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Black Hole as a Baryon-Reactor — Rapid Baryon Number Violation in Black Hole

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Abstract

We find out that baryon numbers of the matter fallen into black holes are rapidly washed-out by investigating the radiation-ball description of the black holes. The radiation-ball solution, which was derived by analyzing the backreaction of the Hawking radiation into space-time and is identified as a black hole, consists of the radiation gravitationally-trapped into the ball and of a singularity. The baryon number of the black hole is defined as that of the radiation in the ball. The sphaleron processes of the Standard Model work in the ball because the proper temperature of the radiation is Planck scale and the Higgs vev becomes zero. The decay-rate of the baryon number becomes $\dot{B}/B \approx -\alpha_{\rm W}^4/r_{\rm BH}$ for the Schwarzschild black hole of radius $r_{\rm BH}$. When we assume the baryon number violating processes of the GUT, we find more rapid decay-rate $\dot{B}/B \approx -\alpha_{\rm GUT}^2/r_{\rm BH}$. We can regard the black holes as the baryon-reactors which convert the baryonic matter into energy of radiation.

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1 Introduction

Whether information of the matters which have fallen into black holes is conserved or not is one of the most important problems on the black hole physics. Especially baryon numbers of the black holes are quite interesting because the solar-massive black holes created by the supernova as a consequence of star-evolutions consist from the baryonic matter and seem to have huge baryon numbers. The Hawking radiations do not radiate the net baryon number because they are thermal in the ordinary arguments [1, 2]. Are the baryon numbers lost, do the Hawking radiations carry out or are remnants carrying the baryon number left after the completion of the radiation?

In this paper we consider this problem by investigating the radiation-ball description of the black holes. The radiation-ball was derived by solving the Einstein equation including the backreaction of the Hawking radiation into space-time by using the proper temperature ansatz [3, 4]. The ball consists of radiation trapped into the ball by deep gravitational potential and of a repulsive singularity. The balls are identified as black holes with quantum mechanical properties including the Hawking radiation [1, 2] and the Bekenstein entropy [5]. The radius of the ball equals to the horizon radius of the corresponding black hole. We understand that the horizon vanishes by the backreaction and the radiation-ball appears. The Hawking radiation is regarded as a leak-out of the radiation from the ball. The Bekenstein entropy is considered as being carried by the inside radiation because the total entropy of the ball reproduces the area-law and is near the Bekenstein entropy. There arises no information paradox because there is no horizon. The inside radiation of the ball has Planck temperature.

The picture of the radiation-ball allows us to analyze the time-evolution of the baryon number of the black hole concretely because the baryon number of the black hole is simply carried by the inside radiation of the ball and the radiation has finite temperature. The Higgs vacuum expectation value (vev) in the ball is zero and the electroweak (EW) symmetry is restored because the inside temperature is much higher than the EW scale. The baryon number in the ball is not conserved because the sphaleron processes work in the ball. The time-evolution of the baryon number is computed by the Boltzmann equation.

We find that the baryon number of the black hole exponentially decreases with the decay-rate $\dot{B}/B \approx -\alpha_{\rm W}^4/r_{\rm BH}$, where $r_{\rm BH}$ is radius of the Schwarzschild black hole and $\alpha_{\rm W}$ is the weak gauge coupling constant. The time scale of the decay-rate is much shorter than the lifetime of the black hole. The time scale becomes order of several seconds for the solar-massive black hole. We have omitted effects of the grand unified theory (GUT). If we take account the baryon-number violating interactions in the GUT, we expect more rapid rate $\dot{B}/B \approx -\alpha_{\rm GUT}^2/r_{\rm BH}$. Therefore the baryon number of the black holes is rapidly washed-out and we can regard the black holes as the baryon-reactors which convert the baryonic matter

into energy of radiation.

The paper is organized as follows. In the next section we review the radiation-ball solution. In Section 3, we consider the time-evolution of the baryon number. In the final section we provide a conclusion and discussions.

2 Radiation Ball

We briefly review the radiation-ball solution, which is a spherically symmetric static solution of the Einstein equation with the radiation obeying the local proper temperature ansatz [3, 4]. The generic metric of the spherically symmetric static space-time parameterized by the time coordinate t and the polar coordinates r, θ and φ is

$$ds^2 = F(r)dt^2 - G(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2. \tag{1}$$

The unknown functions $F(r)$ and $G(r)$ are determined by the Einstein equation. We will consider the situation of the equilibrium between a Schwarzschild black hole with Hawking radiation and a background of thermal radiation. The temperature of the background-radiation should be chosen to correspond with the Hawking temperature $T_{\rm BH} := \frac{1}{4\pi}\frac{1}{r_{\rm BH}}$ for the equilibrium of the system, where $r_{\rm BH}$ is the Schwarzschild radius of the black hole.

The local proper temperature ansatz means that the temperature-distribution of the radiation around the black hole obeys

$$T(r) = \frac{T_{\rm BH}}{\sqrt{F(r)}}. \tag{2}$$

We expect that the radiation which consists from interacting particles in the gravitational potential obeys the ansatz [6, 7]. The analytic continuation of the angular-momentum distribution of the Hawking radiation without the particle-interactions also reproduces the ansatz [8]. The energy-momentum tensor of the radiation becomes

$$\begin{aligned} \rho(r) &= \frac{\pi^2}{30} g_* T^4(r), \qquad P(r) = \frac{1}{3}\rho(r), \\ {T_{\rm rad}}_\mu{}^\nu &= {\rm diag}\Big[\rho, -P, -P, -P\Big], \end{aligned} \tag{3}$$

where g_* is the massless degree of freedom of the theory. We should introduce the positive cosmological constant $\Lambda = (8\pi/m_{\rm pl}^2)\rho_{\rm vac}$ to stabilize the background universe from the background energy density $\rho_{\rm BG} := \frac{\pi^2}{30} g_* T_{\rm BH}^4$. The background space-time becomes the Einstein static universe of radiation by choosing $\rho_{\rm vac} = \rho_{\rm BG}$. By solving the Einstein equation

$$R_\mu{}^\nu - \frac{1}{2}R\delta_\mu{}^\nu - \Lambda\delta_\mu{}^\nu = \frac{8\pi}{m_{\rm pl}^2}{T_{\rm rad}}_\mu{}^\nu, \tag{4}$$

the radiation-ball solution is derived, where $m_{\rm pl}$ is the Planck mass.

We can numerically solve the Einstein equation (4). The Mathematica code for the numerical calculation can be downloaded on [9] and the concrete forms of the numerical solution were shown in [3, 4]. We find the solution whose exterior part ($r \gtrsim r_{\rm BH} + l_{\rm pl}$) corresponds with the Schwarzschild solution. We also find that the horizon $r = r_{\rm BH}$ of the black hole vanishes and the radiation gravitationally trapped in the ball of radius $r_{\rm BH}$ arises instead of the ordinary black hole. This is the radiation-ball solution and it can be identified as the ordinary black hole with the Hawking radiation.

When $r_{\rm BH} \gg l_{\rm pl}$, we can approximately derive the analytic form of the solution which is divided into three parts, namely, the exterior of the ball $r \gtrsim r_{\rm BH} + l_{\rm pl}$, the transitional region $r_{\rm BH} - l_{\rm pl} \lesssim r \lesssim r_{\rm BH} + l_{\rm pl}$ and the interior $r \lesssim r_{\rm BH} - l_{\rm pl}$.

The metric elements of the exterior part ($r \gtrsim r_{\rm BH} + l_{\rm pl}$) become

$$F_{\rm ext}(r) = 1 - \frac{r_{\rm BH}}{r}, \tag{5}$$

$$G_{\rm ext}(r) = \left[F_{\rm ext}(r) + \frac{8\pi}{3m_{\rm pl}^2} r^2 \rho_{\rm BG} \left\{F_{\rm ext}^{-1}(r) - 3F_{\rm ext}(r)\right\}\right]^{-1}. \tag{6}$$

The metric is identified as the Schwarzschild black hole in the Einstein static universe of radiation. The temperature distribution becomes

$$T_{\rm ext}(r) = T_{\rm BH} \times \left(1 - \frac{r_{\rm BH}}{r}\right)^{-1/2}. \tag{7}$$

Therefore the exterior of the ball is identified as the ordinary black hole with Hawking radiation in the Einstein static universe of the radiation.

For the interior ($r \lesssim r_{\rm BH} - l_{\rm pl}$) of the ball the elements of the metric become

$$F_{\rm int}(r) = \frac{g_*}{720\sqrt{2\pi}} \frac{1}{m_{\rm pl}^2 r_{\rm BH}^2} \left(\frac{r_{\rm BH}}{r}\right) \left[1 - \left(\frac{r}{r_{\rm BH}}\right)^5\right]^{-1}, \tag{8}$$

$$G_{\rm int}(r) = \frac{g_*}{72} \frac{1}{m_{\rm pl}^2 r_{\rm BH}^2} \left(\frac{r}{r_{\rm BH}}\right) \left[1 - \left(\frac{r}{r_{\rm BH}}\right)^5\right]^{-3} \tag{9}$$

and the temperature distribution of the radiation becomes

$$T_{\rm int}(r) = \frac{3\sqrt{5} \cdot 2^{1/4}}{\pi^{3/4}} \frac{m_{\rm pl}}{g_*^{1/2}} \left(\frac{r}{r_{\rm BH}}\right)^{1/2} \left[1 - \left(\frac{r}{r_{\rm BH}}\right)^5\right]^{1/2}. \tag{10}$$

The temperature $T_{\rm int}(r)$ has the maximum value $T_{\rm max} = (5 \cdot 3^{2/5})/(2^{7/20}\pi^{3/4})(m_{\rm pl}/\sqrt{g_*}) \simeq 2.580 \times m_{\rm pl}/\sqrt{g_*}$ on the radius $r_{\rho\text{-peak}} = 6^{-1/5} r_{\rm BH} \simeq 0.6988 \times r_{\rm BH}$. On the same radius, $F(r)$ has the minimum value $F_{\rm min} = g_*/(200 \cdot 2^{3/10} \cdot 9^{4/5}\sqrt{\pi} m_{\rm pl}^2 r_{\rm BH}^2) \simeq 9.515 \times 10^{-4} g_*/(m_{\rm pl}^2 r_{\rm BH}^2)$.

Therefore the temperature of the radiation in the ball is on the scale of the Planck energy. The leak of the inside radiation is regarded as the Hawking radiation.

By the thermodynamical relation of the entropy $s(T) = \frac{2\pi^2}{45} g_* T^3$, the total entropy of the radiation in the ball is calculated as

$$S_{\rm int} \simeq \int_0^{r_{\rm BH}} 4\pi r^2 dr \sqrt{G_{\rm int}(r)}\, s\left(T_{\rm int}(r)\right) = \frac{(8\pi)^{3/4}}{\sqrt{5}} m_{\rm pl}^2 r_{\rm BH}^2 \simeq 5.0199 \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2}. \tag{11}$$

The entropy (11) is proportional to the surface-area of the ball and is a little greater than the Bekenstein entropy [5]: $S_{\rm Bekenstein} = \frac{1}{4}\frac{\text{(Horizon Area)}}{l_{\rm pl}^2} = \pi \times \frac{r_{\rm BH}^2}{l_{\rm pl}^2}$. The ratio becomes $S_{\rm int}/S_{\rm Bekenstein} \simeq 1.5978$. Therefore the black hole entropy is regarded as the total entropy of the radiation in the ball.

3 Baryon Number Violation

In the radiation-ball description of the black hole, the baryon number is carried by the internal radiation of the ball in the same way of the entropy. Therefore we can define the baryon number density $b(r)$ and the total baryon number of the ball as

$$B := \int_0^{r_{\rm BH}} 4\pi r^2 dr \sqrt{G_{\rm int}(r)}\, b(r) \tag{12}$$

which is regarded as *the baryon number of the black hole.* This is one of the interesting features of the radiation-ball description of the black hole.

The Higgs scalar vev $\langle\phi\rangle$ in the radiation-ball vanishes and the symmetry of the electroweak (EW) theory restores because the temperature in the ball ($\sim$ Planck scale) is much greater than the EW scale ($\sim$ 100 GeV) and the thermal phase transition of the EW theory arises. Therefore the sphaleron processes are working in the ball and are changing the baryon number. In the EW symmetric phase, the sphaleron transition rate at temperature T is given by

$$\Gamma_{\rm sph} = \kappa \alpha_{\rm W}^4 T^4, \tag{13}$$

where $\kappa \sim O(1)$ is a numerical constant and $\alpha_{\rm W}$ is the weak gauge coupling constant [10, 11, 12]. We have assumed no GUT and $\alpha_{\rm W}$ is the coupling constant at the Planck energy. We should note that the rate (13) is defined for the proper-time rather than the coordinate time t. We should take account of the red-shift effect in the ball to consider the rate for the coordinate-time. The time-evolution of the baryon number density on the coordinate-time is given by the Boltzmann-like equation:

$$\frac{d}{dt} b = \sqrt{F}\left[\frac{\Gamma_{\rm sph}}{T}\mu_B - \frac{39}{2}\frac{\Gamma_{\rm sph}}{T^3} b\right], \tag{14}$$

where μ_B is the chemical potential for the baryon number [13]. The hyper-charge density of the radiation makes the finite chemical potential

$$\mu_B = N\frac{q_Y}{T^2}, \tag{15}$$

where $N \sim O(1)$ is a model-dependent constant and q_Y is the hyper-charge density of the radiation [14]. By substituting the temperature relation (2), (14) becomes

$$\frac{d}{dt}b = \kappa\alpha_W^4 T_{\rm BH}\left[Nq_Y - \frac{39}{2}b\right]. \tag{16}$$

We introduce the total hyper-charge of the ball as

$$Q_Y := \int_0^{r_{\rm BH}} 4\pi r^2 dr\sqrt{G_{\rm int}(r)}\, q_Y(r). \tag{17}$$

It is natural to assume that the hyper-charge Q_Y is much smaller than the extremal charge $Q_Y^* := \alpha_Y/(m_{\rm pl}r_{\rm BH})$ of the black hole because black holes loose their charges spontaneously due to the electric repulsion and cannot have huge charges [15, 16, 17]. The time-evolution of the total baryon number becomes

$$\frac{d}{dt}B = R_{\rm BV}\left[\frac{2N}{39}Q_Y - B\right], \tag{18}$$

where we have defined the decay rate of baryon number of the radiation-ball:

$$R_{\rm BV} := \frac{39}{2}\kappa\alpha_W^4 T_{\rm BH} = \frac{39}{8\pi}\kappa\alpha_W^4\frac{1}{r_{\rm BH}} \simeq \frac{10^{-6}}{r_{\rm BH}}. \tag{19}$$

For the solar-massive black holes $r_{\rm BH} \simeq 1$km, which are created by the gravitational collapse of stars and are formed by huge baryonic matter, the inverse of the rate $R_{\rm BV}^{-1}$ becomes order of seconds. Such a black hole cannot obtain macroscopic hyper-charge. Therefore we conclude that the baryon number of the black holes is rapidly washed-out by the sphaleron processes in the ball.

When we assume that the GUT interactions also violate the baryon number, the GUT process dominates over the sphaleron process because the GUT process is perturbative and the rate of the GUT process becomes $\Gamma_{\rm GUT} \approx \alpha_{\rm GUT}^2 T^4$. In the case the baryon-number decay-rate becomes

$$R_{\rm BV} \simeq \frac{39}{8\pi}\alpha_{\rm GUT}^2\frac{1}{r_{\rm BH}} \simeq \frac{10^{-3}}{r_{\rm BH}}. \tag{20}$$

The rate becomes order of millisecond for the solar-massive black holes.

4 Conclusion and Discussion

The radiation-ball description of black holes allow us to consider time-evolution of the baryon-number of the black holes concretely. Because the entropy of the black hole is carried by the inside radiation of the ball, the baryon number of the black hole is also carried by the radiation and we can define the the total baryon number of the black hole as (12). The sphaleron process in the Standard Model violates the baryon number at least. We can assume the baryon number violating process of the GUT. The time-evolution of the baryon number of the black hole is described by (18). We find that the baryon number of the black hole is exponentially decrease when the hyper-charge of the black hole is small. The decay-rate of the baryon number without the GUT process is evaluated as (19) and the rate including the GUT process becomes (20). The rates depend on the Schwarzschild radius and on the coupling constants of the relevant processes. The time scale of the decay-rates is much shorter than the lifetime of the black hole $\tau_{\rm BH} = 1280\pi g_*^{-1} m_{\rm pl}^2 r_{\rm BH}^3$. For the solar-massive black holes which are formed by the collapse of the stars including huge baryonic matter, the decay-rates become order of seconds without the GUT process and order of milliseconds with the GUT process. Therefore we conclude that the baryon number of the matter which has formed the black hole or has fallen into the black hole rapidly decays.

The Hawking radiation on the radiation-ball description is regarded as a leak-out of the inside radiation of the ball. Therefore the radiation from the black holes includes the net baryon number flux when the black holes have nonzero baryon number (12) and their Hawking temperature is greater than mass of the proton ~ 1 GeV as the lightest baryon. If we throw strong proton beams into such a small black hole, we observe the flux of the net baryon number in the Hawking radiation. When we stop the proton beams, we find the baryon number flux is exponentially dumping according to (18).

We also find that the hyper-charged black holes produce net baryon number. If the hyper-charge Q_Y of the black hole is constant, the radiation-ball tries to keep its the baryon number to $B = \frac{2N}{39} Q_Y$ due to (18). This is a kind of the homeostasis. When the baryon number is taken away from the ball by the Hawking radiation, the mechanism produces the net baryon number to cover the loss. By combining the mechanism of the spontaneous charging-up of the black hole [18], we expect that the baryon number is spontaneously radiated by the Hawking radiation. This mechanism is similar to the electroweak baryogenesis by black holes [19, 20].

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