hep-th/0311133
WIS/31/03-NOV-DPP

Radiation Ball for a Charged Black Hole

Yukinori Nagatani*

Department of Particle Physics,
The Weizmann Institute of Science, Rehovot 76100, Israel

arXiv:hep-th/0311133 v1 16 Nov 2003

Abstract

A radiation-ball solution which is identified as a Reissner-Nordström black hole is found out. The radiation-ball, which is derived by analyzing the backreaction of the Hawking radiation into space-time, consists of radiation trapped in a ball by a deep gravitational potential and of a singularity. The Hawking radiation is regarded as a leak-out of the radiation from the ball. The gravitational potential becomes deep as the charge becomes large, however, the basic structure of the ball is independent of the charge. The extremal-charged black hole corresponds with the fully frozen ball by the infinite red-shift. The total entropy of the radiation in the ball, which is independent of the charge, obeys the area-law and is near the Bekenstein entropy.

*e-mail: yukinori.nagatani@weizmann.ac.il

1 Introduction

Quantum mechanical properties of a black hole — the Hawking radiation [1, 2] and the Bekenstein entropy [3] — are most interesting and mysterious in modern field theory. One of natural approaches is considering the structure of the black hole. The membrane-descriptions [4], the stretched horizon models [5, 6], the Planck solid ball model [7] and the quasi-particles model [8] were proposed as this kind of approach.

Recently the radiation-ball structure which can be identified as the Schwarzschild black hole with the quantum properties was derived by investigating the backreaction of the Hawking radiation into space-time [9]. The structure consists of radiation and of a singularity. The radiation is trapped into the ball by a deep gravitational potential and has the Planck temperature. The gravitational potential is self-consistently produced. The exterior of the ball is fully corresponding with the Schwarzschild black hole. The Hawking radiation is reproduced as a leak-out of the radiation from the surface of the ball. The information paradox of the black hole does not arise because the structure has no horizon. The Bekenstein entropy is regarded as being carried by the internal radiation of the ball because the total entropy of the radiation is proportional to the surface-area of the ball and is near the Bekenstein entropy.

The D-brane descriptions of the (near) extremal-charged black hole succeeded in deriving the entropy with the correct coefficient of the area-law [10] and the Hawking radiation [11]. These approaches are just considering the dual structure of the black hole. To consider the relation between the radiation-ball and the D-brane description, we should derive the radiation-ball for the charged black hole.

In this paper we will extend the radiation-ball to a charged black hole. We assume that the only singularity carries the charge and the radiation is neutral. We will find that the basic structure of the radiation-ball is not changed by the extension. The exterior of the ball corresponds with the Reissner-Nordström black hole. The area-law of the entropy is also not changed. The effect of the extension mainly arises in the gravitational potential in the ball. The gravitational potential in the ball becomes deep as the charge becomes large. In the case of the extremal-charged black hole, the radiation-ball is fully frozen up for a observer at the infinite distance because the red-shift effect becomes infinitely large.

The plan of the rest of the paper is the following. In the next section we present a framework of the investigation. In Section 3, we derive the solution of the radiation-ball. In Section 4, we consider the properties of the ball. In the final section we give discussions.

2 Framework

We will consider the spherically symmetric static space-time parameterized by the time coordinate t and the polar coordinates r, θ and φ. The generic metric is

$$ds^2 = F(r)dt^2 - G(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2, \tag{1}$$

where the elements $F(r)$ and $G(r)$ are functions depending only on r. In the space-time the solution of the spherically symmetric static electromagnetic field becomes $\mathcal{F} = E(r)\sqrt{FG}dt \wedge dr + B(r)r^2 \sin\theta d\theta \wedge d\varphi$, where

$$E(r) := \frac{Q_{\rm e}}{r^2}, \qquad B(r) := \frac{Q_{\rm m}}{r^2} \tag{2}$$

are the radial elements of the field. $Q_{\rm e}$ and $Q_{\rm m}$ are the electric charge and the magnetic charge of the center ($r = 0$) respectively. We define $Q^2 := Q_{\rm e}^2 + Q_{\rm m}^2$.

When we do not consider the backreaction of the Hawking radiation and assume the asymptotic flatness, the space-time becomes the Reissner-Nordström (RN) black hole. The elements of the RN metric are given by:

$$F_{\rm BH}(r) = \left(1 - \frac{r_{\rm BH}}{r}\right)\left(1 - q^2\frac{r_{\rm BH}}{r}\right), \tag{3}$$

$$G_{\rm BH}(r) = 1/F_{\rm BH}(r), \tag{4}$$

where $r_{\rm BH}$ is the radius of the outer horizon and $q := Q/(m_{\rm pl} r_{\rm BH})$ is a charge-parameter which has a value from 0 to 1.

The Hawking temperature of the charged black hole becomes

$$T_{\rm BH} = \frac{1}{4\pi}\frac{1}{r_{\rm BH}}(1 - q^2). \tag{5}$$

When $q = 1$, the black hole becomes the extremal and its Hawking temperature becomes zero. We put the black hole into the background of the radiation with the temperature $T_{\rm BH}$ to consider the stationary situation and the equilibrium of the system [12]. The energy density of the background radiation ($r \to \infty$) is given by the thermodynamical relation:

$$\rho_{\rm BG} = \frac{\pi^2}{30} g_* T_{\rm BH}^4, \tag{6}$$

where g_* is the degree of the freedom of the radiation. We assume that g_* is a constant for simplicity.

Here we should introduce the positive cosmological constant $\Lambda = (8\pi/m_{\rm pl}^2)\rho_{\rm vac}$ to stabilize the background universe from the effect of constant energy density $\rho_{\rm BG}$. The background space-time becomes the Einstein static universe by choosing $\rho_{\rm vac} = \rho_{\rm BG}$.

In the situation we expect the local temperature distribution of the radiation as

$$T(r) \;:=\; \frac{T_{\rm BH}}{\sqrt{F(r)}} \tag{7}$$

due to the effect of the gravitational potential $F(r) = g_{tt}(r)$ [7, 8, 13]. Therefore the energy-density and the pressures of the radiation become

$$\rho(r) \;=\; \frac{\rho_{\rm BG}}{F^2(r)} \tag{8}$$

$$P_r(r) \;=\; \frac{1}{3}\frac{\rho_{\rm BG}}{F^2(r)}, \tag{9}$$

$$P_{\tan}(r) \;=\; \frac{1}{3}\frac{\rho_{\rm BG}}{F^2(r)}, \tag{10}$$

respectively. $P_r(r)$ is the pressure in the r-direction and $P_{\tan}(r)$ is the pressure in the tangential direction (θ- and φ- direction).

3 Radiation-Ball Solution

The space-time structure of the black hole including the backreaction from the radiation is computed by solving the Einstein equation

$$R_\mu{}^\nu - \frac{1}{2}R\delta_\mu{}^\nu - \Lambda\delta_\mu{}^\nu \;=\; \frac{8\pi}{m_{\rm pl}^2}\left\{T_{{\rm rad}\,\mu}{}^\nu + T_{{\rm em}\,\mu}{}^\nu\right\} \tag{11}$$

for the metric (1). Elements of the energy-momentum-tensor for the radiation $T_{{\rm rad}\,\mu}{}^\nu(r) = {\rm diag}\,(\,\rho(r), -P_r(r), -P_{\tan}(r), -P_{\tan}(r)\,)$ are given by (8), (9) and (10). $T_{{\rm em}\,\mu}{}^\nu$ is the energy-momentum-tensor for the electromagnetic field (2). The Einstein equation becomes following three equations:

$$\frac{-G + G^2 + rG'}{r^2G^2} \;=\; \frac{8\pi}{m_{\rm pl}^2}\left\{\rho(r) + \rho_{\rm BG} + \frac{Q^2}{8\pi}\frac{1}{r^4}\right\}, \tag{12}$$

$$\frac{F - FG + rF'}{r^2FG} \;=\; \frac{8\pi}{m_{\rm pl}^2}\left\{P_r(r) - \rho_{\rm BG} - \frac{Q^2}{8\pi}\frac{1}{r^4}\right\}, \tag{13}$$

$$\frac{-r(F')^2G - 2F^2G' - rFF'G' + 2FG(F' + rF'')}{4rF^2G^2} \;=\; \frac{8\pi}{m_{\rm pl}^2}\left\{P_{\tan}(r) - \rho_{\rm BG} + \frac{Q^2}{8\pi}\frac{1}{r^4}\right\}, \tag{14}$$

where $m_{\rm pl}$ is the Planck mass. We obtain the relation

$$G(r) \;=\; \frac{1 - \frac{r}{2}\frac{\rho'(r)}{\rho(r)}}{1 + \frac{8\pi}{m_{\rm pl}^2}r^2\left\{\frac{1}{3}\rho(r) - \rho_{\rm BG} - \frac{Q^2}{8\pi}\frac{1}{r^2}\right\}} \tag{15}$$

from the equation (13) with (9). By substituting (15) into the equation (12) with (8), we obtain the differential equation for the energy density $\rho(r)$ as

$$\begin{aligned}
&-24r\rho^2\left\{\rho-\rho_{\rm BG}+\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\} \;+\; 12r^2\rho\rho'\left\{\rho+2\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\}\\
&+r^3\rho'^2\left\{\rho-9\rho_{\rm BG}-9\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\} \;-\; 2r^3\rho\rho''\left\{\rho-3\rho_{\rm BG}-3\tfrac{Q^2}{8\pi}\tfrac{1}{r^4}\right\}\\
&+\frac{3m_{\rm pl}^2}{8\pi}\left\{-4\rho\rho'+3r\rho'^2-2r\rho\rho''\right\} \;=\; 0. \qquad (16)
\end{aligned}$$

We also obtain the same differential equation (16) by substituting (15) into the equation (14) with (10), therefore, the Einstein equation in (12), (13) and (14) and the assumption of the energy-momentum tensor in (8), (9) and (10) are consistent.

The differential equation (16) is numerically solved and we find out the solution whose exterior part corresponds with the charged black hole in the Einstein static universe. The Mathematica code for the numerical calculation can be downloaded on [14]. The solution is parameterized by $r_{\rm BH}$ and q which are the outer horizon radius and the charge-parameter of the correspondent RN black hole respectively. The numerical solutions for various $r_{\rm BH}$ are displayed in Figure 1. The element of the metric $F(r)$ is derived by (8) and $G(r)$ is derived by (15). Typical forms of the metric elements are displayed in Figure 2. The solution indicates that most of the radiation is trapped in the ball by the gravitational potential $F(r)$ and the radius of the ball is given by $r_{\rm BH}$. This is certainly the charge-extension of the radiation-ball [9].

4 Properties of the Radiation-Ball

4.1 Exterior of the Radiation-Ball

For the external region ($r \gtrsim r_{\rm BH}+l_{\rm pl}$) the differential equation (16) is approximated to

$$\begin{aligned}
&\frac{Q^2}{8\pi}\frac{1}{r^4}\left\{-24r\rho^2+24r^2\rho\rho'-9r^3\rho'^2+6r^3\rho\rho''\right\}\\
&+\frac{3m_{\rm pl}^2}{8\pi}\left\{-4\rho\rho'+3r\rho'^2-2r\rho\rho''\right\} \;=\; 0 \qquad (17)
\end{aligned}$$

when $r_{\rm BH}$ is much greater than the Planck length $l_{\rm pl} := m_{\rm pl}^{-1}$. The approximated equation has a solution

$$\rho_{\rm ext}(r) \;=\; \rho_{\rm BG}\times\left(1-\frac{r_{\rm BH}}{r}\right)^{-2}\left(1-q^2\frac{r_{\rm BH}}{r}\right)^{-2}, \qquad (18)$$

and the elements of the metric become

$$F_{\rm ext}(r) \;=\; \left(1-\frac{r_{\rm BH}}{r}\right)\left(1-q^2\frac{r_{\rm BH}}{r}\right), \qquad (19)$$

$$G_{\rm ext}(r) \;=\; \left[F_{\rm ext}(r)+\frac{8\pi}{3m_{\rm pl}^2}r^2\rho_{\rm BG}\frac{F_{\rm ext}^{-1}(r)-3F_{\rm ext}(r)}{1-q^2\frac{r_{\rm BH}^2}{r^2}}\right]^{-1}. \qquad (20)$$

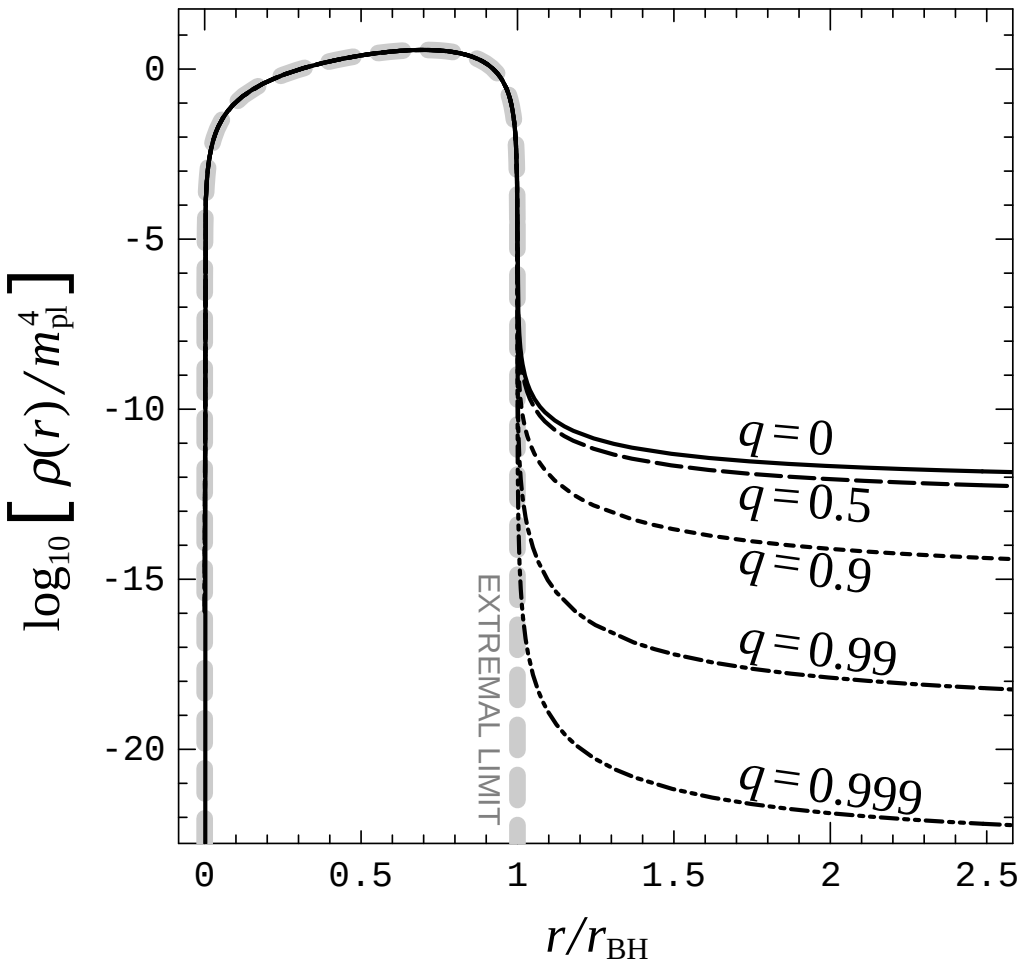

Figure 1: Distributions of the radiation-energy-density $\rho(r)$ in the solution of the radiation-ball for the radius $r_{\rm BH} = 100 \times l_{\rm pl}$, $g_* = 4$ and for various charges q. The horizontal axis is the coordinate r normalized by $r_{\rm BH}$. The solutions for $q := Q/(r_{\rm BH} m_{\rm pl}) = 0, 0.5, 0.9, 0.99$ and 0.999 are displayed. The thick dotted gray curve is the solution for the extremal limit ($q \to 1$). We find that the distribution in the ball ($r < r_{\rm BH}$) has a universal form. The density $\rho(r)$ approaches to the background density $\rho_{\rm BG}$ defined in (6) when r becomes large.

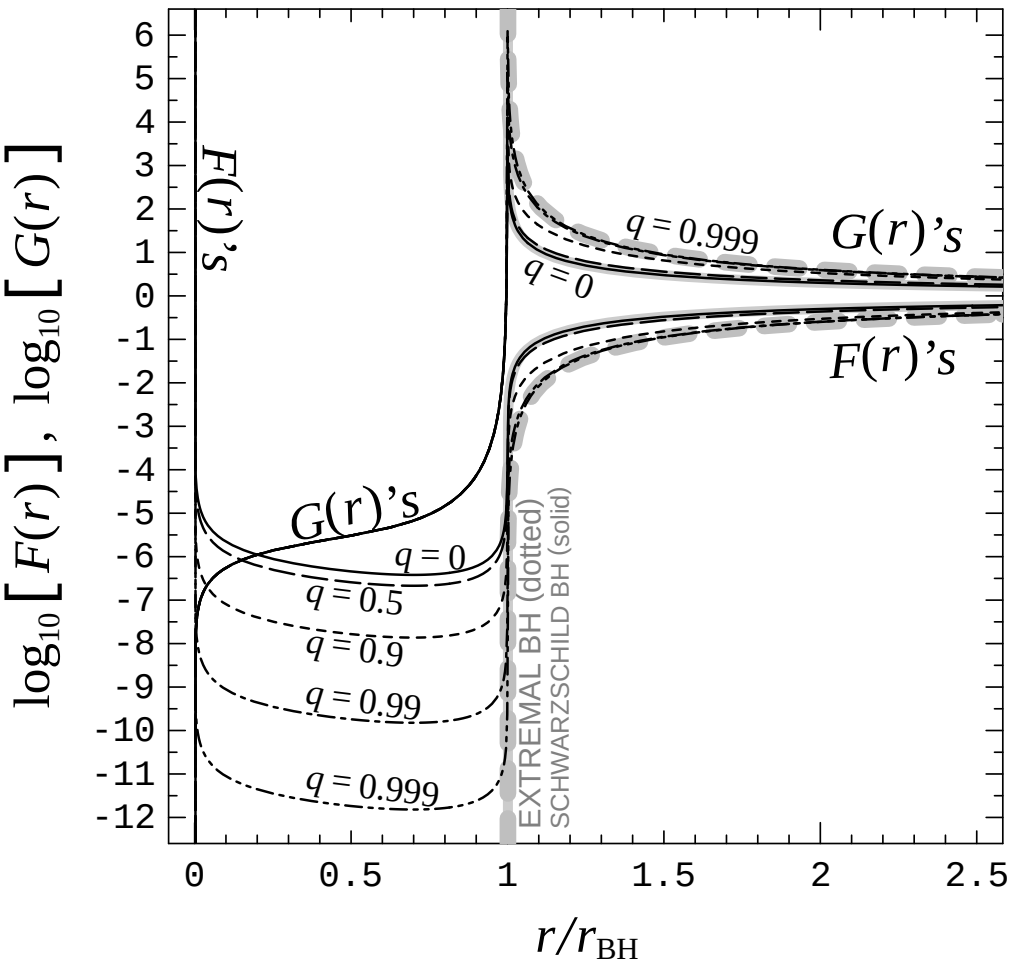

Figure 2: Elements of the metric $F(r)$ and $G(r)$ in the solution of the radiation-ball with $r_{\rm BH} = 100 \times l_{\rm pl}$, $g_* = 4$ and for various charges $q = 0, 0.5, 0.9, 0.99$ and 0.999. The thick dotted gray curves indicate the metric elements $F_{\rm BH}(r)$ and $G_{\rm BH}(r)$ of the exterior part of the extremal-charged RN black hole ($q = 1$). The thick solid gray curves indicate the exterior part of the Schwarzschild metric ($q = 0$). We find that the function form of $G(r)$ in the ball is almost independent of q.

The approximated solution $\rho_{\rm ext}(r)$ is corresponding to (8) with the RN metric (3). The resultant metric ($F_{\rm ext}(r)$ and $G_{\rm ext}(r)$) is also consistent with the external part of the RN metric ((3) and (4)) with a background correction of the Einstein static universe.

4.2 Interior of the Radiation-Ball

For the internal region ($l_{\rm pl} \lesssim r \lesssim r_{\rm BH} - l_{\rm pl}$) the differential equation (16) is approximated to

$$-24\rho(r)^2 \; + \; 12r\rho(r)\rho'(r) \; + \; r^2\rho'(r)^2 \; - \; 2r^2\rho(r)\rho''(r) \;\; = \;\; 0 \tag{21}$$

for $r_{\rm BH} \gg l_{\rm pl}$. We obtain the solution of the equation (21) as

$$\rho_{\rm int}(r) \;\; = \;\; \frac{135}{\pi}\frac{1}{g_*}m_{\rm pl}^4\left(\frac{r}{r_{\rm BH}}\right)^2\left[1-\left(\frac{r}{r_{\rm BH}}\right)^5\right]^2, \tag{22}$$

where the coefficients of the solution are determined by matching to the numerical solution. This does not depend on the charge q because ${T_{\rm em}}_\mu^{\ \nu} \sim q^2 m_{\rm pl}^2/r_{\rm BH}^2$ is much smaller than ${T_{\rm rad}}_\mu^{\ \nu} \sim m_{\rm pl}^4$ in the region. The elements of the metric become

$$F_{\rm int}(r) \;\; = \;\; \frac{g_*}{720\sqrt{2\pi}}\frac{(1-q^2)^2}{m_{\rm pl}^2 r_{\rm BH}^2}\left(\frac{r_{\rm BH}}{r}\right)\left[1-\left(\frac{r}{r_{\rm BH}}\right)^5\right]^{-1}, \tag{23}$$

$$G_{\rm int}(r) \;\; \simeq \;\; \frac{g_*}{72}\frac{1}{m_{\rm pl}^2 r_{\rm BH}^2}\left(\frac{r}{r_{\rm BH}}\right)\left[1-\left(\frac{r}{r_{\rm BH}}\right)^5\right]^{-3}. \tag{24}$$

$G_{\rm int}(r)$ does not depend on q. $F_{\rm int}(r)$ is proportional to $(1-q^2)^2$. When we consider the extremal limit $q \to 1$, $F_{\rm int}(r)$ goes to zero, namely, the red-shift of the radiation-ball goes to infinity and the radiation seems to be frozen for a observer at the infinite distance. The density $\rho_{\rm int}(r)$ has the maximum value $\rho_{\rm max} = (125\cdot 3^{3/5})/(4\pi\cdot 2^{2/5})\, m_{\rm pl}^4/g_* \simeq 14.57 \times m_{\rm pl}^4/g_*$ on the radius $r_{\rho\text{-peak}} = 6^{-1/5} r_{\rm BH} \simeq 0.6988 \times r_{\rm BH}$. On the same radius, $F(r)$ has the minimum value $F_{\rm min} = g_*(1-q^2)^2/(200\cdot 2^{3/10}\cdot 9^{4/5}\sqrt{\pi} m_{\rm pl}^2 r_{\rm BH}^2) \simeq 9.515\times 10^{-4} g_*(1-q^2)^2/(m_{\rm pl}^2 r_{\rm BH}^2)$.

On the transitional region ($r_{\rm BH} - l_{\rm pl} \lesssim r \lesssim r_{\rm BH} + l_{\rm pl}$), the differential equation (16) is approximated to

$$\begin{aligned} &+ r_{\rm BH}^3\rho'^2\left\{\rho - 9\tfrac{Q^2}{8\pi}\tfrac{1}{r_{\rm BH}^4}\right\} \; - \; 2r_{\rm BH}^3\rho\rho''\left\{\rho - 3\tfrac{Q^2}{8\pi}\tfrac{1}{r_{\rm BH}^4}\right\} \\ &+ \frac{3m_{\rm pl}^2}{8\pi}\left\{+3r_{\rm BH}\rho'^2 - 2r_{\rm BH}\rho\rho''\right\} = 0. \end{aligned} \tag{25}$$

The approximated form derived by (25) is a little complicated:

$$\begin{aligned} \rho_{\rm trans}(r) \;\; = \;\; & \frac{3}{8\pi}\frac{m_{\rm pl}^2}{r_{\rm BH}^2}(1-q^2) + \frac{(825)^2}{128\pi^2}\frac{m_{\rm pl}^4}{g_*}\frac{r-r_{\rm BH}}{r_{\rm BH}^2} \\ & \times\left[(r-r_{\rm BH}) - \sqrt{(r-r_{\rm BH})^2 + \frac{96\pi}{(825)^2}\frac{g_*}{m_{\rm pl}^2}(1-q^2)}\right]. \end{aligned} \tag{26}$$

The element of the metric $G(r)$ has the maximum value $G_{\rm peak}$ at the radius $r_{G\text{-peak}}$ which is quite slightly greater than $r_{\rm BH}$ (see Figure 2). In the case of the near extremal ($q \sim 1$), the peak value is approximately given by $G_{\rm peak} \simeq 50.6 \times g_*^{-1/2}(1-q^2)^{-3/2} m_{\rm pl} r_{\rm BH}$ and the radius becomes $r_{G\text{-peak}} \simeq_{\rm BH}$.

4.3 Singularity of the Radiation-Ball

When $q \neq 0$, the asymptotic solution of the singularity ($r = 0$) becomes

$$F_{\rm sing}(r) = \frac{g_*(1-q^2)^2}{12960\sqrt{2\pi} m_{\rm pl}^4 r_{\rm BH}^4} \times q^2 \left(\frac{r_{\rm BH}}{r}\right)^2, \tag{27}$$

$$G_{\rm sing}(r) = +\frac{1}{q^2}\left(\frac{r}{r_{\rm BH}}\right)^2. \tag{28}$$

$G_{\rm sing}(r)$ corresponds to the $r \to 0$ limit of the RN metric (4). The power of r in $F_{\rm sing}(r)$ corresponds with the RN metric (3). The factor of $F_{\rm sing}(r)$ becomes much smaller than that of $F_{\rm BH}(r)$ because of the strong red-shift in the ball. The singularity is time-like, repulsive and naked. Therefore the singularity qualitatively corresponds with that of the over-extremal RN metric.

4.4 Entropy of the Radiation in the Ball

The total entropy of the radiation in the ball is the same as that in the chargeless radiation-ball [9] because $\rho_{\rm int}(r)$ and $G_{\rm int}(r)$ do not depend on q. By combining the entropy density of the radiation $s = \frac{2\pi^2}{45} g_* T^3$ and the energy density $\rho = \frac{\pi^2}{30} g_* T^4$, the entropy density is described as a function of the energy density. The entropy becomes

$$\begin{aligned} S_{\rm int} &\simeq \int_0^{r_{\rm BH}} 4\pi r^2 \sqrt{G_{\rm int}(r)}\; s\left(\rho_{\rm int}(r)\right) \\ &= \frac{(8\pi)^{3/4}}{\sqrt{5}} m_{\rm pl}^2 r_{\rm BH}^2 \;\simeq\; 5.0199 \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2}. \end{aligned} \tag{29}$$

The entropy (29) is proportional to the surface-area of the ball and is a little greater than the Bekenstein entropy [3]:

$$S_{\rm Bekenstein} = \frac{1}{4}\frac{(\text{Horizon Area})}{l_{\rm pl}^2} = \pi \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2,}. \tag{30}$$

The ratio becomes $S_{\rm int}/S_{\rm Bekenstein} \simeq 1.5978$. Therefore the origin of the black hole entropy is regarded as the entropy of the radiation in the ball. This picture is also effective on the extremal-charged black hole.

In this derivation we have assumed the relativistic thermodynamics of the radiation. We expect that treatments of the field dynamics, e.g. the mode-expansion and the state-counting of the field in the ball, reproduces the Bekenstein entropy.

5 Conclusion and Discussion

The structure of the radiation-ball which is identified as the Reissner-Nordström (RN) black hole is derived by investigating the backreaction of the the Hawking radiation into space-time. There arises no horizon. The both the outer horizon $r = r_{\rm BH}$ and the inner horizon $r = q^2 r_{\rm BH}$ of the RN black hole disappear by the backreaction. The radius of the radiation-ball corresponds to that of the outer horizon. The radius of the inner horizon becomes meaningless. The distribution of the radiation and the metric element $G_{\rm int}(r)$ in the ball do not depend on the charge q, then the area-law of the entropy in the radiation-ball is not affected by the extension of the charge. The main effect of the charge appears in the red-shift factor of the ball, namely, $\sqrt{F(r)_{\rm int}}$ is proportional to $(1-q^2)$. The Hawking radiation whose temperature is proportion to $(1-q^2)$ is explained as a leak-out of the radiation from the ball with the red-shift.

When we consider the extremal limit $q \to 1$, the metric element $F(r)$ goes to zero, i.e., the infinite red-shift arises in the ball. However the structure of the radiation-ball, including $\rho(r)$ and $G(r)$, is conserved. Therefore the radiation ball corresponding to the extremal-charged black hole seems to be fully frozen with keeping its structure for the observer at the infinite distance.

In this paper we have assumed that the only singularity carries the charge and the radiation is neutral. One may consider extending the ball such as the radiation carries the charge. We expect that the extension does not change the structure of the radiation-ball because the energy density of the electromagnetic field is much smaller than that of the radiation in the ball except for $r \lesssim l_{\rm pl}$. When the radiation carries the hyper charge, we expect that the ball produces the net baryon number by the sphaleron process or the GUT interaction with the baryon-number chemical potential [15, 16, 17]. The consideration of the ball with the charged-radiation is quite interesting because the phenomenon of the spontaneous charge-transportation into the black hole is known [18].

ACKNOWLEDGMENTS

I would like to thank Ofer Aharony, Micha Berkooz, Nadav Drukker, Bartomeu Fiol, Hikaru Kawai, Barak Kol, Joan Simon and Leonard Susskind for useful discussions. I am particularly grateful to Peter Fischer for pointing out a confusing point in the argument. I would also like to thank the ITP at Stanford university and the organizers of the Stanford-Weizmann workshop for their hospitality at the early stage of the project. I am grateful to Kei Shigetomi for helpful advice and also for careful reading of the manuscript. The work has been supported by the Koshland Postdoctoral Fellowship of the Weizmann Institute of Science.

References

[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975);

[2] S. W. Hawking, Nature **248**, 30 (1974).

[3] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).

[4] K. S. Thorne, R. H. Price and D. A. Macdonald, *Black Holes: The Membrane Paradigm*, Yale University Press, 1986.

[5] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D **48**, 3743 (1993) [arXiv:hep-th/9306069].

[6] L. Susskind and J. Uglum, Phys. Rev. D **50**, 2700 (1994) [arXiv:hep-th/9401070].

[7] K. Hotta, Prog. Theor. Phys. **99**, 427 (1998) [arXiv:hep-th/9705100].

[8] N. Iizuka, D. Kabat, G. Lifschytz and D. A. Lowe, arXiv:hep-th/0306209.

[9] Y. Nagatani, arXiv:hep-th/0310185.

[10] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996) [arXiv:hep-th/9601029].

[11] G. T. Horowitz and A. Strominger, Phys. Rev. Lett. **77**, 2368 (1996) [arXiv:hep-th/9602051].

[12] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).

[13] Y. Nagatani, arXiv:hep-th/0307295.

[14] Y. Nagatani, The Mathematica code "`CRBall100.math`" on the internet web site: `http://www.weizmann.ac.il/~nagatani/RadBall/`.

[15] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Nucl. Phys. B **349**, 727 (1991).

[16] Y. Nagatani, Phys. Rev. D **59**, 041301 (1999) [arXiv:hep-ph/9811485].

[17] Y. Nagatani, arXiv:hep-ph/0104160.

[18] Y. Nagatani, arXiv:hep-th/0307294.