}^2 \simeq 5.0199 \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2}, \end{aligned} \quad (26)$$

where $l_{\rm pl} := m_{\rm pl}^{-1}$ is the Planck length. The entropy (26) is proportional to the surface-area of the ball, i.e., the area-law of the black-hole-entropy is reproduced. The entropy (26) is a little greater than the Bekenstein entropy [3]:

$$S_{\rm Bekenstein} = \frac{1}{4}\frac{(\text{Horizon Area})}{l_{\rm pl}^2} = \pi \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2}. \quad (27)$$

The ratio becomes $S_{\rm int}/S_{\rm Bekenstein} \simeq 1.5978$. Therefore the origin of the black hole entropy is regarded as the entropy of the radiation in the ball.

The information paradox of the black hole does not arise because the structure of the radiation-ball has no horizon. The information of the matter which has fallen into the black hole is carried by the internal radiation of the ball. The picture that the entropy of the black hole is carried by radiation is similar to the quasi particle description in the stretch horizon model [7] and the Planck solid ball model [10]. In these models the entropy is carried by the particles around the spherical surface. On the other hand the particles in the spherical body carries the entropy in our model.

5 Singularity

The asymptotic solution of the singularity ($r = 0$) becomes

$$F_{\rm sing}(r) = \frac{g_*}{720\sqrt{2\pi}} \frac{1}{m_{\rm pl}^2 r_{\rm BH}^2} \left(\frac{r_{\rm BH}}{r}\right), \tag{28}$$

$$G_{\rm sing}(r) = \frac{g_*}{72 m_{\rm pl}^2 r_{\rm BH}^2} \left(\frac{r}{r_{\rm BH}}\right). \tag{29}$$

The mass of the singularity is evaluated as

$$m_{\rm sing} := \lim_{r\to 0} \frac{m_{\rm pl}^2}{2} r \left[1 - \frac{1}{G(r)}\right] = -\frac{36}{g_*} m_{\rm pl}^4 r_{\rm BH}^3, \tag{30}$$

which is negative and whose absolute value is much greater than the mass of the correspondent black hole $m_{\rm BH} = (m_{\rm pl}^2/2) r_{\rm BH}$. The gravitational mass of the radiation in the ball becomes

$$m_{\rm rad} := \int_0^{r_{\rm BH}} 4\pi r^2 dr \rho(r) \simeq \frac{36}{g_*} m_{\rm pl}^4 r_{\rm BH}^3 + m_{\rm BH} \tag{31}$$

which is also much greater than $m_{\rm BH}$. As a consequence of the cancellation of the large negative singularity-mass $m_{\rm sing}$ and the large positive radiation-energy $m_{\rm rad}$, the total mass of the structure becomes the ordinary mass of the black hole

$$m_{\rm sing} + m_{\rm rad} \simeq m_{\rm BH}. \tag{32}$$

This property is consistent with that the external part ($r > r_{\rm BH}$) of the structure is essentially given by the Schwarzschild black hole with mass $m_{\rm BH}$.

6 Particle Motion in the Solution

The radial motion of a test particles in the solution is described by the geodesic equation

$$\dot{r}^2(t) = -W_{\rm eff}\left(r(t)\right), \tag{33}$$

for the particle trajectory $(t, r(t))$, where we have defined the effective potential

$$W_{\rm eff}(r) := -\frac{F(r)}{G(r)} \left[1 - \left(\frac{m}{E}\right)^2 F(r)\right]. \tag{34}$$

The parameter E is the energy of the particle and the parameter m is the mass of the particle. There is repulsive force in the region $0 < r < r_{\rho\text{-peak}}$ and attractive force in the region $r_{\rho\text{-peak}} < r$. If there is enough interaction among the particles to thermalize the radiation in the ball, we expect that the thermodynamics in the gravitational potential $F(r)$ reproduces the temperature distribution (4) which we have assumed. This treatment of the radiation is a kind of the mean-field approximation and the derived solution is self-consistent. Most of the particles are trapped in the ball and the leak-out from the ball is regarded as the Hawking radiation.

7 Conclusion and Discussion

The structure of the radiation-ball which we have derived by the consideration of the backreaction of the the Hawking radiation is identified as the Schwarzschild black hole. The origin of the Hawking radiation is explained as a leak-out of the radiation from the ball. There arises no information paradox because the structure has no horizon and the the inside radiation of the ball carries the entropy.

However the structure has several strange features. There arises a naked time-like singularity. The mass of the singularity $m_{\rm sing} \sim -m_{\rm pl}^4 r_{\rm BH}^3$ is negative and its absolute value is much larger than the mass of the black hole $m_{\rm BH} \sim m_{\rm pl}^2 r_{\rm BH}$. On the other hand the total mass of the radiation is positive and is also much larger than $m_{\rm BH}$. By the cancelation of the large masses, the total mass becomes the ordinary mass $m_{\rm BH}$. When the mass of the black hole increases, it seems that the energy of the singularity is transfered into the energy of the radiation. We expect these properties of the singularity is explained by quantum mechanical treatments of the singularity. The string theory or quantum gravity may be required to understand the properties because the temperature of the radiation around the singularity becomes the Planck energy scale ($\sim m_{\rm pl}$).

The entropy of the radiation in the ball is about 1.6 times of the Bekenstein entropy. The quantum entanglement of the particles in the radiation can reduce the radiation-entropy, therefore, the quantum mechanical treatment of the radiation may be effective and the relations of the entropy in (24) will be deformed.

Phase transitions arise in (or around) the radiation-ball because the temperature of the radiation in the ball is very high [11, 14, 15, 16, 17]. The sphaleron process is not suppressed in the ball because the Higgs vev becomes zero, then the radiation-ball does not conserve the baryon number. The hyper charged radiation-ball produces the net baryon number by the effect of the baryon chemical potential [18]. The phase transitions are accompanied by the formation of the spherical domain walls whose radii nearly equal to (or are greater than) $r_{\rm BH}$. The generation of the baryon number [15, 16] and the spontaneous charge-up of the black hole [17] are expected by the effect of the formed wall. In this paper we have assumed the degree of the freedom g_* as a constant for simplicity. We should relax this assumption to consider the phase transitions because the transitions are accompanied by the change of g_*.

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